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Numerical solutions for phase noise due to pointing jitter with the LISA telescope

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Abstract

The aim of the Laser Interferometer Space Antenna (LISA) is to detect gravitational waves through a phase modulation in long (2.5 Mkm) laser light links between spacecraft. Among other noise sources to be addressed are the phase fluctuations caused by a possible angular jitter of the emitted beam. The present paper follows our preceding one (Vinet *et al* 2019 *Class. Quant. Grav.* **36**, 205 003) based on an analytical study of the far field phase. We address here a numerical treatment of the phase, to first order in the emitted wavefront aberrations, but without any assumptions on the static bias term. We verify that, in the phase change, the higher order terms in the static mispointing are consistent with the results found in our preceding paper.

1. Introduction

After the successful observations by the ground based gravitational wave (GW) detectors operated by the LIGO Scientific Collaboration and the Virgo Collaboration in US and in Europe [1], a new impulse has been given to an old and ambitious project, supported successively by the NASA and the ESA, called LISA (Laser Interferometer Space Antenna) [2]. LISA aims at detecting GWs in the very low frequency band (milliHertz), where a number of signals are expected, particularly from events involving directly or indirectly supermassive black holes. Continuous signals emitted by binary compact stars or pulsars are also in that low frequency band.

The principle of the LISA Mission [3] is to read the GW signal in the propagation time (or equivalently phase) of a light beam from an emitter laser to a receiver photodetector, both in heliocentric orbits, through a 2.5Mkm path in space. It is clear that addressing a target of meter size at such a distance is by itself a challenge. Among all possible spurious effects able to perturb the measurement, there is thus a possible misalignment of the emitted beam due to any permanent (mechanical) or transient (intrinsic laser jitter) mispointing. We have addressed the question in a recent paper [4], where we used an analytical approach. We have shown that spurious effects are due to the conjunction of three elements:

- Some imperfections (aberrations) in the telescope used to enlarge the laser beam, which is necessary to temper the diffraction at a long distance. These aberrations are scaled by a length parameter σ [m] that expresses how much the emitted wavefront departs from an ideal plane;
- A static pointing error θ_0 ;
- A jitter of the laser beam $\theta_1(t)$.

Analytical approaches are based on a first order theory in σ/λ , which seems reasonable, regarding the present state-of-the-art in mirror technology (stimulated, for example, by ground based GW detectors' demands). In our preceding approach, we furthermore adopted a first order treatment of θ_0 . Higher orders can be addressed, but at the price an increasing complexity of the formulas, reducing their practical interest.

In the present paper, we relax the first order approximation condition on θ_0 , which is possible using numerical methods. This numerical approach allows for the presentation of only some special examples, but

provides a check on how the preceding (analytically obtained) numerical conclusions are relevant. This study should also be useful for any system where a laser beam is emitted and then detected at a very large distance.

The organization of this paper is as follows. In section 2 we present the theoretical derivation for the phase error introduced by static and transient mispointing of the LISA telescope. Section 3 contains the numerical calculation of the phase. A conclusion is given in section 4. Section 5 is an Annex that provides a comparison with the results presented in our previous publication [4].

2. Theory

We consider a special link from an emitter spacecraft (SC) to a receiver SC. The emitter SC emits light of wavelength λ through a telescope of aperture (half-diameter) a . The aim of this telescope is to increase the width of an initial Gaussian beam up to the final Gaussian radius parameter w . (x, y, z) represents the coordinate system in which z is along the light propagation path; if $(x, y, 0)$ are the coordinates in the plane containing the emitting aperture, the emitted amplitude A_0 of the assumed Gaussian beam would be (with $r \equiv \sqrt{x^2 + y^2}$):

$$A_0(x, y, 0) = \exp(-r^2/w^2) \quad (r \leq a), \quad A_0(x, y, 0) = 0 \quad (r > a) \quad (1)$$

under ideal conditions. Unfortunately, the mirrors building the telescope are not perfect and distortions of the phase surface, called aberrations, exist. It has been shown [4] that those defects, coupled with mispointing and fluctuations (jitter) can cause a phase noise at reception. It is essential to assess this noise and compare it to the sensitivity level required by LISA.

We may represent the global wavefront aberration as a phase factor $kF(x, y)$ applied to A_0 , in such a way that the actual emitted amplitude is now

$$A(x, y, 0) = A_0(x, y, 0) \times \exp[ikF(x, y)] \quad (k \equiv 2\pi/\lambda). \quad (2)$$

It is conventional to expand aberrations in a circular pupil in a series of Zernike functions [5]. The Zernike functions are:

$$Z_n^{(m)}(\rho, \phi) = c_n^{(m)} R_n^{(m)}(\rho) \cos(m\phi), \quad (3)$$

$$Z_n^{(-m)}(\rho, \phi) = c_n^{(m)} R_n^{(m)}(\rho) \sin(m\phi), \quad (\rho \equiv r/a), \quad (4)$$

where the $R_n^{(m)}$ are the Zernike polynomials [5]:

$$R_n^{(m)}(\rho) = \sum_{s=0}^{(n-m)/2} (-1)^s \frac{(n-s)!}{s![(n+m)/2-s]![(n-m)/2-s]!} \rho^{n-2s} \quad (5)$$

and

$$c_n^{(m)} \equiv \sqrt{\frac{2(n+1)}{\pi(1+\delta_{m,0})}}.$$

The polar coordinates are defined by $(x = a\rho \cos \phi, y = a\rho \sin \phi)$. An expansion of F in a series of Zernike functions is thus:

$$F(x, y) = \sum_{n,m} c_n^{(m)} [\sigma_{n,m} \cos m\phi + \sigma'_{n,m} \sin m\phi] R_n^{(m)}(r/a) \quad (6)$$

The $\sigma_{n,m}$, $\sigma'_{n,m}$ have the following definitions:

$$\sigma_{n,m} \equiv \int_{\Delta} R_n^{(m)}(\rho) F(x, y) \cos(m\phi) \rho \, d\rho d\phi, \quad (7)$$

and

$$\sigma'_{n,m} \equiv \int_{\Delta} R_n^{(m)}(\rho) F(x, y) \sin(m\phi) \rho \, d\rho d\phi. \quad (8)$$

The σ 's have thus the dimension of a length. Δ is the disk of radius a in the plane $z = 0$. We now consider, as we did in our preceding paper [4], that the beam is emitted with a mispointing error that contains both a static value defined by the angles (θ_0, ψ_0) , and a dynamic jitter defined by angles $(\theta_1(t), \psi_1(t))$. This gives an additional phase factor in equation (2):

$$\exp [ikx(\theta_0 \cos \psi_0 + \theta_1 \cos \psi_1) +iky(\theta_0 \sin \psi_0 + \theta_1 \sin \psi_1)], \quad (9)$$

which finally with $(x \equiv r \cos \phi, y \equiv r \sin \phi)$ leads to the aberrated and mispointed amplitude:

$$A(x, y, 0) = A_0(x, y, 0) \times e^{ikF(x,y)} \times e^{ikr\theta \cos(\phi-\psi)}, \quad (10)$$

where

$$\theta \equiv \sqrt{\theta_0^2 + \theta_1^2 + 2\theta_0\theta_1 \cos \delta\psi}, \quad \psi \equiv \arctan \left[\frac{\theta_0 \sin \psi_0 + \theta_1 \sin \psi_1}{\theta_0 \cos \psi_0 + \theta_1 \cos \psi_1} \right], \quad (11)$$

with $\delta\psi \equiv \psi_0 - \psi_1$. Now, if we consider the far propagated field amplitude $B(x, y, L)$ at a distance L , it is well known that it amounts (Fraunhofer regime) to computing the Fourier transform of $A(x, y, 0)$ namely:

$$B(x, y, L) = -\frac{i}{\lambda L} \exp \left[i\pi \frac{r^2}{\lambda L} \right] \int_{\mathbb{R}^2} e^{ipx' + iqy'} A(x', y', 0) dx' dy' \quad (12)$$

with ($p \equiv kx/L$, $q \equiv ky/L$). For a geometry like that for LISA where the beam exiting one telescope has a waist $\sim a$, and the beam observed by the receiving telescope has an aperture radius also of a , the condition for the amplitude of the electric field to be uniform to better than $x\%$ across this area is

$$z > \frac{10^{4/3} \pi a^2}{\lambda x^{2/3}}. \quad (13)$$

At the very long distance for the LISA arms, $L \sim 2.5$ Mkm, the amplitude inside a disk of radius a (~ 15 cm), is practically constant and given simply by

$$B(0, 0, L) = -\frac{i}{\lambda L} \int_{\mathbb{R}^2} A(x, y, 0) dx dy, \quad (14)$$

or, after a change $(x, y) \rightarrow (r, \phi)$:

$$B(0, 0, L) = -\frac{i}{\lambda L} \int_0^\infty r dr \int_0^{2\pi} d\phi A_0(r, \phi, 0) e^{ikF(r, \phi)} e^{ikr\theta \cos(\phi - \psi)}. \quad (15)$$

If we assume very weak aberration amplitudes ($\sigma_{n,m}, \sigma'_{n,m} \ll \lambda$), we can expand to first order the first exponential factor, and write:

$$B(0, 0, L) = -\frac{i}{\lambda L} \int_0^\infty r dr \int_0^{2\pi} d\phi e^{ikr\theta \cos(\phi - \psi)} A_0(r, \phi, 0) [1 + ikF(r, \phi)] \quad (16)$$

which yields

$$B(0, 0, L) = -\frac{ika^2}{\lambda L} \left\{ D + ik \sum_{n,m} i^m c_n^{(m)} [\sigma_{n,m} \cos(m\psi) + \sigma'_{n,m} \sin(m\psi)] N_{n,m} \right\} \quad (17)$$

where

$$D(\Omega, \nu) \equiv \int_0^1 J_0(\Omega\rho) e^{-\nu\rho^2} \rho d\rho \quad (18)$$

with $\Omega \equiv ka\theta$, $\nu \equiv a^2/w^2$, and

$$N_{n,m}(\Omega, \nu) \equiv \int_0^1 J_m(\Omega\rho) e^{-\nu\rho^2} R_n^{(m)}(\rho) \rho d\rho, \quad (19)$$

the J_m being the Bessel functions of the 1st kind. The parameter w (the telescope's aperture a being fixed) has an optimum value resulting from a compromise between diffraction losses (too small w) and clipping losses (too large w). The optimum value (see [4]) is $a/w = 1.12$. However clipping losses result in scattered light issues which are attenuated by taking a slightly suboptimal value $a/w = 1.5$. This is why in this sequel article we compare the results obtained with these two possible options. From equation (17) we see that (to first order in F/λ) only even terms in m will contribute to the phase of $B(0, 0, L)$, and thus both m and n are even. Eventually the spurious phase is:

$$\delta\Phi = k \sum_{n,m} (-1)^m c_{2n}^{(2m)} [\sigma_{2n,2m} \cos(2m\psi) + \sigma'_{2n,2m} \sin(2m\psi)] \frac{N_{2n,2m}(\Omega, \nu)}{D(\Omega, \nu)} \quad (20)$$

3. Numerical

As described above, for the evaluation of the noisy phase we essentially have to compute:

$$G_{n,m}(\Omega, \nu) \equiv \frac{N_{2n,2m}(\Omega, \nu)}{D(\Omega, \nu)} \quad (21)$$

Integrals of the kind (equations (18) and (19)) can be easily numerically integrated by Simpson's rule. An excellent precision is reached with a sampling rate of 1000. We have checked that integrals similar to the ones in equations (18) and (19), but for which we have an analytical expression, can be computed with a relative accuracy

much better than 10^{-9} . We recall that the parameter Ω is $\Omega = ka(\theta_0 + \theta_1 \cos \delta\psi) = \Omega_0 + ka\theta_1 \cos \delta\psi$, (see equation (11) at first order in θ_1) where $\Omega_0 = ka\theta_0$ is due to the static mispointing (we use the value 700 nRad as in [4]) and θ_1 the (much smaller) jitter. In order to obtain a first order expansion in $ka\theta_1$, we need the derivative G' (with respect to Ω) of $G = N/D$ for $\Omega_0 \sim 0.62$. Owing to the well known properties of Bessel functions, we have

$$N'_{2n,2m}(\Omega_0, \nu) \equiv \frac{\partial N_{2n,2m}}{\partial \Omega}(\Omega_0, \nu) = - \int_0^1 J_{2m+1}(\Omega_0 \rho) R_{2n}^{(2m)}(\rho) e^{-\nu \rho^2} \rho^2 d\rho + \frac{2m}{\Omega_0} \int_0^1 J_{2m}(\Omega_0 \rho) Z_{2n}^{(2m)}(\rho) e^{-\nu \rho^2} \rho d\rho$$

and

$$D'(\Omega_0, \nu) \equiv \frac{\partial D}{\partial \Omega}(\Omega_0, \nu) = - \int_0^1 J_1(\Omega_0) e^{-\nu \rho^2} \rho^2 d\rho,$$

so that the coefficient we need to evaluate the spectral density of the noise due to the jitter is determined (for Zernike indices $(2n, 2m)$) by:

$$\frac{\partial}{\partial \Omega} \delta\Phi_{2n,2m}(\Omega_0) = \left[\frac{\sigma_{2n,2m}}{\lambda} \cos 2m\psi + \frac{\sigma'_{2n,2m}}{\lambda} \sin 2m\psi \right] \times 2\pi c_{2n}^{(2m)} ka G'_{n,m} \theta_1$$

with (see equation (21)) $G'_{n,m} = (N'_{2n,2m} D - N_{2n,2m} D')/D^2$. The desired spectral density of the noise $S_{\delta\Phi}^{1/2}$ due to the jitter θ_1 is related to the spectral density $S_{\theta_1}^{1/2}$ by:

$$S_{\delta\Phi}^{1/2}(f) = \sum_{n,m} \left[\frac{\sigma_{2n,2m}}{\lambda} \cos 2m\psi + \frac{\sigma'_{2n,2m}}{\lambda} \sin 2m\psi \right] k \gamma_{2n}^{(2m)}(\Omega_0) \times S_{\theta_1}^{1/2}(f)$$

with the scaling lengths:

$$\gamma_{2n}^{(2m)}(\Omega_0) \equiv 2\pi c_{2n}^{(2m)} a G'_{n,m}.$$

If now we need the spectral density of the equivalent displacement δL , we simply have

$$S_{\delta L}^{1/2}(f) = \frac{\lambda}{2\pi} S_{\delta\Phi}^{1/2}(f) = \sum_{n,m} \left[\frac{\sigma_{2n,2m}}{\lambda} \cos 2m\psi + \frac{\sigma'_{2n,2m}}{\lambda} \sin 2m\psi \right] \gamma_{2n}^{(2m)}(\Omega_0) \times S_{\theta_1}^{1/2}(f).$$

Table 1 gives the coefficients $\gamma_{2n}^{(2m)}(\Omega_0)$ for the first Noll indices (units are [m]). Noll indices are frequently employed to have a one-index list of Zernike functions [6]. We give the corresponding (n, m) Zernike indices. The coefficients are computed for the two reference values of a/w : 1.12 and 1.50.

We also show in figure 1 the same data plotted on a logarithmic scale. Figure 2 gives an idea of the dependence of the most significant scale factors $\gamma_{2n}^{(2m)}$ with respect to the ratio a/w .

In the very worst case, if all aberrations accumulate with the same magnitude σ and identical signs, and assuming $\psi = 0$, the resulting global scaling factor would be:

$$\gamma \equiv \sum_{n,m \geq 0} \gamma_{2n}^{(2m)}.$$

The sum is formally infinite, but converges in practice because of the rapid decrease of the $\gamma_{2n}^{(2m)}$. For example the result obtained for Noll indices $N \leq 28$ and with a ratio $a/w = 1.12$ is $\gamma \sim 0.13\text{m}$; with a ratio $a/w = 1.5$ the result is similar.

With a noise spectral density of angular jitter $S_{\theta_1}^{1/2} \sim 10 \text{ nRd}/\sqrt{\text{Hz}}$, the spectral density of wavefront displacement would be roughly:

$$S_{\delta L}^{1/2}(f) = \frac{\sigma}{\lambda} \times \gamma \times 10^{-8} \text{m}/\sqrt{\text{Hz}} \simeq \frac{\sigma}{\lambda} \times 1300 \text{ pm}/\sqrt{\text{Hz}}, \tag{22}$$

slightly larger than the result given in [4], equation (48), where the factor was $1200 \text{ m}/\sqrt{\text{Hz}}$ instead of $1300 \text{ m}/\sqrt{\text{Hz}}$. See in the following Annex, section 5, a detailed comparison between the present numerical results and the estimations given in our preceding paper [4].

We can also compare our order of magnitude calculations with the estimates given by Sasso *et al* [7]. Their estimate, given by their table 3, is $\delta L = 0.07 \text{ pm/nrad}$ for an amplitude of Zernike of $\lambda/20$. If we look at our equation (22), and ignoring the spectral densities, with $\sigma/\lambda \sim 1/20$, one has $\delta L \sim 65 \text{ pm}$ for $\theta_1 = 10 \text{ nrad}$, or 6.5 pm/nrad , which is two orders of magnitude higher. Also, we have used 700 nrad for the static mispointing where a value of 10 nrad is used in [7]. However we must keep in mind that we have cumulated all aberrations in a worst case scenario, whereas their approach is based on Monte-Carlo methods, in which some defects can randomly cancel others.

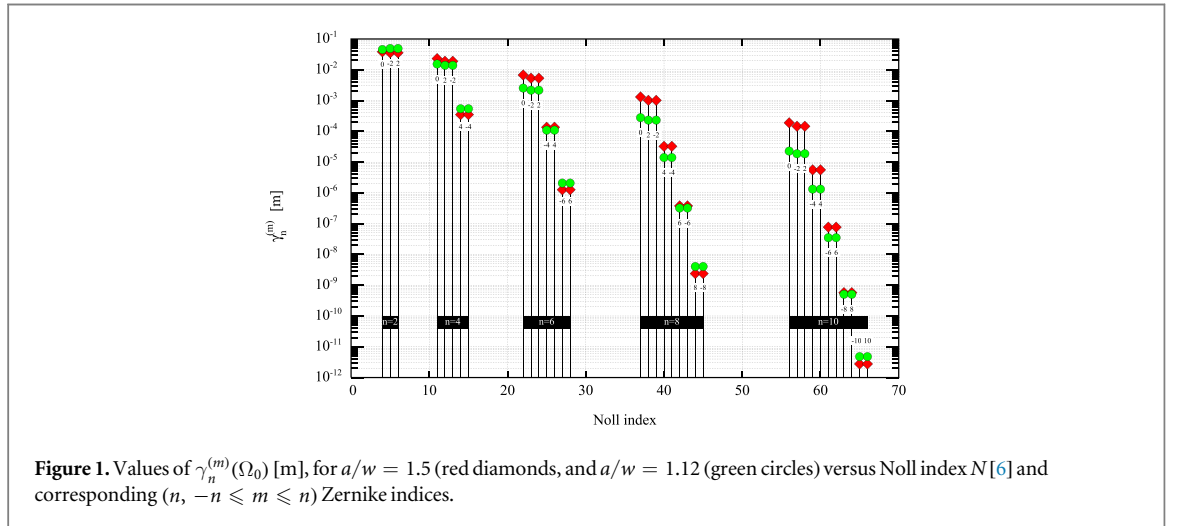


Table 1. Scaling lengths $\gamma_n^{(m)}$ (units = [m]) for Noll indices N [6] equivalent to Zernike indices (n, m) for two different a/w ratios.

N :	(n, m)	$\gamma_n^{(m)}$ [m] $a/w = 1.12$	$\gamma_n^{(m)}$ [m] $a/w = 1.5$
4	(2, 0)	0.4553E-01	0.3843E-01
6	(2, 2)	0.4913E-01	0.3558E-01
11	(4, 0)	0.1537E-01	0.2304E-01
12	(4, 2)	0.1391E-01	0.1889E-01
14	(4, 4)	0.5422E-03	0.3494E-03
22	(6, 0)	0.2542E-02	0.6750E-02
24	(6, 2)	0.2169E-02	0.5327E-02
26	(6, 4)	0.1091E-03	0.1356E-03
28	(6, 6)	0.2078E-05	0.1266E-05
37	(8, 0)	0.2783E-03	0.1307E-02
38	(8, 2)	0.2313E-03	0.1015E-02
40	(8, 4)	0.1408E-04	0.3251E-04
42	(8, 6)	0.3214E-06	0.3772E-06
44	(8, 8)	0.4053E-08	0.2389E-08
56	(10, 0)	0.2276E-04	0.1891E-03
58	(10, 2)	0.1869E-04	0.1458E-03
60	(10, 4)	0.1324E-05	0.5589E-05
62	(10, 6)	0.3504E-07	0.7695E-07
64	(10, 8)	0.5068E-09	0.5723E-09
66	(10, 10)	0.4785E-11	0.2762E-11

4. Conclusion

It appears that the estimations based on an expansion in Ω_0 limited to the second order, as presented in our preceding paper [4], slightly underestimate the noise by a few percent for the first Zernike contributions. The global result obtained for worst case conditions is also slightly higher, due to contributions of several (small) higher order terms not taken into account in our preceding theoretical presentation. On the other hand, our preceding paper gave no estimations of scaling factors values for $m \geq 4$, whereas the numerical treatment presented here does, showing that those values are much smaller for higher order Zernike $n > 2$, as expected.

The attitude jitter of a SC can induce a noise in the distance determination of the associated laser link. This is not the only example of tilt-to-length coupling, which can also arise in the interferometric system for a LISA test mass because of SC jitter with respect to the reflected laser beam from the test mass. In addition, noise can come from the jitter at the local SC with respect to the laser beam from a far SC [8]. Consequently, wavefront error, as discussed in this article, contributes only to a part of the total tilt-to-length coupling for LISA.

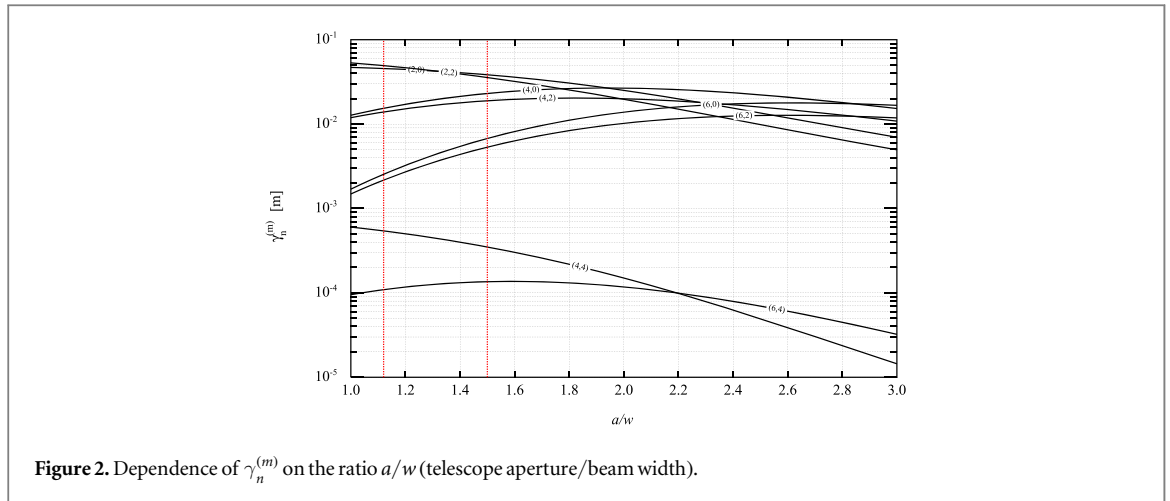


Figure 2. Dependence of $\gamma_n^{(m)}$ on the ratio a/w (telescope aperture/beam width).

The subject of wavefront errors and beam jitter in the LISA optical system has, of course, been the subject of other studies [7, 9]. For many years this has been recognized as an important problem for LISA [10].

As discussed in this paper, the analysis and study of the far-field laser phase and intensity are critical for ensuring that the LISA GW detector operates at its desired sensitivity. The numerical results presented in this paper extend our previous analytic work [4], and help to display how aberrations in the LISA telescope plus pointing errors can create phase noise after the beam has traveled 2.5 Mkm.

5. Annex

In order to compare the numerical results presented here with the analytic results of our preceding paper [4], simply take the very first Noll indices [6] and a ratio $a/w = 1.5$. For $n = 2, m = 0$, and with a noise spectral density of angular jitter of $10 \text{ nRad}/\sqrt{\text{Hz}}$, we find

$$S_{\delta L, (2,0)}^{1/2}(f) = \frac{\sigma_{2,0}}{\lambda} \times 3.843 \times 10^{-2} \times 10^{-8} \text{ m}/\sqrt{\text{Hz}} = \frac{\sigma_{2,0}}{\lambda} \times 3.843 \times 10^{-10} \text{ m}/\sqrt{\text{Hz}}.$$

In our preceding paper [4], we had (equation (47)) with a coefficient $\alpha_1 = 2.425$:

$$S_{\delta L, (2,0)}^{1/2}(f) = \frac{\sigma_{2,0}}{\lambda} \times 1.55 \times 10^{-10} \times 2.425 \text{ m}/\sqrt{\text{Hz}} = \frac{\sigma_{2,0}}{\lambda} \times 3.759 \times 10^{-10} \text{ m}/\sqrt{\text{Hz}}.$$

For $n = 2, m = 2$, the same way:

$$S_{\delta L, (2,2)}^{1/2}(f) = \frac{\sigma_{2,2}}{\lambda} \times 3.558 \times 10^{-2} \times 10^{-8} \text{ m}/\sqrt{\text{Hz}} = \frac{\sigma_{2,2}}{\lambda} \times 3.558 \times 10^{-10} \text{ m}/\sqrt{\text{Hz}},$$

in our preceding paper, with a coefficient $\beta_1 = 2.247$:

$$S_{\delta L, (2,2)}^{1/2}(f) = \frac{\sigma_{2,2}}{\lambda} \times 1.55 \times 10^{-10} \times 2.247 \text{ m}/\sqrt{\text{Hz}} = \frac{\sigma_{2,2}}{\lambda} \times 3.483 \times 10^{-10} \text{ m}/\sqrt{\text{Hz}}.$$

It can be seen that the results are very similar, up to a few percent. This small difference is due to a better (numerical) evaluation of integrals involving Bessel functions.

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