



HAL
open science

An asynchronous double auction market to study the formation of financial bubbles and crashes

Sadek Benhammada, Frédéric Amblard, Salim Chikhi

► **To cite this version:**

Sadek Benhammada, Frédéric Amblard, Salim Chikhi. An asynchronous double auction market to study the formation of financial bubbles and crashes. *New Generation Computing*, 2017, 35 (2), pp.129-156. 10.1007/s00354-017-0010-6 . hal-02538633

HAL Id: hal-02538633

<https://hal.science/hal-02538633v1>

Submitted on 9 Apr 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Open Archive Toulouse Archive Ouverte


OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible

This is an author's version published in: <http://oatao.univ-toulouse.fr/22278>

Official URL:

<https://doi.org/10.1007/s00354-017-0010-6>

To cite this version:

Benhammada, Sadek and Amblard, Frédéric  and Chikhi, Salim *An asynchronous double auction market to study the formation of financial bubbles and crashes.* (2017) *New Generation Computing*, 35 (2). 129-156. ISSN 0288-3635.

Any correspondence concerning this service should be sent to the repository administrator: tech-oatao@listes-diff.inp-toulouse.fr

An Asynchronous Double Auction Market to Study the Formation of Financial Bubbles and Crashes

Sadek Benhammada¹ · Frédéric Amblard² ·
Salim Chikhi¹

Abstract Stock market is a complex system composed from heterogeneous traders with highly non-linear interactions from which emerge a phenomenon of speculative bubble. To understand the role of heterogeneous behaviors of traders and interactions between them in the emergence of bubbles, we propose an agent-based model of double auction market, with asynchronous time management, where traders act asynchronously and take different times to make decisions. The market is populated by heterogeneous traders. In addition to fundamentalist, noise, and technical (chartist) traders, we propose a hybrid trader, which can switch between technical (chartist) and fundamentalist strategies integrating panicking behavior. We find that when market is populated by a majority of hybrid traders, we observe quite realistic bubble formation characterized by a boom phase when hybrid traders switch to technical behavior, followed by a relatively shorter burst phase when hybrid traders return to fundamentalist strategy and change to panicked state. The aim is to design agents which act asynchronously, with simple behaviors, but complex enough to produce realistic price dynamics, which provide a basis for developing agents with sophisticated decision-making processes.

Keywords Artificial stock market · Multi-agent simulation · Agent-based computational economic

✉ Sadek Benhammada
s.benhammada@centre.univ.mila.dz

Frédéric Amblard
frederic.amblard@ut-capitole.fr

Salim Chikhi
chikhi@umc.edu.dz

¹ MISC Laboratory, Computer Science Department, University of Constantine 2, Constantine, Algeria

² UMR 5505 CNRS IRIT, Université Toulouse 1 Capitole, Toulouse, France

Introduction

Nowadays, financial markets play a central role in the economies across the world. Therefore, studying and understanding their underlying mechanisms are a major scientific and societal challenge. One of the traditional approaches to study such systems consists in using analytical models. However, this approach has already shown its limitations. In fact, most analytical models make unrealistic assumptions such as the perfect rationality of homogeneous operators implied by the efficient market hypothesis [16], which threatens the validity of their results and motivates the proposal of alternative methods [31]. In this context, we suggest to use artificial stock markets (ASM), as an attempt to provide a better understanding of market dynamics by computer simulations based on multi-agent systems. Models such as artificial stock markets are designed to capture the essential properties of real stock markets to reproduce, analyze, and understand market dynamics through computational experiments [14]. Economic agents are represented by (software) agents interacting in an artificial environment. Since the first artificial stock market was developed in the early nineties at the Santa-Fe Institute [36], many market models have been developed. Though most of them aim at understanding market dynamics with the same multi-agents simulation methods, various artificial stock markets proposed are created using different strategies and are customized for specific requirements. Some are synchronous, whereas others are asynchronous. Some are populated by homogeneous traders, whereas others are populated by different heterogeneous traders. Good and recent overview of ASMs can be found in [8, 10, 27, 28].

The formation of speculative bubbles is an important phenomenon that occurs in stock markets. It is typically understood as an increase in prices which deviates from fundamental values, followed by sudden drop in prices called crash. Kindleberger and Aliber [25] define a bubble as “any significant increase in the price of an asset or a security or a commodity that cannot be explained by the fundamentals”. Similarly, Barlevy [3] highlighted that most economists would define a bubble “as a situation where an asset’s price exceeds the fundamental value of the asset”. [40] affirm that “a financial bubble is a temporary situation where prices become elevated beyond any realistic fundamental valuations”.

The importance of studying speculative bubble formation comes from the fact that they are an immediate cause for financial crises [43]. Research on financial bubbles has a rich literature of theoretical studies ([6, 15] and others), as well as experimental studies where [38] proposed a standard design for experimental asset-market to perform empirical studies of bubbles in laboratory, after which a proposed standard design was used in many works to perform experimental studies of bubble such as in [18, 29] and others.

Experimental studies are performed with of number of investors in laboratory endowed with the initial cash and a number of assets. Investors can buy and sell assets during a defined number of periods. This method has limitations on the number of traders, the cash available to traders, traded assets, and duration of experiments. Artificial markets do not have these limitations and can present an

alternative. In the literature, we find some works that use artificial markets to study the phenomenon of speculative bubble. In [19], authors propose a model inspired from the Santa Fe Artificial Stock Market and the Minority Game with agents who can choose between different strategies, authors found in their experiments the appearance of an oscillating phase with bubbles and crashes. In [13], authors developed a simplified fundamentalists and technical trader. They found that when technical agents represent the main part of the agents' population, during some periods, the system reaches a critical state characterized by an increase in prices that deviates from fundamental values (bubble) before falling back violently to its fundamental value (crash). In [24], they found that the market with agents who follow prospect theory and exponential discounting tended to cause a bubble and crash. However, models used in these latter works are inspired from the original Santa Fe artificial stock market [36], which is a discrete time and synchronous model. The market evolves with a continuous time and traders act asynchronously by sending independently buy and sell orders. However, [7, 34] have shown that prices dynamics of discrete-time models are significantly different from those of continuous-time models; consequently, the asynchronous nature of trading in real markets should be explicitly considered in agent-based models.

In addition, one of the bubble characteristics is that the boom duration is longer than burst duration (see "[Measure of bubbles magnitude](#)" for more details). Thus, time simulation is essential to compare duration of boom and burst phases.

In addition, these works have not used measures of bubbles magnitude to analyze results of experiments, such those used in laboratory stock market experiments, e.g., [12, 21]. Those measures of bubbles magnitude are crucial to be more precise and complete in analysis of results.

To understand the role of heterogeneous behaviors of traders and interactions from which emerge a phenomenon of speculative bubble, we propose an agent-based model of double auction market, with asynchronous time management, when traders act asynchronously and take different times to make decisions, thus we can study time-related properties of bubbles. We develop heterogeneous agents, namely technical traders (also called chartists or speculators) which use historical prices of an asset to forecast its future trends and make decisions, fundamental traders form rational expectations about the fundamental value of an asset, and make decisions according to this value, noise traders representing traders unable to estimate the fundamental value of an asset. In addition to these classical types in literature, we propose a hybrid trader, which can switch between technical and fundamental strategies, integrating panicking behavior.

To evaluate the model, we perform a series of experiments, we analyze statistical properties of generated prices series, and we compare them with real price series. We use some measures used in experimental economy to analyze the magnitude of bubbles in performed experiments.

This paper is organized as follows. Section "[Description of the model](#)" presents a description of the proposed model, its components, description of different traders' behavior, and choices regarding time management. In "[Results and discussion](#)", we perform a series of experiments and we discuss the results. Finally, "[Conclusions](#)" section concludes and outlines open research directions.

Description of the Model

We choose to propose a model of double auction market, because it is the most widely used method of price formation in modern financial markets as well as the dominant institution for the real-world trading of equities, energy, derivatives, etc. [45].

The proposed model has three main components, (i) *Market* which allows buyers and sellers of assets to interact to facilitate transactions; (ii) *Traders* who exchange assets through the market; and (iii) *External world* which generates fundamental values of assets used by traders to make decisions.

The Market

The role of market is receiving and executing orders placed by traders using orderdriven system. An order $x = (d_x, q_x, p_x, t_x)$ submitted by one trader is described by a direction d_x (buy or sell), a quantity q_x of assets to buy or sell, a quoted price p_x , and a time of arrival in the order book t_x . In a buy order, quote price p_x is the maximum limit price above which the order should not be executed. Conversely, in a sell limit order, p_x specifies a minimum limit price under which the order should not be executed. The unexecuted orders are placed in the *order book*. At a specific time t , the order book is described by two lists, the list of buy orders $B_t = (b_n, \dots, b_2, b_1)$, and the list of sell orders $S_t = (s_1, s_2, \dots, s_m)$. Where b_i represents buy limit orders (bids) and s_j sell limit orders (offers or asks). They are all waiting in a queue to eventually get executed. The list of buy orders B_t is arranged in decreasing order of prices (i.e., $p_{b_n} \leq \dots \leq p_{b_2} \leq p_{b_1}$), while the list of sell orders S_t is arranged in increasing order of price (i.e., $p_{s_1} \leq p_{s_2} \leq \dots \leq p_{s_m}$). The highest bid p_{b_1} , also called best bid, and the lowest offer p_{s_1} , or best offer, define the spread $(p_{s_1} - p_{b_1})$. b_1 will be executed only if the book receives a limit sell order s with a lower quoted price (i.e., $p_s \leq p_{b_1}$). Idem, s_1 , will be executed only if the book receives a limit buy order b with a higher quoted price (i.e., $p_b \geq p_{s_1}$).

If a submitted order finds a matching order of the opposite side in the book, a trade is generated. When a trade is generated, the Market (i) deletes satisfied orders from the order book (orders can be satisfied completely or partially (depending on quantity), so the unsatisfied part of a partially satisfied order is placed in the book), (ii) calculates the amount of a transaction, and (iii) notifies corresponding buyer and seller. Once notified, the buyer makes payment and receives assets. Conversely, the seller delivers assets and receives payment.

External World

The fundamental value is the expected discounted value of the asset dividend stream [35]. It is expressed as follows:

$$FV(t) = \sum_{h=t}^{\infty} \frac{d(h)}{(1+r)^{(h-t)}} \quad (1)$$

where $d(h)$ is a dividend at time h , and r the risk-adjusted discount rate.

To estimate the fundamental value of the asset, traders use the information coming from the external world; in this case, the estimated fundamental value can be considered as a quantification of news with respect to the stock traded [7].

In our model, the role of *External world* is to generate fundamental values of the asset that will be used by traders to make decisions. In our experiments, we will use different jump processes to simulate fundamental values. Thus, to generate fundamental values without trend, we use the following process:

$$Fv(t+1) = Fv(t) + \omega_t \quad (2)$$

where $\omega_t \sim \mathcal{N}(0, \sigma)$ is a white noise from a normal distribution with mean 0 and variance $\sigma^2 = 1$.

To generate fundamental values with trend, we use following process:

$$Fv(t+1) = Fv(t) + b + \omega_t \quad (3)$$

where $b > 0$ is a constant.

Traders

At each time t , each trader $i \in \{1, 2, \dots, N\}$, has some number of assets $q_i(t)$ and units of cash $c_i(t)$ which compose his wealth $W_i(t)$ calculated as follows:

$$W_i(t) = q_i(t) \cdot p(t) + c_i(t), \quad (4)$$

where $p(t)$ is the market price of an asset at time t .

To make a decision, each trader i is able to ask the *Market* some information about its current state (prices history and order book), he can also ask *External world* if his decision-making process requires exogenous information, which can be used to estimate a fundamental value $f_{v_i}(t)$ of the asset at time t . Traders then can use prices and fundamental values' history to make decision. Three types of decisions are allowed in the ASM: (1) submit a buy order, (2) submit a sell order, and (3) wait. A trader i can borrow cash to buy assets, i.e., $c_i(t) \leq 0$ is allowed. However, short-selling is not allowed, i.e., trader i cannot sell if $q_i(t) \leq 0$. In finance, a trading strategy is a predefined set of rules for making trading decisions. An investor uses trading strategy to assist in making wiser investment decisions and eliminating the emotional aspect of trading [11]. In artificial stock market literature, we can find different types of traders according to their trading strategies. We will describe each type of traders that we integrated into our artificial stock market. We chose to develop agents behaviors with limited cognitive ability, to determine minimal conditions leading to the emergence of speculative bubbles. To the exception of hybrid traders that we propose in this paper, the three other types of traders are often encountered in literature; however, their implementations can vary considerably.

Noise Trade

The noise traders will make a decision to submit a buy order, a sell order or wait with corresponding probabilities $Prob_{buy}$, $Prob_{sell}$ and $Prob_{wait}$, respectively. Such probabilities are defined as the parameter of the simulations and remain constant during a given simulation experiment. If a trader i decides to submit a buy or sell order denoted by $x = (d_x, q_x, p_x, t_x)$, the limit price p_x is calculated from the current market price $p(t)$ as follows:

$$p_x = p(t) + p(t) * \varepsilon_i(t) \quad (5)$$

$\varepsilon_i(t) \sim \mathcal{U}(-\alpha_i, +\alpha_i)$,¹ where α_i is the maximal difference rate between current price and limit price for trader i . The desired quantity $q_x \sim \mathcal{U}(1, Qmax_i)$, where $Qmax_i$ is the maximum quantity that a trader i may request to buy or offer to sell.

Fundamental Trade

The basic idea behind the strategy of such traders is that they form rational expectations about the fundamental value of the asset and change their decision according to this value [31]. The decision rules of the fundamentalist are shown in Table 1. To make decision, a fundamental trader, i , estimates the fundamental value of the asset as follows:

$$fv_i(t) = FV(t) + FV(t) * \eta_i(t) \quad (6)$$

where $FV(t)$ is a correct fundamental value of asset received from external world $\eta_i(t) \sim \mathcal{U}(-\theta_i, \theta_i)$, where θ_i is the error margin of trader i in estimating the fundamental value.

If $p(t) < fv_i(t)$, the fundamental trader i considers that assets are undervalued, and so decides to submit a buy order $b = (d_b, q_b, p_b, t_b)$, the corresponding limit price is calculated as follows:

$$p_b = p(t) + \delta_i(t) \quad (7)$$

where $\delta_i(t) \sim \mathcal{U}(p(t), fv_i(t))$;

If $p(t) > fv_i(t)$, fundamental trader i considers that asset is overestimated and places a sell order $s = (d_s, q_s, p_s, t_s)$, with the price:

$$p_s = p(t) - \delta_i(t) \quad (8)$$

where $\delta_i(t) \sim \mathcal{U}(fv_i(t), p(t))$.

Technical Trader

Speculation is traditionally defined as *The purchase (or sale) of goods aims at reselling them at a later date, where the motivation behind such action is the expectation of a change in the relevant prices relatively to the ruling price and not a*

¹ The notation $a \sim \mathcal{U}(x, y)$ expresses that a is a uniformly selected in $[x, y]$.

Table 1 Decision rules of the fundamental trader

State of the market	Decision type	Limit price
$p(t) < fv_i(t)$	Submit buy order b	$p_b \quad p(t) + \delta_i(t)$
$p(t) > fv_i(t)$	Submit sell order s	$p_s \quad p(t) - \delta_i(t)$
$p(t) = fv_i(t)$	Wait	

gain accruing through their use, or any kind of transformation effected in them, or their transfer between markets [23]. Traders who use this strategy are called technicals, chartists, or speculators. We develop a simplified technical trader based on historical prices analysis for forecasting the direction of prices, and estimating market liquidity. The decision rules of the technical are shown in Table 2. To analyze the trend of prices, technical trader i use moving average [31] defined as follows:

$$MA(L, t) = \frac{p(t) - \left(\frac{1}{L} \sum_{i=1}^L p(t-i)\right)}{\frac{1}{L} \sum_{i=1}^L p(t-i)} \quad (9)$$

where t is the current time and L is a period length.

To estimate market liquidity, technical i estimate effective half spread [5], which is based on the actual trade price and estimated fundamental value. Effective half spread at time t ($ES_i(t)$) is calculated as follows:

$$ES_i(t) = \frac{p(t) - fv_i(t)}{fv_i(t)} \quad (10)$$

where $p(t)$ is the last price of the asset at time t , and $fv_i(t)$ is the estimated fundamental value by trader i at time t (Eq. 6)

To make a decision, a technical trader i calculates the moving average (Eq. 9), and compares it to a threshold β_i . Trader i estimates also the liquidity (Eq. 10) and compares it with $MaxES_i$ which is the threshold above which market is considered not sufficiently liquid. If $MA(L, t) > \beta_i$ and, $ES_i(t) < MaxES_i$, trader i , therefore, assumes that prices increase, and market has sufficient liquidity, and decides to submit a buy order $b = (d_b, q_b, p_b, t_b)$. The limit price is calculated as follows:

$$p_b = p(t) + p(t) * \tau_i(t) \quad (11)$$

where $\tau_i(t)$ is randomly chosen from a uniform distribution in the range $[0, \alpha_i]$ (i.e., the limit price of buy order is slightly higher than the current price). When trader i assumes that price decreases ($MA(L, t) < \beta_i$) or market liquidity ($ES_i(t) > MaxES_i$), he decides to submit sell order $s = (d_s, q_s, p_s, t_s)$. The corresponding limit price is calculated as follows:

$$p_s = p(t) - p(t) * \tau_i(t) \quad (12)$$

i.e., the limit price of sell order is slightly lower than the current price. Otherwise, trader i decides to wait for a given time.

Table 2 Decision rules of the technical trader

State of the market	Decision type	Limit price
$MA(L, t) > \beta_i$ (Prices increase) $ES_i(t) < MaxES_i$ (Market has sufficient liquidity)	Submit buy order	$p_b \quad p(t) + p(t) * \tau_i(t)$
$MA(L, t) < \beta_i$ (Prices decrease)	Submit sell order	$p_s \quad p(t) \quad p(t) * \tau_i(t)$
$ES_i(t) > MaxES_i$ (Market has not sufficient liquidity)	Submit sell order	$p_s \quad p(t) \quad p(t) * \tau_i(t)$
Otherwise	Wait	

Hybrid Trade

According to [20], bubbles are caused by a progressive diffusion of speculative behavior in investors' segments usually unaware of such practices but who, becoming aware of gains they can make, are led to adopt this behavior. In addition, in [35], the author states that once market liquidity decreases, any speculation would be rendered impossible, and technicals return to fundamentalist behavior. In other words, market traders do not necessarily adopt a single investment strategy, it can switch between the fundamentalist and technical (chartist) behavior. This idea already exists in analytically models of financial markets (see [17, 42]). Many artificial market models implement also agents that are able to switch between the two strategies or refer to fundamental and technical factors to make decisions (see [2, 22, 30, 32]).

Therefore, we designed a hybrid trader, which can switch between fundamental and technical strategy. When trader adopts fundamentalist (technical) strategy, it makes decisions in the same way as fundamentalist (technical) trader presented previously (see Tables 1, 2).

Hybrid trader i can be also in two states, *panicked* and *not panicked*. When it is *not panicked*, the time required for traders i to take decision is uniformly selected $T_i(t) \sim \mathcal{U}(T_{min}, T_{max})$, where T_{min} (respect. T_{max}) is a minimal (respect. maximal) time necessary for trader to make decision. When it is in *panicked* state, he tries to sell assets as quickly as possible [4], i.e., the decision-making time is less than when it is in the *not panicked* state, so $T_i(t) \sim \mathcal{U}(panicT_{min}, panicT_{max})$, where, $panicT_{min} < T_{min}$, and $panicT_{max} < T_{max}$.

A Hybrid trader i is initially in *not panicked* state, and adopts fundamental strategy. It has a desire $D_i(t)$ to become technical, initialized at $InitD_i$. It continuously observes prices trend using moving average (Eq. 9) and market liquidity with effective half spread (Eq. 10). Every time, it detects an uptrend in prices ($MA(L, t) > \beta_i$) associated with sufficient market liquidity ($ES_i(t) < MaxES_i$), it can switch to technical behavior with a probability proportional to its desire $D_i(t)$. If it persists in fundamentalist behavior, it increases his desire of speculation $D_i(t + 1)$ as follows:

$$D_i(t + 1) = D_i(t) + \lambda_i. \quad (13)$$

In other words, whenever trader i detects that conditions of speculation are satisfied, it becomes technical or increases its desires of speculation. When trader i adopts

technical strategy, it continuously observes prices trend and market liquidity. If price tends to decline ($MA(L, t) < -\beta_i$) or market liquidity is insufficient ($ES_i(t) \geq MaxES_i$), he switches to the fundamental strategy and *panicked state*, and resets desire $D_i(t)$ to its initial value $InitD_i$. Initializing desire $D_i(t)$ allows trader i to be more cautious towards technical behavior in future. When trader is in *panicked state*, he cannot switch to technical behavior until it returns to the *not panicked state*. Activity diagram of Fig. 1 summarizes algorithm of selecting strategy and state by hybrid trader.

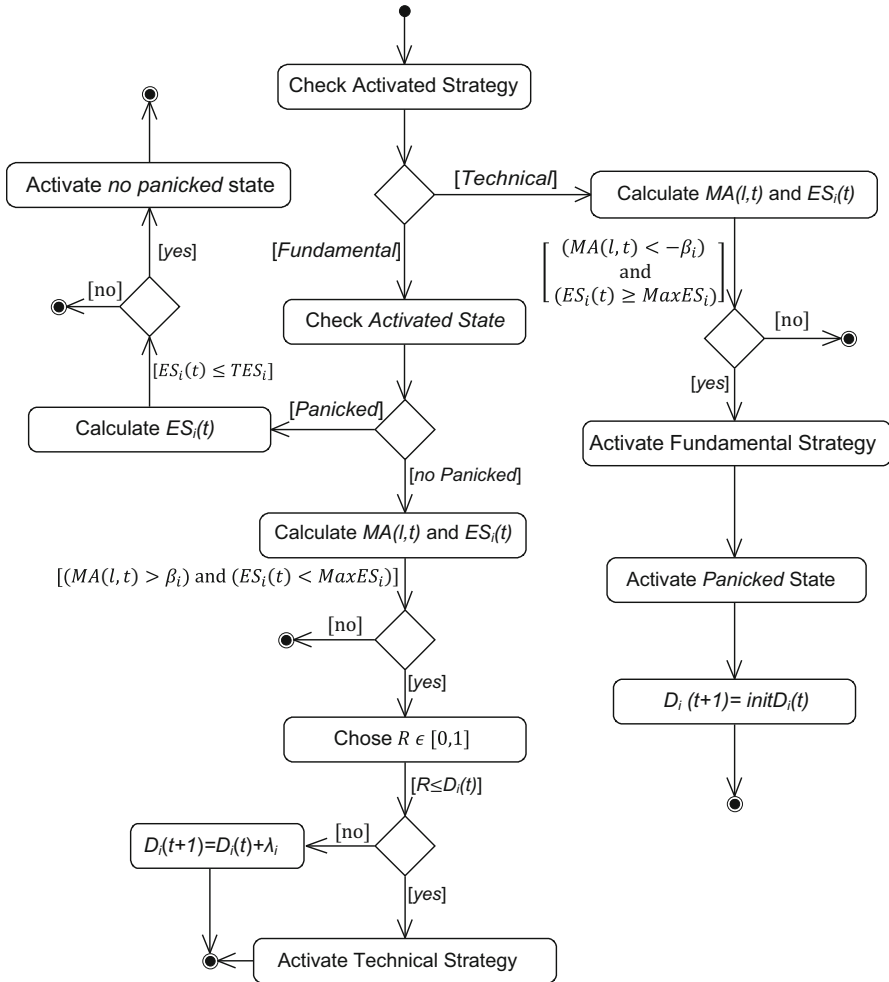


Fig. 1 Activity diagram of selecting strategy and changing state by hybrid trader

Time Management

Most of the results in the field were obtained simulating artificial stock markets with a discrete-time schedule, synchronous agents, or even both. On the other hand, real markets evolve in a continuous time with traders acting asynchronously by sending independently bid and ask orders. Artificial stock markets can evolve using either discrete or continuous time. For discrete time, time is divided in periods: at each period t , one trader or selected groups of traders make simultaneously (synchronously) their decision and send their orders to the market. Therefore, the market is cleared and the new price announced. Traders then update their cash and assets, and the simulation jumps to the next period $t + 1$. In continuous time models, time is not discretized and traders can make their decisions at any time in a completely asynchronous way.

In [34], authors explain that a change in the sequence of arrival to the market of 2 orders may change the actual price by a significant value and eventually lead to completely different subsequent market development (even if their arrival time difference is arbitrarily small). To implement the asynchronous action of agents and the continuous-time scheduling in our model, we used the concept of Markov Net. A Markov Net [34] is a set of events that happen to a set of agents, the events are linked by causal relationships, i.e., an event (effect event) is caused by another (causal event), with a certain lag of time between the cause and the effect. At the time of the arrival of the causal event, the effect event is only “potential”, it can be affected by another happening event in the lag of time.

In our model, events can be either information received by trader from the market or external world, or either decisions made by traders. Information generated by assets market and external world is the events that cause decisions made by traders. The time of causation then corresponds to the time required for trader to make a decision (see Fig. 2).

Results and Discussion

The aim of this section is first to compare statistical properties of price series generated by performed experiments with statistical properties of prices dynamic in real markets, and then, to determine the necessary conditions for the emergence of

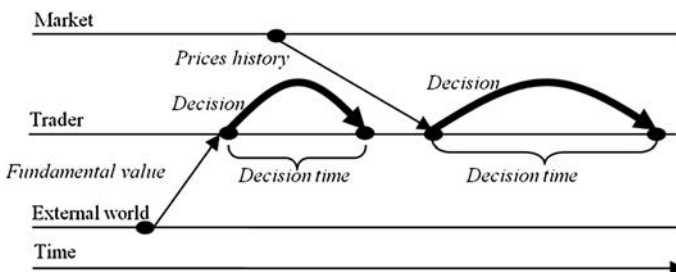


Fig. 2 Principle of Markov networks

Table 3 Experimental design

	<i>Exp.1</i>	<i>Exp.2</i>	<i>Exp.3</i>	<i>Exp.4</i>	<i>Exp.5</i>	<i>Exp.6</i>	<i>Exp.7</i>	<i>Exp.8</i>
General parameters of market								
Number of transactions	5000	N/A	N/A	5000	5000	5000	5000	5000
Number of traders	500	100	100	500	500	500	500	500
Open price	1000	300	300	1000	1000	1000	1000	1000
Rate of noise traders	0%	0 100%	0 100%	10%	10%	10%	10%	10%
Rate of fundamental traders	100%	0 100%	0 100%	90%	45%	10%	10%	10%
Rate of technical traders	0%	0%	0%	0%	45%	80%	0%	0%
Rate of hybrid traders	0%	0%	0%	0%	0%	0%	80%	80%
Common parameters of traders								
Initial cash	10000	10000	10000	10000	10000	10000	10000	10000
Initial stocks	200	200	200	200	200	200	200	200
Max. quantity (in order)	10	10	10	10	10	10	10	10
Min. decision time	5	5	5	5	5	5	5	5
Max. decision time	10	10	10	10	10	10	10	10
L (Eq. 9)	N/A	N/A	N/A	N/A	100	100	100	100
Min. panic decision time	N/A	N/A	N/A	N/A	N/A	N/A	5	2.5
Max. panic decision time	N/A	N/A	N/A	N/A	N/A	N/A	10	5
Parameters of generated fundamental values								
Initial fundamental value	1000	300	300	1000	1000	1000	1000	1000
b (Eq. 3)	0.5	0	2	0.5	0.5	0.5	0.5	0.5
k (Sect. “External world”)	150	0	200	150	150	150	150	150

realistic speculative bubbles and crashes. The experimental design aims to explore gradually the proposed model; thus, we explore different proportions for the different types of traders and we observe the impact on price dynamics. Table 3 summarizes settings of all performed experiments. In experiment *Exp.1*, we test a model with fundamentalist traders only; then, in *Exp.2* and *Exp.3*, we introduce noise traders and discuss their role on market liquidity.² In *Exp.5* and *Exp.6*, we introduce technical traders and we observe their influence on price dynamics. Then, we replace technical traders by hybrid traders, without panic in *Exp.7*, and with panic behavior in *Exp.8*. To observe the impact of uptrend fundamental value on the behavior of traders and on bubble formation, we generate a fundamental value with an increasing trend ($b = 0.5$ in Eq. 3) during the first $k = 150$ transactions, and without (Eq. 2) for the remaining transactions. Finally, we conduct the sensitivity tests of parameters that play a significant role in bubbles formation.

In the following, we present measures of bubbles magnitude we used. Then, we observe the statistical properties of the experiments and we compare them with real prices series. Finally, we analyze and discuss results of each experiment regarding

² *Exp.2* and *Exp.3* are the particular experiments performed to analyze the liquidity, while the rest of experiments aims at comparing price dynamics with fundamental value.

the price dynamics compared to fundamental value and its relationship with the behavior of traders.

Measure of Bubbles Magnitude

In this work, we give special attention to study speculative bubble, so we use some measures introduced in experimental economics literature to analyze bubbles magnitude. In the following, we present each of used measures.

Relative absolute deviation (RAD), introduced by [39], is the average of absolute deviation of price from fundamental value relative to the average fundamental value.

$$RAD = \frac{1}{T} \sum_{t=1}^T \frac{|P(t) - FV(t)|}{\overline{FV}} \quad (14)$$

where $\overline{FV} = \frac{1}{T} \sum_{t=1}^T FV(t)$.

A relative absolute deviation of 0.15 indicates prices on average to deviate about 15% from average fundamental value in either direction (henceforth, we say that the market is mispriced by 15%) [18].

Relative deviation (RD), is similar to *RAD*, but instead of considering absolute price deviations, it relies on (raw) price deviations [39].

$$RD = \frac{1}{T} \sum_{t=1}^T \frac{(P(t) - FV(t))}{\overline{FV}}. \quad (15)$$

A positive value for *RD* is an indicator that the market on average overvalues the asset, whereas negative values are an indicator for undervaluation, e.g., the value of $+(-)0.10$ for *RD* means that the asset is on average overvalued (undervalued) by 10% compared to *FVs* [39]. *RD* around zero indicates prices to track fundamental value [18]. *RD* is by definition equal to or smaller than *RAD*. Large deviations between *RD* and *RAD* indicate the presence of positive and negative deviations of prices from fundamental values.

Boom duration is the greatest number of consecutive periods when prices increase relatively to fundamental value [21]. It is measured as follows:

$$Boom = \max \left\{ m : \tilde{P}_i - \widetilde{FV}_i < \tilde{P}_{i+1} - \widetilde{FV}_{i+1} < \dots < \tilde{P}_{i+m} - \widetilde{FV}_{i+m} \right\} \quad (16)$$

where \tilde{P}_i (respect. \widetilde{FV}_i)³ equals the median price (respect. the median fundamental value) in period *i*.

Bust duration is the greatest number of consecutive periods when price is below the fundamental value [21]. It is measured as follows:

³ In experimental economics, time is divided into periods; in each period, the fundamental value has a fixed value. In our model, time is not divided into periods, therefore, to calculate *Boom* and *Burst* durations, we divide the time of each experiment to 150 periods, and we use the median of fundamental values in each period.

$$Burst = \max\left\{m : \tilde{P}_i - \tilde{FV}_i > \tilde{P}_{i+1} - \tilde{FV}_{i+1} > \dots > \tilde{P}_{i+m} - \tilde{FV}_{i+m}\right\}. \quad (17)$$

Positive deviation (PD), a deviation of the price from the fundamental value if prices are above (below) fundamental value [18].

$$PD = \sum_{P(t) > FV(t)} |P(t) - FV(t)|. \quad (18)$$

Negative deviation (ND), defined as deviation of the price from the fundamental value if prices are below fundamental value [18]:

$$ND = \sum_{P(t) < FV(t)} |P(t) - FV(t)|. \quad (19)$$

According to authors in [18], a bubble is characterized as (1) the positive *Relative Deviation* of prices (*RD* is not below or at zero), (2) long *Boom Duration* and short *Bust Duration* (*Boom* > *Burst*), and (3) high *Positive Deviation* and low *Negative Deviation* (*PD* > *ND*). Thus, to measure the magnitude of bubbles in our experiments, we will test the following null hypothesis:

$H_0 : \text{Relative Deviation} \leq 0$

$H_0 : \text{Boom} \leq \text{Burst}$

$H_0 : PD \leq ND$.

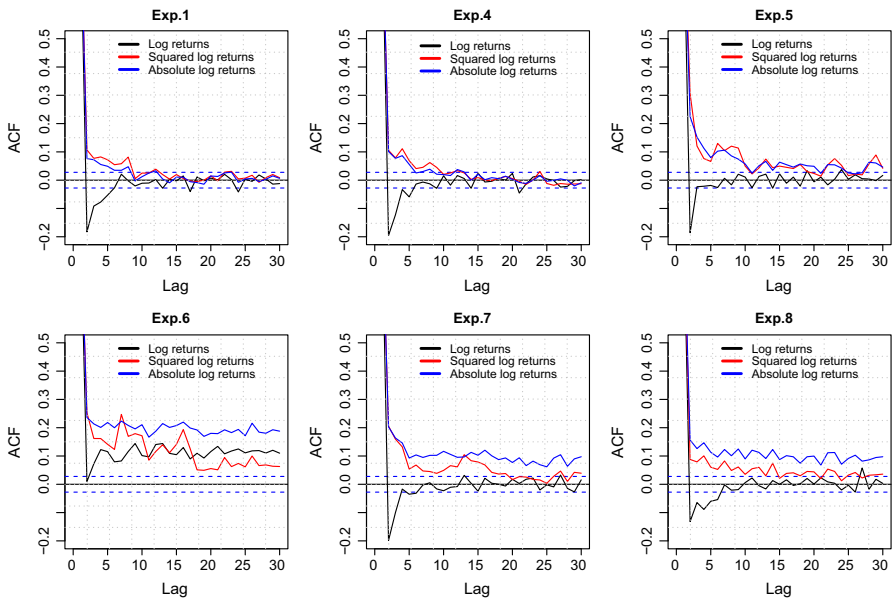
Statistical Properties of the Experiments

First, we will observe the statistical properties of the experiments and compare it with two real series. The aim is to evaluate to what extent the proposed model replicates statistical properties of real-price series, e.g., “stylized facts”. We performed ten runs for each experiment, we calculated statistical measures of each run, and we then report the corresponding mean and standard deviation. Table 4 shows results of performed experiments and those of two real series (FTSE100 index and Barclays banks [31]). We can see that price series generated by performed experiments have statistical properties that are close to the real series. More specifically, it has been found on real financial markets that log return of prices exhibits a *kurtosis* > 3, which indicates a *leptokurtic* distribution of return. This phenomenon is known as heavy tails of log return distribution [31]. We can see that *kurtosis* is higher than 3 in *Exp.7* and *Exp.8* when the market is dominated by the hybrid traders, and so exhibits a phenomenon of heavy tails. It is the same in the other experiments, except in *Exp.6* where the market is dominated by technical traders that we implemented for comparison purpose with hybrid traders.

We also measured the volatility clustering which is observed in real stock markets, and characterized by a positive and significant autocorrelation of the absolute and squared returns [31, 37]. Thus, we report, in Fig. 3, the autocorrelations of the log returns, the absolute log returns, and the squared log returns of performed experiments. In *Exp.7* and *Exp.8*, when the market is dominated by hybrid traders,

Table 4 Statistical properties of log return in *Exp.1*, *Exp.4*, *Exp.5*, *Exp.6*, *Exp.7*, and *Exp.8*

	Mean	Median	Minimum	Maximum	Kurtosis	Skewness
FTSE100	0.0000341	0	0.058853	0.059026	5.13829	0.13
Barclays	0.0002035	0	0.089806	0.09374	4.62582	0.113409
EXP.1	0.0000245	0	0.067427	0.0667924	3.5152825	0.0390313
EXP.4	0.0000058	0	0.0629901	0.0649818	3.5455669	0.0015712
EXP.5	0.0000148	0	0.0490494	0.0563786	10.0921407	0.2092250
EXP.6	0.0023504	0.000368	0.0429592	0.0381872	2.7219738	0.1220623
EXP.7	0.0000239	0	0.060795	0.0610120	5.4510302	0.0604129
EXP.8	0.0000158	0	0.0611785	0.0596754	5.5936151	0.1406656

**Fig. 3** Autocorrelations for different lags of the log returns, absolute log returns, and squared log returns on *Exp.1*, *Exp.4*, *Exp.5*, *Exp.6*, *Exp.7*, and *Exp.8*

the values of the autocorrelations are somehow acceptable. In fact, the autocorrelation of log return decreases and becomes close to zero, while the autocorrelation of absolute return decreases slowly, and remains positive and significant. In *Exp.1* and *Exp.4*, when the market is dominated by fundamentalists, and *Exp.5* when the market is populated by technical traders and fundamentalists, we can observe a lack of autocorrelation of log returns, when autocorrelations of absolute and log return decay until they become close to 0. In *Exp.6*, when the market is dominated by technical traders, the autocorrelation of log returns is always positive and significant which is far from realistic.

Fundamental Trading

To study the prices dynamics with fundamentalist traders, we perform some experiments using this behavior. The first experiment (*Exp.1*) was performed with population $N = 500$ of fundamental traders who are endowed with an initial cash and a number of assets. Fundamentalists estimate fundamental values with an error margin fixed at $\theta_i = 0.05$ for all traders. Figure 4 shows price series and fundamental values generated by one run of *Exp.1*. We can observe that prices follow very closely fundamental values.

Table 5 shows bubble measures of bubbles magnitude for every experiment. Concerning *Exp.1*, we can see that $RD = -0.00025$ is close to 0; in addition, we cannot reject the null hypothesis that $RD \leq 0$, which indicates that prices follow closely fundamental values [18]. A null hypothesis that $Boom \leq Burst$ is not rejected; in addition, $Boom = 2.5$ and $Burst = 2.6$ are not significantly higher compared to the number of periods (150). Then, we cannot reject null hypothesis that $PD > ND$. The tree null hypothesis is rejected, which confirms the absence of bubble in experiment *Exp.1*. The explanation of this is that fundamentalists representing 100% of traders in *Exp.1* estimate the fundamental value of assets with a small error margin ($\theta_i = 0.05$), and place orders with limit prices between the current price and the estimated fundamental value (Eqs. 7 and 8); therefore, the asset prices remained close to their fundamental value and no bubble is observed with these traders.

Introduction of Noise Traders

The aim of this section is to show the necessity to include noise traders in the population used in experiments to ensure market liquidity. The previous works such as [26] has shown that noise traders can improve the liquidity of a market by maintaining the balance of supply and demand. In this section, we first study the role of noise traders in improving market liquidity (*Exp.2* and *Exp.3*), and then, we analyze the influence of noise traders on statistical properties of prices and measures

Fig. 4 Evolution of prices and fundamental values with 100% of fundamentalists (*Exp.1*)

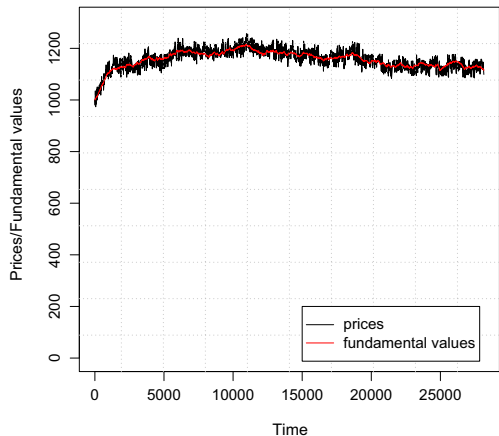


Table 5 Means of observed bubble measures and null hypothesis statistical significance testing in *Exp.1*, *Exp.4*, *Exp.5*, *Exp.6*, *Exp.7*, and *Exp.8*

	<i>Exp.1</i>	<i>Exp.4</i>	<i>Exp.5</i>	<i>Exp.6</i>	<i>Exp.7</i>	<i>Exp.8</i>
<i>RAD</i>	0.00657	0.00678	0.05314	0.81016	0.10228	0.07916
<i>RD</i>	0.00025	0.00010	0.00628	0.70436	0.09794	0.07537
<i>Boom</i>	2.5	2.3	7	7.8	18.4	19.8
<i>Burst</i>	2.6	2.4	3.4	7	17.1	10.8
<i>PD</i>	36.0 k	36.4 k	173.8 k	825.7 k	612.6 k	569 k
<i>ND</i>	37.5 k	35.9 k	137.3 k	3210.2 k	25.0 k	22.4 k
$H_0 : RD \leq 0$	<i>NR</i>	<i>NR</i>	<i>NR</i>	<i>NR</i>	^a	^a
$H_0 : Boom \leq Burst$	<i>NR</i>	<i>NR</i>	^a	^a	<i>NR</i>	^a
$H_0 : PD \leq ND$	<i>NR</i>	<i>NR</i>	<i>NR</i>	<i>NR</i>	^a	^a

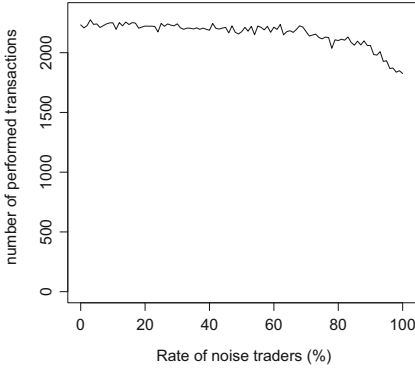
NR indicates that null hypothesis is not rejected

^a Indicates that null hypothesis is rejected

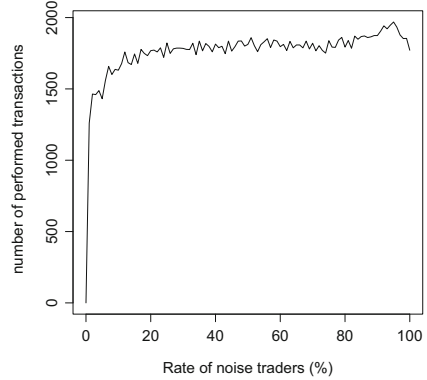
of bubble magnitude (*Exp.4*). Liquidity has three important dimensions: price, quantity, and immediacy. A market is considered liquid if an investor can quickly execute a significant quantity at a price near the fundamental value [9]. There are different approaches to measure liquidity in the literature; here, we will focus on the measurement of immediacy, i.e., the speed at which a transaction can be completed. Immediacy can be approximated by average time between two consecutive transactions. However, conducting experiments with our model, we found that there are experiments where no transaction has been executed; therefore, it will not be always possible to calculate the average time between two transactions. To measure immediacy, we perform experiments over a fixed period and we observe the number of executed transactions.

To study the role of noise traders in improving liquidity of a market, we conduct experiments with a population $N = 500$, composed of fundamental traders who are able to estimate the fundamental value with an error margin $\theta_i = 0.05$ and noise traders. We change the ratios of noise traders between 0 and 100%, and we observe the number of transactions performed for each ratio. Experiments were performed during 10 000 simulated time units, This is largely sufficient for traders to take enough decisions, since the time required for trader to make decision is randomly selected in the range [5,10] (see “[Time management](#)”).

First, we conduct experiments (*Exp.2*) with the fundamental value that varies randomly in time (see Eq. 2). Figure 5a shows obtained results. We can observe that the number of performed transactions decreases slightly while increasing the rate of noise traders. This is confirmed by measuring Pearson and Spearman correlation coefficients, respectively, -0.7911118 and -0.8559429 . The two coefficients are negative, which implies a negative correlation between the rate of noise traders and the number of performed transactions. However, market remains liquid even with 100% of noise traders (the number of performed transactions with 100% of noise traders is 1826). In other words, when the fundamental value varies randomly in time, the market is liquid with or without noise traders. We can explain why market



(a) Fundamental value without uptrend
(Exp.2)



(b) Fundamental value with increasing trend
(Exp.3)

Fig. 5 Number of transactions against the rate of noise traders in *Exp.2* and *Exp.3*

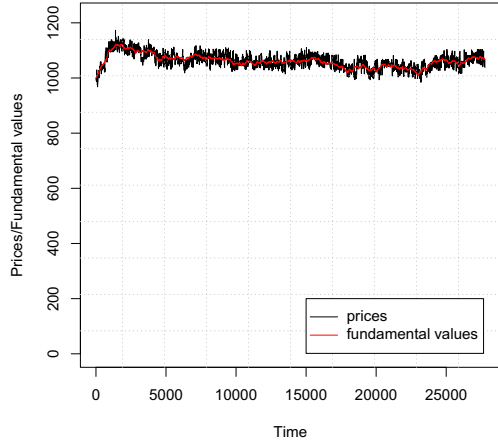
is liquid without noise traders when the fundamental value changes randomly as follows: The fundamental value changes randomly in time, thus, at one time, the fundamental value may be higher, lower, or close to the market price; therefore, the fundamental value estimated by fundamentalists ($fv_i(t)$) with a small error margin $\eta_i(t) \in [-0.05, 0.05]$ (see Eq. 6) may be higher than the market price which implies submitting buy orders, or lower than the market price, which implies submissions of sell orders. Thus, the market remains liquid even without noise traders, because it continually receives buy and sell orders submitted by fundamentalists.

Then, we conduct experiments (*Exp.3*) with an increasing fundamental value in time (see Eq. 3). Figure 5b show obtained results. We can observe that the number of performed transactions generally increase while increasing of the rate of noise traders. This is confirmed by the positive values of Pearson and Spearman correlation coefficients, where, respectively, 0.5239376 and 0.7985928. In the experiment performed with 0% of noise traders, no transaction has been executed.

We can clarify why the noise traders promote liquidity when the fundamental value increases in time as follows: when market is populated only with fundamental traders and the fundamental value $FV(t)$ increases in time and exceeds the market price $p(t)$. When the difference between fundamental value and market price exceeds noise $\eta_i(t)$ in estimating fundamental values for all traders i (see Eq. 6), all fundamental traders i estimate fundamental values $fv_i(t)$ higher than market price, and so decide all to submit buy orders (see Table 1). Therefore, the list of sell orders remains empty and no transaction will take place. In the case with noise traders, they submit sell and buy orders irrespective to the market price and the fundamental value; thus, the market remains liquid, because it continually receives buy and sell orders.

To explore the influence of noise traders on statistical properties of prices series and bubble magnitude, we performed an experiment *Exp.4* with 10% of noise traders and 90% fundamental traders, and then we compare the dynamics of prices with those generated in *Exp.1*.

Fig. 6 Evolution of prices and fundamental value with 90% of fundamentalists and 10% noise traders (*Exp.4*)



We can see in Table 4 that price series generated by *Exp.4* have statistical properties which is very well related to *Exp.1* and real series FTSE100 and Barclays. Figure 6 shows the price series generated by one run of *Exp.4*. We can see that prices follow very closely the exogenous fundamental value in the same way than experiment *Exp.1*.

In Table 5, we see that $RD = -0.00010$ is close to 0 and we cannot reject the null hypothesis that $RD \leq 0$, and the same for null hypothesis that $PD < ND$. Only a null hypothesis that $Boom \leq Burst$ is rejected, but $Boom = 2.3$ and $Burst = 2.4$ are not significantly higher. Thus, as in *Exp.1*, there is no significant deviation of prices from fundamental values in *Exp.4*.

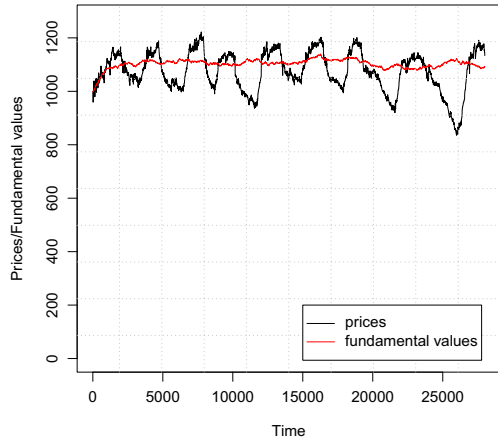
We conclude that noise traders promote market liquidity when informed traders can in some situations take similar decision to buy or sell. In addition, the presence of noise traders does not seem to have an influence on the properties of price series. Thus, we will include 10% of noise traders in all next experiments to ensure market liquidity, i.e., to run the number of transactions fixed in advance.

Introduction of Technical Behavior

To study the impact of technical trading, we performed some experiments using this behavior. First, we performed an experiment *Exp.5*, with the population of 500 traders composed of 45% of fundamentalists, 45% of technical traders, and 10% of noise traders. To compare the results with the previous experiments, we use the same values of parameters in experiments *Exp.1* and *Exp.4*. Figure 7 shows the price series generated by one run of *Exp.5*. If we compare with Fig. 6, we can see that prices are not as close to fundamental values than in experiment *Exp.4* when population was composed of 90% of fundamentalists and 10% of noise traders.

In Table 5, we can see that we cannot reject all null hypotheses except for $Boom \leq Burst$, which indicates the absence of bubble; however, we observe some significant changes in some measures of bubble magnitude compared to *Exp.4*, $RAD = 0.05314$ is more significant than in *Exp.4*, and the same for $RD = 0.00628$

Fig. 7 Evolution of prices and fundamental value with 45% fundamentalists, 45% technicals, and 10% noises (*Exp.5*)



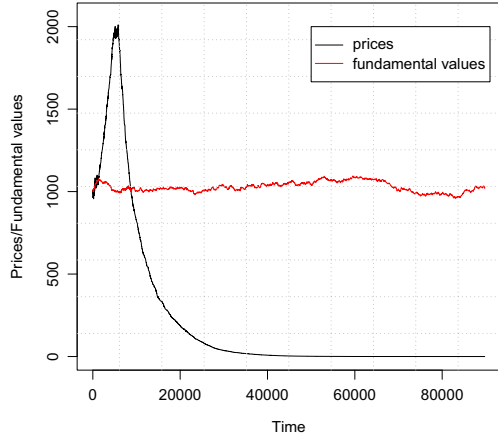
which is positive, this indicate a positive deviation of prices; therefore, there is a deviation of prices from fundamental values. This is confirmed by the ND value that is significantly higher than PD , unlike *Exp.4* where values of two measures are close. We conclude that the introduction of technical behavior has a significant influence on price dynamics, and amplifies significantly the deviation of prices from fundamental values.

Therefore, we study the price dynamics and bubble magnitude when technical traders represent the majority of traders in market, we performed *Exp.6* with 80% of technical traders, 10% of fundamentalists, and 10% of noise traders. Figure 8 shows price series and fundamental values generated by one run of *Exp.6*. We can clearly observe a deviation of prices from fundamentals followed by a crash. However, after crash, prices continue to fall below the fundamental value, rather than staying around this value.

In Table 5, we can see that characteristics of bubble are not satisfied, since we cannot reject all null hypotheses; however, we observe significant changes in measures compared to previous experiments, thus, $Boom$ and $BustDuration$ are more significant than *Exp.5*; in addition, $RD = -0.70436$ is significantly negative, and, the difference between RD and RAD in *Exp.6* is large, which indicates that prices are more undervalued than in *Exp.5*. PD is significant which indicates that there is a period when prices are overestimated (bubble formation); in addition, ND are significantly high, which confirm that there are a periods when prices fall below the fundamental value.

We can explain this results as follows: Technical traders wait (do not take decisions) until execution of $L = 100$ transactions to be able to analyze trend of prices using moving average (see “[Technical trader](#)” and Table 3), only fundamentalists and noise traders are active in first time; thus, prices fluctuate around the fundamental value such we will see in *Exp.4*, as FV is in uptrend during the first 150 transactions (see Table 3); thus, prices have an uptrend at the start, without deviating from the fundamental value. When technical traders analyses prices and detect this uptrend, they submit sell orders which cause the deviation of

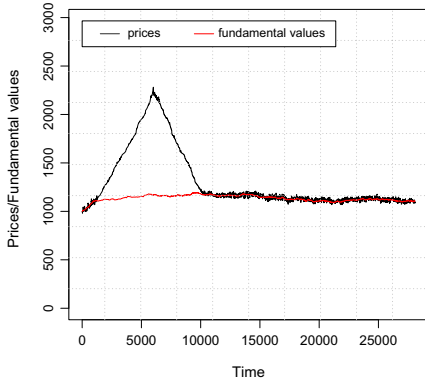
Fig. 8 Evolution of prices and fundamental value with 10% of fundamentalists, 10% of noise traders, and 80% of technical traders (*Exp.6*)



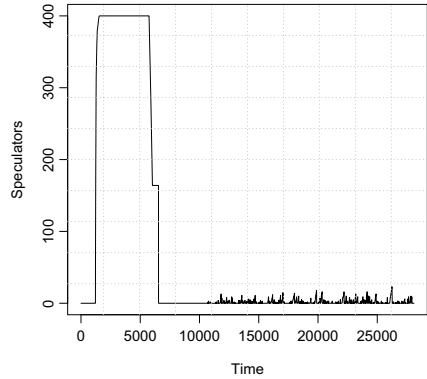
prices from fundamental values and the formation of bubble, when liquidity becomes insufficient, technical traders submit buy orders which cause the fall of prices below the fundamental value, because technical traders do not consider this value to make decisions.

Replacement of Technical Traders by Hybrid Traders

Now, we replace the technical traders from (*Exp.6*) with hybrid traders who can switch between fundamental and technical behaviors, and can also be in one of the two states, *panicked* or *notpanicked* (see Fig. 1). We then use a population of 80% hybrid traders, 10% of fundamental traders, and 10% of noise traders. To analyze the role of panic behavior, we performed the experiment *Exp.7* with hybrid traders without panic. Hereafter, we performed *Exp.8* with hybrid traders able to be either *panicked* or *notpanicked*. Figure 9a shows price series and fundamental values generated by one run of *Exp.7*. We can observe the formation of a bubble with a boom phase followed by a crash. After the crash, prices remain close to the fundamental value, unlike *Exp.6* where prices continued to fall after the crash. The results in Table 5 shows that $RD = 0.09794$ is positive, and very close to $RAD = 0.10228$, which implies that asset prices are only overvalued (compared to fundamental values). This is confirmed by the rejection of null hypothesis $RD \leq 0$ and $PD \leq ND$. However, it is not clear in Fig. 9a whether or not the burst phase is shorter than the boom phase. The results in Table 5 show that the difference between boom and burst duration of *Exp.7* is not important, and the null hypothesis $Boom \leq Burst$ is not rejected. We explain the results of *Exp.7* by the behavior of hybrid traders which use fundamentalist strategy until the execution of $L = 100$ transactions to be able to analyze trend of prices (see “Hybrid trade” and Table 3). Only fundamentalist and noise behaviors are in the market before the execution of these transactions. Thus, prices fluctuate around the fundamental value and follow its uptrend. When hybrids traders analyze prices history and detect this uptrend, they switch to technical strategy (see Fig. 9b). The switching of hybrid traders to technical strategy led to a boom phase when prices increase relatively to



(a) Evolution of prices and fundamental value

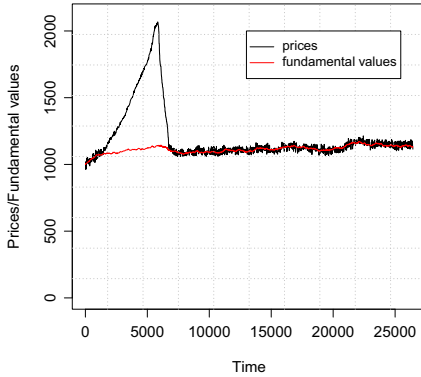


(b) Evolution of traders following a technical strategy

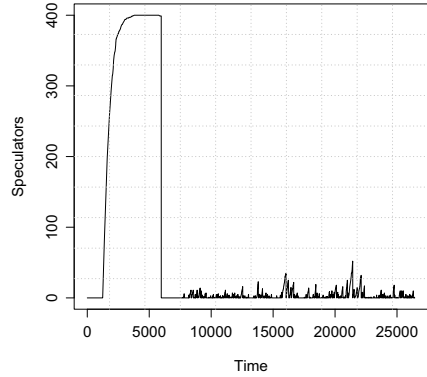
Fig. 9 Evolution of prices and the number of hybrid agents following a technical strategy in *Exp.7*

fundamental value. When fundamental traders estimate that there is no sufficient liquidity, they return to fundamentalist behavior, which led to a burst phase when prices decrease and remain close to fundamental value. However, this deviation of prices does not have all the characteristics of a bubble, more precisely the fact that crash phase duration is not short compared to the boom phase duration.

Now, in experiment (*Exp.8*), we use hybrid traders that can be either *panicked* or *notpanicked*. They may switch to *panicked* state when prices decrease or market liquidity is insufficient (see Fig. 1). Figure 10a shows price series and fundamental values generated by one run of *Exp.8*. We can observe the formation of a realistic bubble with a boom phase and burst phase. After crash, prices remain close to fundamentals values. A boom phase is larger than burst phase unlike in *Exp.7*. Figure 10b shows the evolution of hybrid agents who follow a technical strategy in experiment *Exp.8*. The comparison with Fig. 10a shows a direct relation between the number of traders who adopt technical behaviors, and the formation of the bubble. The more technical traders increase, the more prices deviate from the fundamental value. To confirm this correlation, we measure Pearson and Spearman coefficients of prices and the number of technical traders during boom phase. The values of Pearson and Spearman coefficients are, respectively, 0.7892675 and 0.9737146. Both coefficients were positive and significantly large which implies a positive correlation between technical traders and deviation of prices from fundamental values. We see in Table 5 that we can reject a third null hypothesis unlike in the previous experiments, where the three characteristics were never verified. Thus, (1) $RD = 0.07537$ is significantly positive; in addition, it is very close to $RAD = 0.07916$, which implies that asset prices are only overvalued (compared to fundamental values), i.e., there are no period where the asset was undervalued. (2) The *Boom* is significantly higher than the *Burst*. (3) PD is higher than ND which is not significant regarding PD , which confirms a positive deviation of prices from fundamental values. We, therefore, conclude that *Exp.8* has allowed the reproduction of a realistic bubble. Such as in *Exp.7*, the switching of hybrid



(a) Evolution of prices and fundamental value



(b) Evolution of traders following a technical strategy

Fig. 10 Evolution of prices and the number of hybrid agents following a technical strategy in *Exp.8*

traders to technical strategy led to a boom phase, and the return to fundamentalist behavior led to a burst phase. While the switching of hybrid traders to *panicked* state at the same time than the return to fundamentalist strategy has allowed to have a burst phase duration shorter than boom phase duration; in fact, when hybrid traders are in *panicked* state, they take less time to make decisions and submit buy order, which accelerates the execution of transactions, and leads to a sharp decline in prices.

Sensitivity Analysis

The aim of the sensitivity analysis is to determine the influence of parameters on the output of the model [1]. Thus, to test the sensitivity of one parameter, we fixed the other parameters to their reference value (i.e., the values that have allowed the emergence of realistic bubble in *Exp.8*), and we performed experiments to a broad range of values of the tested parameter, and then we analyze the output of experiments to estimate the sensitivity of the model to the concerned parameter [1]. The model contains a rather large number of parameters; however, we will only test the sensitivity of some parameters which play a significant role in bubble formation. The bubble is defined as the difference between stock price and fundamental value, so we observe the output of experiments the value of *Relative Deviation (RD)*, which measures the deviation of prices from fundamental values (Eq. 15). When *RD* is positive and away to 0, this indicates the formation of bubble in experiment. For each parameter, we tested a range of 100 values, except for the rate of hybrid traders, where we tested 91 values in the range [0,90], because we need 10% of noise traders to ensure liquidity of market (see “[Introduction of noise traders](#)”), the remainder of the population is fundamentalists (i.e., $Rate\ Fundamentalists = 100 - Rate\ Hybrids - Rate\ Noises$). The test of each parameter is performed 10 times, i.e., for each value tested, we perform 10 experiments.

To analyze the sensitivity of model to different parameters, we build graphs representing the output value of *Relative Deviation* with each parameter (Fig. 11); in addition, we estimate the correlation of each parameter with the output value using Pearsons and Spearmans coefficients (Table 6), and we test null hypothesis that no correlation of parameter with *RD*.

In Table 6, we can see that p value of correlation test is less than 0.05 for all parameters, whether for coefficient of *Pearson* or *Spearman*. Consequently, null hypothesis that no association with *RD* is rejected for all tested parameters. All parameters have a relation with *RD*; however, this relation is different from one to another.

The model has a high sensitivity to the hybrid traders ratio. In Fig. 11a, we can see that the values of *RD* remain close to 0 when the rate of hybrid traders is less than 60%. This means that when hybrids are less than 60%, their switching to speculation is insufficient to deviate prices from fundamental values. From 60%, *RD* starts to deviate from 0 and becomes significant. This is explained by the switching of hybrids to the speculative behavior, which causes a deviation of price from the fundamental value and the formation of a bubble (see “[Replacement of technical traders by hybrid traders](#)” for more details). From 70%, *RD* begins to decrease but remains positive and significant; this can be explained as follow: with a high rate of hybrids, there will be a high rate of buyers during the boom phase, thus, prices increase quickly, so the boom phase takes less time, and consequently, bubble duration decreases. *RD* is the average of deviations of prices from fundamental values (see Eq. 15), and thus, *RD* decreases also with the decrease of the bubble duration. It means that the bubble is going faster when increasing the number of hybrids number.

The model is particularly sensitive to $MaxES_i$ (Fig. 11b). $MaxES_i$ is threshold of effective half spread (Eq. 10) from which hybrid traders assume that market is no longer liquid (see Fig. 1), after they switch to fundamental strategy and stop pushing prices to deviate from fundamental values. The deviation of prices of fundamental values increases when increasing $MaxES_i$, which explains the strong correlation between *RelativeDeviation* and $MaxES_i$.

RD has a low positive sensitivity to parameter $InitD_i$, the initial value of $D_i(t)$ which represents the desire of hybrid trader to adopt technical strategy. We can see in Fig. 11c that only with values of $InitD_i$ close to 0, hybrid trader persists in fundamentalist strategy and prices remain around fundamental values.

RD has a positive correlation with λ_i that is the value used to increase a desire $D_i(t)$ of hybrid trader i (Eq. 13 and Fig. 1). In Fig. 11d, we can see that prices deviate from fundamental values even with small values of λ_i , the reason is possibly the value of $InitD_i$ that is sufficient for hybrid traders to become technicals when conditions are satisfied.

The model has a high sensitivity to parameter L which is a period length used to calculate moving average of prices by hybrid traders (Eq. 9). Figure 11e shows that with small values of L (lower than 200), hybrid traders cannot detect the uptrend of prices and remain in fundamentalist strategy, consequently, prices fluctuate around the fundamental value, and *RD* remains close to 0. When

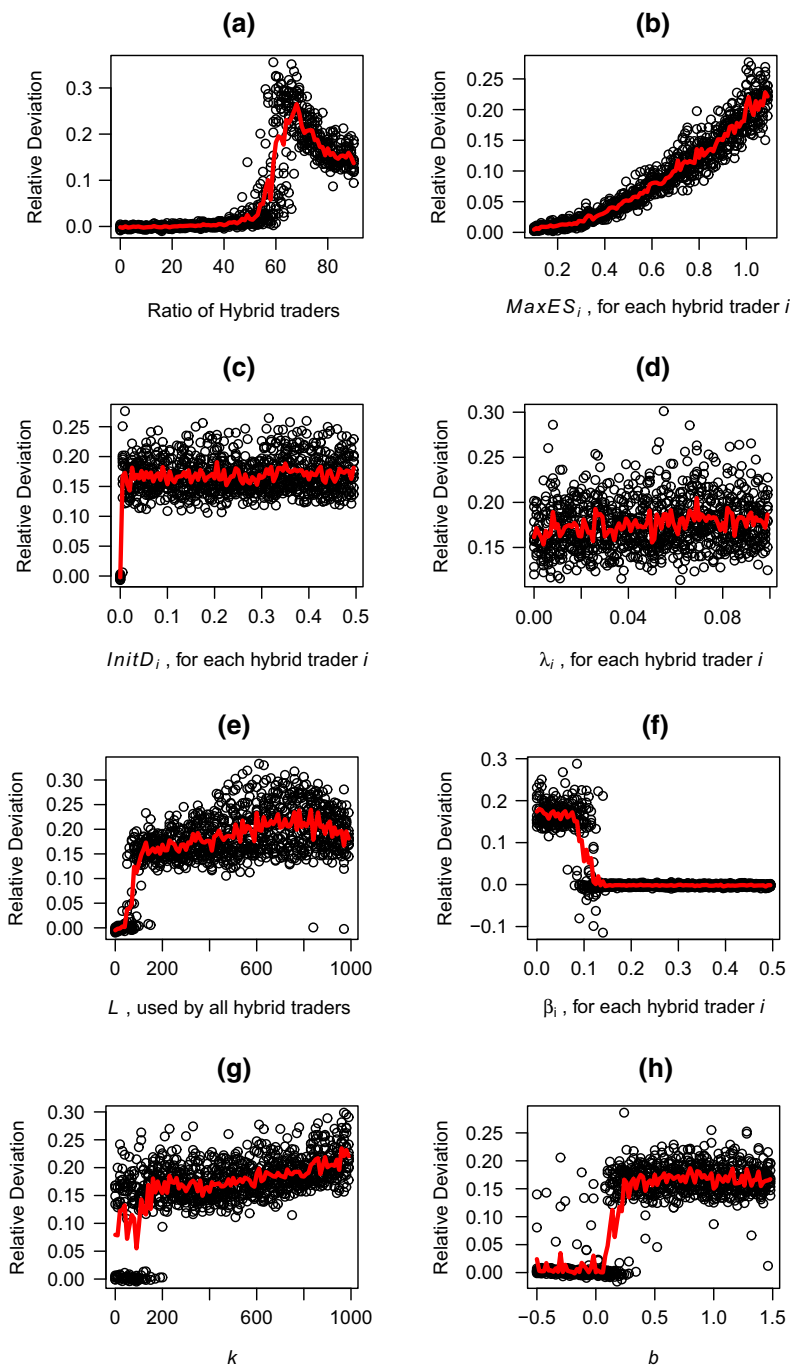


Fig. 11 Sensitivity analysis of the most important parameters

Table 6 Correlation test of different parameters with the *Relative Deviation (RD)*

	Coef. of Pearson	Coef. of Spearman
% of hybrids	0.823 ^a	0.916 ^a
$MaxES_i$	0.977 ^a	0.997 ^a
$InitD_i$	0.272 ^a	0.281 ^a
λ_i	0.471 ^a	0.482 ^a
L	0.716 ^a	0.801 ^a
β_i	0.741 ^a	0.655 ^a
k	0.836 ^a	0.902 ^a
b	0.845 ^a	0.732 ^a

^a Indicate that p value is below 0.05, i.e., correlation is significant

$L > 200$, hybrids detect the uptrend in prices and switch to technical strategy (see Fig. 1).

RD has a negative correlation with β_i which is a threshold of moving average (Eq. 9) from which hybrid trader assumes that prices increase. In Fig. 11f, $\beta_i < 0.1$, hybrids detect uptrend in prices and switch to technical strategy (see Fig. 1), which causes a deviation of prices and bubble formation. When $\beta_i > 0.1$, hybrids do not detect uptrend of prices and thus remain in the fundamentalist strategy; therefore, prices remain close to fundamental values.

RD has a positive correlation with k that is the number of transactions during which the fundamental value increases (see “External world”). We can see that RD is close to 0 in some experiments ($k \leq 200$). Figure 11g shows that with small value of k , the uptrend in prices (following the uptrend of the fundamental value) is not detected in some experiments by hybrids traders that persist in the fundamentalist strategy, and therefore, prices do not deviate and no bubbles observed.

Table 6 shows that RD has a positive correlation with b that is a parameter used to generate a fundamental value with trend (see Eq. 3). In Fig. 11h, we can see that RD remains close to 0 when b is negative or slightly higher than 0, when the value of b is sufficiently high, RD becomes positive and significant which indicates the formation of a bubble. A negative and small value of b generates a downtrend or a slight uptrend of the fundamental values ignored by hybrids that persist in the fundamentalist strategy. When the value of b is sufficiently high, the uptrend of prices that fluctuates around the growing fundamental value is detected by hybrids, which switch, therefore, to speculative behavior. This leads to a deviation of prices from fundamental values and the formation of a bubble.

Conclusions

In this paper, we presented an asynchronous artificial double auction market composed of heterogeneous traders, and then, we used it to reproduce market dynamics. A first result of these experiments is the important role of noise traders to ensure the liquidity of market. In the absence of noise traders, informed traders can

all make the same decision (buy or sell), which reduces market liquidity. When fundamentalists are dominant in the market, prices follow closely fundamental values. When market is populated with fundamental traders, prices follow the exogenous fundamental values very closely. When market is populated by a majority of traders who adopt a purely technical strategy, this leads to deviation of prices from fundamental values; however, this deviation does not have all the characteristics of a realistic bubble. When the market is populated by hybrid traders who can switch between fundamental behavior and technical behavior as well as panicked and not panicked state, we get a far more realistic speculative bubble formation, hereafter followed by a crash. After the crash, prices tend to stabilize around the fundamental value.

Now that we obtained a model able to reproduce speculative bubbles with simple behaviors of traders, we aim at introducing other kinds of agents and corresponding rationalities to find their potential influence on market dynamics. This latter point is especially crucial nowadays that most transactions taking place on markets are, in fact, achieved by computerized trading agents whose rationality is, indeed, inferred from algorithms. In addition, as the previous studies evaluated the impact of regulation policies on financial markets by the artificial market model (see [33, 41, 44]), one interesting application of this work consists in testing and evaluating the impact of regulation policies on real markets.

References

1. Amblard, F., Phan, D.: *Modélisation et simulation multi agents*. Hermes Lavoisier, Paris (2006)
2. Arthur, W.B., Holland, J.H., LeBaron, B., Palmer, R.G., Tayler, P.: Asset pricing under endogenous expectations in an artificial stock market. SSRN 2252 (1996)
3. Barlevy, G.: Economic theory and asset bubbles. *Econ. Perspect.* **31**(3) (2007)
4. Bernanke, B.S.: *The Federal Reserve and the financial crisis*. Princeton University Press (2013)
5. Bessembinder, H., Venkataraman, K.: Bid ask spreads: measuring trade execution costs in financial markets. *Encycl. Quant. Finance* (2009)
6. Blanchard, O.J., Watson, M.W.: *Bubbles, rational expectations and financial markets* (1982)
7. Boer, K., Kaymak, U., Spiering, J.: From discrete time models to continuous time, asynchronous modeling of financial markets. *Comput. Intell.* **23**(2), 142–161 (2007)
8. Boer Sorban, K.: *Agent based simulation of financial markets: a modular, continuous time approach*. Number EPS 2008 119 LIS. Erasmus Research Institute of Management (ERIM) (2008)
9. Chacko, G., Jurek, J.W., Stafford, E.: Pricing liquidity: the quantity structure of immediacy prices. Division of Research, Harvard Business School (2006)
10. Chen, S. H., Chang, C. L., Du, Y. R.: Agent based economic models and econometrics. *Knowl. Eng. Rev.* **27**(02), 187–219 (2012)
11. Cui, B., Wang, H., Ye, K., Yan, J.: Intelligent agent assisted adaptive order simulation system in the artificial stock market. *Expert Syst. Appl.* **39**(10), 8890–8898 (2012)
12. Deck, C., Porter, D., Smith, V.: Double bubbles in assets markets with multiple generations. *J. Behav. Finance* **15**(2), 79–88 (2014)
13. Derveeuw, J.: Market dynamics and agents behaviors: a computational approach. In: *Artificial Economics*, pp. 15–26. Springer (2006)
14. Derveeuw, J., Beaufils, B., Mathieu, P., and Brandouy, O.: Testing double auction as a component within a generic market model architecture. In: *Artificial Markets Modeling*, pp. 47–61. Springer (2007)
15. Diba, B.T., Grossman, H.I.: Rational inflationary bubbles. *J. Monet. Econ.* **21**(1), 35–46 (1988)

16. Fama, E.F.: Efficient capital markets: a review of theory and empirical work. *J Finance* **25**(2), 383–417 (1970)
17. Frankel, J.A., Froot, K.: Chartists, fundamentalists and the demand for dollars. NBER Working Paper, (R1655) (1991)
18. Füllbrunn, S., Neugebauer, T.: Margin trading bans in experimental asset markets. Technical report, Jena Economic Research Papers (2012)
19. Giardina, I., Bouchaud, J. P.: Bubbles, crashes and intermittency in agent based market models. *Eur. Phys. J. B Condens. Matter Complex Syst.* **31**(3), 421–437 (2003)
20. Gravereau, J., Trauman, J.: Crises financières. *Economica* (2001)
21. Haruvy, E., Noussair, C.N.: The effect of short selling on bubbles and crashes in experimental spot asset markets. *J. Finance* **61**(3), 1119–1157 (2006)
22. Izumi, K., Okatsu, T.: An artificial market analysis of exchange rate dynamics. In: *Evolutionary Programming*, pp. 27–36. Citeseer (1996)
23. Kaldor, N.: Speculation and economic stability. *Rev. Econ. Stud.* **7**(1), 1–27 (1939)
24. Karino, Y., Kawagoe, T.: Bubble and crash in the artificial financial market. In: *Artificial Economics*, pp. 159–170. Springer (2009)
25. Kindleberger, C.P., Aliber, R.Z.: *Manias, panics and crashes: a history of financial crises*. Palgrave Macmillan (2011)
26. Kobayashi, S., Hashimoto, T.: Analysis of random agents for improving market liquidity using artificial stock market. In: *Proceedings of The Fourth Conference of The European Social Simulation Association (ESSA 4th07)*, pp. 315–317 (2007)
27. Kumar, P., Amritha, T., Gowtham Ram, S., Hari Krishna, S., Karthika, M., Mohandas, V.: A survey of agent based artificial stock markets (continuous session models). *Int. Res. J. Finance Econ.* **64**, 126–139 (2011)
28. Kumar, P., Jha, A., Gautham, T., Mohan, J., Rama Subramanian, A., Mohandas, V.: A survey of call market (discrete) agent based artificial stock markets. *Int. J. Comput. Sci. Eng.* **2**(09), 3025–3032 (2010)
29. Lugovskyy, V., Puzzello, D., Tucker, S., Williams, A.: Asset holdings caps and bubbles in experimental asset markets. *J. Econ. Behav. Organ.* **107**, 781–797 (2014)
30. Lux, T., Marchesi, M.: Scaling and criticality in a stochastic multi agent model of a financial market. *Nature* **397**(6719), 498–500 (1999)
31. Martinez, S., Tsang, E.P.: An heterogeneous, endogenous and coevolutionary agent based financial market. *Evol. Comput. IEEE Trans.* **13**(1), 33–55 (2009)
32. Mizuta, T., Izumi, K., Yoshimura, S.: Price variation limits and financial market bubbles: artificial market simulations with agents' learning process. In: *Computational Intelligence for Financial Engineering & Economics (CIFER), 2013 IEEE Conference*, pp. 1–7. IEEE (2013)
33. Mizuta, T., Kosugi, S., Kusumoto, T., Matsumoto, W., Izumi, K., Yagi, I., Yoshimura, S.: Effects of price regulations and dark pools on financial market stability: an investigation by multiagent simulations. *Intell. Syst. Account. Finance Manag.* (2015)
34. Muchnik, L., Solomon, S.: Markov nets and the netlab platform: application to continuous double auction. In: *Complexity hints for economic policy*, pp. 157–180. Springer (2007)
35. Orléan, A.: L'auto référence dans la théorie keynésienne de la spéculation. *Cahiers d'économie politique/Papers in Political Economy*, pp. 229–242 (1988)
36. Palmer, R.G., Arthur, W.B., Holland, J.H., LeBaron, B., Tayler, P.: Artificial economic life: a simple model of a stockmarket. *Phys. D Nonlinear Phenom.* **75**(1), 264–274 (1994)
37. Sewell, M.: Characterization of financial time series. Research Note. University College London (2011)
38. Smith, V.L., Suchanek, G.L., Williams, A.W.: Bubbles, crashes, and endogenous expectations in experimental spot asset markets. *Econometrica*, pp. 1119–1151 (1988)
39. Stöckl, T., Huber, J., Kirchler, M.: Bubble measures in experimental asset markets. *Exp. Econ.* **13**(3), 284–298 (2010)
40. Tan, J.P., David, M.: Study of property market in johor: an investigation of property bubble phenomenon (2012)
41. Thurner, S., Farmer, J.D., Geanakoplos, J.: Leverage causes fat tails and clustered volatility. *Quant. Finance* **12**(5), 695–707 (2012)
42. Vigfusson, R.: Switching between chartists and fundamentalists: a markov regime switching approach (1996)

43. Wang, P., Wen, Y.: Speculative bubbles and financial crises. *Am. Econ. J. Macroecon.* **4**(3), 184–221 (2012)
44. Westerhoff, F.H.: The use of agent based financial market models to test the effectiveness of regulatory policies. *Jahrbücher für Nationalökonomie und Statistik*, pp. 195–227 (2008)
45. Zovko, I.I.: *Topics in market microstructure*. Amsterdam University Press (2008)

Sadek Benhammada received the B.Eng. and M.Sc. degrees in computer science from the Mentouri University of Constantine (Algeria) in 2003 and 2007, respectively. He is currently a Ph.D. Student in computer science at Constantine 2 University (Algeria). He is member of MISC laboratory (Modeling and implementation of complex systems). His current research work focuses on agent based computational economics, complex systems, social networks, and machine learning.

Frederic Amblard is an Associate Professor of Computer Science at University Toulouse 1 Capitole and associated researcher at the Institut de Recherche en Informatique de Toulouse (IRIT, UMR CNRS 5505) in the SMAC (Cooperative Multi Agent Systems) team. He holds M.S. degree from the Montpellier University and Ph.D. degrees in computer science from Blaise Pascal University. His works concern mostly agent based social simulation of influence dynamics, opinion dynamics, and social networks.

Salim Chikhi received his Ph.D. in computer science from Mentouri University, in 2005. He is currently a full Professor at Constantine2 Absentmind Gehri University, Algeria. He is the director of MISC (Modelling and Implementation of Complex Systems) laboratory. His research areas include soft computing and artificial life techniques and their application in several domains, machine learning, complex systems, and agent based computational economics.