



## Discussion on "Graphical models for extremes" by Sebastian Engelke and Adrien Hitz

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Discussion on  
*Graphical models for extremes*  
 by Sebastian Engelke and Adrien Hitz

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Independence and conditional independence require distributions supported on product spaces. Engelke and Hitz define a notion of conditional independence for a multivariate Pareto distribution  $\mathbf{Y} = \lim_{u \rightarrow \infty} \mathbf{X}/u \mid \|\mathbf{X}\|_\infty > u$  when

$$\mathbb{P}(\mathbf{Y} \in \tilde{\mathcal{E}}) = 1, \quad (1)$$

where  $\tilde{\mathcal{E}} = (0, \infty)^d = \{\mathbf{x} \in [0, \infty)^d, \min_{1 \leq k \leq d} x_k > 0\}$ . By conditioning the limit vector  $\mathbf{Y}$  on the event that  $\{Y_k > 1\}$ , the authors actually work on the product space  $\mathcal{L}^k = \{\mathbf{x} \in \mathcal{E}, x_k > 1\}$  with  $\mathcal{E} = [0, \infty)^d \setminus \{\mathbf{0}\}$ .

The definition of  $\tilde{\mathcal{E}}$  encourages to use a characterization of regularly varying random vectors in terms of the minimum of their marginals. A random vector  $\mathbf{X} \in \mathcal{E}$  is regularly varying on the space  $\tilde{\mathcal{E}}$  if and only if

$$\min_{1 \leq k \leq d} X_k \text{ is regularly varying} \quad \text{and} \quad (u^{-1}\mathbf{X} \mid \min_{1 \leq k \leq d} X_k > u) \xrightarrow{d} \mathbf{Y}', \quad u \rightarrow \infty, \quad (2)$$

where  $\mathbf{Y}'$  takes values in the product space

$$\{\mathbf{x} \in \mathcal{E}, \min_{1 \leq k \leq d} x_k > 1\} = (1, \infty)^d = \bigcap_{1 \leq k \leq d} \mathcal{L}^k,$$

see Segers et al. (2017), Proposition 3.1. Restricting the regular variation condition of  $\mathbf{X}$  to the space  $\tilde{\mathcal{E}}$  is necessary to capture its asymptotic behavior through the events  $\{\min_{1 \leq k \leq d} X_k > u\}$  for  $u > 0$ .

In the framework of Engelke and Hitz (i.e. regular variation on  $\mathcal{E}$  and assumption (1)), both conditions in (2) hold and the limit vector  $\mathbf{Y}'$  corresponds to the vector  $\mathbf{Y}$  conditioned on the event that  $\{\min_{1 \leq k \leq d} Y_k > 1\}$ , see Figure 1. Conversely, the two assumptions in (2) are more general since they include for instance the case where the marginals of  $\mathbf{X}$  are independent (and then  $\mathbf{Y}$  exists and its distribution concentrates on the axes).

An alternative notion of conditional independence for a multivariate Pareto distribution can thus be defined through conditional independence of  $\mathbf{Y}'_A$  and  $\mathbf{Y}'_C$  given  $\mathbf{Y}'_B$ . Such property holds if the density  $f_{\mathbf{Y}'}$  of  $\mathbf{Y}'$  factorizes as

$$f_{\mathbf{Y}'}(\mathbf{y}') f_{\mathbf{Y}', B}(\mathbf{y}'_B) = f_{\mathbf{Y}', A \cup B}(\mathbf{y}'_{A \cup B}) f_{\mathbf{Y}', B \cup C}(\mathbf{y}'_{B \cup C}), \quad \mathbf{y}' \in (1, \infty)^d.$$

Regarding the original vector  $\mathbf{X}$ , the study of  $\mathbf{Y}'$  instead of  $\mathbf{Y}$  provides two advantages. First, the vector  $\mathbf{Y}'$  models the extremal behavior of  $\mathbf{X}$  when all its marginals are simultaneously large. Therefore, it provides accurate models for extremal dependent data. Note that if the strong

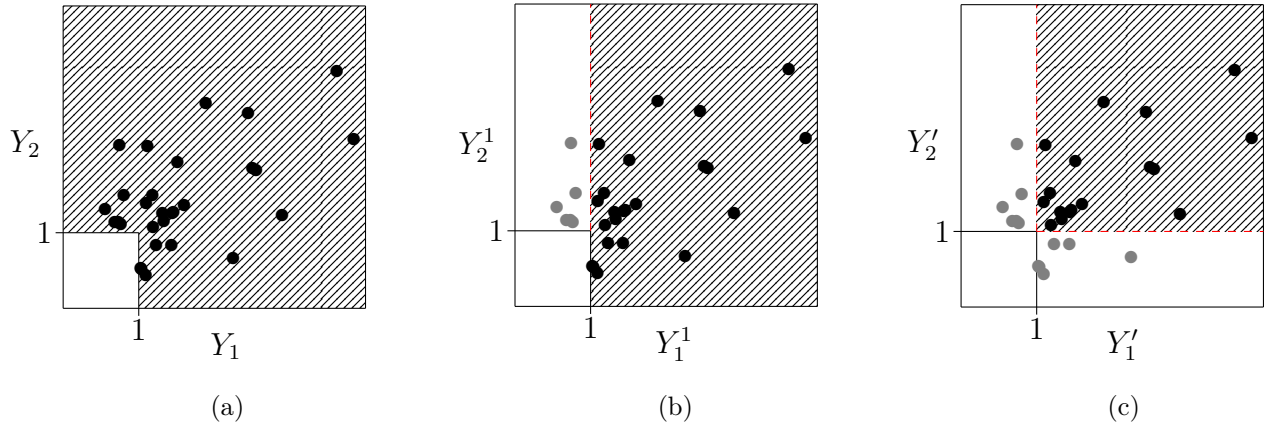


Figure 1: The shaded areas correspond to (a) the support of  $\mathbf{Y}$ , (b) the support of  $\mathbf{Y}^1$  and (c) the support of  $\mathbf{Y}'$ .

condition that all marginals are extreme together is not satisfied, then (2), and thus (1), is very unlikely. Second, conditional independence of  $\mathbf{Y}'$  can be interpreted in terms of  $\mathbf{X}$ . Indeed, if  $\mathbf{X}_A$  is conditionally independent of  $\mathbf{X}_C$  given  $\mathbf{X}_B$ , then  $\mathbf{Y}'_A$  is conditionally independent of  $\mathbf{Y}'_C$  given  $\mathbf{Y}'_B$ . In the case  $B = \emptyset$ , we obtain independence of  $\mathbf{X}_A$  and  $\mathbf{X}_C$  and thus of  $\mathbf{Y}'_A$  and  $\mathbf{Y}'_C$ , whereas  $\mathbf{Y} \in \mathcal{E} \setminus \hat{\mathcal{E}}$  (as soon as  $\mathbf{Y}$  exists). There, (2) is satisfied but not (1).

## References

Segers, J., Zhao, Y., and Meinguet, T. (2017). Polar decomposition of regularly varying time series in star-shaped metric spaces. *Extremes*, 20(3):539–566.