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# IntuiGeo: Interactive tutor for online geometry problems resolution on pen-based tablets

Omar Krichen and Eric Anquetil and Nathalie Girard<sup>1</sup>

## Abstract.

This paper presents "IntuiGeo", a tutoring system for geometry learning in middle-schools on pen-based tablets. The objective is to design an intuitive numerical tool which allows the free drawing of geometric shapes by the simulation of the pen and paper traditional setting. Our approach is based on three main principles. The first one is the online recognition of the user's hand-drawn sketches. The second axis of our approach is the ability of the tutor to supervise the resolution strategy of the pupil in the context of a construction problem resolution. The third principle is the capacity to give real-time visual, corrective and guidance feedback to the pupil to achieve a personalized and autonomous experience. Our system is composed of two main engines. The 2D recognition engine is responsible for the interpretation of the hand-drawn strokes (and has already been validated in [9]). The supervision engine is responsible for the automated generation of a geometry construction problem from a solution example drawn by the teacher, as well as the supervision of the resolution strategies chosen by the pupil. This paper will focus on the concepts associated with the new supervision engine. The first user experiments on the tutor have been conducted in pilot middle-schools in the region of Brittany, France. The analysis of the collected data demonstrates the tutor performance in terms of supervision of the pupils production and its ability to provide pertinent and personalized corrective and guidance feedback.

## 1 Introduction

Our work is in the context of the *Actif* project, which aims at fostering active learning by introducing pen-based tablets in the classrooms. The digital device can be a support to the child's learning abilities (*e.g.* the Instuiscript project for handwriting learning [5]). In this paper, we present IntuiGeo, an intelligent tutoring system for learning geometry in middle-school. IntuiGeo's interface is illustrated in figure 1. Pen and touch interaction allows the free elaboration of geometric shapes, by drawing with the stylus and manipulating virtual tools with the fingers (*e.g.* compass, ruler, etc). The objective is to provide an intuitive experience for the user, who can draw freely, while our **2D engine**, based on bi-dimensional formalism, analyses on the fly the stylus input [9]. We are therefore able to simulate the traditional pen and paper setting,

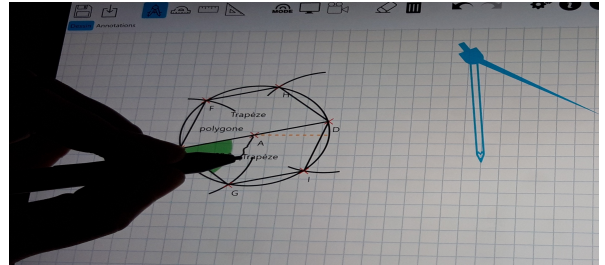


Figure 1: IntuiGeo's interface

making the digital support transparent to the pupil. This allows us to overcome the limitations of most dynamic geometry software used in the classrooms, such as Geogebra [2], Cabri-Geometry [10], or Geometer's SketchPad [16], that require a fastidious training phase for the pupils to get accustomed to the digital support. The *tutoring* aspect of our system comes from the ability to supervise, **in real-time**, the pupil's resolution strategies in the context of a construction problem, in order to provide correction and guidance feedback. In [4], Chickering and Gamson demonstrate the importance of providing prompt feedback as a principle for good practice in undergraduate education. In [8], Kluger and DeNisi show the impact of feedback intervention on learning performance. To our knowledge, there are no dynamic geometry software that offer the possibility to guide, in real-time, the pupil in his/her resolution steps. We designed a **supervision engine** that is able to interpret semantically the pupil's actions relatively to a given problem. The handwritten strokes recognized by the 2D engine are fed in real-time to the supervision engine that will be able to assess the learner knowledge state. We also offer the teacher the possibility to create construction problems by feeding the system with one solution example. From this example, the system should be able to synthesize different resolution strategies since there are different ways to solve an exercise. This ability is primordial in order to recognize in real-time the child's strategy or plan in the context of the resolution process. The 2D engine, being already validated in [9], will be presented briefly for clarity purposes. We will focus in this paper on presenting the new supervision engine and the tutoring aspect of our system. We define our supervision engine with three main objectives:

- The automated generation of the construction problem (*i.e.* exercise) from the example drawn by the teacher;
- Synthesis of strategies to solve the generated exercises;
- Supervision of the child resolution to provide feedback.

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This paper is organized as follows. Sec. 2 presents an overview of related works on intelligent tutoring systems for geometry learning support. IntuiGeo’s architecture is presented in Sec. 3. The 2D recognition engine and the analysis process are explained briefly in Sec. 4. We will focus on the description of the new supervision engine in Sec. 5. To validate this engine, experiments and results are presented in Sec. 6. Conclusion and future works are given in Sec. 7.

## 2 Related works

Our approach is shaped by two domains, pattern recognition and intelligent tutoring. As mentioned before, our system uses a 2D engine that we designed to recognize and interpret structurally, and in real time, the handwritten features. Since this engine has already been validated [9], we will focus in this section on the inner workings of *intelligent tutoring systems*.

The most common architecture of such systems is the tuple *domain module - tutoring module - learner module*, alongside the user interface (cf. figure 2). *The domain module* encom-

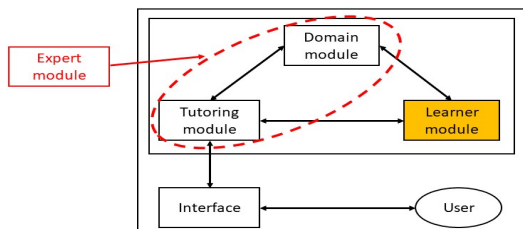


Figure 2: The four component architecture [13]

passes the domain knowledge, in the form of rules, for example. *The learner module* represents the state of knowledge or skills achieved by the student at any given point during the problem resolution. *The tutoring module* is responsible of the correction and supervision of the chosen strategies.

### 2.1 Planning and plan recognition for strategy synthesis

Intelligent tutoring systems vary widely in their architecture [13] [14]. An interesting one is Mentoniez [15], a tutor for geometry proof demonstrations. The system is divided into two main components: the *expert module* (by merging domain and tutoring modules, cf. figure 2) and the *learner module*. *The expert module* is then responsible for the strategy synthesis while *the learner module* is responsible for the interpretation and supervision of the pupil resolution strategies. The authors consider the strategy synthesis problem as a *planning problem* [12], such as the goal is to apply a sequence of domain actions (*theorems, facts...*) in order to solve the exercise. The knowledge base is an ensemble of if-then rules, representing the geometric axioms and theorems of middle-school level, and the expert module applies this knowledge on a given problem variables to reach a goal state. The strategy synthesis here is **offline**, such as the expert module generates a set of possible solutions plans for each given problem. The learner module interprets the pupil actions (*i.e.* demonstration steps) by means of matching the plan/strategy the pupil is choosing in real-time with the expert module generated solutions. Recognizing the plan allows feedback generation and student

guidance. **The architecture of our supervision engine is inspired by this work** although our domain, construction of geometric shapes, is different from proof demonstration. Going back to construction problems, in [7], the authors propose a method for synthesizing geometry constructions to solve typical exercises, such as "draw a triangle given the length of its three sides". The authors approach the resolution of such exercises as a program synthesis problem [6], which can be cast as a planning problem. The expert module must then be able to automatically solve the given problem by synthesizing construction plans, where the steps are ruler-compass based drawing actions, leading to the construction of the desired figure. Our strategy synthesis process is different from the two later approaches in the sense that it is **online**. Our planning environment is **dynamic**, *i.e.* it takes into account exogenous actions. In the context of the resolution of a given problem, *our learner module* interprets on the fly the pupil’s production relatively to the given instruction (what element of the problem is being drawn?, what are the satisfied constraints?). *Our expert module* is then aware of the pupil’s actions in real-time, and tries to find a solution plan that is **adapted to his/her resolution strategy**. Hence, the tutor is able to guide the pupil **forwards** by hinting to the next possible steps, or **backwards** if the pupil is stuck.

### 2.2 Automated generation of construction problems and knowledge representation

We are interested in works that offer an intuitive author mode for the teacher to create his own construction problems. In [1], the authors propose a method for the automatic generation of geometry proof problems from analyzing a figure uploaded by the user. This enables the teacher to generate a set of similar exercises from one example. In [17], the authors propose a system for the generation of high-school geometry questions. The user can select a set of concepts from the interface (*e.g.* triangle + perpendicularity), then the algorithm generates a figure related to these concepts. From this figure, a proof problem is generated. Our approach is more straight-forward: we want the teacher to be able to generate a construction problem from drawing one example solution directly with IntuiGeo. We choose to represent the information associated to each problem with a knowledge graph [18], containing the geometric objects and the related concepts extracted from the teacher’s drawn solution. This knowledge graph will be used by the expert module to synthesize resolution strategies, or plans, and by the learner module for the semantic interpretation of the pupil resolution process. We therefore add a new unit to our system, *the problem generation module*, for the interactive creation of a new construction problem.

Based on this study, we propose an original tutor for the interactive resolution of geometry problems. Unlike most Intelligent Tutoring Systems, IntuiGeo combines two forms of domain knowledge:

- pattern recognition to analyze hand-drawn sketches;
- tutoring expertise to create, solve, and correct geometry construction problems.

This combination allows for the real-time interpretation of the pupil’s hand-drawn strokes as well as his/her resolution strategy, in the context of a free sketching scenario.

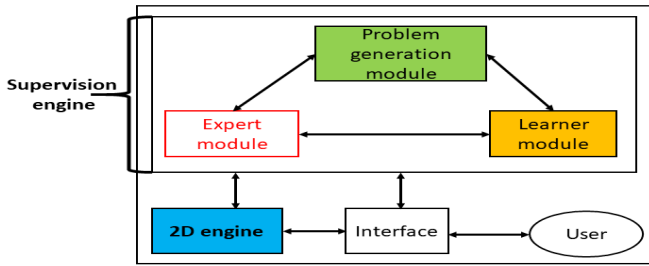


Figure 3: IntuiGeo architecture

### 3 System architecture

Figure 3 illustrates the architecture of our tutoring system. The combination of the two forms of domain knowledge transpires clearly in it as well as in the user-system interaction (cf. figure 4 and figure 5). We present a global vision of the two main engines of IntuiGeo in this section.

#### 3.1 Pattern Recognition expertise

The pattern recognition knowledge is embedded in the **2D engine**. It is called that way since it relies on a bi-dimensional formalism CD-CMG [11] (Context Driven Constraints Multi-Set Grammar) to analyze, on the fly, the stylus input, whereas the tutoring expertise is embedded in the supervision engine. The user, a teacher/pupil, can draw freely on the interface, and the stylus input is interpreted, stroke by stroke, by the 2D engine, then transformed into a geometric element by the CD-CMG grammar. A beautification of the raw stroke is performed and an "idealized" version of the recognized element is rendered by the interface. The user can implicitly validate this interpretation, or redo his drawing. The 2D engine, along with the user interface, can be thought of as a tool for the free drawing of geometric shapes. This engine is presented in more details in Sec. 4.

#### 3.2 Tutoring expertise

The tutoring aspect of IntuiGeo is represented by the **supervision engine**, which is composed of the expert and learner, as well as the problem generation modules.

##### 3.2.1 The problem generation module

Figure 4 illustrates the process of a new construction problem creation. The teacher draws a complete example of the

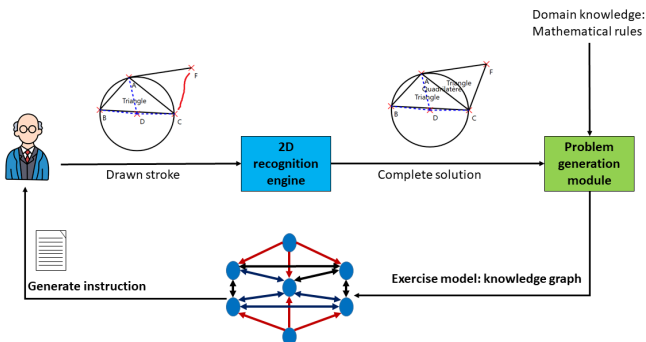


Figure 4: Teacher interaction with the tutor

target figure or production. The problem generation module takes as input the figure interpreted by the the 2D engine and creates a model of the problem, in the form of a knowledge graph, storing all the geometric elements and their links, mathematical constraints such as length, angle, or perpendicularity. The knowledge here is defined by the mathematical rules that are applied on the recognized geometric elements. For example, the teacher has drawn a right-angled triangle ABC, the problem generation module will update the K.G. with the link  $\text{Perpendicular}([AB], [AC])$ . The module translates the knowledge from the graph to plain text to generate the instruction, which can be modified by the teacher.

##### 3.2.2 The learner module

Figure 5 illustrates the child/tutor interaction. The learner module will be responsible of the semantic interpretation of the pupil's strokes, recognized by the 2D engine. The learner module matches each newly interpreted element with the corresponding node in the knowledge graph. Hence the extraction of satisfied and unsatisfied constraints is straight forward. This enables the tutor to generate corrective feedback. To generate guidance feedback, the learner module communicates with the expert module, which will try to synthesize a resolution strategy based on the state of the knowledge graph, that is to say the expert module tries to recognize the solution plan in which the child's strategy lies.

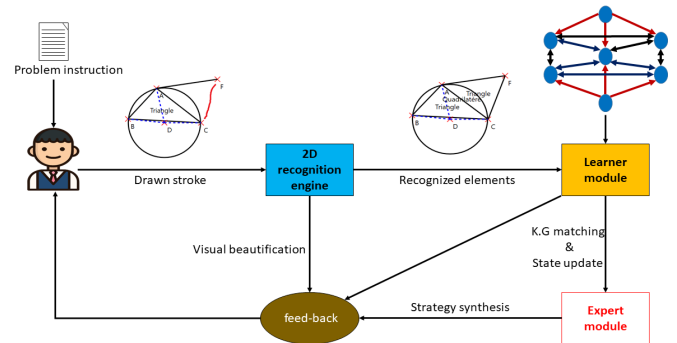


Figure 5: Pupil interaction with the tutor

##### 3.2.3 The expert module

Since all the necessary information about the problem is represented in the K.G. (Knowledge Graph), it is possible to generate alternative strategies to solve an exercise, if we consider the problem as a planning problem, where the initial state is an empty drawing zone, and where the final state is a production/figure where all the mathematical constraints in the links of the K.G. are satisfied. The knowledge here is defined by the set of domain drawing actions that will be the basis of the strategy chosen to solve the problem. More details will be given in Sec. 5.

## 4 The 2D engine

In this section, we present the 2D engine, defined by the CD-CMG grammar [11] and its associated parser.

**Definition 1.** A CD-CMG is a tuple  $G=(V_N, V_T, S, P)$  with:

- $V_N$ : the set of non terminal symbols = symbol classes;
- $V_T$ : the alphabet, here  $V_T = \{\text{stroke}\}$ ;
- $S$ : the first symbol, or axiom;
- $P$ : the set of production rules.

And where a production rule  $p \in P$  is denoted as follows:

$$\alpha \rightarrow \beta \left\{ \begin{array}{l} \text{Preconditions} \\ \text{Constraints} \\ \text{Postconditions} \end{array} \right\} \mid \alpha \in V_N^+, \beta \in (V_T \cup V_N)^+$$

Where preconditions and postconditions are based on the concept of *Document Structural Context* (DSC), which models a zone in the document and the awaited elements in it.

**Definition 2.** A DSC is defined by  $(\lambda)[\text{position}](\gamma)[\text{part}]$  where:

- $\lambda$  is a set of reference elements;
- $\text{position}$  is a zone (i.e. a position) related to  $\lambda$ ;
- $\gamma$  is a set of awaited symbols in this zone;
- $\text{part}$  is a part of the awaited symbol in the zone.

Figure 6 presents the angle production rule. Figure 7 illustrates the roles of the preconditions, postconditions, and constraints blocks. The analysis here is driven by the context. The preconditions, ①, are a set of DSC that model the verification step of the analysis. In this example, the analyzer checks that the two extremities of the stroke  $t$  intersect two existing segments zones (SegmentLength) before checking the constraints block ②. Here, a shape recognizer is called to verify that  $t$  is an arc. The postconditions, ③, are new DSC, or zones, created after the rule reduction that update the document global structure and model the prediction phase. In this example, the production rule of a bisector will be triggered if a segment intersect the new zone [res] [AngleZone].

Based on these definitions, a bi-dimensional analysis process is defined, that is a combination of a bottom-up strategy (guided by the reduced elements) and a top-down strategy (guided by the postconditions DSCs). For each new element, the parser searches the DSC it satisfies and vice versa. A production is triggered if at least one of its preconditions DSC is satisfied and one of its parameters is a new element. The analysis process of a stroke  $t$  is illustrated in figure 8. The root represents the stroke  $t$ . The nodes and the leaves represent the triggered rules. The analysis result is a sequence of reduced productions rules (blue dotted path). The adapted grammar and the associated parser define our 2D engine. This

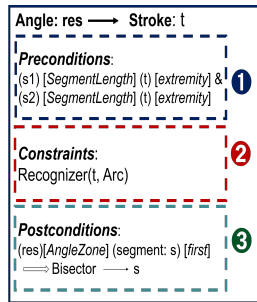


Figure 6: CD-CMG rule

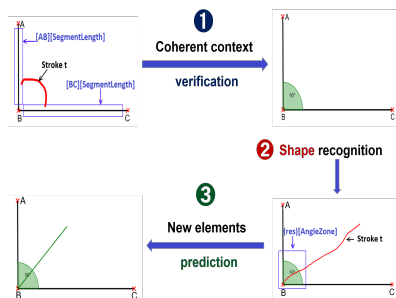


Figure 7: Blocks role

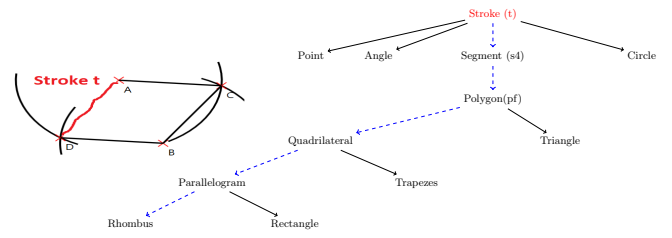


Figure 8: Analysis tree

enables a real-time interaction with user by the interpretation of the handwritten stroke with visual feedback. This engine has been validated in [9], where more details can be found. In the next section, we present the supervision engine, which constitutes the core of this paper.

## 5 The supervision engine

As explained in the previous section, the challenges here are the automated generation of construction problems from the teacher's interpreted strokes, and the supervision of the child's resolution strategies. In this section, we describe in detail the three modules of this engine and their relations.

### 5.1 The problem generation module

This module is responsible for the exercise creation and knowledge graph (K.G.) construction. Once a complete solution example is drawn by the teacher, the module generates the exercise model.

**Definition 3.** The knowledge graph is defined by a set of nodes and edges such as:

- a node  $n$  is defined by
  - $T$ : the recognized element from the 2D engine;
  - $refC$ : reflexive constraints on  $T$ , such as length value;
  - $DefC$ : Constraints depending on the mathematical definition of  $T$ .
- Each edge is denoted by a triple  $e = (Nf, Ns, Rel)$  with
  - $Nf$ : the node corresponding to the tail node;
  - $Ns$ : the node corresponding to the head node;
  - $Rel$ : the set of relative constraints linking  $Ns$  and  $Nf$ .

$DefC$  are directly extracted from the mathematical rules inherent to the geometry domain knowledge. The state of a node, and the state of the K.G. by extrapolation, are defined by the state of the constraints of each node. A node is in the state *complete* if all of its constraints are satisfied. By contrast, it is in the state *partial* if the corresponding geometric element  $T$  has been drawn and at least one of its constraints is not yet satisfied. It is then clear that for the exercise to be solved, all the nodes in the K.G. have to be in the state *completed*. A node is in the *idle state* if its corresponding element is not yet drawn by the pupil. Figure 9 illustrates the process of knowledge graph construction. In this example, the teacher draws two triangles ABC and BCD (see 2. in figure 9). First, the problem generation module constructs the nodes corresponding to the geometric elements present in the document. The  $refC$  constraints are intrinsic to these nodes (e.g.  $\text{length}(AB, 2.5\text{cm})$ ).



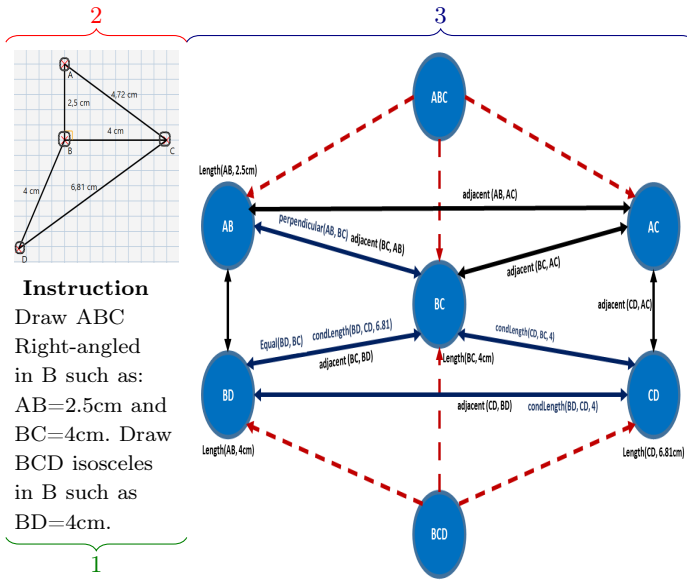


Figure 9: Instruction, Drawn solution, K.G. construction

The logical dependence relations are then added (red arrows in figure 9). These dependence relations enable the module to propagate the *DefC* constraints from a node to its neighbours. For example, BCD isosceles  $\implies$  Equal(BC,BD). Finally, the structural constraints are extracted from the 2D engine, constraints such as adjacency, intersection, that do not depend on a particular figure definition (see 3. in figure 9). Once all this knowledge of the problem is extracted, the generation of the instruction is straight-forward. The problem generation module extracts the necessary and sufficient information from the K.G. to construct the target figure, and translates it to plain text (see 1. in figure 9).

## 5.2 The learner module

The learner module function is to interpret *semantically* the pupil's actions relatively to the resolution of a geometry construction problem. Each stroke, transformed by the 2D engine in real-time into a new geometric element, is fed into the learner module to assess the resolution state of the pupil.

### 5.2.1 Verification and corrective feedback

One basic and necessary function of this module is the verification of the goodness of the pupil's actions. For example, as in the pen/paper setting, the user draws a segment and labels the extremities as point A and point B. The module will simply search the node in K.G. labelled AB, and replace its element *T* by the newly drawn segment. The corrective feedback is then straight-forward, since it is a description of the unsatisfied and satisfied constraints in the K.G. node. In our case, we want to go a step further to fluidify the human-machine interaction by establishing the semantic interpretation of the pupil's actions.

### 5.2.2 Semantic interpretation and matching

Since we are in a free drawing scenario, pupils are not inclined to label their elements each time they draw them. They can

draw a whole figure without naming their points, and add the labels after figure completion, which render the real-time verification obsolete. Therefore, in order to provide feedback on the fly, the module needs to **interpret in real-time what the child intended to draw**, by trying to match the new element with one of the nodes of the Knowledge Graph. This is an interesting and important challenge. To do so, the learner module evaluates the adequacy of the new element with each node of the K.G. and chooses the one with the highest score. Figure 10 presents a step in the resolution of the exercise created in figure 9. The node BC has been already solved by the pupil, since the only constraint to satisfy at that point was the length of BC. The pupil draws a new segment denoted *s1*. The module computes the adequacy score (*Adeg*) depending on the satisfied constraints of each node as follows.

**Definition 4.**  $Adeg(s1, n) =$

$$\begin{cases} 0, & \text{if } class(n) \neq class(s1) \\ 0, & \text{if } |Strut_{sat}(s1, n)| = 0 \\ \frac{|Strut_{sat}(s1, n)| + |Sem_{sat}(s1, n)|}{|Strut(n)| + |Sem(n)|}, & \text{otherwise} \end{cases}$$

such as *Strut* is the set of structural constraints for the node *n*, *Sem* is the set of mathematical/semantic constraints for the node *n*, and  $Strut_{sat}(s1, n)$  is the set of satisfied structural constraints when matching *s1* with the node *n*.

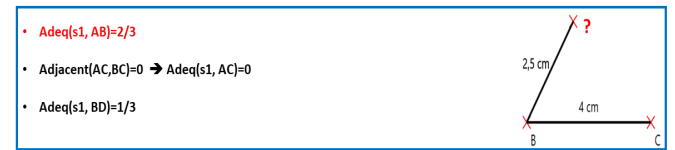


Figure 10: Supervision and matching

We can see in figure 10 an unknown extremity label (? in the figure). When evaluating the adequacy of the new segment *s1* with a node, the learner module matches *s1* with the node label (e.g. ? is labelled A if we are evaluating  $Adeg(s1, AB)$ ).

The node with the highest adequacy score is chosen and the K.G. is updated by replacing the element corresponding to the node by the newly created segment *s1* and updating its state with the satisfied and unsatisfied constraints. In this example, the match is the node AB since *s1* satisfies two of the three constraints of the node (*minus*  $AB \perp BC$ ). After the matching, the corrective feedback is provided to the pupil. The guidance feedback is the responsibility of the expert module. This automated semantic interpretation allows for an efficient pen-based Human Machine Interaction.

## 5.3 The expert module

In order to supervise the pupil's resolution strategies, the tutor must have the necessary geometry domain knowledge to solve automatically these construction problems.

### 5.3.1 Planning domain definition

As mentioned before in Sec.2, a construction problem can be seen as automatic planning problem, where the goal is to

generate a sequence of drawing actions to achieve the target figure. Our approach consists in decomposing the resolution of a construction exercise into an ensemble of sub-problems, such as the resolution of one node is in itself a planning problem. We define our planning environment relatively to each node in the Knowledge Graph. The actions of the pupil (interpreted by the learner module) are seen as exogenous events impacting the planning environment of the expert module, which makes it dynamic. More formally, the definition of this state-transition system is as follows:

**Definition 5.** *Our planning domain is a tuple  $\sigma=(S, A, E, \gamma)$  with:*

- $S=\{\text{idle, partial, complete}\}$ : the set of states for each node in the K.G.;
- $A$ : the set of drawing actions, defining the behaviour of the expert module;
- $E$ : the set of events observed by the learner module;
- $\gamma=S \times (A \cup E) \rightarrow \mathcal{P}^S$  a state transition function. If  $a \in A$  is applicable then the system is in the state  $s'=\gamma(s,a)$ .

The set of actions are mainly ruler-compass based actions, with the possibility of using the protractor to create angles or the try-square tool. Editions actions are also available for the expert module, as well as for the pupil, such as modifying the length of a segment, or the value of an angle.

**Definition 6.** *The actions in our planning domain are defined by a tuple  $a=(\text{parameters, precond, stroke, effects})$  where:*

- *parameters are geometric elements or numeric values;*
- *stroke is the drawing action performed on the document;*
- *preconditions and effects are a set of constraints.*

An example of such actions is as follows.

*Action: MakePerpendicular (Segment: S1, Segment: S2)*

- **precond:** Adjacent(S1, S2);
- **stroke:** OrthoGesture(S1,S2);
- **effects:** Perpendicular(S1,S2).

Here **precond** are the set of constraints that have to be satisfied in the state  $s$  in order for the actions to be applicable. **Stroke** is command gesture realized by the expert on the drawing zone (cf. figure 11, 12). The **effects** are the constraints that will be satisfied in the state  $s'$  after the action is realized. The state transition function  $\gamma$  is then defined as:  $\gamma(s,a) = s \cup \text{effects}(a)$ .

### 5.3.2 Planning algorithm

The planning algorithm is necessary to deliver the correction and guidance to the pupil. The objective is to achieve a goal state  $G$  where all the nodes of K.G. are in the state Complete. Algorithm 1 illustrates this process. The basic principle is that the expert module tries to solve each node on its own. A node is chosen such as its state is not complete. Then the expert searches for the set of applicable actions on this particular node (line 5 in algorithm 1). If the set of applicable actions is

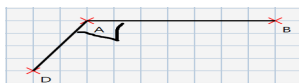


Figure 11: Orthogonal sign

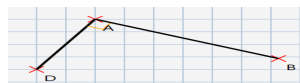


Figure 12: Action result

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### Algorithm 1 Resolution algorithm

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**Require:** State = State of K.G., G=Goal state

```

1: Plan= { }
2: while State != G do
3:   node = n ∈ State | (node.Partial or node.Idle)
4:   while ! node.Complete do
5:     Actions= Ac|∀ ac ∈ Ac, ac.applicable(node)
6:     if applicables = ∅ then
7:       node =chooseAnotherNode(State)
8:       if node=∅ then
9:         Backtrack(State)
10:      end if
11:     else
12:       action= ChooseBestCost(Actions)
13:       State= γ(State,action)
14:     end if
15:   end while
16:   Plan=Plan ∪ action
17: end while

```

---

empty, the node is not solvable at this point and the expert delays its resolution by choosing another one to solve (line 7 in Algorithm 1). For example, we can't draw the hypotenuse [AC] without knowing its length (exercise in figure 9). We should solve the other sides of the triangle first. If there is no more solvable nodes (line 8), we are in an unsolvable state, the expert has then to backtrack to the last state, by undoing the last performed action. This feature is necessary, since it mimics the ability of the pupil to erase an error if it exists. If there are applicable actions for the node  $n$ , the best action is chosen in terms of cost (line 12). The cost of an action is as follows.

$\text{Cost}(a) = |\text{effects}(a)| - \text{effort}(a)$ , where  $\text{effort}$  is the number of drawing operations needed to execute  $a$ . For example, we can consider that the effort of drawing a 60 degree angle with the protractor tool is less than the effort by drawing it by intersecting two compass arcs. The process is repeated until the node is in the state completed. Once it is completed, we repeat the search for another node to solve until all nodes are in the state complete, which means the goal state  $G$  is achieved.

With this expert knowledge, the tutor will be able to guide the pupil **forwards**, with a hint on how to solve the next steps, as well as **backwards**, with a hint to undo the last actions that led him/her to a deadlock. The experiments demonstrating the performance of the tutor are given in the next section.

## 6 Experiments and results

IntuiGeo was tested in five pilot middle-schools in the region of Brittany, France. The experiments were conducted by our partners in ergonomics and cognitive psychology from the LP3C laboratory<sup>2</sup>. We analyze in this section the **performance of the learner module** (is the tutor able to analyze the pupil productions relatively to the instruction?), as well as the **performance of the expert module** (is the tutor able to find solutions plans given any learner state?). 54 pupils

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<sup>2</sup> <https://www.lp3c.fr/>

- were given four exercises with increasing difficulty as follows:
- 1) Draw a rectangle  $ABCD$  given  $AB=4\text{cm}$  and  $BC=9\text{cm}$ ;
  - 2) Draw a parallelogram  $EFGH$  given  $EF=4.5\text{cm}$ ,  $FG=6.5\text{cm}$ , and  $EFG=76^\circ$ ;
  - 3) Draw a rhombus  $ABCD$  given  $AB=4\text{cm}$  and  $AC=7\text{cm}$ ;
  - 4) Draw three triangles such as  $ABC$  is equilateral,  $ABD$  isosceles,  $ADE$  equilateral.

For pedagogical purposes, corrective and guidance feedback are given at the pupil's demand to avoid interference with his/her creative process. Since every action is saved in real-time by the system, we are able to replay exactly, step by the step, the pupil resolution process and feed it to the tutor in a batch fashion. This deployment in the schools had as a result the total of 206 "digital sheets" containing all the saved pupils actions (.e.g handwritten strokes, virtual tools usage) when solving one of the given exercises. This represents the test set we use to analyze our system performance.

## 6.1 The learner module performance

Let's note first that the set is unevenly distributed. This is due to the fact that two versions of the same problem were presented to the pupils in the case of the rectangle and the parallelogram, with a change in the angles values and segments lengths. Only the pupils who had the time to complete these two problems were able to try to solve the triangle and rhombus problems. TABLE 1 presents the supervision performance on each exercise and on the whole set.

Table 1: Learner module performance

| Problem       | Success | Failure | TP | FN | Pre  | Rec    | Acc    |
|---------------|---------|---------|----|----|------|--------|--------|
| Rectangle     | 55      | 20      | 55 | 0  | 100% | 100%   | 100%   |
| Parallelogram | 23      | 71      | 23 | 1  | 100% | 95.65% | 98.94% |
| Rhombus       | 5       | 19      | 5  | 1  | 100% | 83.3%  | 95.8%  |
| Triangle      | 3       | 10      | 3  | 0  | 100% | 100%   | 100%   |
| Total         | 86      | 120     | 86 | 2  | 100% | 97.72% | 99.03% |

The objective here is to see if the learner module represented correctly the state of pupil resolution process: success means the tutor considers that the exercise has been solved by the pupil, failure otherwise. (FN) refers to false negatives, productions that were correct but labelled wrongly by the system as failure. We can observe that Precision (Pre) is 100%, which means the learner module doesn't take risks in assessing the pupil production. However, we can see the presence of 2 false negatives (FN) in the set of 120 negatives (one in the parallelogram set, one in the rhombus set). These FN come from correct strategies that were unknown to the learner module. This will be fixed by enriching the knowledge base of the tutor (.e.g. expanding the mathematical knowledge in the *DefC* constraints and adding new planning domain actions). In any ways, an accuracy (Acc) of 99.03% and a recall (Rec) of 97.72% on a statistically significant set can be considered as an acceptable performance, although there is room for improvement.

## 6.2 The expert module performance

In order to give pertinent and personalized real-time guidance, the expert module should be able to solve the problem at any given learner state. If the state is not solvable, the expert should be able to **backtrack to the last solvable state**

**attained by the pupil**, and propose a solution plan from there. TABLE 2 presents the expert performance on each exercise and on the set of true negatives. *Opt plan* illustrates the

Table 2: Expert module performance

| Problem       | Quantity | Opt plan | Avg plan Length | Avg backtrack | Corrected |
|---------------|----------|----------|-----------------|---------------|-----------|
| Rectangle     | 20       | 6        | 4               | 5             | 100%      |
| Parallelogram | 70       | 5        | 6.49            | 3.86          | 100%      |
| Rhombus       | 18       | 5        | 3.94            | 3.55          | 100%      |
| Triangle      | 10       | 8        | 6.9             | 13.9          | 100%      |

optimal plan length chosen by the expert module to solve each of the exercises from the initial state (empty drawing zone). *Avg plan length* and *Avg backtrack* represent respectively the average length of the correction plan and the average number of backtracking actions performed by the expert module on the "resolution sheets". The expert module was able to provide a correction plan for each data of the set (*Corrected*). This demonstrates that it is always able to find a solution plan, given any learner state, either by direct planning if it is solvable, or by backtracking to the last solvable learner state. The difference between the optimal plan length and the average correction plan length demonstrates on the other hand that the tutor resolution is adaptive to the child production, and doesn't simply backtrack to the initial state and plan from there. This result is quite important, since we need IntuiGeo to provide pertinent and personalized guidance. The fact that the system is able to detect the last solvable state is also interesting, since it could avoid error propagation for the pupils.

## 6.3 Pupils perception and feedback

To see the pupils perception of the tool, and to study its usability, our partners in experimental psychology have conducted a post-test questionnaire. After the experiment, the pupils answered these following questions, illustrated in TABLE 3, with a *Likert scale* [3] from one to seven. The enjoy-

Table 3: Post-test questionnaire results

| Question                 | Mean | Standard deviation |
|--------------------------|------|--------------------|
| Is the tool easy to use? | 5,85 | 1,43               |
| Is the tool useful?      | 4,7  | 1,4                |
| Is the tool enjoyable?   | 5,87 | 1,27               |

ment aspect can be easily explained by the digital support (using a tablet is fun). The fact that the system is easy to use and seen as useful by pupils are also interesting aspects: they show that there is a potential for the tutor to help the pupils enhancing their performances. The evaluation of this potential will be studied in future experiments. More details on the project as a whole, as well as an illustrative video demonstrating the user-tutor interaction, can be found on the IntuiDoc team website<sup>3</sup>.

## 7 Conclusion

In this paper, we propose IntuiGeo, an interactive geometry tutor on pen-based tablets. The combination of pattern recognition with tutoring knowledge allows for our system to be

<sup>3</sup> <https://www-intuidoc.irisa.fr/en/projet-efran-actif/>



quite intuitive. In fact, the teacher only needs to draw one example solution for the tutor to be able to generate the problem model (K.G.), and synthesize alternative resolution strategies. Our learner module interprets in real-time the pupil's actions and assesses his/her knowledge state, allowing an efficient interaction. Our new dynamic planning environment enables the expert module to synthesize strategies in an online fashion, which results in a dynamic and personalized corrective feedback. The experiments showed the ability of our tutor to supervise and guide pupils in the resolution process by synthesizing resolution plans given any learner state. Our future works consist in the improvement of the tutor's knowledge base to include more strategies, as well as a wider variety of construction problems. Another axis will be the study of the pedagogical impact of the tutor on pupils performance, with further experiments that will be conducted in middle-schools.

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