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# Discussion on <br> Graphical models for extremes <br> by Sebastian Engelke and Adrien Hitz 

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Independence and conditional independence require distributions supported on product spaces. Engelke and Hitz define a notion of conditional independence for a multivariate Pareto distribution $\mathbf{Y}=\lim _{u \rightarrow \infty} \mathbf{X} / u \mid\|\mathbf{X}\|_{\infty}>u$ when

$$
\begin{equation*}
\mathbb{P}(\mathbf{Y} \in \tilde{\mathcal{E}})=1 \tag{1}
\end{equation*}
$$

where $\tilde{\mathcal{E}}=(0, \infty)^{d}=\left\{\mathbf{x} \in[0, \infty)^{d}, \min _{1 \leq k \leq d} x_{k}>0\right\}$. By conditioning the limit vector $\mathbf{Y}$ on the event that $\left\{Y_{k}>1\right\}$, the authors actually work on the product space $\mathcal{L}^{k}=\left\{\mathbf{x} \in \mathcal{E}, x_{k}>1\right\}$ with $\mathcal{E}=[0, \infty)^{d} \backslash\{\mathbf{0}\}$.

The definition of $\tilde{\mathcal{E}}$ encourages to use a characterization of regularly varying random vectors in terms of the minimum of their marginals. A random vector $\mathbf{X} \in \mathcal{E}$ is regularly varying on the space $\tilde{\mathcal{E}}$ if and only if

$$
\begin{equation*}
\min _{1 \leq k \leq d} X_{k} \text { is regularly varying and }\left(u^{-1} \mathbf{X} \mid \min _{1 \leq k \leq d} X_{k}>u\right) \xrightarrow{d} \mathbf{Y}^{\prime}, \quad u \rightarrow \infty \tag{2}
\end{equation*}
$$

where $\mathbf{Y}^{\prime}$ takes values in the product space

$$
\left\{\mathbf{x} \in \mathcal{E}, \min _{1 \leq k \leq d} x_{k}>1\right\}=(1, \infty)^{d}=\bigcap_{1 \leq k \leq d} \mathcal{L}^{k}
$$

see Segers et al. (2017), Proposition 3.1. Restricting the regular variation condition of $\mathbf{X}$ to the space $\tilde{\mathcal{E}}$ is necessary to capture its asymptotic behavior through the events $\left\{\min _{1 \leq k \leq d} X_{k}>u\right\}$ for $u>0$.

In the framework of Engelke and Hitz (i.e. regular variation on $\mathcal{E}$ and assumption (1)), both conditions in (2) hold and the limit vector $\mathbf{Y}^{\prime}$ corresponds to the vector $\mathbf{Y}$ conditioned on the event that $\left\{\min _{1 \leq k \leq d} Y_{k}>1\right\}$, see Figure 1. Conversely, the two assumptions in (2) are more general since they include for instance the case where the marginals of $\mathbf{X}$ are independent (and then $\mathbf{Y}$ exists and its distribution concentrates on the axes).

An alternative notion of conditional independence for a multivariate Pareto distribution can thus be defined through conditional independence of $\mathbf{Y}_{A}^{\prime}$ and $\mathbf{Y}_{C}^{\prime}$ given $\mathbf{Y}_{B}^{\prime}$. Such property holds if the density $f_{\mathbf{Y}^{\prime}}$ of $\mathbf{Y}^{\prime}$ factorizes as

$$
f_{\mathbf{Y}^{\prime}}\left(\mathbf{y}^{\prime}\right) f_{\mathbf{Y}^{\prime}, B}\left(\mathbf{y}_{B}^{\prime}\right)=f_{\mathbf{Y}^{\prime}, A \cup B}\left(\mathbf{y}_{A \cup B}^{\prime}\right) f_{\mathbf{Y}^{\prime}, B \cup C}\left(\mathbf{y}_{B \cup C}^{\prime}\right), \quad \mathbf{y}^{\prime} \in(1, \infty)^{d}
$$

Regarding the original vector $\mathbf{X}$, the study of $\mathbf{Y}^{\prime}$ instead of $\mathbf{Y}$ provides two advantages. First, the vector $\mathbf{Y}^{\prime}$ models the extremal behavior of $\mathbf{X}$ when all its marginals are simultaneously large. Therefore, it provides accurate models for extremal dependent data. Note that if the strong


Figure 1: The shaded areas correspond to (a) the support of $\mathbf{Y}$, (b) the support of $\mathbf{Y}^{1}$ and (c) the support of $\mathbf{Y}^{\prime}$.
condition that all marginals are extreme together is not satisfied, then (2), and thus (1), is very unlikely. Second, conditional independence of $\mathbf{Y}^{\prime}$ can be interpreted in terms of $\mathbf{X}$. Indeed, if $\mathbf{X}_{A}$ is conditionally independent of $\mathbf{X}_{C}$ given $\mathbf{X}_{B}$, then $\mathbf{Y}_{A}^{\prime}$ is conditionally independent of $\mathbf{Y}_{C}^{\prime}$ given $\mathbf{Y}_{B}^{\prime}$. In the case $B=\emptyset$, we obtain independence of $\mathbf{X}_{A}$ and $\mathbf{X}_{C}$ and thus of $\mathbf{Y}_{A}^{\prime}$ and $\mathbf{Y}_{C}^{\prime}$, whereas $\mathbf{Y} \in \mathcal{E} \backslash \tilde{\mathcal{E}}$ (as soon as $\mathbf{Y}$ exists). There, (2) is satisfied but not (1).

## References

Segers, J., Zhao, Y., and Meinguet, T. (2017). Polar decomposition of regularly varying time series in star-shaped metric spaces. Extremes, 20(3):539-566.

