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Yuzhu Wang, Roger Serra, Pierre Argoul. Based on the virtual experiment study of the impact of load sequence on the calculation process of random vibration fatigue damage.. *Procedia Structural Integrity*, 2019, 19, pp. 682-687. 10.1016/j.prostr.2019.12.074 . hal-02531526

**HAL Id: hal-02531526**

**<https://hal.science/hal-02531526v1>**

Submitted on 16 Mar 2022

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Article in *Procedia Structural Integrity* · December 2019

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Fatigue Design 2019

# Based on the virtual experiment study of the impact of load sequence on the calculation process of random vibration fatigue damage.

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## Abstract

At present, the linear fatigue accumulation theory proposed by Miner is used to calculate the fatigue problem of random vibration. In this process, the loading order of the loads is ignored, so that the results obtained are riskier. The nonlinear fatigue cumulative damage theory takes into account the effects of the load loading sequence, but the calculation becomes more complicated. Many scholars have shown that the fatigue calculation of alternating loads is not strictly linear, and the high and low order will affect the damage index. This phenomenon will be more obvious in the random vibration process. In this paper, the effects of loading sequence on the calculation of non-Gaussian random vibration fatigue damage are studied by virtual experiments. Through the phase selection method, 7 sets of excitations with different kurtosis were loaded onto the notched specimen. Through the calculation results of fatigue damage in the notch sensitive area, it was confirmed that the damage calculated by linear and nonlinear cumulative damage formula was improved with the increase of kurtosis. The results can make a significant difference.

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Peer-review under responsibility of the Fatigue Design 2019 Organizers.

*Keywords:* Vibration fatigue, Random vibration, Damage cumulative calculation;

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## 1. Introduction

Random vibration fatigue refers to the fatigue damage that occurs when a structure receives a random vibration state and an alternating load. When structural failure occurs, the structure is loaded far below the yield limit. The main

cause of structural failure comes from the cumulative damage caused by alternating loads. In response to this process, the predecessors proposed different fatigue damage accumulation theories in the research process. The more commonly used linear damage accumulation theory proposed by Miner. This theory ignores the stress loading order and simply superimposes the damage values at various levels of stress.

For many mechanical components, the working load is in the form of non-Gaussian random vibration. In this working state, the load subjected to fatigue damage of the structure is an alternating load, and the severity increases as the kurtosis of the excitation increases. If you ignore the effects of the load-loading sequence during this process, the calculation results will deviate significantly from the real value.

The interaction of the load under the alternating load and the effect of the load sequence all have an impact on the fatigue crack growth rate, which in turn affects the remaining life of the structure. The current research results indicate that the residual compressive stress of the crack discontinuity and the crack closure effect caused by the plasticity of the crack tip are the main factors causing the expansion rate of the element. Many experts have proposed some crack propagation rate models to explain this phenomenon. In this paper, the exponential cumulative damage model is selected. (Manson & Halford, 1981)(Gao, Huang, Lv, Zuo, & Wang, 2015)

In this study, through a virtual experiment, a series of acceleration signals with the same kurtosis, which are the same as the RMS, are applied to clamping part of the notched specimen by ABAQUS, and the response stress at the notch position at different kurtosis is obtained. The stress cycle distribution is obtained by rain flow counting. Both linearized. damage accumulation theory and nonlinear damage accumulation theory are used to perform damage calculation.

### Nomenclature

N	Number of cycles
$N_f$	Number of fracture
D	Damage indicator
C,m	The material constants are related to the stress ratio and the stress concentration factor
PSD	Power Spectral Density in $g^2/Hz$
PDF	Probability Distribution Function
$G(f)$	Input acceleration by frequency domain
$W(f)$	Response acceleration by frequency domain
$H(f)$	Frequency Response Function
$L_c$	Failure time
$S_f$	Fatigue limit stress
$a_0$	Initial crack length
$a_f$	Crack length of the final fracture
$n, N_f$	The number of cycles of the load when the corresponding crack length is reached
$a_f$	Crack propagation parameter
$n_{req}$	Equivalent cycle number

## 2. Non-Gaussian signal and Kurtosis control

Typically, a Gaussian signal is modulated to obtain a specific non-Gaussian signal. There are many ways to carry out this process. The phase selection method proposed by Steinwolf is used in this paper (Cornelis, Steinwolf, Troncossi, & Rivola, 2015). By fast Fourier transform, a time domain signal can be described as a superposition of harmonics in the frequency domain.

$$x(t) = \sum_{n=1}^N A_n \cos(2\pi n \Delta f t + \varphi_n) \quad (1)$$

The amplitudes of the harmonics are obtained

$$A_n = \sqrt{2\Delta f S(n\Delta f)} \quad (2)$$

Then the  $n$ th centre moment could be written as

$$M_z = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \{x(t)\}^z dt = \frac{1}{T} \int_0^T \{x(t)\}^z dt, z > 2 \quad (3)$$

After the kurtosis formula could be written as

$$K = \frac{M_4}{M_2^2} = 3 - \frac{3}{2} \left( \frac{\sum_{n=1}^N A_n^4}{(\sum_{n=1}^N A_n^2)^2} \right) + \left( \frac{1}{2} \sum_{n=1}^N A_n^2 \right)^{-2} \left\{ \frac{3}{2} \sum_{\substack{i+j=k \\ i \neq j}} A_i A_j^2 A_k \cos(\varphi_i + 2\varphi_j - \varphi_k) + \frac{3}{2} \sum_{\substack{i+j=2k \\ i < j}} A_i A_k^2 A_j \cos(\varphi_i + \varphi_j - 2\varphi_k) \right. \\ \left. + 3 \sum_{\substack{i+j=k+m \\ i < j, ?k < m, ?l < k}} A_i A_j A_k A_m \cos(\varphi_i + \varphi_j - \varphi_k - \varphi_m) + 3 \sum_{\substack{i+j+k=m \\ i < j < k < m}} A_i A_j A_k A_m \cos(\varphi_i + \varphi_j + \varphi_k - \varphi_m) + \frac{1}{2} \sum_{i=3j} A_i A_j^3 \cos(\varphi_i - 3\varphi_j) \right\} \quad (4)$$

To ensure that the mean and RMS of the non-Gaussian signal are same as before, only the distribution is changed. So to make the K fitted for the requirement by modulating the specific  $\varphi$ , the non-Gaussian distribution time series with specific K could be found. It should be noted that as the bandwidth increases, the amount of calculation becomes very large and also affecting the amount is the sampling frequency and data length. In general, the required non-Gaussian data is obtained in a combination of several methods.

### 3. Theory of fatigue cumulative

What is now commonly used for fatigue calculations is the fatigue damage linear accumulation principle from Miner in 1945 (MA.Miner, 1945).

$$D = \sum \frac{n_i}{N_i} = \frac{1}{C} \sum n_i S_i^m \quad (5)$$

Where  $n_i$  is the number of cycles in the stress amplitude  $S_i$ , which was obtained by rain flow counting.  $N_i$  is the cycles of fatigue failure limit corresponding S-N curve at the same stress amplitude. When  $D=1$ , it means structural failure.

The nonlinear cumulative damage theory holds that there is mutual interference between the load history and the damage. The cumulative rate of damage increases with the number of cycles and is more consistent with the actual situation. The representative one is the damage curve method.

Referring to the crack propagation equation proposed by Manson and Halford in 1981,

$$a = a_0 + (a_f - a_0) \left( \frac{n}{N_f} \right)^{a_f} \quad (6)$$

Crack propagation parameter as follow

$$a_f = \frac{2}{3} N_f^{0.4} \quad (7)$$

Therefore, the structural cumulative damage D can be written as

$$D = \frac{a}{a_f} = \frac{1}{a_f} [a_0 + (a_f - a_0) \left( \frac{n}{N_f} \right)^{a_f}] \quad (8)$$

Using the principle of equivalent damage, the number of cycles at the load level can be equivalent to the number of cycles of the reference load level to calculate the final damage. The maximum load is usually used as the reference load.

$$D_i = \left( \frac{n_i + n_{req}}{N_{i,f}} \right)^{\left( \frac{N_{i,f}}{N_{ref}} \right)^{0.4}} \quad (9)$$

#### 4. Random vibration virtual experiment

According to the phase selection method, non-Gaussian signals at different kurtosis are obtained according to Gaussian signal modulation, and each group of signals is applied to the specimen holding portion by a total of 14 random vibration acceleration excitations by two different time series. This process is completed by the virtual experiment and records the response stress of the critical nodes. (Wang, Serra, & Argoul, 2019)

In order to speed up the experiment and ensure the damage to appear in the observable position, the specimen is designed with two notches and the stress is concentrated at the notch position by changing the cross-sectional area, which means that the crack initiation and expansion area has been known. Through the CAE design, the length of the specimen is adjusted so that the displacement of the second bending mode is at the maximum position in the middle of the first notch.

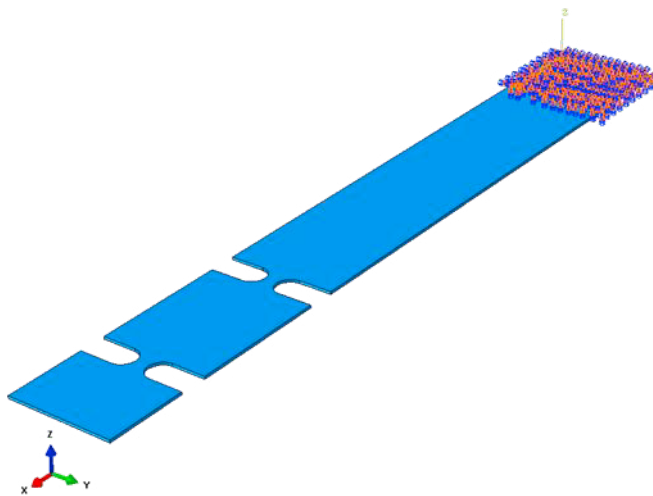


Fig. 1. Two notch specimen & excitation

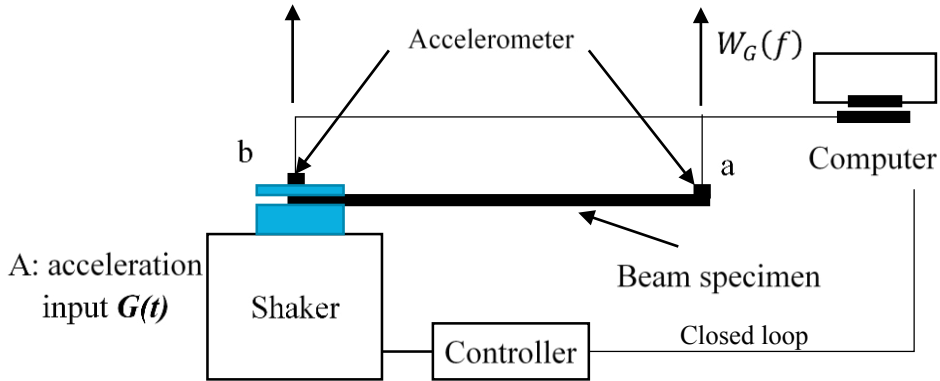


Fig. 2.Schematic diagram of the test

For time length, 10s time series of acceleration were generated with different kurtosis. The bandwidth selection is based on the modal analysis result of the specimen, the second-order bending mode is 51.2 Hz, and 15-95 Hz is selected as the frequency range of the excitation signal. The sampling ratio is 1024Hz according to the band width.

Table 1. The RMS and Kurtosis of acceleration

No.	1	2	3	4	5	6	7	8
RMS(m/s <sup>2</sup> )	31.44	31.32	31.27	31.31	31.10	30.84	31.54	31.44
Kurtosis	3.01	3.50	4.01	4.53	5.02	8.04	10.24	2.85
No.	9	10	11	12	13	14	15	16
RMS(m/s <sup>2</sup> )	31.44	31.44	31.44	31.44	31.44	31.44	31.40	31.52
Kurtosis	2.92	2.80	3.06	2.96	3.12	2.90	3.15	3.21

Before taken simulation, the modal analysis was used to check FE model to do verify and validation according mode frequency and stress distribution. Though the modal dynamic, the response stress of different times series could be obtained. It can be found from the results that with the same RMS acceleration of different kurtosis, the RMS variation of the response stress is not large, but the magnitude and the peak-to-valley value of the stress are very different.

Table 2. The RMS and Kurtosis of response stress

No.	1	2	3	4	5	6	7	8
RMS(MPa)	28.89	29.00	28.88	29.74	29.39	30.21	31.77	29.07
Kurtosis	3.07	3.30	3.52	3.92	3.70	6.11	6.49	2.85
No.	9	10	11	12	13	14	15	16
RMS(MPa)	28.89	29.09	29.17	28.34	28.89	28.45	29.00	28.93
Kurtosis	3.02	2.78	2.90	3.07	3.88	3.14	3.04	3.18

## 5. Results

Rain-flow counting is performed on the time series of response stress, and the stress spectrum of different kurtosis is obtained. The number of spectrum grade is selected. 24. At the same time, according to the material parameters, the linear damage accumulation principle and the nonlinear damage accumulation principle are used to calculate the damage and calculate the relative error. The results are as follows.

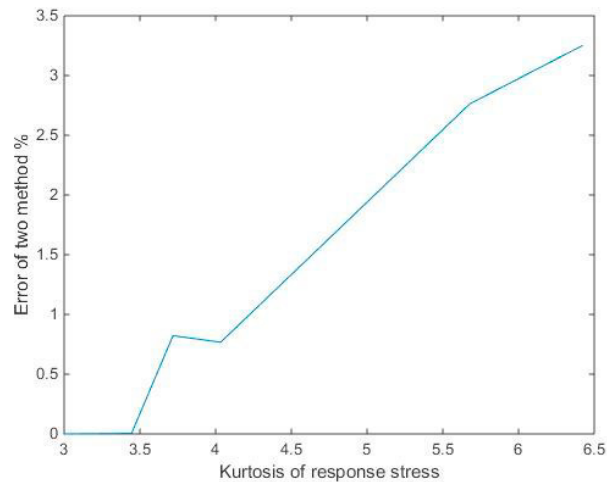


Fig. 3. The difference of two principle

## 6. Conclusion

Through virtual experiments, non-Gaussian time domain signals of different kurtosis are used to verify the effect of kurtosis on the load sequence in the fatigue damage calculation process. From the result analysis, it is known that with the increase of the kurtosis, the linear damage accumulation theory is used, and the neglected damage is gradually increased. Unfortunately, this formula does not take into account the effects of changes in stress amplitude. Part of the future work is to introduce the influence of stress amplitude changes on the calculation of fatigue damage in this process.

## Acknowledgments

This contribution has been elaborated under the China Scholarship Council.

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