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Modeling the Movement of Electrostatic Motors in a 3D Finite Element Code

N. Boukari, Y. Lefèvre, and P. Spitéri

Abstract—When we use a lumped parameter model for electrostatic micromotor, it is necessary to associate it with a 3D finite element code. In order to compute the lumped parameters for different positions of the rotor, we present a method, up to now scarcely employed, which allows us to take into account the movement. To solve the algebraic system thus obtained, an iterative method associated with a SSOR preconditioning is then used. The numerical model is applied to an existing motor.

Index Terms—Finite element method, electrostatic motors, movement modeling, iterative methods, SSOR preconditioning.

I. INTRODUCTION

WITH the recent development of micromechanical technologies (silicon surface machining, LIGA processing, ...) electrostatic motors have now found a domain of interest. To identify their industrial applications it is important to be able to evaluate their electromechanical performances.

In most cases, the electrical behavior of electrostatic structures can be modeled by linear relationships between potential and charge quantities on all conductors. So, it is not necessary to develop a coupled field and circuit model to simulate their dynamic operation. Electromechanical lumped parameter models [1] [2] are favored. The parameters of this model vary with the relative positions of the moving parts and stationary parts of the devices. To evaluate these parameters, according to the thin axial length of these motors, 3D electrostatic field software is required.

Our work is concerned with rotating electrostatic motors. The aim is to present a method for modeling the movement in a 3D FEM code in order to evaluate these parameters for different successive positions of the rotor. This overlapping element method has been used in 2D but not in 3D [3]. We have chosen this method because it does not require local or global remeshing and restrictions on the meshes or the rotor motion. Furthermore the elements at the interface between the moving and fixed parts are not distorted [4]. This method is an alternative to other methods such as the Lagrange multipliers method [5] or the interpolation method [6]. In this paper, first we describe the overlapping element method (OLM). The obtained algebraic system is then solved using the conjugate

gradient method preconditioned by the SSOR (symmetric successive overrelaxation) [7]. The whole numerical model is applied to an existing micromotor [8].

II. THE ELECTROMECHANICAL MODEL

Various research on the electrostatic motors have allowed a lumped parameter model to be established. So, the electric equation which governs the structure is of the type:

$$\{V\}_{ele} = [R]_{ele} \frac{d}{dt} ([C(\theta)]_{ele} \{U\}_{ele}) + \{U\}_{ele}, \quad (1)$$

The mechanical equation is:

$$J_{mec} \frac{d^2\theta}{dt^2} = T_{ele} - T_{res} \quad (2)$$

and the coupling equation, which gives the electrostatic torque, is:

$$T_{ele} = \frac{1}{2} \{U\}_{ele}^T \frac{\partial}{\partial \theta} [C(\theta)] \{U\}_{ele} \quad (3)$$

with: θ , angular attitude of the rotor around its own axis; $\{V\}_{ele}$, voltage applied on the phases of the motor; $\{U\}_{ele}$, potential of electrodes of the motor, $[R]_{ele}$: resistance of phases; $[C(\theta)]_{ele}$, induction coefficients and self capacitances matrix of the rotor; T_{ele} , electric drive torque; J_{mec} , rotational inertia and T_{res} , resisting torque due to viscous and dry friction.

III. ELECTROSTATIC FIELD MODELLING

When we discretize the study domain with finite nodal elements Ω_e , the scalar fields V and ρ , correspond respectively to the generalized vector $\{V\}$ of the values of electric potential V_i at every node i and the generalized vector $\{s\}$ of equivalent electric charges s_i applied on every node i . The total potential energy W can be put in the form:

$$W = \frac{1}{2} \{V\}^T [K] \{V\} - \{s\}^T \{V\} \quad (4)$$

The condition of extremum of the potential energy for each node i is written:

$$\frac{\partial W}{\partial V_i} = 0 \quad (5)$$

which gives:

$$[K] \{V\} = \{s\} \quad (6)$$

The matrix $[K]$ is symmetric and positive definite.

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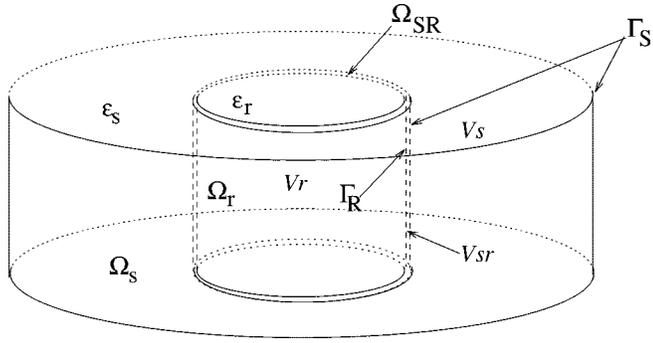


Fig. 1. The volume interface Ω_{SR} .

A. Principle of the Overlapping Element Method (OLM)

In this method, the interface is constituted by a cylindrical volume Ω_{SR} as shown in Fig. 1. In this volume fictitious elements are generated during the movement. These elements are composed of main nodes and fictitious nodes. Main nodes are generated during the movement and consist of the normal projection of the main nodes on the surface of Ω_{SR} opposite to them. It can be shown that the potentials at the fictitious nodes are related linearly to the potentials at the main nodes. This method leads to an algebraic system of equations whose matrix is symmetric and positive definite.

B. Implementation of the Method

On the Fig. 2, we can see an example of overlapping mesh. The lowercase notation (a_i, b_i, c_i) represents the fictitious nodes while the uppercase notation (A_i, B_i, C_i) represents the main nodes. The potentials $\{V_{ev}\}$ at the fictitious nodes are equated to the potentials $\{V_{ep}\}$ at the main nodes by a simple relation:

$$\{V_{ev}\} = [\alpha]\{V_{ep}\} \quad (7)$$

where $[\alpha]$ is a factor matrix. The coefficients are easy to determine according to the relative displacement of the rotor.

IV. SOLUTIONS OF THE ALGEBRAIC SYSTEM OF EQUATIONS

The application of the finite element method to a boundary value problem of the type described above yields a sparse system of linear algebraic equations, usually symmetric and positive definite. Solving such a system is a major computing task in itself. The most common methods for solving such linear algebraic systems are direct methods such as Gaussian elimination and the closely related Cholesky methods, when the matrix arising in the formulation of the algebraic system is symmetric and positive definite. Nevertheless, if the system is ill-conditioned, the computed solution may indeed have large relative errors due to rounding. This is particularly the case for finite element problems in two or three space dimensions, even if the sparsity of the matrix is taken into account. It is one of the reasons why iterative methods are often used to solve linear algebraic systems derived from finite element discretization. Indeed, when the asymptotic rate of convergence is good, these methods are successful. Among such iterative algorithms, one of the most successful is the conjugate gradient method. The

effectiveness of the conjugate gradient method can be much improved by the technique of preconditioning, a topic of current research in applied mathematics. There are two important kinds of preconditioning, one based on the SSOR method and the other one based on an incomplete factorization (ICCG). In the present paper, we use the conjugate gradient method by SSOR [7]. Furthermore, from a theoretical point of view, it can be noted that, the improvement of the condition number is not established in the general case, when ICCG preconditioning are used. This improvement can be shown only by numerical experiments.

V. APPLICATIONS

We have applied the whole numerical model to compute the electromechanical lumped parameters of an existing micro-motor constructed by the LAAS (Laboratory for Analysis and Architecture of Systems) at Toulouse, France. This kind of structure is a 3/2 type and this value represents the ratio between the number of electrodes at the stator and the number of teeth at the rotor.

The Fig. 3 gives us the dimensionnal characteristics of the microactuator. The structure has been meshed with hexahedral nodal elements (Fig. 4). The different cases simulated are presented in Table I.

The first case is considered as the reference case. Fig. 5 shows the results obtained from both the reference case and the last case. In the last case the meshes on each of the cylindrical surfaces of the interface volume Ω_{sr} , are constituted of irregularly spaced main nodes.

A. Capacity Computation

The capacitances are computed by energy consideration. The results in Fig. 5 represent the self capacitances of the rotor (C_{rr}) and of one phase of the stator (C_{ss1}) for a mesh step of 1° and an irregular mesh step, which takes the values 3, 5, or 6° , with a rotation step of 1° . The differences between the two computations do not depend on the angular position of the rotor. This tends to show that the errors are mainly due to the approximation by finite element method, when the characteristic dimension h of the elements increase. The error introduced by the OLM method can be neglected.

B. Torque Computation

We compute the static torque from the derivative of the capacitances with (3), and compare the case described above (irregular mesh) to the reference one. To calculate the derivatives, we used the Euler order 1 and 2 method. We have also implemented the Maxwell stress tensor method. A technique of curve smoothing, analogous to a technique used to take into account the skewed slots of electric machines is also used in order to suppress eventual torque ripples. The slot step is equivalent here to the mesh step.

The results in Fig. 6 show the static torque variation with an irregular mesh step. The static torque is composed of harmonic waves of different frequencies, related to the mesh step when we use an Euler formula. Only the smoothing method is efficient

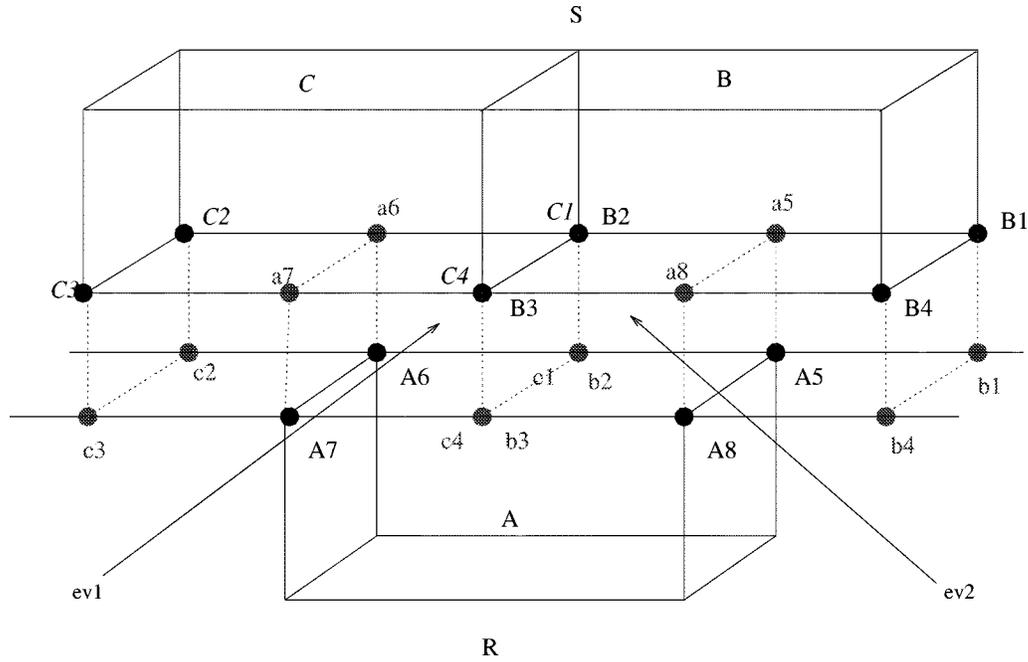


Fig. 2. An example of overlapping elements.

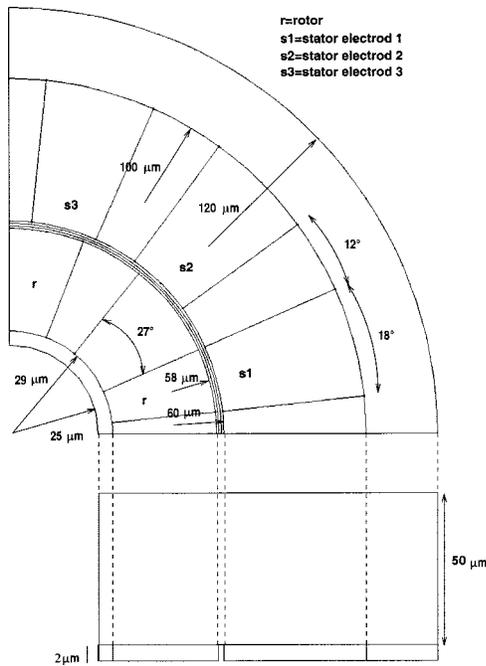


Fig. 3. Dimensions of the micromotor.

TABLE I
DIFFERENT CASES
SIMULATED

α : Mesh step	Number of nodes	Number of elements
β : Rotation step		
$\alpha = 1^\circ, \beta = 1^\circ$	99361	87120
$\alpha = 3^\circ, \beta = 1^\circ$	33120	29040
$\alpha = 3^\circ, \beta = 3^\circ$	33120	29040
$\alpha = 3 \dots 6^\circ, \beta = 1^\circ$	18768	16456

enough to suppress the problem. The torque computed by the Maxwell stress sensor does not present torque ripples.

C. Convergence Analysis

We have compared the performance of two preconditioner for a rotation of the micromotor rotor. The case simulated corresponds to a step mesh of 3° and a mesh rotation of 3° . We have chosen this case because the number of elements is not very high and because the commercial software does not allow the step rotation to be smaller than the mesh step. The results in Figs. 7 and 8 show that the SSOR preconditioning gives better results than ICCG preconditioning.

VI. CONCLUSION

In order to evaluate the electromechanical lumped parameters, a method which takes into account the movement in a 3D finite element code has been presented. This method was until now very scarcely employed and only in 2D. We have shown that even if irregular mesh is employed at the interface the quality of the solution is quite acceptable. This method seems to be a good alternative to the Lagrange multipliers or the interpolation methods. The next stage will be to implement the case where tetrahedral elements are on both side of the interface. We have also evaluated the performances of the numerical model

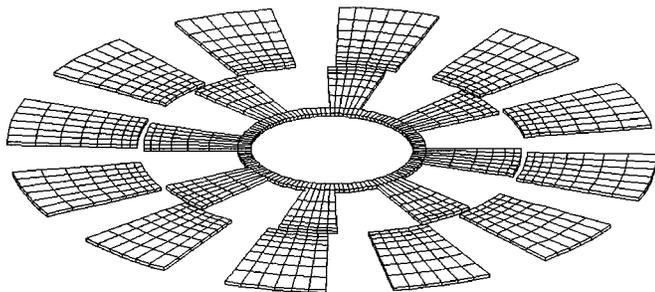


Fig. 4. A view of the meshing.

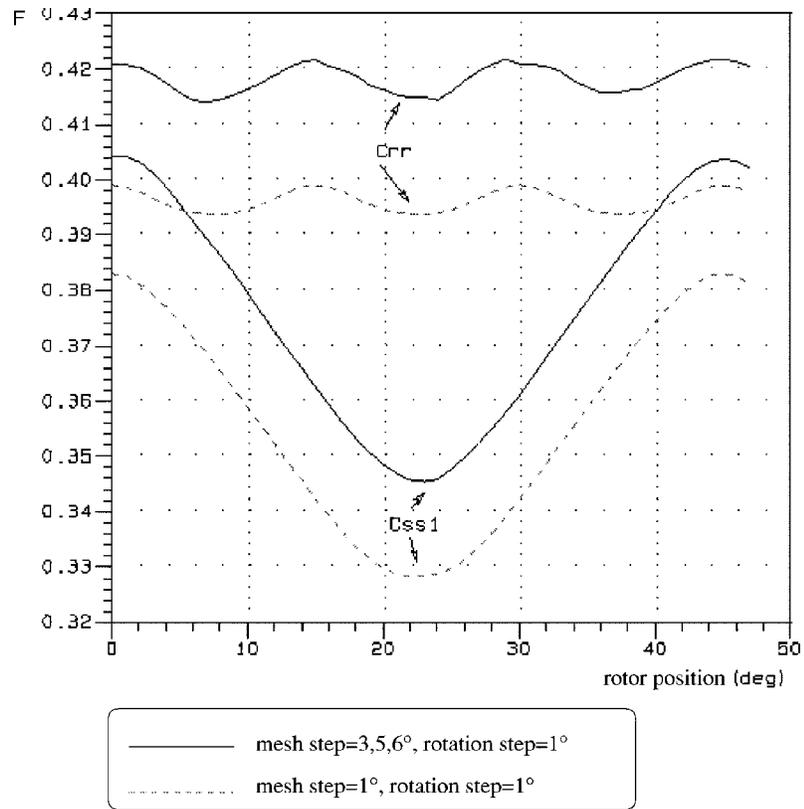


Fig. 5. Irregular mesh results compared to the reference ones.

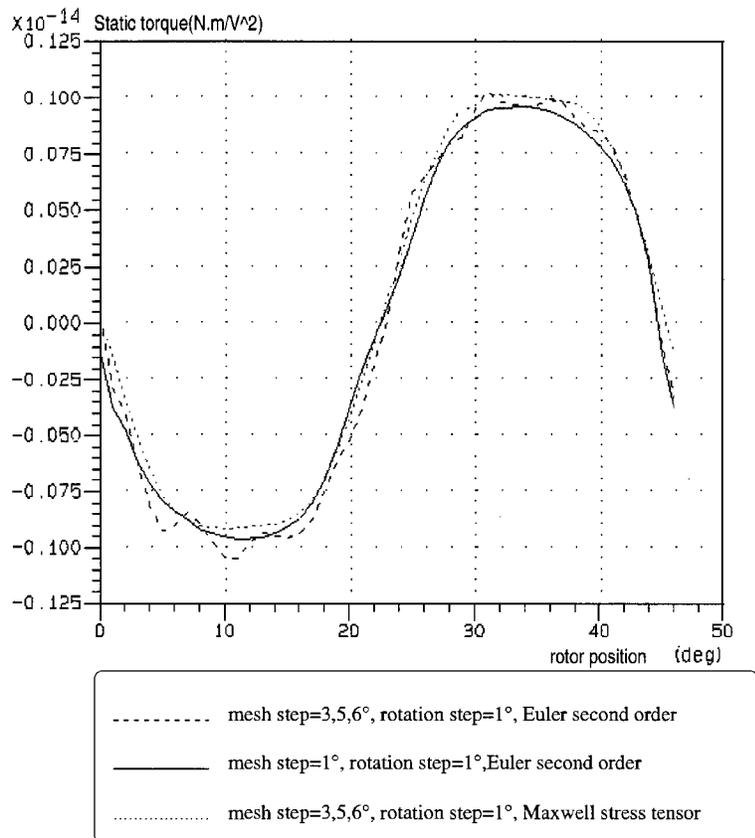


Fig. 6. Torque for irregular mesh.

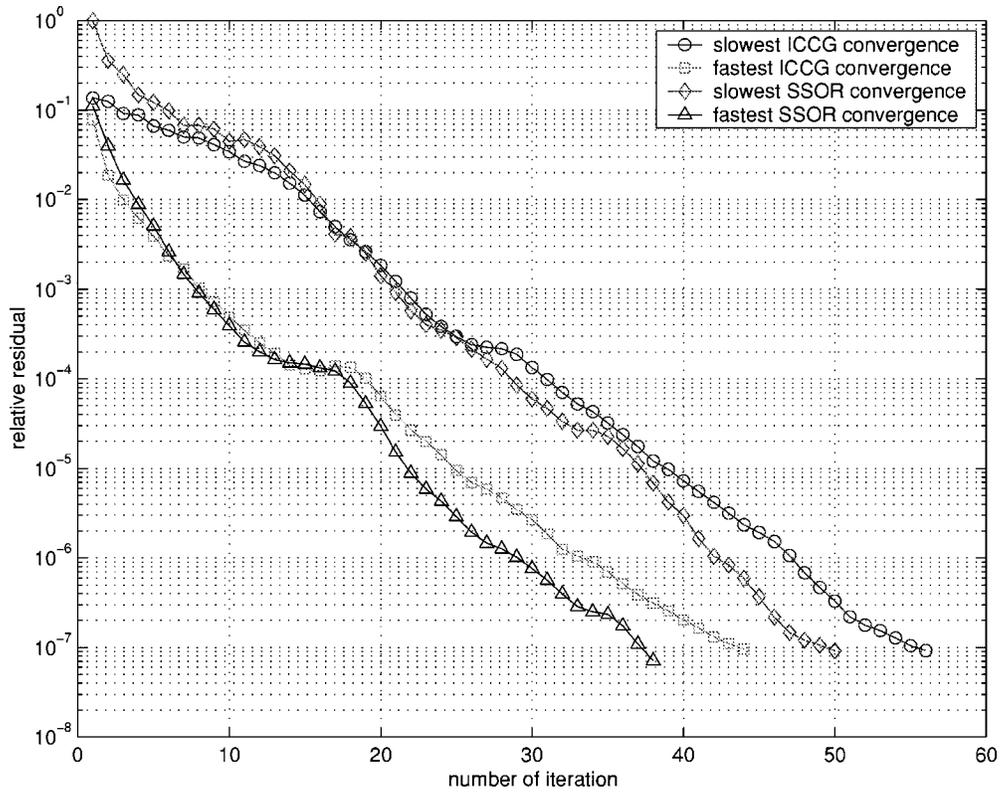


Fig. 7. Speed of convergence between ICGG and SSOR.

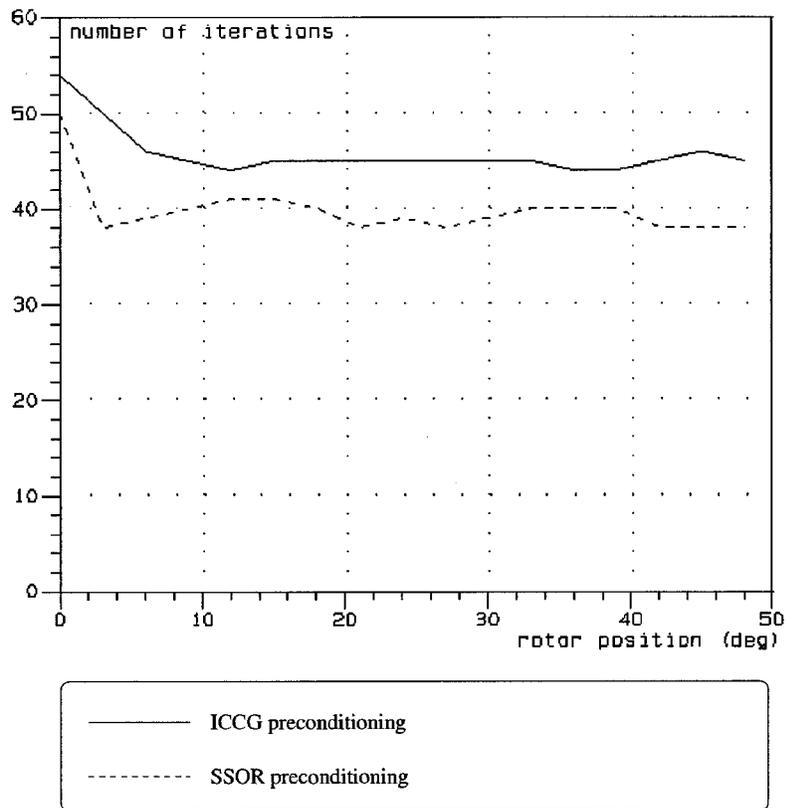


Fig. 8. Number of iterations for ICGG and SSOR preconditioning.

presented here, comparing the OLM method associated to the iterative method based on SSOR, with the one used in commercial

software associating slip surface method with ICGG. The better speed of convergence for SSOR, the predictability of theoretical

results and its relative simplicity of implementation are the main reasons for the choice of this preconditioner.

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