Three-Dimensional Swirl Flow Velocity-Field Reconstruction Using a Neural Network With Radial Basis Functions

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THREE-DIMENSIONAL SWIRL FLOW VELOCITY-FIELD RECONSTRUCTION USING A NEURAL NETWORK WITH RADIAL BASIS FUNCTIONS

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Abstract

For many studies, knowledge of continuous evolution of hydrodynamic characteristics is useful but generally measurement techniques provide only discrete information. In the case of complex flows, usual numerical interpolating methods appear to be not adapted, as for the free decaying swirling flow presented in this study. The three-dimensional motion involved induces a spatial dependent velocity-field. Thus, the interpolating method has to be three-dimensional and to take into account possible flow non-linearity, making common methods unsuitable. A different interpolation method is thus proposed, based on a neural network algorithm with Radial Basis Functions.

1 - Introduction

Except for simple cases having analytical representations, a flow-field is often described as a discrete distribution of hydrodynamical specific characteristics. For example, experimental measurements provide only local information. Interpolation schemes, like linear or cubic interpolation, are thus usually employed in fluids engineering to determine unknown values at desired positions, as for pathlines determination for instance. The same problem arises if special hydrodynamical features have to be determined, like stress-fields or vorticity-fields, that imply the use of derivative approximation with finite difference methods. Results are then biased by the systematic error induced by the method. A way to improve the determination of such results is to employ a fitting function of the hydrodynamical features evolutions. This solution provides a continuous mathematical expression, which can be then employed to calculate values at unknown locations. If the mathematical function is chosen derivable, special hydrodynamical features, like stress-fields or vorticity-fields, can then be easily deduced without adding approximation of derivative schemes. For example, Marshall and Grant [1] have developed a Lagrangian vorticity collocation method where a polynomial fitting is used to determine a continuous vorticity-field using local values defined on a non-uniform grid, allowing the exact differentiation of the vorticity-field which is needed to calculate a viscous diffusion term [1]. But, naturally, the accuracy of such a method depends on the ability of the retained function to fit the local values distribution.

When the flow becomes complex and three-dimensional, hydrodynamical features can be characterized by non-linear evolutions. If the flow exhibits a great spatial dependency, polynomial fitting or common interpolation schemes are then not adapted, because a large amount of data is requested to represent with accuracy the flow in the entire geometry. Another method based on a neural network representation is proposed in this study. By considering non-linear evolutions, such technique appears interesting to reconstruct complex flows, such as the three-dimensional swirling flows investigated here. The neural network method will provide a continuous mathematical expression of the flow-field, by representing it as a sum of Gaussian functions, each function being derivable, that allows further calculation of specific hydrodynamical features involving differentiation.

Swirling flows are often used in chemical engineering for the design of new hydrocyclones or heat exchangers for example [2]. By involving a three-dimensional motion and increasing turbulent intensities, such flows appear as an efficient way to enhance transport phenomena, like mass or heat transfer [3-5]. But the generated hydrodynamics remains difficult to be fully characterized, owing to the spatial dependency of the flow-field [6-9]. A tangential inlet can be used to induce a stationary swirling flow in tubes or annular geometries with a rather simple design [2-10]. The swirl motion is achieved at the inlet of the device and then decays freely along the flow path (Fig. 1). Hydrodynamic characteristics are not only a function of the radial position, but also depend on the axial location, due to the free decay of the swirl intensity, and on the angular position, the flow generated not being axisymmetric. Thus, a full characterization of the flow requests a high amount of experimental data, especially when local measurement techniques are employed, like laser Doppler anemometry or hot-wires techniques [11]. A previous study has shown the interest of spatial measurement techniques, like the Particle Image Velocimetry (PIV) which allows faster investigation by measuring the instantaneous velocity-field in a designed area [10]. But two problems remain : (i) the obtained velocity-field is not continuous, (ii) the acquisition area remains limited and a complete investigation of the flow-field would request numerous measurement areas.

Thus, an experimental investigation can require a very long time to obtain a sufficient amount of data. Because neural networks can express non-linearities, they seem an efficient tool for the reconstruction of data linked to multiple parameters, and thus an interesting alternative solution to common interpolation schemes. Neural networks with Radial Basis Functions are especially designed for this particular kind of application [12-14]. This method can thus be employed to determine a mathematical expression of the flow-field defined on the entire geometry, using only a small amount of experimental local measurements. To evaluate the accuracy of this interpolating technique, a large set of data has first been measured using PIV technique. Next, the set has been divided into two parts : the first being used by the neural network in order to reconstruct the velocity-field during the learning step, and the second to estimate the reconstruction efficiency by comparing values obtained with the neural method to experimental measurements. The number of data requested to achieve an accurate calculation of the continuous flow-field has been determined.

2 - Experimental PIV investigation

In a previous work, we have shown that a usual PIV technique can be used to characterize the main characteristics of a swirling decaying flow induced by a tangential inlet in an annular geometry. The three components of the velocity and turbulent intensity have been investigated. The main features of the experimental investigation are described in details by Pruvost et al. [10].

It has been shown that, by allowing spatial investigation, PIV measurements appear to be well-adapted to this particular flow which was found to be very complex and space dependent. But, to fully characterize hydrodynamics in the geometry under study, a large set of data is needed. Furthermore, because the employed PIV system allows the simultaneous acquisition of only two velocity components, two series of measurements are requested to determine the complete velocity-field : the first allows acquisition of axial and radial components of the velocity vector and the second, the circumferential one. For the last series, the measurements cannot be done for several radial positions simultaneously, contrary to axial and radial velocity components acquisitions. The measurement time is thus greatly increased to obtain an accurate spatial characterization of each velocity component, especially for the circumferential one.

The hydraulic test rig used for experimental investigation is described by Pruvost et al. [10]. It consists of an annular test section (Fig. 1) made of two concentric Plexiglas tubes having a total length of 1.5 m. The internal radius of the outer cylinder, R_o, is 50 mm, the outer radius of the inner tube, R_i, being 20 mm. The diameter of the tangential inlet is 30 mm, equal to the annular gap width, e, corresponding to pure swirling flow according to Legentilhomme and Legrand [4]. The outlet of the annulus is of axial type, so as not to disturb the flow. For all experimental measurements, the water flow rate is Q = 1.27 m^3/h corresponding to a mean axial velocity in the annulus, $\overline{U}$, of 5 cm/s and a Reynolds number, Re, equal to 3000, where Re is calculated by :

$$Re = \frac{2\, e\, \overline{U}}{\nu} \tag{1}$$

The PIV Dantec system used consists of a Spectra Physics Model 2020 laser source with an emission wavelength of 488 nm for a maximum emission power of 5 Watt. The laser beam is shutter

gated to induce laser light pulses and a thin light sheet is generated by means of an optical lens. The electro-optical shutter is synchronized with a CCD camera Double Image 700, the camera sensor includes 768 x 484 light-sensitive pixels. Tracer particles are of TiO_2 type. A representation of the experimental test rig set with the PIV measurement technique is shown in Fig. 2. Additional parameters of the PIV set-up can be found in Pruvost et al. [10].

To investigate the neural network reconstruction capacity, a large set of experimental data has been collected. The reconstruction accuracy is analyzed from the influence of two parameters : the number of values requested along the axial direction to represent the decrease of the swirl intensity and the number of needed angular positions to characterize the asymmetric behavior of the flow. Measurements positions for mean components are defined in Table 1-a and 1-b, and in Table 1-c for turbulent intensities. The measurement of the circumferential velocity component and the evaluation of turbulent intensities requesting longer time of investigation, measurements were made for only three angular positions. This particular point will be discussed further. The various investigated axial positions show a non uniform distribution of the velocity-field. Thus, the number of axial locations has to be increased near the inlet where the flow is found to be very spatial dependent, and then spaced out for locations further in the annulus where inlet disturbances vanish.

3 - Neural network reconstruction method

3 – 1 Global method

Neural networks are used in various research areas, one of them being data interpolation. Because of their efficiency in this field, a special kind of neural network is devoted to this application : neural network with Radial Basis Functions (RBF) [15]. The use of functions with local spatial influence like RBF instead of the common sigmoidal functions improves their spatial representation capacity, thereby enabling this makes this kind of networks to interpolate any kind of *a priori* unknown function depending on n-parameters [16]. In this study, each hydrodynamical characteristic to be approximated, named V, is spatial-dependent, and thus a function of the three spatial coordinates x, r and ξ.

RBF are directly linked to the function approximation theory [17]. In fact, with mathematical formulation, the goal of RBF is to find the best approximation F(E,P) of a function f(P) which depends on a set of N - parameters $P=(p_1,p_2,p_3,...,p_N)$, E being a set of fitting parameters to determine. Because f(P) can be of any type, the exact solution cannot always be provided, but an approximated solution can always be found, when F(E,P) is expressed as a weighted-sum of elementary functions of a base $\Phi=\{\phi_i\}_{i=1}^{m}$. If weights are named $W=\{W_i\}_{i=1}^{m}$, F(E,P) can be expressed by the following equation:

$$F(E,P) = \sum_{i=1}^{m} W_i \phi_i(P) \tag{2}$$

This expression can be represented using a neural network architecture. This kind of net has three layers:

- an input layer, where the set of function parameters P are given to the net, with one unit per parameter. In this study, the goal function is $V(x,r,\xi)$ and the net is thus defined with three input units (x,r,ξ),
- an output layer, with one unit, corresponding to the function to reconstruct f(P), which is in this case $V(x,r,\xi)$,
- a hidden layer, of m-unit with activation function $\phi_i(x,r,\xi)_{i=1,m}$, where the parameter m has to be found.

The neural network principle is rather simple: for given values in input, corresponding to a location (x,r,ξ) in the geometry, the net has to provide the desired response, $V(x,r,\xi)$. Neural networks are interconnected as shown in Fig. 3. The input signal is directly given to each unit of the hidden layer activated by a function chosen in Φ: each unit calculates $\phi_i(x,r,\xi)_{i=1,m}$. Results are next transferred using weight-links to the output units, where they are finally summed. Thus, this kind of structure is the exact representation of Eq. (2).

With the right set of functions Φ, the objective function named $V_{app}(x,r,\xi)$, can be interpolated. Local functions, as one of gaussian type, have been retained because they can approximate any kind of

function [18], and they remain derivable for further investigation of hydrodynamical feature deduced from differentiation. For the three-dimensional reconstruction, this kind of functions is defined as follows :

$$\phi(x,r,\xi) = \exp\left(-\frac{(x - C_x)^2}{\tau_x{}^2} + \frac{(r - C_r)^2}{\tau_r{}^2} + \frac{(r\xi - C_\xi)^2}{\tau_\xi{}^2}\right) \tag{3}$$

where C and τ are respectively centers and radii associated to each direction of the reconstruction problem. To obtain a homogeneous formulation, the circumferential coordinate ξ has been expressed as the product of the radial position by the circumferential one in Eq. (3).

Thus, the objective is to find the best approximation $V_{app}(x,r,\xi)$ of $V(x,r,\xi)$:

$$V(x,r,\xi) \approx V_{app}(x,r,\xi) = \sum_{i=1}^{m} W_i \exp\left(-\left(\frac{(x - c_{xi})^2}{\tau_{xi}{}^2} + \frac{(r - c_{ri})^2}{\tau_{ri}{}^2} + \frac{(r\xi - c_{\xi i})^2}{\tau_{\xi i}{}^2}\right)\right) \tag{4}$$

where C_{ji} and τ_{ji} are respectively centers and radii of the i^{th} neuron in the hidden layer of size m in direction j.

A solution of Eq. (4) is obtained by minimizing the following function, called the cost function :

$$E_1 = \sum_{i=1}^{p} \left[V_i(x,r,\xi) - V_{app_i}(x,r,\xi)\right]^2 \tag{5}$$

where p is the size of the subset of a given hydrodynamical characteristic V used for the net learning step.

But, minimizing only the function E_1 is an ill-posed solution because, when interpolating data, the objective function has to represent both available and unknown data. Accurate results are not necessarily obtained if only Eq. (5) is employed, and especially if original data are noisy, as for PIV measurements. This is the so-called “representation problem”. This shortcoming is solved by estimating the neural net capacity to find accurate values where no experimental data are available and by introducing an additional regularization term in Eq. (5) [19,20] :

$$C = E_1 + \lambda\, \varphi(W) \tag{6}$$

where λ is the regularization parameter and φ a function to define.

In this study, the estimator employed is the Cross Validation or CV described in Wahba [21]. To determine the CV-value, experimental data employed during the learning step are divided into two parts. Each part is used in turn to determine the neural net parameters, while the remaining amount of data is employed to calculate the error between net results and experimental values using Eq. (6). Next, the mean of reconstruction errors gives an estimation of the neural net accuracy, CV, which has to be minimized.

To allow CV calculation, function φ has to be defined in Eq. (6). An overall review of such functions can be found in Poggio et al. [22]. The chosen function is related to experimental data by assuming noise to be equally shared between all data [17]. The final cost function to be minimized is thus :

$$C = \sum_{i=1}^{p} \left[V(x, r, \xi) - V_{app}(x, x, \xi) \right]^2 + \lambda \sum_{j=1}^{m} (w_j)^2 \tag{7}$$

Dedicated algorithms are available to solve this non-linear problem of finding the best solution $V_{app}(x,r,\xi)$. Each net parameter is determined during the learning step. This step can be divided into two successive parts : the neural net architecture design, and the weight-values determination. Both are well-known and a complete description can be found in Orr [23]. In this application, weight-values are calculated following the “standard ridge regression” algorithm and the neural net is designed using the “forward selection” scheme [23], which presents an incremental construction of the neural network structure: each unit of the hidden layer is added step by step, until the net can provide the requested response. After the learning step, when a new value is presented to the net, its response is close to the desired one: the net is thus able to reconstruct and interpolate data.

In this study, experimental values are obtained for different locations (185 mm< x <1500 mm, 20 mm< r <50 mm, $0 < \xi < 2\pi$). Learning values are first transformed in order to follow a normal

statistical distribution (zero mean and variance one) to prevent numerical approximation problem during the learning step due to the use of data having different order of magnitude.

3 – 2 Learning the neural network

Algorithms of “standard ridge regression” and “forward selection” are implemented using Matlab software on a personal computer Pentium II 333 MHz. To initialize the neural network method, only radii of activation functions ϕ have to be previously defined. They are arbitrarily initialized to give an a priori good representation of the desired evolution. Though this step is not essential because these values will be next optimized during the neural network learning step, computation-time can be decreased in this way. Initial values of radii are given in Table 2.

3 – 3 Evaluations criteria of the neural network accuracy

The initial set of experimental data is divided into two parts. The first is used during the learning step and the second, containing remaining data, is next employed to evaluate the reconstruction accuracy. For each experimental value, the three coordinates are given as input to the net, and the result is next calculated and compared to the experimental data. To evaluate the neural network reconstruction accuracy, two parameters are calculated: a mean error criterion Err_{abs} and a more sensitive parameter giving an evaluation of the error distribution around its mean value, the associated root mean square τ_{err}, defined by:

$$Err_{abs} = \frac{1}{n}\sum_{i=1}^{n}\left|V_{app\,i} - V_i\right| \tag{8}$$

$$\tau_{err} = \left(\frac{1}{n}\sum_{i=1}^{n}(\left|V_{app\,i} - V_i\right| - Err_{abs})^2\right)^{1/2} \tag{9}$$

where n is the total number of experimental values available for a given hydrodynamical characteristic V.

Although error of reconstruction is fully determined using these two parameters, neural network accuracy is not well represented. Error is indeed evaluated by comparing the neural network

result to the experimental one, which presents experimental uncertainty. Thus, having null values of error does not mean a perfect reconstruction, but only that experimental uncertainty is fully integrated to the neural network reconstruction, making results biased too. Values calculated following Eq. (8) and Eq. (9) are thus not fully representative of the reconstruction accuracy and, because the measured velocity-field cannot be experimentally noise free, they can be considered only as estimators.

Finally, three kinds of data sets are defined : (i) the total set of size n, called the generalization set, used to estimate the reconstruction accuracy on the entire geometry, (ii) the learning set of size p, composed of values used during the learning step and which is a part of the generalization set and (iii) the test set of size n-p, composed of values not employed in the learning set. By changing the size of the learning and of the test sets, the minimum number of requested values can be deduced. The test set will be used to evaluate the neural network interpolation accuracy for unknown locations.

4 – Results and discussion

4 – 1 Validation of the neural method

Neural network method efficiency depends on data employed during the learning step. As it often appears, increasing the size of the learning set will improve the resulting accuracy. But another parameter is the distribution of data employed. For example, in the particular case of the swirling decaying flow, the velocity-field has been found asymmetric and not established along the axial coordinate. Thus, before using the neural network method to reconstruct hydrodynamical characteristics, interpolation accuracy has been first evaluated following two parameters : (i) the number of experimental values needed to represent swirling flow evolution along the axial coordinate, and (ii) the number of angular positions requested to represent the flow asymmetry following the circumferential coordinate. Axial mean velocity representation has been chosen for this preliminary evaluation because of its important dependence on spatial coordinates. The mean distribution of axial velocity deduced from the average of PIV measurements was thus employed during the learning step (see section 4-2 for the averaging procedure applied to PIV acquisitions).

4 – 1 – 1 Axial coordinate influence

The effect of the number of axial positions has been studied in order to determine the size of the learning set to interpolate the flow-field with accuracy. The learning step was performed on five sets containing different numbers of data representing the distribution of the mean axial velocity. Efficiency was next estimated using the generalization set and the test set. Results are given in Fig. 4.

Figure 4 shows that a minimal error of 0.46 cm/s is obtained, but as indicated in section 3-3, error free values cannot be reached, because an experimental uncertainty about 10% of the mean axial velocity is included in reference data. This value, which corresponds to the experimental error represents the limit of the method efficiency. These results reveal the number of experimental data necessary to reach an accurate reconstruction of the axial velocity component. An experimental set of 19096 values has been measured, but as shown in Fig. 4, only 2000 values are needed. Increasing the size of the learning set does not provide any improvement in neural reconstruction.

4 – 1 – 2 Influence of the number of angular positions

Because the swirling flow induced by a tangential inlet is not axisymmetric, experimental measurements have to be repeated for several angular positions. The influence of the number of angular positions given for the neural network learning step has been studied in order to show if neural method was able to represent the flow asymmetry and to determine the number of circumferential positions requested to obtain accurate results.

Reconstruction efficiency as a function of the number of angular positions is shown in Fig. 5. The number of angular positions experimentally investigated appears to be sufficient to give a good representation of the flow asymmetry. Values of τ_{err} can be considered as minimal using only three angular positions. These results show the efficiency of the neural method, which is able to represent non-linear evolutions of the swirling flow asymmetry. With this method, only little information is needed to allow an efficient reconstruction of the data evolution.

4 – 2 Hydrodynamical parameters reconstruction

As shown by previous results, the number of measured data is sufficient to obtain an efficient reconstruction of the spatial evolution of hydrodynamical parameters. The neural network will be used in this section to reconstruct mean velocities and turbulent intensities of the flow-field. Mean velocities and turbulent intensities employed in the learning step are obtained with PIV technique. The averaging procedure of instantaneous PIV acquisitions is described as follows:

$$U_x = \frac{1}{N}\sum_{i=1}^{N} u_{x_i}, \qquad U_r = \frac{1}{N}\sum_{i=1}^{N} u_{r_i}, \qquad U_\xi = \frac{1}{N}\sum_{i=1}^{N} u_{\xi_i} \tag{10}$$

$$T_x = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(u_{x_i} - U_x)^2}}{\overline{U}}, \quad T_r = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(u_{r_i} - U_r)^2}}{\overline{U}}, \quad T_\zeta = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(u_{\xi_i} - U_\zeta)^2}}{\overline{U}} \tag{11}$$

To achieve a statistically stable mean spatial distribution of experimental hydrodynamical quantities, N=200 instantaneous acquisitions were used for mean values, and N=1000 for turbulent intensities, for each of the investigated position listed in Table 1.

The neural network method has next been applied for each flow characteristic. Fig. 6 shows the reconstruction obtained for $\xi = 0$ for the three mean components of the velocity, whereas Fig. 7 is dedicated to turbulent intensities characteristics. The neural network is able to represent their spatial evolution efficiently and to give a good representation of the main features of the swirling flow as its three-dimensional behavior and the free decay of the swirling motion along the axial coordinate. Thus, this method appears as a universal approximation tool, which is able to reconstruct any kind of data and hydrodynamical feature. This is confirmed by final errors Err_{abs} obtained for each component of the velocity-field (see Table 3).

Following Eq. (4), the neural network results are the exact representation of a sum of elementary continuous functions. After the learning step, when neural network is defined, a continuous expression of each hydrodynamical parameter is obtained. Any location in the investigated geometry can be provided to the net input, a solution will be determined in output. Two examples of velocity-field deduced from initial PIV measurements are compared to the neural network results. These examples are given in Fig. 8 for two plane projections, in the axial-radial (Fig. 8-a) and radial-

circumferential one (Fig. 8-b). Neural network results have been obtained by providing a set of some locations corresponding to a finest mesh than the one used for PIV measurements. These results show the neural method interest which transforms the discrete velocity-field experimentally measured into a mathematical expression defined on the whole domain, despite the very complex behavior of the swirling decaying flow. Thanks to the neural network method, hydrodynamical characteristics are now determined at any location in the studied geometry.

5 - Conclusion

A neural network method using RBF has been used to express hydrodynamical parameters obtained using PIV on the entire geometry of an annular test-cell involving a swirling decaying flow induced by a tangential inlet. The neural method being able to express non-linear evolutions of a three-dimensional flow, this kind of technique appears to be an efficient tool to accurately represent any hydrodynamical characteristics of this complex flow, as well for the three components of the mean velocity as for turbulent intensity. This universal approximation behavior is very interesting when no a priori idea of what kind of evolution has to be obtained. Results show that this method can be used where hydrodynamical conditions are found to be very disturbed and three-dimensional. But, the neural network is not restricted to this particular flow and the algorithm remains adaptive. For example, it can be simplified either for two-dimensional flows, giving the net only two inputs, or adapted to complex non stationary flows. In the latter case, the neural network has four inputs (three for the spatial coordinates, and one for the time dependency). Assuming that the number of experimental data is sufficient, the neural network will give accurate results, this method being able to reconstruct any function depending on n-parameters. Thus, this technique is thus interesting when complex data bases have to be interpolated, and especially experimental data which present measurements uncertainty, neural networks with RBF functions being found to be robust interpolating methods when data are noisy. But the comparison presented in this study between experimental measurements and neural network results does not allow to conclude about noise influence on the reconstruction efficiency, because the exact flow-field is unknown. This could be achieved by a further study where, for instance, results of a CFD simulation would be used instead of experimental

PIV measurements. Reconstruction accuracy could then be determined as a function of an artificial noise added to CFD results.

The second main advantage of such neural network is to provide a derivable mathematical expression of each hydrodynamical feature valid for the whole domain. This is helpful, for example, for the determination of boundary conditions in CFD simulations using local experimental measurements, or for the calculation of additional hydrodynamical information of the flow, like stress-fields, vorticity-fields or pathlines. When values are locally defined, derivative methods are requested, which implies the use of numerical approximation scheme (finite difference techniques). Defining the flow-field on the whole domain by means of a derivable mathematical function will prevent the employment of such methods. Therefore, the use of this neural method seems an interesting tool for flow-field data post-processing, especially for three-dimensional flows, where common numerical methods are difficult to apply. In conclusion, such a technique provides two main advantages: it reduces the investigation time by decreasing the number of requested experimental measurements, and it leads to a better understanding of the flow-field by allowing the numerical determination of pertinent hydrodynamical characteristics.

List of symbols

Capital letters

C_{in} : center of the n-gaussian activation function associated to the i-direction of the reconstruction problem

Err_{abs} : error criterion for neural reconstruction efficiency evaluation (m/s)

Q : flow rate in the annulus (m^3/s)

R_i : external radius of the inner cylinder (m)

$Re = 2e\overline{U} / \nu$: Reynolds number (eq. 1)

R_o : internal radius of the outer cylinder (m)

T : turbulent intensity (eq. 11)

$\overline{U} = Q / (\pi (R_o^2 - R_i^2))$: average velocity in the annulus (m/s)

U : mean velocity (m/s)

V : generic hydrodynamical characteristic to be reconstructed

V_{app} : neural approximation of V

W: set of weights in the neural network

Lowercase letters

e : thickness of the annular gap (m)

r : radial position with respect to the annulus axis (m)

u : instantaneous velocity (m/s)

x : axial position with respect to the tangential inlet (m)

w_n : weight of the n-radial basis function

Greek symbols

ξ : circumferential position with respect to the tangential inlet axis (radians)

Φ : base of elementary functions

ϕ_n : elementary function of activation from the n-neuron in the hidden layer

λ : regularization parameter

ν : kinematic viscosity of water (m^2/s)

τ_{err} : root mean square of error provided by the neural network (m/s)

τ_{in} : radius of the n-gaussian activation function associated to the i-direction of the reconstruction problem

Subscripts

x : axial component of mean velocity, velocity fluctuation or turbulent intensity

r : radial component of mean velocity, velocity fluctuation or turbulent intensity

ξ : circumferential component of mean velocity, velocity fluctuation or turbulent intensity

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