



HAL
open science

Discussion on "Graphical models for extremes" by Sebastian Engelke and Adrien Hitz

Nicolas Meyer, Olivier Wintenberger

► **To cite this version:**

Nicolas Meyer, Olivier Wintenberger. Discussion on "Graphical models for extremes" by Sebastian Engelke and Adrien Hitz. *Journal of the Royal Statistical Society: Series B*, 2020. hal-02537123

HAL Id: hal-02537123

<https://hal.science/hal-02537123v1>

Submitted on 8 Apr 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Discussion on
Graphical models for extremes
 by Sebastian Engelke and Adrien Hitz

Nicolas Meyer* and Olivier Wintenberger†
Sorbonne Université, LPSM, F-75005, Paris, France
 **nicolas.meyer@upmc.fr* †*olivier.wintenberger@upmc.fr*

Independence and conditional independence require distributions supported on product spaces. Engelke and Hitz define a notion of conditional independence for a multivariate Pareto distribution $\mathbf{Y} = \lim_{u \rightarrow \infty} \mathbf{X}/u \mid \|\mathbf{X}\|_\infty > u$ when

$$\mathbb{P}(\mathbf{Y} \in \tilde{\mathcal{E}}) = 1, \tag{1}$$

where $\tilde{\mathcal{E}} = (0, \infty)^d = \{\mathbf{x} \in [0, \infty)^d, \min_{1 \leq k \leq d} x_k > 0\}$. By conditioning the limit vector \mathbf{Y} on the event that $\{Y_k > 1\}$, the authors actually work on the product space $\mathcal{L}^k = \{\mathbf{x} \in \mathcal{E}, x_k > 1\}$ with $\mathcal{E} = [0, \infty)^d \setminus \{\mathbf{0}\}$.

The definition of $\tilde{\mathcal{E}}$ encourages to use a characterization of regularly varying random vectors in terms of the minimum of their marginals. A random vector $\mathbf{X} \in \mathcal{E}$ is regularly varying on the space $\tilde{\mathcal{E}}$ if and only if

$$\min_{1 \leq k \leq d} X_k \text{ is regularly varying} \quad \text{and} \quad (u^{-1} \mathbf{X} \mid \min_{1 \leq k \leq d} X_k > u) \xrightarrow{d} \mathbf{Y}', \quad u \rightarrow \infty, \tag{2}$$

where \mathbf{Y}' takes values in the product space

$$\{\mathbf{x} \in \mathcal{E}, \min_{1 \leq k \leq d} x_k > 1\} = (1, \infty)^d = \bigcap_{1 \leq k \leq d} \mathcal{L}^k,$$

see Segers et al. (2017), Proposition 3.1. Restricting the regular variation condition of \mathbf{X} to the space $\tilde{\mathcal{E}}$ is necessary to capture its asymptotic behavior through the events $\{\min_{1 \leq k \leq d} X_k > u\}$ for $u > 0$.

In the framework of Engelke and Hitz (i.e. regular variation on \mathcal{E} and assumption (1)), both conditions in (2) hold and the limit vector \mathbf{Y}' corresponds to the vector \mathbf{Y} conditioned on the event that $\{\min_{1 \leq k \leq d} Y_k > 1\}$, see Figure 1. Conversely, the two assumptions in (2) are more general since they include for instance the case where the marginals of \mathbf{X} are independent (and then \mathbf{Y} exists and its distribution concentrates on the axes).

An alternative notion of conditional independence for a multivariate Pareto distribution can thus be defined through conditional independence of \mathbf{Y}'_A and \mathbf{Y}'_C given \mathbf{Y}'_B . Such property holds if the density $f_{\mathbf{Y}'}$ of \mathbf{Y}' factorizes as

$$f_{\mathbf{Y}'}(\mathbf{y}') f_{\mathbf{Y}', B}(\mathbf{y}'_B) = f_{\mathbf{Y}', A \cup B}(\mathbf{y}'_{A \cup B}) f_{\mathbf{Y}', B \cup C}(\mathbf{y}'_{B \cup C}), \quad \mathbf{y}' \in (1, \infty)^d.$$

Regarding the original vector \mathbf{X} , the study of \mathbf{Y}' instead of \mathbf{Y} provides two advantages. First, the vector \mathbf{Y}' models the extremal behavior of \mathbf{X} when all its marginals are simultaneously large. Therefore, it provides accurate models for extremal dependent data. Note that if the strong

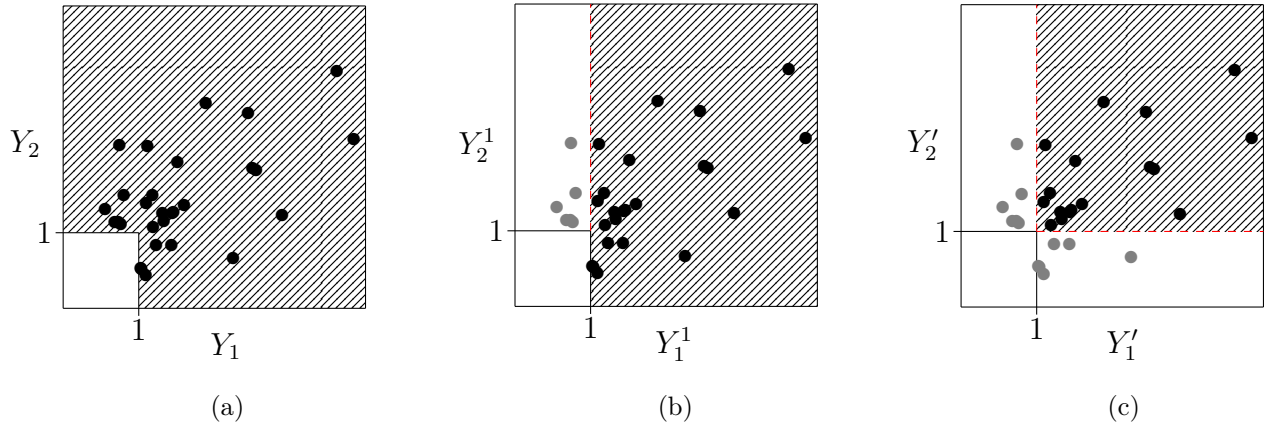


Figure 1: The shaded areas correspond to (a) the support of \mathbf{Y} , (b) the support of \mathbf{Y}^1 and (c) the support of \mathbf{Y}' .

condition that all marginals are extreme together is not satisfied, then (2), and thus (1), is very unlikely. Second, conditional independence of \mathbf{Y}' can be interpreted in terms of \mathbf{X} . Indeed, if \mathbf{X}_A is conditionally independent of \mathbf{X}_C given \mathbf{X}_B , then \mathbf{Y}'_A is conditionally independent of \mathbf{Y}'_C given \mathbf{Y}'_B . In the case $B = \emptyset$, we obtain independence of \mathbf{X}_A and \mathbf{X}_C and thus of \mathbf{Y}'_A and \mathbf{Y}'_C , whereas $\mathbf{Y} \in \mathcal{E} \setminus \hat{\mathcal{E}}$ (as soon as \mathbf{Y} exists). There, (2) is satisfied but not (1).

References

Segers, J., Zhao, Y., and Meinguet, T. (2017). Polar decomposition of regularly varying time series in star-shaped metric spaces. *Extremes*, 20(3):539–566.