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Optimal design of helical heat/mass exchangers under laminar 1 flow: CFD investigation and correlations for maximal transfer 2 efficiency and process intensification performances 3

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Abstract

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5 7 9 10 11 12 13 Highly curved helical pipes (HCHPs) offer tremendous potentialities for intensified heat/mass transfer performances as they 14 generate intense Dean-type vortices. However, these designs have not been explored so far in the literature, probably because 15 they are difficult to build using traditional manufacturing techniques. Nowadays, thanks to a witnessed progress in 3D-16 printing, the fabrication of HCHPs has become achievable. Therefore, investigating their performance in terms of heat and 17 mass transfer intensification presents significant interest from both academic and industrial points of view.

18 19 In this paper, CFD simulations are carried out to determine the heat/mass transfer efficiency in helical pipes (particularly highly curved ones) under laminar flow conditions. The packing density (i.e. interfacial area) of these geometries is evaluated 20 using a CAD software. The results reveal that HCHPs not only allow achieving much higher transfer rates than straight and classical helical pipes, but they can also be densely packed. Therefore, when appropriate designs are selected, impressive process intensification factors are achievable, with up to 8-fold volume reductions.

21 22 23 24 25 26 27 28 29 Finally, correlations are developed for evaluating the interfacial area and the heat/mass transfer efficiency in classical and highly curved helical pipes. In future works, these correlations will be used in model-based optimization for determining the optimal designs of helically coiled heat/mass exchangers.

Keywords: CFD; Correlation; Heat transfer; Helical pipe; Mass transfer; Packing density

1. Introduction

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31 Economic issues and continuously strengthening environmental regulations force the industry to develop 32 increasingly compact reactors and heat and mass transfer equipment. Such units allow improving the processes 33 efficiency while reducing their overall energy requirements, costs, environmental impacts, weight and footprint. 34 More specifically, minimizing volume and/or weight is of primary importance in applications where space is 35 limited, e.g. decentralized energy production, domestic applications, space and offshore processes.

36 Various strategies to enhance heat and mass transfer efficiencies have been proposed and employed [1, 2]. 37 They include the use of rotating devices [3], pulsating flow [4], twisted tape inserts [5], vortex generators [6] or 38 helical pipes, etc. Compared to other alternatives, helical pipes present the advantage of not involving any internals or moving parts. Therefore, they are less subject to failure, breakdown, fouling and clogging, and thus, 39 40 require lower maintenance costs.

41 Compared to straight tubes, due to centrifugal effects, the flow in curved and helical pipes develops 42 secondary flow structures called Dean cells (Figure 1). They consist in a pair of counter-rotating vortices in the 43 cross-stream direction, perpendicularly to the primary flow. These vortices allow a great improvement of the 44 transfer efficiency. Indeed, if we consider fully developed laminar flow conditions, in the case of straight tubes, 45 fluid particles move along straight streamlines parallel to the walls. Therefore, lateral mixing, i.e. radial 46 heat/mass transfer, occurs under the sole effect of conduction/diffusion, which leads to poor transfer efficiencies 47 when the pipe diameter is not too small. On the other hand, in helical pipes, Dean cells provide efficient 48 advective transport of the fluid particles between the pipe walls and its centerline, leading to increased transfer 49 performance. Accordingly, these geometries have been suggested and/or employed in several applications and 50 industrial processes: (1) Heat transfer enhancement for nuclear plants [7] and many other applications [8, 9]. (2) 51 Increased mass transfer rates in catalytic reactors [10], dense [11] and porous [2] membrane contactors, etc. (3) 52 Improvement of mixing efficiency [12].



Figure 1: Schematic representation of Dean cells in a helical pipe.

57 The shape of a helical pipe is described by two geometric parameters, the dimensionless pitch, $p^* = p/d$, and 58 the dimensionless helix radius, $R_H^* = R_H/d$, where d is the pipe diameter, p the helix pitch and R_H the helix 59 radius. Figure 2 illustrates some representative helical tube geometries and the limits of the so-called forbidden 60 region. The equation of this frontier has been determined by Przybył and Pierański [13]: it corresponds to the 61 limit beyond which it is not possible to further decrease the helix pitch because the consecutive turns of the helix 62 would overlap one with another. In other terms, the forbidden region corresponds to the set of p^* and R_H^* for 63 which it is no more possible to design helical shapes. Figure 3 is a contour plot of the dimensionless helix curvature, $\kappa^* = \kappa d = d/\gamma$, in the (R_H^*, p^*) space, where κ is the pipe curvature and γ its curvature radius. The 64 65 mathematical expression of κ is as follows:

$$\kappa = \frac{1}{R_H \left[1 + \left(\frac{p}{2\pi R_H}\right)^2 \right]}$$
Eq. 1

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Figure 3 illustrates the fact that the helix curvature tends toward zero at three asymptotic limits: (1) when the dimensionless pitch, $p^* = p/d$, tends to infinity. (2) when the dimensionless helix radius $R_H^* = R_H/d$ tends to zero. (3) when R_H^* tends to infinity. Indeed, as shown in Figure 2, at the first two limits, the helix geometry tends toward that of a straight tube, while at the third limit, the helical tube straightens and becomes locally similar to a straight pipe.



Figure 2: Limit of the forbidden region in the (R_H^*, p^*) space (adapted from [13]) and some representative helix geometries [14].





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80 Most importantly, Figure 3 reveals that the highest pipe curvatures are associated to helixes of low pitch and 81 relatively low helical radius. These geometries will be referred to as Highly Curved Helical Pipes (HCHPs) 82 further on in this paper. Given their low radius of curvature, they exhibit the highest centrifugal effects. Thus, 83 HCHPs are expected to produce the most intense Dean vortices, which should lead to the best heat/mass transfer 84 performance. Nonetheless, to the authors' best knowledge, no previous study has focused yet on the transfer 85 efficiency in such geometries. As can be noticed from Table 1 for example, only 'classical' helical geometries 86 have been considered thus far in literature. This is probably because HCHPs are difficult to fabricate using 87 traditional manufacturing techniques.

Studies dealing with classical helical pipes have shown that these geometries allow a significant enhancement of the heat/mass transfer. Indeed, Nusselt and Sherwood numbers up to 7 times greater than in straight tubes were reported in literature [19, 24, 25]. However, it is noteworthy that helical pipes cannot be as densely packed as straight ones. Therefore, the transfer efficiency improvement they allow is - at least partially counterbalanced by their lower packing density. For example, Kaufhold et al. [26] reported that if R_H^* is not sufficiently small, classical helixes lead to lower performance than straight tubes in terms of volumetric mass transfer rates (i.e. mass flux per unit volume).

95 Interestingly, HCHPs are not only expected to lead to better transfer rates than classical helixes, but also, 96 their elongated shape (Figure 2) allow them to be densely packed. Hence they should allow a significant 97 intensification of the overall transfer rate, i.e. a meaningful unit volume reduction of reactors, heat exchangers 98 and membrane contactors. Nowadays, with the development of 3D-printing technologies, manufacturing HCHPs 99 has become achievable [14]. Therefore, investigating their performance in terms of heat and mass transfer 9100 efficiency offers significant interest from both academic and industrial points of view.

101 This paper focuses on transport phenomena in helical pipes, particularly highly curved ones (HCHPs), under 102 fully developed laminar flow conditions. The laminar flow regime is encountered in many applications of 103 practical interest, in particular small-scale devices, e.g. microfluidics, micro-structured heat exchangers and 104 reactors, micro-mixers and hollow fiber membranes. CFD (computational fluid dynamics) simulations are 105 reported hereafter in order to determine the Nusselt number in helical pipes, and by analogy, the Sherwood 106 number. Different operating conditions, described by the Reynolds number, and various helix designs are 107 examined. Prandtl numbers (and by analogy, Schmidt numbers) ranging from 1 to 10 are considered. This range 108 of fluid properties covers many existing processes. Additionally, for evaluating the volumetric transfer rates 109 allowed, helixes packing density as a function of the helixes geometry has been determined using a CAD 110 (computer-aided design) software.

CFD results revealed that among helical pipes, highly curved ones allow achieving the maximal Nusselt and Sherwood numbers, which can be nearly an order of magnitude higher than in straight ones. Moreover, numerical data confirmed that HCHPs can be densely packed. The volumetric transfer rates they allow is up to 8 times higher than that reached with straight tubes. On the other hand, given their low packing densities, classical helical pipes may lead to lower overall performance than straight ones.

116 It is also shown that available correlations fail in estimating the heat/mass transfer rates in HCHPs and the 117 helixes packing density with a sufficient accuracy. Therefore, two new correlations are proposed in this paper. 118 The first one allows predicting the Nusselt and Sherwood numbers over a wide range of helical pipe geometries 119 and operating condition, and the second allows determining the optimal packing density as a function of the 120 helix geometry. This set of correlations, along with that proposed by Abushammala et al. [14] for calculating the 121 friction coefficient in helical pipes, can be used in model-based optimization of helically coiled heat exchangers / 122 membrane contactors. This procedure allows determining the optimal helix geometry, i.e. the one leading to the 123 most lucrative trade-off between the process intensification (unit volume reduction) and energy efficiency (extra 124 pressure drop).

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	Reference	Correlation	Correlation's validity range according to its authors	Geometric parameters of the helical pipes investigated
	Schmidt [15]	$Nu_{H}^{\infty} = 3.65 + 0.08 \left[1 + 0.8 \left(\frac{1}{2R_{H}^{*}} \right)^{0.9} \right] Pr^{\frac{1}{3}} Re^{m}$ $m = 0.5 + 0.2903 \left(\frac{1}{2R_{H}^{*}} \right)^{0.194}$	Laminar regime with $Re \ge 100$ Large range of Pr (air, water and oil were used as test fluids)	$2.5 \le R_H^* \le 42$ $3 \le p^* \le 137$
	Dravid et al. [16]	$Nu_{H}^{\infty} = (0.76 + 0.65 De_{R_{H}}^{0.5}) Pr^{0.175}$	$50 \le De_{R_H} \le 2\ 000$ $5 \le Pr \le 175$	$10 \le R_H^* \le 50$ The range of p^* is not reported.
	Kalb and Seader [17]	$Nu_{H}^{\infty} = 0.836 De_{R_{H}}^{0.5} Pr^{0.1}$	$De_{R_H} \ge 80$ $0.7 \le Pr \le 5$	$5 \le R_H^* \le 50$ The range of p^* is not reported.
nencai pipes.	Manlapaz and Churchill [18]	$Nu_{H}^{\infty} = \left[\left(3.657 + \frac{4.343}{z_{1}} \right)^{3} + 1.158 \left(\frac{De_{R_{H}}}{z_{2}} \right)^{3/2} \right]^{1/3}$ $z_{1} = \left(1 + \frac{957}{De_{R_{H}}^{2} Pr} \right)^{2} ; z_{2} = 1 + \frac{0.477}{Pr}$	Entire laminar regime	$R_H^* \ge 2.5$ The range of p^* is not reported.
	Moulin et al. [19]	$Sh_H^{\infty} = 0.14 De_{\gamma}^{0.75} Sc^{0.33}$	$150 \le Re \le 2\ 000$	$\begin{array}{l} 11.5 \leq R_{H}^{*} \leq 12.4 \\ 9.9 \leq p^{*} \leq 15.4 \end{array}$
	Yildiz et al. [20]	$Nu_{H}^{\infty} = 0.0551 \ De_{R_{H}}^{0.864} \ Pr^{0.4}$	$1\ 265 \le De_{R_H} \le 2\ 000$ $5 \le Pr \le 175$	$120 \le p^* \le 360$ $R_H^* = 7.5$
	Xin and Ebadion [21]	$Nu_{H}^{\infty} = (0.318 De_{R_{H}}^{0.643} + 2.153) Pr^{0.177}$	$20 \le De_{R_H} \le 2\ 000$ $0.7 \le Pr \le 175$	$22.6 \le R_H^* \le 75$ The value of p^* was not reported.
	Ghobadi and Muzychka [22]	$Nu_{H}^{\infty} = [3.66^{4} + (0.91375 De_{R_{H}}^{0.5} Pr^{-0.1})^{4}]^{1/4}$	$40 \le De_{R_H} \le 700$ $5 \le Pr \le 15$	$6.1 \le R_H^* \le 24.2$ The range of p^* is not reported.
	Sheeba et al. [23]	$Nu_{H}^{\infty} = 3.6063 De_{R_{H}}^{0.2216} Pr^{0.0540} \left(\frac{p *}{2\pi R_{H}^{*}}\right)^{0.0472}$	$200 \le De_{R_H} \le 700$	$R_H^* = 29.7$ $2 \le p^* \le 18.2$

Table 1: Set of commonly used correlations for predicting Nu_H^{∞} and Sh_H^{∞} for laminar flow in helical pipes under uniform wall temperature/concentration. Nu_H^{∞} and Sh_H^{∞} respectively correspond to the asymptotic Nusselt and Sherwood numbers in

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2. CFD computation of Nusselt (and Sherwood) number in helical pipe flows

147 2.1. Nusselt (and Sherwood) number in helical pipe flows

In helical and curved pipes undergoing heat transfer between the fluid flow and the wall, the convective heat transfer is not uniform over the wall, but varies in both radial and axial directions. However, in most engineering applications, only the heat transfer coefficient averaged over the pipe circumference (i.e. in the radial direction) is of practical interest. It will be simply referred to as convective heat transfer coefficient, h, in the remaining part of this paper.

154 Considering a cross-section of the tube, the fluid mixing-cup temperature (i.e. mass flow rate weighted 155 temperature), $T_{f,MC}$, at this position is calculated as follows:

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$$T_{f,MC} = \frac{\iint_{S} \rho \ T \ \vec{u} \cdot \vec{n} \ dS}{\iint_{S} \rho \ \vec{u} \cdot \vec{n} \ dS} = \frac{\iint_{S} \rho \ T \ \vec{u} \cdot \vec{n} \ dS}{\dot{m}}$$
Eq. 2

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where S is the surface of the circular flow section (Figure 4) and \vec{n} its unit normal vector. ρ denotes the fluid density, T the local fluid temperature, \vec{u} its velocity and \vec{m} its total mass flow rate. The convective heat transfer coefficient at this position is defined and can be calculated as follows:

$$h = \lim_{S_w \to 0} \frac{\Phi}{S_w \left(T_w - T_{f,MC}\right)}$$
 Eq. 3

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where S_w is an annular surface on the wall (Figure 4) inclosing the flow section, *S*. T_w is the average temperature of this wall element, or simply the wall temperature when an isothermal wall is considered as in this paper. Φ denotes the heat flux transferred between the wall element and the flow. The Nusselt number, *Nu*, which can be regarded as a dimensionless expression of *h*, is defined as follows in both helical and straight pipes:

 $Nu = \frac{h \, d}{\lambda}$ Eq. 4

169 where *d* is the pipe internal diameter and λ the fluid thermal conductivity.

Figure 4: Typical shape of a wall element (in red) over which h is averaged.

Figure 5a illustrates a qualitative variation of Nu (and thus of h) along a helical or a straight tube, the abscissa axis being the - curvilinear - position along the pipe centerline. The Nusselt number is the highest at the pipe entrance and decreases over a distance called 'thermal entrance length' before reaching an asymptotic value, Nu^{∞} . For a laminar flow in a straight pipe under uniform wall temperature conditions, the length of the thermal entrance region, L_{th} , can be estimated as follows [27]:

$$L_{th} = Maximum (0.0565 \, Re \, d; 0.037 \, Re \, Pr \, d)$$
 Eq. 5

181 Re = Ud/v is the Reynolds number, where U is the mean velocity of the primary flow and v the fluid kinematic 182 viscosity, $Pr = v/\alpha$ is the Prandtl number where α is the fluid thermal diffusivity. The first term on the right 183 hand side of Eq. 5 represents the length required for the flow to fully develop, i.e. for the velocity field and the 184 local friction factor to become axially invariant, while the second term corresponds to the distance required for 185 the flow to become thermally developed.





Figure 5: Typical variation of the Nusselt (Sherwood) number from the entrance of a straight or a helical pipe.

189 Equation 5 reveals that the thermal entrance length may become very important for large values of Pr. 190 However, in the current study, only Pr numbers ranging from 1 to 10 are considered. With such fluids, entrance 191 effects are usually negligible in the case of pipes of small diameter and/or sufficient length, which is generally 192 the case in the applications targeted in this study, namely heat exchangers (and catalytic reactors and hollow 193 fiber membranes in the case of mass transfer). This is particularly true when dealing with helical geometries. 194 Indeed, under similar operating conditions, the entrance length in helical pipes is generally shorter than in straight ones [28]. Therefore, the asymptotic Nusselt number, Nu^{∞} , is sufficient for characterizing the effective 195 196 heat transfer in helical shape exchangers. Accordingly, in this paper, determination of Nu^{∞} in helical pipes via 197 CFD simulations will be emphasized. Thus, although the flow and heat transfer characteristics over the entire 198 pipe are simulated, only Nu^{∞} results are reported; entrance effects data are considered beyond the scope of this 199 paper and will not be reported here.

200 With a laminar flow regime and an isothermal wall, under ideal conditions (see Section 2.2), the asymptotic 201 Nusselt number in straight pipes, Nu_S^{∞} , equals 3.657 [29]. On the other hand, contrary to Nu_S^{∞} , the asymptotic 202 Nusselt number in helical pipes, Nu_H^{∞} , is not constant, but depends on both Reynolds and Prandtl numbers as 203 well as on the pipe geometry.

Assuming an incompressible flow with uniform fluid properties, the heat transfer coefficient under thermally 204 205 developed flow conditions would depend on the helical pipe geometric parameters (d, R_H and p), the fluid 206 properties (ρ , v, λ and α) and the mean velocity of the primary flow, U. Therefore, according to the Buckingham 207 π theorem, Nu_{H}^{μ} is a functional relation of the following four independent dimensionless parameters: (1) The 208 dimensionless helix pitch, $p^* = p/d$. (2) The dimensionless helix radius, $R_H^* = R_H/d$. These first two 209 parameters characterize the helix shape. (3) The Reynolds number, Re = Ud/v, which accounts for the 210 operating conditions. (4) The Prandtl number, Pr, which accounts for the fluid properties. As a result of the heat transfer enhancement by the Dean vortices, Nu_H^{∞} is always greater than 3.657. Table 1 shows a summary of 211 commonly used correlations for evaluating Nu_H^∞ under laminar flow conditions. 212

Finally, it is worthy to note that the heat transfer analysis carried out in this section can be transposed and applied to mass transfer phenomena. In fact, it has been long recognized that heat and mass transfer processes present similar behaviours. Indeed, in many practical situations, the equations describing mass and heat transfer are mathematically analogous (see Sections 2.2 and 2.3 for details). Accordingly, heat transfer results can be converted to mass transfer results and vice versa.

Hence, in the framework of the present study, the heat/mass analogy stipulates that if the same geometric (R_H^* and p^*) and operating conditions (*Re*) are considered, then:

$$Sh_{H}^{\infty} = Nu_{H}^{\infty}$$
 (and $Sh = Nu$) if $Sc = Pr$ Eq. 6

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where *Sh* is the Sherwood number (the mass transfer analogue to Nu) and Sh_H^∞ its asymptotic value in helical pipes. *Sh* is defined as follows:

$$Sh = \frac{k d}{D}$$
 Eq. 7

where *k* is the convective mass transfer coefficient between the wall and the fluid and *D* the mass diffusivity of the involved species. Under laminar flow conditions and uniform wall concentration, Sh_H^{∞} equals 3.657 in virtue 228 of the heat/mass transfer analogy. Sc = v/D is the Schmidt number, the mass transfer analogue of Pr. It 229 represents the ratio of momentum diffusivity (kinematic viscosity) to mass diffusivity. Based on the above 230 discussion about Nu_{H}^{α} and the heat/mass transfer analogy, it can be argued that Sh_{H}^{α} depends on the following 231 four dimensionless numbers: R_H^* , p^* , Re and Sc.

233 2.2. CFD modeling and simulation of heat transfer in helical pipes under laminar flow conditions

235 CFD simulations were carried out for various helical pipe designs and operating conditions. The geometries 236 were drawn using Autodesk Inventor Professional 2018 software, based on a sufficient tube length to attain the 237 thermally developed flow region. The 3D numerical domain was meshed using the ANSYS Meshing software. 238 The grid consisted of hexahedral cells only, with a boundary layer mesh in the near-wall region for an accurate 239 calculation of the steep gradients prevailing in this zone. Based on preliminary simulations, the cells size and 240 density were chosen so at to ensure a mesh-independent solution for all the cases studied.

241 CFD simulations were conducted using the commercial code ANSYS Fluent 16. The flow field in the pipe 242 was determined by solving the continuity and Navier-Stokes equations assuming a Newtonian, incompressible, 243 and steady flow and uniform fluid properties (density and viscosity):

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$$div(\vec{u}) = 0$$

$$\vec{div}(\vec{u} \otimes \vec{u}) = -\frac{1}{\rho} \vec{\nabla}P + \nu \vec{\Delta}\vec{u}$$

Eqs. 8

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246 As boundary conditions, a uniform velocity profile was set at the pipe inlet: as the flow rate is imposed and 247 since a uniform fluid density is considered, the gravity force has no effect on the velocity field and therefore this 248 body force term was not included in the Navier-Stokes equations. At the pipe outlet, a uniform pressure 249 condition was used and the no-slip condition was set at the pipe wall. 250

The temperature field was determined by solving the energy balance equation:

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$$div(\vec{u} T) = \alpha \Delta T$$
 Eq. 9

253 As boundary conditions, a uniform fluid temperature is enforced at the inlet, an isothermal wall is considered and 254 the zero temperature gradient is imposed at the pipe outlet. It is noteworthy that the velocity and temperature 255 profiles at the inlet affect the flow and heat transfer phenomena in the entrance region only. Therefore, as this 256 paper focuses on heat transfer in the thermally developed zone (i.e. Nu_H^{∞}), the velocity and temperature 257 distributions at the inlet were simply considered as uniform.

258 The CFD model (i.e. Equations 8 and 9 and their boundary conditions) assumes: (1) Steady-state flow and 259 heat transfer phenomena. (2) An incompressible Newtonian fluid with uniform properties. (3) The absence of 260 phase transition phenomena. (4) A purely forced convection. Indeed, natural convection effects are neglected 261 since the fluid density is supposed uniform. This assumption is relevant as long as the Reynolds number is not 262 extremely low and the Grashof number (i.e. buoyancy effects) not extremely high, which is generally the case in 263 heat exchangers. (5) Radiative phenomena are ignored. (6) Heat generation by viscous dissipation is neglected in 264 Equation 9. This hypothesis is relevant apart in few particular situations where highly viscous fluids are used 265 under high strain rate conditions. It is worthy to note that, if in addition the axial heat conduction effects are 266 neglected, these assumptions lead to the theoretical result of $Nu_s^{\infty} = 3.657$ in the case of straight pipes [29].

267 The hydrodynamics and heat equations were iteratively solved until convergence. The advective terms were 268 discretized using the QUICK scheme while the diffusive ones were central-differenced. Pressure interpolation 269 was carried out using a second order scheme.

270 A total of nearly 280 CFD simulations was carried out. More than 25 helix designs (particularly highly 271 curved ones) were investigated within the following range of geometric conditions: $1.25 \le p^* \le 15$ and $0.05 \le 10^{-10}$ 272 $R_{H}^{*} \leq 10$. Eight Reynolds number values, ranging from 10 to 2 000, were examined. Three Prandtl numbers 273 were considered in the simulations: 1, 5 and 10. Fluids used in many industrial applications fall within this range 274 of Pr, e.g. most gases, water, refrigerants, many light organic liquids, molten salts and some oils under very high 275 temperature conditions.

276 For each simulation, the local Nu was calculated at different longitudinal positions across the pipe length 277 using Equations 3 and 4 (and considering a sufficiently small wall element as that depicted on Figure 4), and its 278 asymptotic value, Nu_H^{∞} , was determined. These CFD results were used to correlate Nu_H^{∞} as a function of the 279 helix design (p^* and R_H^*), the fluid properties (*Pr*) and the flow conditions (*Re*). They are provided in an Excel 280 sheet as supplementary material to this paper.

282 2.3. Heat and mass transfer analogy

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The procedure described in Section 2.2 allows determining the Nusselt number characterizing the heat transfer between the fluid and the pipe wall for Pr ranging from 1 to 10. Under some conditions discussed thereafter, the obtained results may be transposed to describe mass transfer between the fluid and the wall (e.g. mass transfer in membranes, adsorption or heterogeneous reaction processes). Obviously, this is possible for Schmidt numbers in the range of 1 to 10, which is the case in most gas-gas systems.

In binary mixtures with uniform density, the diffusive mass flux of a component A, \vec{n}_A (in kg m⁻² s⁻¹), is given by Fick's law:

$$\vec{n}_A = -D \ \overline{grad}(\rho_A) + \rho_A \vec{v}$$
 Eq. 10

293 where ρ_A is the mass concentration of A and D its mass diffusivity. \vec{v} is the velocity resulting from the diffusive 294 motion of A and the other species within the mixture. The second term on the right-hand side of Eq.10 is 295 generally referred to as the 'bulk motion contribution'. It represents the advective mass flux resulting from the 296 diffusion of the different species which induces a local motion of the mixture. When this term is negligible, 297 Fick's law becomes analogue to Fourier's law of heat conduction. This is the case under equimolar counter 298 diffusion of species of similar molecular weights or under dilute mixture conditions. Such conditions are 299 encountered in many important gas-gas applications such as tritium removal, recovery of volatile organic 300 compounds and adsorption processes in isotropic mixtures, etc. Most importantly, the bulk motion contribution 301 can be generally neglected in forced convection situations when the Reynolds number is not very low and the 302 flow field three-dimensional (as in the presence of Dean vortices). This significantly broadens the range of 303 conditions for which the heat/mass transfer analogy can be applied in helical geometries.

When the bulk motion contribution is negligible, in the absence of any homogeneous chemical reaction, if uniform fluid properties are considered, the steady-state the mass transfer equation becomes analogue to Eq. 9 and is given by: 307

$$div(\vec{u} C_A) = D \Delta C_A$$
 Eq. 11

309 where C_A is the molar concentration of component A.

310 Another restriction to the validity of the heat/mass transfer analogy concerns the boundary condition at the 311 wall. Indeed, in heat transfer simulations (Section 2.2), the no-slip condition applies at the pipe wall, while in 312 mass transfer processes, the mass flux at the wall obviously results in a non-zero wall-normal velocity. However, 313 in many situations, this velocity component has negligible effects on the flow field, and thus, the heat/mass 314 transfer analogy remains valid. This is the case in equimolar counter diffusion of species of similar molecular 315 weights or under dilute mixture conditions, even in the presence of a heterogeneous reaction or adsorption 316 phenomena at the wall. In any case, in situations where the heat/mass transfer analogy is not strictly valid, 317 although the results reported in this paper do not allow a precise evaluation of the Sherwood number, they still 318 provide a first insight about the optimal shape of helical tubes in mass transfer devices.

320 **3.** Optimal packing density of helixes

Helical pipes allow higher transfer efficiencies per unit surface than straight ones. However, they cannot be as densely packed as straight tubes. Thus, the transfer efficiency improvement they provide can be - at least partially - counterbalanced by their lower packing density. Therefore, in order to determine the overall volumetric transfer rate (i.e. heat/mass flux per unit volume) that helical pipes allow achieving, it is necessary to calculate their optimal packing density. To the authors' best knowledge, no analytical solution for this problem has been published thus far in literature. Only Kaufhold et al. [26] have reported a correlation for estimating helixes packing density, however, as discussed in Section 4.5, their model is highly inaccurate.

The ideal arrangement for non-overlapping straight tubes or cylinders is an - equilateral - triangular packing (also known as dense hexagonal lattice) (Figure 6a) where the distance between the axes of two neighboring cylinders equals their diameter. This configuration allows the straight tubes to best cover the available volume: it can been analytically shown that, $\phi_{S,max}$, the volume fraction they fill, equals $\pi/[4 \sin(\pi/3)] \approx 90.7\%$.

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340 Figure 6: Top view of: (a) ideally packed cylinders. (b) ideally packed helixes of $R_H^* = 2.5$ and $p^* = 1.25$. The triangular **341** (or dense hexagonal) lattice is illustrated by dashed lines. The black dots represent the axes of the helical or straight tubes. **342**

The CAD commercial software Autodesk Inventor Professional 2018 was used for determining the ideal packing of identical helixes under a triangular arrangement. a_{min} will denote the minimum achievable distance between two neighboring non-overlapping helixes (Figure 6b) and $\phi_{H,max}$ the volume fraction occupied by these closely packed helixes.

347 a_{min} was determined for more than 240 helix designs in the following range of geometric conditions: $1 \le p^* \le 20$ and $0.05 \le R_H^* \le 10$. Given the periodicity of the lattice, only three helixes disposed on an equilateral 349 triangular pattern were drawn using the CAD software. Their diameter, *d*, was arbitrarily taken as 1 mm. The 350 distance separating these helixes was iteratively decreased until determining a_{min} with an absolute tolerance of 351 0.01 mm. As the lowest possible value for a_{min} is the pipe diameter (situation that occurs in the case of straight 352 pipes), i.e. 1 mm in the present case, thus, the maximum relative error on the computed a_{min} values is about 1%.

These results allowed calculating, $a_{min}^* = a_{min}/d$, the dimensionless minimum distance separating the closely packed helixes. Indeed, dimensionless analysis shows that a_{min}^* is a functional relation of p^* and R_H^* . The CAD results were used to correlate a_{min}^* as a function of these two geometric parameters. They are provided in an Excel sheet as supplementary material to this paper.

For a given helix design, once the a_{min}^* value is known, $\phi_{H,max}$ can be easily calculated. Indeed, let's consider a triangular prism of height equal to *p*, the pitch of the helix, and which base is the equilateral triangle that connects the axes of the three closely packed helixes. This volume encloses 1/6th of each of the three helices, which corresponds to the volume of half a helix. Thus, $\phi_{H,max}$ equals to the half volume of a helix over the volume of the prism element:

$$\phi_{H,max} = \frac{\frac{1}{2} V_{Helix}}{V_{Prism}} = \frac{\frac{1}{2} \frac{\pi d^2}{4} \sqrt{p^2 + (2\pi R_H)^2}}{\frac{1}{2} a_{min}^2 \sin\left(\frac{\pi}{3}\right) p} = \frac{\pi \sqrt{1 + (2\pi R_H^*/p^*)^2}}{4 a_{min}^{*^2} \sin\left(\frac{\pi}{3}\right)}$$
Eq. 12

363364 4. Results and discussion

366 *4.1. CFD results*

In the following sections, the CFD results will be mainly presented in terms of Nusselt number and heat transfer enhancement. However, as discussed in Section 2.3, based on the heat/mass transfer analogy, these same results can be interpreted in terms of Sherwood number and mass transfer enhancement.

371 As mentioned earlier, for each CFD simulation performed, the circumference-averaged Nu was calculated at 372 different positions along the helical pipe length so as to determine its asymptotic value, Nu_H^{∞} . This value, divided 373 by the Nusselt number in a straight pipe, $Nu_S^{\infty} = 3.657$, represents the heat transfer enhancement allowed by a 374 given helical pipe.

Figure 7 shows contour plots of the Nu_{H}^{∞} to Nu_{S}^{∞} ratio in the (R_{H}^{*}, p^{*}) space. These contours were obtained using a triangulation-based cubic interpolation of the CFD results. The geometric parameters for which simulations were performed are represented by black dots. It is worthy to note that, at these points, the values of Nu_{H}^{∞} (divided by Nu_{S}^{∞}) that are displayed on the contour plots are exactly the same than those provided by CFD. On the other hand, the results predicted between these points are generated by interpolation. Therefore, they may be expected to not be very accurate since they are quite sensitive to the interpolation scheme used. Indeed, as can be noticed from Figure 7, the contour plots exhibit some relatively irregular variations.

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Figure 7: Contour plots of the Nu_H^{∞} to Nu_S^{∞} (respectively Sh_H^{∞} to Sh_S^{∞}) ratio at: (a) Re = 400 and Pr (resp. Sc) = 1. (b) Re = 400 and Pr (resp. Sc) = 10. (c) $Re = 2\ 000$ and Pr (resp. Sc) = 1. (d) $Re = 2\ 000$ and Pr (resp. Sc) = 10. The black dots represent the geometric conditions for which CFD simulations have been performed. The contours are derived by interpolating these data.

Nonetheless, these contours allow deriving several major conclusions:

(1) HCHPs allow a great improvement of the heat transfer efficiency compared to straight tubes. Indeed, given
 their high curvatures (Figure 3), the flow in HCHPs undergoes strong centrifugal effects leading to intense Dean type vortices that greatly improve the heat transfer rates.

393 (2) As *Re* increases, centrifugal forces become more intense, which further enhances the heat transfer efficiency. 394 Hence, in the case where Pr = 1, for Re = 400, the maximal Nu_H^{∞} achieved in HCHPs is about 3.5 times higher 395 than in straight tubes (Figure 7a), and its gets nearly 8 times greater than Nu_S^{∞} for $Re = 2\,000$ (Figure 7c).

396 (3) Nu_H^∞ increases when Pr is increased. For example, under a Re of 2 000, the maximal Nu_H^∞ is about 8 times 397 higher than in straight tubes for Pr = 1 (Figure 7c), and becomes more than 9 times greater than Nu_S^∞ for Pr = 10398 (Figure 7d).

(4) The geometric parameters for which Nu_H^∞ is maximal are not significantly affected by the Reynolds and Prandtl numbers values. However, when *Re* and/or *Pr* are increased, the value of Nu_H^∞ becomes more sensitive to the helix geometry.

402 (5) For any given dimensionless pitch, p^* , there exists an optimal dimensionless helix radius, R_H^* , at which Nu_H^{∞} 403 is maximal. The Nu_H^{∞} to Nu_S^{∞} ratio sharply decreases toward unity when R_H^* tends to zero as the helix geometry 404 approaches that of a straight pipe. This ratio also decreases to unity when R_H^* tends to infinity as the helical pipe 405 straightens (its curvature tends to zero as can be seen from Figure 3) and becomes locally similar to a straight 406 tube. However, these effects are not always clearly noticeable from the contour plots of Figure 7 since

407 simulations were performed in the range of $0.05 \le R_H^* \le 10$ only.

408 (6) For any given R_H^* , the Nu_H^∞ to Nu_S^∞ ratio decreases when p^* increases and tends towards unity at infinite p^* as 409 the helix design approaches a straight pipe, although this effect is not always noticeable in Figure 7 as no 410 simulations were performed for p^* values higher than 15.

411 (7) Although Nu_H^{∞} decreases when p^* is increased, as can be seen from Figure 7, this effect is only significant in

the case of HCHPs. Indeed, for relatively high R_H^* values, the curvature is not much sensitive to the value of p^* (Figure 3). As only helical pipes with rather large R_H^* were investigated in the literature, most authors concluded

413 (Figure 3). As only helical pipes with rather large R_H^* were investigated in the literature, most authors concluded 414 that the helix pitch has no significant effect on Nu_H^∞ as can be noticed from the correlations reported in Table 1.

- 415 Thus, it is obvious that these correlations cannot accurately predict the Nusselt number in HCHPs as detailed in
- 416 Section 4.3.

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418 4.2. Correlation for predicting Nusselt (and Sherwood) numbers in helical pipe laminar flows

420 The CFD results revealed that, for given Re and Pr numbers, the Nu_H^{∞} field in the (R_H^*, p^*) space (Figure 7) 421 presents a single maximum (no local optima). And apart the particular case of straight pipes, for any helix 422 geometry, Nu_H^{∞} increases with increasing Re and Pr.

423 Despite such a relatively regular behavior, finding a mathematical model that correctly fits the CFD data was 424 extremely challenging and tedious. Indeed, Nu_H^{∞} exhibits highly nonlinear variations, especially with respect to 425 the geometric parameters. In fact, the Nu_H^{∞} field undergoes steep variations in the HCHPs region, with highly 426 non-uniform and anisotropic gradients, which magnitudes are very sensitive to the helix geometry. Moreover, the 427 mathematical formulation should ensure that the model predicts that $Nu_H^{\infty} = Nu_S^{\infty} = 3.657$ at all of the three 428 asymptotic limits where the pipe curvature vanishes and the helix geometry tends toward that of a straight tube, 429 i.e. when R_H^* tends to zero or infinity and when p^* tends to infinity. Otherwise, the derived correlation cannot be 430 reliably used in any model-based optimization for determining the optimal pipe design in heat/mass transfer 431 devices.

432 The development of the regression model for fitting the Nu_H^{∞} data has been conducted by trial-and-error. The 433 following complex expression has been found to provide the best fit of the CFD results:

where:

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$$Nu_{H}^{\infty} = 3.657 + p_{1} A^{p_{2}} Re^{B} Pr^{p_{7}} exp(-C)$$

$$A = \left[R_{H}^{*} \left(1 + \left(\frac{p^{*}}{2\pi R_{H}^{*p_{3}}} \right)^{p_{4}} \right) \right]^{-1}$$
Eqs. 13
$$B = p_{5} Pr^{p_{6}}$$

$$C = p_{8} R_{H}^{*p_{9}} Pr^{p_{10}}$$

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The present model includes 10 regression parameters denoted p_i (*i* being an integer ranging from 1 to 10) which values were determined using an optimization procedure. The term 3.657 corresponds to the asymptotic Nusselt number in a straight tube, Nu_S^{∞} . The second right-hand side term in the mathematical model is formulated so as to be always positive, hence, the present correlation guarantees that the predicted Nu_H^{∞} value remains greater (or equal) than Nu_S^{∞} .

The term denoted *A* is analogous to the helix dimensionless curvature κ^* (see Eq. 1), with the difference that p_3 and p_4 are treated as optimization variables instead of assigning their values to 1 and 2 respectively. The term A vanishes when R_H^* tends to zero or infinity or when p^* tends to infinity, i.e. when the helical pipe geometry tends toward that of a straight one. Thus, the model correctly predicts that $Nu_H^\infty = Nu_S^\infty = 3.657$ at these three limits.

446 The mathematical expression presented in Equations 13 was used to correlate the CFD data. It should be 447 recalled that, as mentioned in Section 2.2, these results were obtained in the following range of dimensionless 448 parameters: $1.25 \le p^* \le 15, 0.05 \le R_H^* \le 10, 10 \le Re \le 2\,000$ and $1 \le Pr \le 10$. The optimization problem 449 was formulated as a minimization of the maximum relative difference between the model predictions and the 450 numerical data. For a better accuracy, two sets of parameter values were calculated, the first one for *Re* ranging 451 from 10 to 400, and the second one for Re between 400 and 2 000. The optimal sets of parameters p_i are reported 452 in Table 2. They were determined using a genetic algorithm and the results were further refined using a local 453 optimizer, GRG2. An Excel sheet with the present correlation is available as supplementary material to this 454 paper.

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Table 2: Optimal sets for the parameters of the correlation expressed by Equations 13.

Validity range $10 \le Re \le 400$ p_1 p_2 p_3 p_4 p_5 3.73×10^{-2} 3.81×10^{-1} 9.50×10^{-1} 2.64 9.38×10^{-1} p_6 p_7 p_8 p_9 p_{10} -7.09×10^{-2} 5.71×10^{-1} 6.43×10^{-2} -1.15 3.84×10^{-1} $400 \le Re \le 2\ 000$ p_1 p_2 p_3 p_4 p_5 3.03×10^{-2} 2.82×10^{-1} 7.19×10^{-1} 2.62 5.77×10^{-1} p_6 p_7 p_8 p_9 p_{10} -9.01×10^{-2} 4.35×10^{-1} 1.01×10^{-2} -3.13 -1.32×10^{-1}

Figure 8 compares the correlation predictions and the present CFD results (represented by brown disks). It shows that the proposed correlation correctly fits the Nu_H^{∞} data as most points are within an error margin of 15%. Indeed, the maximum relative difference between the numerical results and the correlation predictions is respectively 16.9% for *Re* below and 400 and 14.8% for *Re* above 400.

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Figure 8: Parity diagram of Nu_H^{∞} : CFD and experimental data versus correlation predictions.

466 To check the robustness of the correlation, additional CFD simulations were performed for p^* up to 60 and 467 R_{H}^{*} up to 20. Although the correlation was built upon CFD results acquired for $p^{*} \leq 15$ and $R_{H}^{*} \leq 10$, it successfully predicts these additional Nu_H^{∞} data (represented by blue triangles in Figure 8) within an error 468 469 margin below 15%. This predictive capacity of the present correlation is due to its mathematical formulation 470 (Eqs. 13) which as discussed previously, guarantees correct results at the asymptotic limits where the helical pipe 471 geometry tends toward that of a straight one, i.e. ensures results to be correctly bounded. Therefore, the present 472 correlation is believed to lead to realistic and valid Nu_H^{∞} values even beyond the range of geometric conditions 473 investigated in this paper.

Figure 9 shows the contour plots of the Nu_H^{∞} to Nu_S^{∞} ratio that are calculated using the present correlation for *Re* values of 400 and 2 000 and *Pr* values of 1 and 10. A good agreement with Figure 7 is observed, although the model data are much smoother since they are not affected by interpolation approximations. The maximal values of Nu_H^{∞} and the geometric parameters for which they occur are correctly estimated by the model, and the Nu_H^{∞} variations with respect to the helix geometry, operating conditions and fluid properties are successfully predicted.





Figure 9: Contour plots of the Nu_{H}^{∞} to Nu_{S}^{∞} (respectively Sh_{H}^{∞} to Sh_{S}^{∞}) ratio calculated using the present correlation (Eqs. 13): (a) Re = 400 and Pr (resp. Sc) = 1. (b) Re = 400 and Pr (resp. Sc) = 10. (c) $Re = 2\,000$ and Pr (resp. Sc) = 1. (d) Re = 2000 and Pr (resp. Sc) = 10. The contour plots at Re = 400 were generated using the set of parameters valid for $10 \le Re \le$ 400 (Table 2).

486 4.3. Comparison between the current and literature correlations

488 In order to further assess the potentiality of the new correlation, its predictions are compared to that of 489 literature correlations in Figures 10, 11 and 12. It is noteworthy that most available correlations (see Table 1) 490 account for both flow and geometry effects via a single dimensionless parameter, the Dean number, which is 491 defined as follows: 492

$$De_{R_H} = \frac{\rho dU}{\mu} \sqrt{\frac{d}{2R_H}} = Re \sqrt{\frac{1}{2R_H^*}}$$
 Eq. 14

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494 However, this formulation leads to systematic errors in the HCHPs region. First, as shown in Figure 10 for 495 the correlations of Dravid et al. [16] and Kalb and Seader [17], when R_H^* tends to zero, models based on De_{R_H} 496 predict an infinite Nusselt number instead of 3.657. Moreover, these correlations ignore the effects of the helix 497 pitch on the achieved Nu_{H}^{ω} . Indeed, as reported in Table 1, so far, literature studies has only dealt with 'classical helixes geometries', i.e. helical designs with a relatively high R_H^* . For such geometries, the helix pitch has little 498 499 effect on Nu_H^{∞} values as can been noticed from Figures 7 and 9, which explains why this parameter has not been 500 considered in most previous correlations. Nonetheless, since these correlations were derived by regressing 501 experimental measurements acquired on non-highly curved helixes, they are expected to accurately predict Nu_{H}^{α} 502 in such geometries. Therefore, the good agreement between the current correlation and the former ones at large 503 R_H^* values (Figure 9) validates the current CFD simulations and correlation for the case of classical helixes. 504





508 The correlation provided by Sheeba et al. [23] (Table 1) accounts for the helix pitch effects on the achieved 509 Nu_{H}^{∞} . However, it presents two major weaknesses. First, it leads to an infinite Nusselt number when R_{H}^{*} tends to 510 zero (Figure 10), and second, it predicts a slight increase of Nu_H^{∞} when the helix pitch is increased (Figure 11) 511 contrary to what is obtained by CFD and observed experimentally in other studies (e.g. [30]).

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517 The correlation of Moulin et al. [19] (Table 1), built upon mass transfer experimental data, accounts for the
518 helix geometry and operating conditions effects via a Dean number defined as follows:
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$$De_{\gamma} = Re \sqrt{\frac{d}{2\gamma}} = Re \sqrt{\frac{\kappa^*}{2}} = Re \sqrt{\frac{1}{2R_H^* \left[1 + \left(\frac{p^*}{2\pi R_H^*}\right)^2\right]}}$$
Eq. 15

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521 where κ^* is the dimensionless helix curvature. This definition of the Dean number is more appropriate than that 522 presented in Eq. 14. In particular, De_{γ} vanishes at the three asymptotic limits for which the helix geometry 523 approaches a straight pipe. This is why the correlation of Moulin et al. [19] correctly reproduces the shape of the 524 curves showing Nu_{μ}^{∞} versus R_{H}^{*} (Figure 10) and captures the decrease of Nu_{μ}^{∞} when p^{*} is increased (Figure 11). 525 However, it also suffers from several drawbacks. Indeed, it leads to Nu_H^{∞} equal zero instead of 3.657 at the limits at which the helix curvature vanishes, i.e. when R_H^* tends to zero (Figure 10) or infinity and when p^* tends to 526 infinity. More importantly, it largely overestimates the Nu_H^{∞} value in HCHPs when Re is increased. As shown in 527 528 Figure 12, the present model correctly captures the *Re* effects and is way more accurate than the correlation of 529 Moulin et al. [19]. Note that the discontinuity in the actual correlation curves is due to the use of two different 530 sets of parameter values depending on whether Re is higher or lower than 400 (Table 2).







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Figure 12: Nu_H^{∞} (respectively Sh_H^{∞}) variation with *Re* for four different helixes at *Pr* (respectively *Sc*) = 10.

535 *4.4. Correlation and CFD data validation using experimental data from literature*

In addition to CFD results, for a more trustworthy validation, the correlation results were compared to the experimental data of Ghobadi and Muzychka [22] and Sheeba et al. [23]. Table 3 summarizes the geometric and operating conditions over which these measurements were obtained. It is noteworthy that even the experiments performed for a *Re* higher than 2 300 correspond to a laminar flow regime. Indeed, transition to turbulence is delayed in helical pipe flows due to the stabilizing effects of centrifugal forces [24, 31].

 Table 3: Ranges of geometric and operating conditions over which the literature experimental data were acquired and the present correlation was built.

	Sheeba et al. [23]	Ghobadi and Muzychka [22]: helical pipe 1	Ghobadi and Muzychka [22]: helical pipe 2	Ghobadi and Muzychka [22]: helical pipe 3	Range of parameters investigated using CFD and upon which the present correlation was built
R_H^*	29.8	6.1	12.1	24.2	[0.05 - 10]
p^*	6.1	nearly 1	nearly 1	nearly 1	[1.25 – 15]
Pr	[3 – 5]	[5 – 8.5]	[5 - 8.5]	[5 - 8.5]	[1 - 10]
Re	[940 – 3 200]	[440 – 2 510]	[960 – 2 530]	[700 – 2 020]	$[10 - 2\ 000]$

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547 The last column in Table 3 recalls the range of conditions over which the present CFD data were performed, 548 and hence, upon which the current correlation was built. It can be noticed that most of the experimental results 549 were acquired for conditions beyond the validity range of the present correlation. Nevertheless, as shown in 550 Figures 8 and 13, an excellent agreement is obtained between the correlation and Sheeba et al. [23] results 551 (represented by green squares) which were all predicted within an error margin of 9%. Figure 13 shows that the 552 correlation correctly captures the effects of the Reynolds number. A good agreement is also obtained with the 553 data of Ghobadi and Muzychka [22] (represented by red diamonds in Figure 8) as most of their experimental 554 results are predicted with a relative error less than 25%.



Figure 13: Comparison between the present correlation predictions and the experimental data of Sheeba and coworkers [23].

560 Differences between the correlation and the experimental results are due to several reasons, mainly the 561 assumption of uniform fluid properties in CFD simulations, the accuracy of the current correlation and 562 experimental errors. However, as can be noticed from Figure 8, the data of Ghobadi and Muzychka [22] are 563 systematically overestimated by the current model. This is probably due to the experimental procedure they 564 followed. Indeed, Ghobadi and Muzychka [22] placed their helical coil within a non-agitated water bath which 565 temperature was maintained at 40 °C. For calculating the convective heat transfer coefficient and the Nusselt 566 number, they assumed the wall temperature of their helical pipe to be equal to 40°C. However, in the absence of 567 an efficient agitation, the wall temperature can significantly deviate from that of the water bath, especially 568 because the heat transfer coefficient in the internal flow is large. This leads to an underestimation of the 569 experimentally measured Nu_{H}^{∞} and probably explains the systematic deviation between their measurements and 570 the actual correlation predictions.

572 4.5. Optimal packing density of helixes: results and correlation

As noted in Section 3, the dimensionless distance between closely packed helixes, a_{min}^* , was determined using a CAD software for different helixes geometries. Figure 14 shows the contour plot of the interpolated results. The lowest possible value for a_{min}^* is 1, and is achieved with straight tubes, i.e. when R_H^* is zero or when p^* tends to infinity. a_{min}^* obviously increases when R_H^* is increased as can be noticed from Figure 6 for example. When p^* is increased, the spacing between the helixes turns enlarges. Hence, the helixes can be brought closer and imbricate, which leads to a decrease of a_{min}^* (as can be noticed from Figure 15).



Figure 14: Contour plot of a_{min}^* in the (R_H^*, p^*) space. The black dots represent the geometric conditions for which CAD results are acquired.

To the authors' knowledge, the only available correlation for determining a_{min}^* or the helixes packing density is that reported in Kaufhold et al. [26]. However, the effects of the helix pitch are not taken into account in this correlation. Thus, it obviously lacks accuracy since a_{min}^* can be very sensitive to p^* value as can be noticed from Figure 14. Therefore, a regression model for a_{min}^* is proposed in this paper. The following mathematical expression, constructed by trial-and-error, was found to provide the best fit of the CAD data:

where:

$$a_{min}^{*} = 1 + A \tanh (B \ p^{*C})$$

 $A = p_1 R_H^{*} + p_2$
 $B = p_3 R_H^{* p_4}$
 $C = p_5 \ln (R_H^{*}) + p_6$
Eq. 16

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The present expression includes 6 regression parameters denoted p_i (*i* being an integer ranging from 1 to 6) which values were determined using an optimization procedure in order to minimize the maximum relative difference between the correlation outputs and the CAD data. The hyperbolic tangent term is formulated so as to vanish when R_H^* tends to zero or when p^* tends to infinity. Accordingly, the correlation correctly predicts that a_{min}^* equals one at these limits.

The mathematical expression presented in Equations 16 was used to correlate the CAD data which were obtained in the following range of dimensionless parameters: $1 \le p^* \le 20$ and $0.05 \le R_H^* \le 10$. The optimization problem was solved using a genetic algorithm and the results were further refined using a local optimizer, GRG2. For a better accuracy, two correlations were derived, depending on whether R_H^* is lower or higher than 2 respectively. The optimal sets of the parameters p_i are reported in Table 4. An Excel sheet where the present correlation is implemented is provided as supplementary material to this paper.

604 Validity range
 Table 4: Optimal sets of the parameters of the correlation expressed by Equations 16.

$0 \le R_H^* \le 2$	p 1	p ₂	<i>p</i> ₃	<i>p</i> ₄	p 5	<i>p</i> ₆
	1.88	-5.54×10 ⁻²	3.50	5.65×10 ⁻¹	-1.65×10 ⁻²	-1.50
$2 \le R_H^* \le 10$	<i>p</i> ₁	p ₂	p 3	<i>p</i> ₄	<i>p</i> ₅	p 6
	2.04	-3.65×10 ⁻¹	4.44	0	1.35×10 ⁻¹	-1.52

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606 An excellent agreement between the model and the CAD results is obtained (Figure 15) as their maximal 607 relative difference is 2.80% for R_H^* below 2 and 4.30% for R_H^* above 2 respectively. To check the robustness of 608 the correlation, additional a_{min}^* computations (provided as supplementary material) were performed for p^* up to 609 1 000 and R_H^* up to 500. The model was able to predict all of these data with within a maximum error margin of 610 4.5% which demonstrates its predictive capacity.

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(b)



Figure 15: a_{min}^* versus p^* at (a) $R_H^* = 0.2$ and (b) $R_H^* = 4$: comparison between the CAD results and the present correlation (Equations 16).

621 The contour plot of a_{min}^* was calculated using the present model (Equations 16) and is given in Figure 16a. 622 The current correlation along with Equation 12 were used to determine the helixes optimal packing density, 623 $\phi_{H,max}$, in the (R_H^*, p^*) space. The contour plot of the $\phi_{H,max}$ to $\phi_{S,max}$ ratio is given in Figure 16b. $\phi_{S,max}$ is 624 the optimal packing density of straight tubes which approximately equals 90.7%.



Figure 16: Contour plots in the (R_H^*, p^*) space of (a) a_{min}^* and (b) of the $\phi_{H,max}$ to $\phi_{S,max}$ ratio as calculated using the present model (Equations 16).

Figure 16b shows that the $\phi_{H,max}$ to $\phi_{S,max}$ ratio is always lower than unity as helixes cannot be as efficiently packed as straight tubes. However, it can be noticed that highly curved helixes allow considerably higher packing densities than classical ones, and thus, they provide much larger specific surface areas for heat/mass transfer. As HCHPs also lead to higher transfer efficiencies, they are by far more advantageous than classical ones in term of process intensification.

Figure 16b also reveals that apart for very low R_H^* , when p^* is increased, $\phi_{H,max}$ first decreases to a minimum before starting to increase. This is explained by the fact that increasing the helix pitch engenders additional void between the helix turns on the one hand, and on the other hand, the larger space between the helix turns allows the different helixes to be brought closer together. However, at very low pitches, this additional spacing enables the helixes to slightly approach only but not to imbricate. Therefore, the overall volume fraction filled by the helixes decreases. Once a sufficient p^* is reached, further increasing the helix pitch allow the helixes to better imbricate (Figure 15) which increases their optimal packing density.

642 4.6. Overall intensification factor and potentiality of highly curved helical pipes designs643

644 As shown in the previous sections, HCHPs lead to great transfer efficiencies but cannot be as densely packed 645 as straight tubes. These two aspects should be combined together for quantifying the volumetric transfer rate 646 enhancement that these designs allow achieving when used in packed modules, i.e. for assessing the enabled unit 647 volume reduction of reactors, heat exchangers or membrane contactors. Accordingly, the following 648 intensification factor, ε , is proposed:

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$$\varepsilon = \frac{Nu_H^{\infty} \phi_{H,max}}{Nu_S^{\infty} \phi_{S,max}} \quad \text{or} \ \varepsilon = \frac{Sh_H^{\infty} \phi_{H,max}}{Sh_S^{\infty} \phi_{S,max}}$$
Eq. 17

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For a same tube diameter, it can be analytically shown that the $\phi_{H,max}$ to $\phi_{S,max}$ ratio equals the ratio of the specific surface area of helical and straight pipes. For a same fluid, the Nu_H^∞ to Nu_S^∞ (resp. Sh_H^∞ to Sh_S^∞) ratio is 653 equal to the ratio of the heat (resp. mass) transfer coefficients in helical and straight tubes respectively. Thus, ε 654 physically represents the ratio of the heat/mass flux achieved in helical pipes to that achieved in straight ones. For example, an $\varepsilon = 2$ indicates that the use of HCHPs allows obtaining similar transfer performance than 655 656 straight tubes while reducing the volume of the heat/mass transfer device by nearly a factor 2. This is true 657 whenever the heat/mass transfer resistance in the internal fluid side is the one that limits the overall transfer rate. 658 This situation is commonly encountered in heat exchangers. It also occurs in many mass transfer devices, e.g. 659 fluid degassing in membrane contactors under vacuum operation and fluid dispensing systems based on 660 vaporization (e.g. pure liquid on the shell side of a contactor).

661 Figure 17 shows the contour plot of ε in the (R_H^*, p^*) plane for different fluid properties and operating 662 conditions. Note that different scales are used for each subfigure. Figures 17a and 17b reveal that, at very low 663 *Re*, HCHPs allow a moderate improvement of the transfer rates only, while classical helixes perform even worse 664 than straight tubes. The reason is that, at low *Re*, centrifugal effects are moderate and thus generate Dean 665 vortices of low intensity. Therefore, the provided heat/mass transfer enhancement remains limited and barely 666 compensates the effects of the decrease of the packing density.

The intensification factor improves with increasing Re and Pr (resp. Sc) numbers. For $Re = 2\ 000$ for example, HCHPs achieve volumetric transfer rates nearly 6.5 times higher than straight tubes for Pr (resp. Sc) = 1 (Figure 17c), and the intensification factor exceeds 8 for Pr (resp. Sc) = 10 (Figure 17d). These results demonstrate the huge potential of HCHPs for process intensification.





Figure 17: Contour plots of the intensification factor, ε , for: (a) Re = 10 and Pr (resp. Sc) = 1. (b) Re = 10 and Pr (resp. Sc) = 10. (c) $Re = 2\ 000$ and Pr (resp. Sc) = 1. (d) $Re = 2\ 000$ and Pr (resp. Sc) = 10.

5. Conclusion and perspectives

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> 677 This paper focused on the heat/mass transfer enhancement in helical pipes, especially highly curved ones, 678 under laminar flow conditions. Novel designs, offering huge potentialities for intensified mass and/or heat 679 transfer performance have been identified: on the one hand, they generate intense Dean-type vortices, and on the 680 other hand, they can be densely packed. The results demonstrate that highly curved helixes allow intensifying the 681 volumetric transfer rate up to a factor 8, which should enable a massive unit volume reduction of reactors, heat 682 exchangers and membrane contactors, as soon as the mass or heat transfer resistance is mostly located in the 683 internal fluid (bore side). This situation is likely to occur in gas-liquid heat exchangers, catalytic reactors with 684 large reaction rates, fluid degassing in membrane contactors under vacuum operation, or fluid dispensing 685 systems based on vaporization (e.g. pure liquid on the shell side of a contactor).

Future studies by the authors will concentrate on the following points:

687 1- Experimental measurements of the heat/mass transfer efficiency in HCHPs (highly curved helical pipes)
 688 for a trustworthy validation of the present numerical results. Indeed, as explained in this paper, such data lack in
 689 the literature since HCHPs are difficult to manufacture using traditional manufacturing techniques. However,

690 nowadays, thanks to a witnessed progress in 3D-printing, the fabrication of such designs has become achievable691 [14].

692 2- The current paper showed that the heat/mass transfer efficiency in HCHPs considerably improves when
693 the Prandtl/Schmidt number is increased. Therefore, it is worthwhile investigating the performance of these
694 designs for fluids with higher Prandtl (e.g. oils) or Schmidt (e.g. liquids) numbers since enormous intensification
695 factors may be expected [32].

3- Although intense Dean vortices in HCHPs lead to improved heat and mass transfer efficiencies compared
to straight tubes, they also induce higher pressure drops and hence involve greater pumping costs. Therefore a
multi-objective optimization is necessary to compare the benefits provided by HCHPs with the additional
operating costs involved, and thus, assessing the potential of these designs. The correlations provided in this
paper and in Abushammala et al. [14] allow determining the Nusselt/Schmidt number and the friction factor in
helical pipes, and can thus be used as surrogates in this intended model-based optimization.

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707 Nomenclature

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a_{min}	Distance separating two closely packed helixes (m)
C_A	Molar concentration of A (mol m ⁻³)
d	Pipe diameter (m)
D	Mass diffusivity (m ² s ⁻¹)
De_{R_H}	Dean number following the definition of Eq. 14 (-)
De_{γ}	Dean number following the definition of Eq. 15 (-)
h	Heat transfer coefficient (W m ⁻² K ⁻¹)
k	Mass transfer coefficient (m s ⁻¹)
$L_{ m th}$	Length of the thermal entrance region (m)
'n	Mass flow rate (kg s ⁻¹)
$ec{n}_A$	Mass flux of component A (kg m ⁻² s ⁻¹)
Nu	Nusselt number (-)
Nu_H^∞	Asymptotic Nusselt number in helical pipes (-)
Nu_S^∞	Asymptotic Nusselt number in straight pipes (-)
р	Helix pitch (m)
Р	Pressure (Pa)
Pr	Prandtl number (-)
R_H	Helix radius (m)
Re	Reynolds number (-)
Sc	Schmidt number (-)
Sh	Sherwood number (-)
Sh_{H}^{∞}	Asymptotic Sherwood number in helical pipes (-)
Sh_S^∞	Asymptotic Sherwood number in straight pipes (-)
Т	Temperature (K)
$T_{f,MC}$	Mixing-cup temperature (K)
T_w	Temperature of the wall (K)
\vec{u}	Fluid velocity vector (m s ⁻¹)
U	Mean velocity of the primary flow (m s ⁻¹)
$ec{ u}$	Bulk motion velocity (m s ⁻¹)
V	Volume (m ³)
G 11.4	
Greek letters	The sum of $1 + \frac{1}{2} +$
α	Dedice of connections of the pine (m)
γ	Radius of curvature of the pipe (m)
ρ	Letensification factor ()
8	Dire sumstand (mal)
ĸ	Pipe curvature (m^2)
λ	Demonstration of the state of t
μ	Dynamic viscosity (Pa S) K_{in} substitution $(m^2 - 1)$
ν	Kinematic viscosity (m^2 s ⁻¹)
ρ_A	Mass concentration of A (kg m^{-3})
$\varphi_{H,max}$	Optimal packing density of helical pipes (-)
$\phi_{S,max}$	Optimal packing density of straight pipes (-)

Φ Heat flux (W)

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712 Superscripts

Designates a dimensionless number

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