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# Some considerations about a potential application of the abstract concept of asymmetric distance in the field of astrophysics

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## Abstract

We examine the fundamental question of velocity measurements in astrophysics. We show that following the way these measurements are interpreted, a lot of phenomena appealing for various and distinct paradigms such as dark matter, wave density for the spiral galaxies, superluminal velocities in jets of active galaxies and "sleeping" black holes could strongly be correlated between them and eventually could stem from an unique paradigm. A test to validate (or not) this paradigm is supplied. The new paradigm does not alter the phenomena at the scale of a planetary system, nor does it modify the conclusions based on the standard model of cosmology as for the age of the Universe, its size, the chain of processes leading to the formation of first stars and galaxies. The interest of this paradigm is that it does not introduce a new physics (in the sense that the physics laws are essentially local and very well supported at the laboratory level), nor a new particle species as usually proposed elsewhere. The non-appeal to various exotic physics with no support in Earth laboratory experiments seems to be a positivist view. The present model is also a nice application of the notion of asymmetric distances to the field of astrophysics.

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## 1 Introduction

Today, following the orthodox point of view of astrophysics, it is admitted that fundamental problems, such as 1. the flat rotation curves of galaxies, 2. the persistence of the spiral substructures, 3. the superluminal velocities of some jets in active galactic nuclei (AGN) and 4. the very high velocities for stars measured near the center of the Milky Way, are definitively solved except for "some points of detail". However :

The statement 1. is explained by the presence of a rather gigantic and even possibly unrealistic quantity of dark matter. However this proposal admitted by most astrophysicists could be controversial. Other alternative theories have also been proposed<sup>1</sup>. Two of them are very important for historical reasons and

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<sup>1</sup>We exclude obviously here some isolated suggestions that the observational estimations of velocity and density distributions in the galaxies could be strongly biased, such as in Křížek, 2018.

also because they have inspired most of the other theories. In 1983, Milgrom introduces the paradigm MOND to account the mass discrepancies without dark matter by a slight modification of the Newtonian law of motion at extremely small accelerations of less than  $10^{-10} \text{ m.s}^{-2}$  (Milgrom, 1983; Milgrom, 2019<sup>2</sup>). On the other hand, modified gravity of Moffat (2006) belongs to the large class of models with variable fundamental constants.

Today to eliminate the dark matter, a very impressive number of models have also been produced which suggest to modify the Einstein-Hilbert action following various ways (Moffat, 1995, 2005; Mannheim, 2006; Capozziello and De Laurentis, 2012, Austin, 2015).

The statement 2. is explained by an artificial mechanism : a stationary density wave (Shu, 2016), a subtle mechanism but which seems difficult to maintain for a very long time (the mean age of a galaxy  $\sim 10^{10} \text{ years}$ ).

The statement 3. is explained by appeal to the Relativity, but the mechanism is well operating if and only if we admit that the jets are mainly directed toward the observer, in some cases within an angle smaller than  $20^\circ$  degrees (Rees, 1966).

The statement 4. requires the presence of a non-active black hole (a so-called sleeping black hole) at the center of "normal" galaxies.

We know from the special Relativity that the velocities and lengths are relative and depend on the observer motion. Here the question of how the length measurements in space are conducted is raised.

First let us notice that the ratio between the so-called "classical" electron radius and one meter is the same that the ratio between one meter and the distance which separates us from the closest star, i.e.  $\sim 10^{-17}$  ! We know that the physics is not the same at the scale of an electron than at the scale of one meter. However we apply the same physics without any change at the scale of interstellar distances than at the scale of one meter ! While at the nanoscale the physics is drastically changed (classical mechanics  $\Rightarrow$  quantum mechanics), at the interstellar scale, we will see that the physics laws remain exactly the same than at the scale of one meter, but just with the addition of a simple scale factor which is introduced in the equations.

In physics it is well known that in order to make a length measurement an unit of length must in advance be defined (for instance the standard meter). The current definition is : a. one meter is exactly equal to the length of the path travelled by light in a vacuum in  $\frac{1}{299\,792\,458}$  of a second, b. the speed of light is an universal fundamental constant. The problem with this (circular) definition is that the vacuum is not itself properly defined ! For an observer located near the galactic center, the definition would be exactly the same but is the vacuum over there the same ? In this paper we propose a new definition.

Def : Let  $l_H$  the radius of a hydrogen atom and  $c_l$  the speed of light. These quantities are linked by the relation :

$$\Delta t_H = \frac{l_H}{c_l} = 1.76 \cdot 10^{-16} \text{ s} \quad (1)$$

---

<sup>2</sup>There exists different relativistic versions of this theory, especially the one built by Bekenstein (2004).

We assume that  $\Delta t_H$  is an absolute constant in time and the same everywhere in the Universe. This is the duration for the light to travel a distance equal to the radius of a hydrogen atom. The quantity  $l_H$  is hidden, because measured relatively to itself (likewise for  $c_l$ ); but by abuse in our solar system we put  $l_H = r_H$  and  $c_l = c$ . Both the quantities  $r_H$  or  $c$  are respectively the conventional radius of the hydrogen atom and the speed of light. Any reference length corresponds to  $\mathcal{N}$  hydrogen atoms arranged tight next to each other along a straight line. Once again the meter defined in this way (i.e. the number  $\mathcal{N} = 1.89 \cdot 10^{10}$ ) is universal. In astrophysics the reference lengths are the astronomical unit (AU) or the parsec (pc). These lengths are expressed as  $\mathcal{N}r_H$ , with respectively  $\mathcal{N} = 2.83 \cdot 10^{21}$  and  $\mathcal{N} = 5.58 \cdot 10^{26}$ .

We admit throughout this paper that the lengths which are currently measured by the observers are accurate (they are obtained by trigonometry for the close distances and by luminosity measurements for more distant objects). We add the important requirement that these distances are conventional and relative to the standard unit length locally defined by us.

We raise also the question : for two observers very far away from each other and respectively located at the points  $x$  and  $y$ , is the distance  $d(x, y)$  equal to  $d(y, x)$  ? It is implicitly admitted that this statement is true but nobody has checked it (unfortunately we have only one reference point in the Universe : the Earth located in a very small open set attached to it, i.e. the solar system). Following Gromov (2007) "Besides, one insists that the distance function be symmetric, that is  $d(x, y) = d(y, x)$  (this unpleasantly limits many applications)" (for a presentation of an asymmetric metric theory, see for instance, Mennucci, 2013). What would be the point of view of an observer located near the galactic centre compared to ours ? It is maybe possible that the strangeness in the velocity measurements in some situations (e.g. the flat rotation curve for the galaxies and the superluminal velocities observed in some quasar jets) be a clear proof of the relativity of the length measurements. Our aim is to revisit the notion of length for the very large distance measurements,  $\gtrsim 1 \text{ pc}$ , i.e. the interstellar distances. We state here that :

- i. For any observer in the Universe making local measurements (at the scale of a solar system), the physics laws are the same as those known at the terrestrial level. The fundamental constants are the same everywhere. No new local physics is introduced, nor new species of exotic particle. The essential difference is rather in the way whereby distinct observers correlate their measurements of distances.
- ii. the measurement of a length is always attached to a local device (e.g. the radius of an atom or even a device of the size of a few meters) which is always infinitesimally small compared to the interstellar lengths.
- iii. without a pre-existing matter a length remains undefined. A length measurement is depending on a reference unit taken locally (i.e. depending on a local observer and on the quantity of matter (the mass density) surrounding it). It is admitted here that only relative lengths (i.e. related to a given observer) can be defined in the vacuum at a very large scale.
- iv. a set of observers assumed to be distributed in the Universe can only transmit a multiple (integer or not) of the number  $\mathcal{N}$  (and not a "distance" *stricto sensu*).

The mechanisms described in the statements 1., 2., 3., 4. listed above are not antagonist with the new paradigm, but can now be moderated by the latter one :

The statement 1 : We admit the existence of a (maybe large but not still observed) quantity of invisible matter but in a baryonic form : rogue planets, brown dwarfs, neutron stars, stellar black holes, some of them may be far more massive than expected (Liu, 2019).

The statement 2 : in a self-gravitating gaseous disc the formation of a series of pieces of spiral arms is possible, but in the form of a short-lived, recurrent, transient pattern (Binney and Tremaine, 2008)).

The statement 3 : the relativistic effect explaining in an usual way the apparent superluminal velocities in some jets in AGNs is there, but the direction of the jet can now be fully arbitrary or at least less constrained.

The statement 4 : the sleeping black holes are existing at the center of "normal" (non-active) galaxies, but with a weaker mass than usually admitted.

We will see that this new paradigm gives the impression to lead to a modification of the acceleration term of the newtonian equation of motion. Such a modification has been initially proposed by Milgrom (1983), even though in a very different manner (here this modification only applies for a remote observer, not for a local observer, being the local physics is expected to remain unchanged). Likewise a variable  $G$  (the gravitational constant) also appears as in the Moffat's proposal (2006), even though the variability is now just apparent and not real (the constants  $G$  and  $c$  are the same everywhere). In some sense the present paradigm could appear as an hybrid one, borrowing some elements in both previous works.

## 2 Algebraic details

### 2.1 Structures on the space vector $\mathbb{R}^3$

1. A vector space structure is obtain on  $\mathbb{R}^3$  once it is equipped with an addition,  $+$ , and a multiplication by a scalar  $\cdot$  (Pascoli and Pernas, 2019). For two real triplets  $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$  and  $\boldsymbol{\sigma}' = (\sigma'_x, \sigma'_y, \sigma'_z)$  the sum is defined by

$$\boldsymbol{\sigma} + \boldsymbol{\sigma}' = (\sigma_x, \sigma_y, \sigma_z) + (\sigma'_x, \sigma'_y, \sigma'_z) = (\sigma_x + \sigma'_x, \sigma_y + \sigma'_y, \sigma_z + \sigma'_z)$$

and the multiplication of  $\boldsymbol{\sigma}$  by a scalar  $\lambda$  by

$$\lambda \cdot \boldsymbol{\sigma} = \lambda \cdot (\sigma_x, \sigma_y, \sigma_z) = (\lambda\sigma_x, \lambda\sigma_y, \lambda\sigma_z)$$

The neutral element for the addition is  $\mathbf{0} = (0, 0, 0)$ , the symmetric for the addition of  $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$  is  $-\boldsymbol{\sigma} = (-\sigma_x, -\sigma_y, -\sigma_z)$ .

If we set  $\mathbf{e}_x = (1, 0, 0)$ ,  $\mathbf{e}_y = (0, 1, 0)$  and  $\mathbf{e}_z = (0, 0, 1)$  we obtain the so called  $\mathbb{R}^3$  canonical basis.

Two basis define the same orientation of the space when the determinant of the matrix representing one of the basis in the second is strictly positive.

2. A Euclidean structure is obtained on  $\mathbb{R}^3$  when one had chosen a definite positive bilinear symmetric form on the vector space  $\mathbb{R}^3$ . Such an object is an application

$$B : \mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}$$

fulfilling the condition

$$\forall \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\psi} \in \mathbb{R}^3, \quad \forall \lambda, \lambda' \in \mathbb{R}, B(\lambda \cdot \boldsymbol{\sigma} + \lambda' \cdot \boldsymbol{\sigma}', \boldsymbol{\psi}) = \lambda B(\boldsymbol{\sigma}, \boldsymbol{\psi}) + \lambda' B(\boldsymbol{\sigma}', \boldsymbol{\psi})$$

$$\forall \boldsymbol{\sigma}, \boldsymbol{\psi} \in \mathbb{R}^3, \quad B(\boldsymbol{\sigma}, \boldsymbol{\psi}) = B(\boldsymbol{\psi}, \boldsymbol{\sigma})$$

$$\forall \boldsymbol{\sigma} \in \mathbb{R}^3 \setminus \{\mathbf{0}\}, \quad B(\boldsymbol{\sigma}, \boldsymbol{\sigma}) > 0$$

The formula  $(\boldsymbol{\sigma}|\boldsymbol{\sigma}') = \sigma_x \sigma'_x + \sigma_y \sigma'_y + \sigma_z \sigma'_z$  defines an Euclidean structure on  $\mathbb{R}^3$ . However there are plenty of different formulae defining such a structure; nevertheless each time an Euclidean structure is given on  $\mathbb{R}^3$  there exist a basis  $(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$  of  $\mathbb{R}^3$  such that in the coordinates associated to that basis the expression of  $B(\boldsymbol{\sigma}, \boldsymbol{\sigma}')$  is

$$B(\boldsymbol{\sigma}, \boldsymbol{\sigma}') = \sigma_1 \sigma'_1 + \sigma_2 \sigma'_2 + \sigma_3 \sigma'_3$$

Such a basis is said orthonormal for  $B$ .

– Any euclidean structure induces a "norm" on  $\mathbb{R}^3$  :

$$\forall \boldsymbol{\sigma} \in \Sigma, \quad \|\boldsymbol{\sigma}\|_B = \sqrt{B(\boldsymbol{\sigma}, \boldsymbol{\sigma})}$$

A norm on a vector space  $E$  is an application  $N : E \rightarrow \mathbb{R}$  satisfying

$$- \forall x \in E, \quad N(x) \geq 0$$

$$- \forall x \in E, \quad N(x) = 0 \iff x = 0$$

$$- \forall x, y \in E, \quad N(x + y) \leq N(x) + N(y)$$

$$- \forall x \in E, \quad \forall \lambda \in \mathbb{R}, N(\lambda \cdot x) = |\lambda| N(x)$$

– To a Euclidean structure is associated a notion of angle between two non null element of  $\mathbb{R}^3$ . For  $\boldsymbol{\sigma}$  et  $\boldsymbol{\sigma}'$  in  $\mathbb{R}^3$ , we have

$$B(\boldsymbol{\sigma}, \boldsymbol{\sigma}') = \|\boldsymbol{\sigma}\|_B \cdot \|\boldsymbol{\sigma}'\|_B \cos(\theta)$$

– When a vector space  $E$  is equipped with a norm, a function  $f : E \rightarrow \mathbb{R}$  is différentiable (with respect to the norm  $N$ ) at the point  $\boldsymbol{\sigma}$  when

– The function  $f$  is define on a neighborhood of  $\boldsymbol{\sigma}$  (i.e at least on a  $N$ -ball centered

$$\text{on } \boldsymbol{\sigma}, B_N(\boldsymbol{\sigma}, r) = \{\boldsymbol{\psi} \text{ such as } N(\boldsymbol{\sigma} - \boldsymbol{\psi}) < r\}$$

– There exists a function  $\varepsilon$  defined on  $B_N(\mathbf{0}, r)$  continuous at  $\mathbf{0}$  and satisfying  $\varepsilon(\mathbf{0}) = 0$ .

– There exists a continuous linear form  $df/\boldsymbol{\sigma}$  on  $E$  such that

$$f(\boldsymbol{\sigma} + \boldsymbol{\eta}) = f(\boldsymbol{\sigma}) + df/\boldsymbol{\sigma}(\boldsymbol{\eta}) + N(\boldsymbol{\eta})\varepsilon(\boldsymbol{\eta})$$

The condition of continuity of the linear form  $df/\boldsymbol{\sigma}$  will be always fulfilled in the space we will be working on, for it being of finite dimension.

## 2.2 Action of $\mathbb{R}_*^+$

Let  $B$  be a Euclidean structure on  $\mathbb{R}^3$  and  $k \in \mathbb{R}_*^+$ , the form

$$(\boldsymbol{\sigma}, \boldsymbol{\sigma}') \mapsto B_k(\boldsymbol{\sigma}, \boldsymbol{\sigma}') = B(k \cdot \boldsymbol{\sigma}, k \cdot \boldsymbol{\sigma}')$$

defines another euclidean structure on  $\mathbb{R}^3$ .

– The norms associated to  $B$  and  $B_k$  are different, explicitly  $\|\boldsymbol{\sigma}\|_{B_k} = k \cdot \|\boldsymbol{\sigma}\|_B$ , but the notions of angles defined by means of those two structures coincides.

– Another fact of importance is that the norms associated to  $B$  and to  $B_k$  are equivalent. As we are working on a finite dimension space, it is trivial in our case, but it would be still true in non finite dimension because of the special relation between  $B$  and  $B_k$ . This implies that the differentiability and the value of the differential of a given function are the same using the structure  $B$  or  $B_k$ . Transforming  $B$  into  $B_k$  mimicks an angular magnification of the space  $\mathbb{R}^3$ .

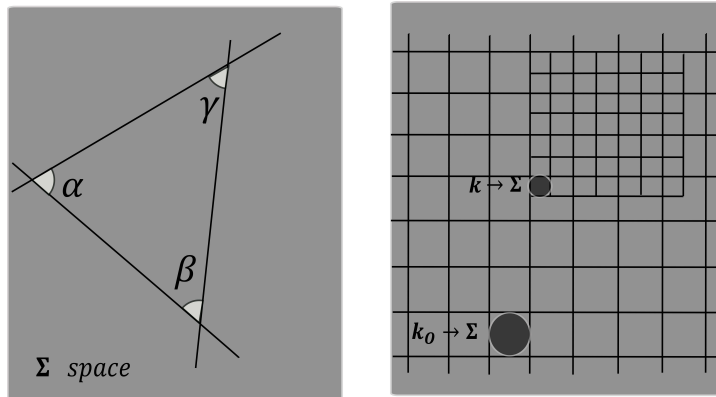


Figure 1

Thus the direction of a  $\boldsymbol{\sigma} \in \mathbb{R}^3$  is the same for each structure  $B_k$  and likewise for the angle  $\theta$  between two distinct directions. For any triangle drawn in  $\mathbb{R}^3$  one measures the same top angles using any of the structures  $B_k$ , while the side lengths are different but are in the same proportion (similar triangles) (fig. 1). The coefficient  $k$  mimicks an angular magnification of the space  $\Sigma$ . In a practical way each observer is equipped with a "perfect lens free from any aberration". However the term lens should not be taken at face value. Observing the Moon with a telescope has no physical impact on its true distance ! Another analogy derived from the electrostatics can also be made where the coefficient  $k$  bears resemblance to a kind of "static permittivity of vacuum" (see the paragraph "applications"). Once again this second analogy should no longer taken too literally.

– **The physics behind the function  $k \equiv k(\sigma)$**

As we will see in the paragraph "applications", the function  $k$  seems linked to the density in any very large structure : a galaxy (schematically composed of a central supermassive black hole, a bulge and a disk for a spiral galaxy), a galaxy cluster or around a black hole (this one being accompanied or not by an accretion disk). In fig. 3 for a spiral galaxy  $k$  decreases as a function of  $r$  (the

distance to the galactic center), from the bulge (high density) to the outskirts (weak density). Likewise we observe a similar situation for an active galactic nucleus, fig. 4). More high is the density, more high is  $k$ . Just around a sleeping black hole we can assume that this one has created a large empty cavity around it and the density is weak (and  $k$  is weak).

## 2.3 The space $\Sigma$

The space  $\Sigma$  is the set  $\mathbb{R}^3$  equipped with its vector space structure, a Euclidean structure  $B$  and an application  $k : \mathbb{R}^3 \rightarrow \mathbb{R}_*^+$ . In the sequel we will consider  $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$  an orthonormal basis so the expression of  $B$  in the associated coordinate system will be  $B(\boldsymbol{\sigma}, \boldsymbol{\sigma}') = \sigma_x \sigma'_x + \sigma_y \sigma'_y + \sigma_z \sigma'_z$ .

### 2.3.1 The space $\Sigma$ seen by an observer

To a point  $O$  of  $\Sigma$  is associated the Euclidean structure  $B_{k(O)}$ , when equipped with this structure we will say that the space is "seen by an observer sitting at the point  $O$ ". The Euclidean structure  $B$  is a Euclidean structure on  $\mathbb{R}^3$  hidden to any observer or at least partially hidden as angles measures and determination of differentials associated to  $B$  are accessible through the use of  $B_{k(O)}$ . Therefore, the regularity of any function defined on  $\Sigma$  as well as angles are independent of the observer.

### 2.3.2 Lengths

Let  $\boldsymbol{\sigma}$  and  $\boldsymbol{\sigma}'$  in  $\Sigma$ , the segment  $[\boldsymbol{\sigma}, \boldsymbol{\sigma}']$  is the convex hull of those two points, as the set  $[\boldsymbol{\sigma}, \boldsymbol{\sigma}']$  depends only on the vector space structure of  $\Sigma$ , the segments do not depend on the choice of a particular observer.

An observer  $O$  will measure a length

$$L_O([\boldsymbol{\sigma}, \boldsymbol{\sigma}']) = k(O) \|\boldsymbol{\sigma} - \boldsymbol{\sigma}'\|$$

Next to this segment's length determined by the choice of an  $O$ , there is another way of defining the length of  $[\boldsymbol{\sigma}, \boldsymbol{\sigma}']$ : its "laser length"

$$L^{laser}([\boldsymbol{\sigma}, \boldsymbol{\sigma}']) = \int_{\boldsymbol{\sigma}}^{\boldsymbol{\sigma}'} k.d\boldsymbol{\sigma}$$

The segment  $[\boldsymbol{\sigma}, \boldsymbol{\sigma}']$  is the same for each observer. Then it makes sense to consider a subdivision of the segment  $[\boldsymbol{\sigma}, \boldsymbol{\sigma}']$ ,  $\boldsymbol{\sigma} = \boldsymbol{\sigma}_1 < \boldsymbol{\sigma}_2 < \dots < \boldsymbol{\sigma}_n = \boldsymbol{\sigma}'$  where the points  $\boldsymbol{\sigma}_k$  are in  $[\boldsymbol{\sigma}, \boldsymbol{\sigma}']$ , the length associated to that subdivision would be the sum of the lengths  $L_{\boldsymbol{\sigma}_i}([\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_{i+1}])$ . The "path" of such a subdivision depends on the observer who computes it, but all agree when it comes to say that the path tends to 0. The integral (the laser ranging distance  $d^{laser}$ ) is obtained as the limit value of this sum when the path of the subdivision tends to 0, the condition of existence for such a limit being the riemann-integrability of the function  $k$  restricted to the segment. Another way to compute this integral, maybe more satisfactory, would be to interpret  $\Sigma$  as  $\mathbb{R}^3$  endowed with some metric defined by the choice of the function  $k$  (if  $k$  is regular enough), and use this metric to compute the length of the segment, the segments (straight Euclidean lines) do not need however to be geodesic segments.



### 2.3.3 Computing and comparing lengths

Let  $[A, B]$  and  $[C, D]$  two distinct segments. An observer  $O$  can compare these two segments by applying to them the operator  $k(O)$ . Thus it makes sense to say that  $L_O([A, B]) = k(O)\|A - B\|$  can be larger, smaller or equal to  $L_O([C, D])$ . We can also compare the lengths of a segment  $[A, B]$  measured by two different observers  $O_1$  and  $O_2$  by applying the relation

$$L_{O_1}([A, B]) = k(O_1)\|A - B\| = \frac{k(O_1)}{k(O_2)}L_{O_2}([A, B])$$

As an observer  $O$  by himself does not have access to the Euclidean structure  $B$ , then he is, de facto, unable to compute any distances, to do so he will need help of a second observer. Let  $O$  and  $O'$  two "nearby" observers, i.e.  $O' = O + \Delta\sigma$ . The distances between  $O$  and  $O'$  measured by  $O$  and  $O'$  are

$$d_O(O, O') = k(O)\|\Delta\sigma\| = \mathcal{N}r_H \quad (2)$$

and

$$d_{O'}(O, O') = k(O')\|\Delta\sigma\| = k(O)\|\Delta\sigma\| + \left(\frac{dk}{d\sigma} \cdot \Delta\sigma\right)\|\Delta\sigma\| \quad (3)$$

(assuming here that  $k$  is a differentiable function of  $\sigma$ ). At the first order in  $\Delta\sigma$  we have  $d_O(O, O') = d_{O'}(O', O)$  and this distance is symmetric. This is the base calibrated by  $O$  (with help from  $O'$ ) of a parallactic triangle  $O, O', P$  where  $P$  is a distant point. By this procedure each observer can calculate the distance  $d_O(O, P)$  of any distant point  $P$  in space.

We now have to set an "universal helper". Let  $O$  be a point and  $O_H$  another point such that the radius of a hydrogen atom in its fundamental state would fit exactly into  $[O, O_H]$ . We denote  $O - O_H$  by  $\sigma_H(O)$  the element of  $\Sigma$  representing the radius of a hydrogen atom. Let  $\ell_H(O) = d_O(O, O_H)$ , this quantity is "the same everywhere" (this is  $r_H$ ) and locally all measurements of lengths are reported to it. The expression "the same everywhere" is taken in the sense that if one transports an atom of hydrogen from a point  $O_1$  to another very distant point  $O_2$  and compare it to a local hydrogen atom, the radii are the same, but to say the "same everywhere" before transport has no signification because  $\ell_H$  is a length defined by reference to itself !

Another way of saying this is that the duration taken by a beam of light to travel any segment  $[O, O_H]$  is the same everywhere (the time is assumed to be universal in absence of gravity).

By contrast the differences appear if an observer at a position  $O_1$  decides to measure, from where he is, the radius of a hydrogen atom located very far from him at a position  $O_2$ . In this case from the position  $O_2$  the segment  $[O_2, O_{2H}]$  has length

$$r_H = \ell_{O_2}([O_2, O_{2H}]) = k(O_2) \cdot \|O_2 - O_{2H}\|$$

While from the position  $O_1$  the observer finds an apparent length

$$\ell_{O_1}^{app}([O_2, O_{2H}]) = k(O_1) \cdot \|O_2 - O_{2H}\|$$

with the ratio  $\frac{\ell_{O_1}^{app}([O_2, O_{2H}])}{r_H} = \frac{k(O_1) \cdot \|O_2 - O_{2H}\|}{k(O_1) \cdot \|O_1 - O_{1H}\|} = \frac{k(O_2)k(O_1) \cdot \|O_2 - O_{2H}\|}{k(O_2)k(O_1) \cdot \|O_1 - O_{1H}\|} = \frac{k(O_1)}{k(O_2)} \neq 1$ .

This seems to contradict the preceding statement that the radius  $r_H$  is "invariable". In fact there is no contradiction. In reality the observer at the position 1 does not measure the atom at the position 2 itself but attempts to provide a value to length of the segment  $[O_2, O_{2H}]$  where it fits. Very unconsciously he then applies the operator  $k(O_1)$  on it and the result of this operation is  $\ell_{O_1}^{app}([O_2, O_{2H}])$ . It is an apparent length. To make the measurement of the radius of a hydrogen atom at a very large distance is obviously not practicable; but maybe a planet with a determined radius (such as for instance a transportable and well calibrated Earth from a point to another point) would be a more adequate reference. Unfortunately we know that the exoplanets are of all sizes and this precludes a reference length to be defined. The fact that we have no reference length in space for measuring very large lengths authorizes various speculative proposals (dark matter hypothesis, modified gravity theories, this work, etc).

In Figure 2  $\mathcal{N}$  is universal, i.e. another observer will see the same number  $\mathcal{N}$ , but he will attribute a different value to  $\Delta\sigma$  applying his proper  $k$ . For instance an observer located at  $P$  measures the base  $OO'$  and attributes to it the length  $k(P)\Delta\sigma$ .

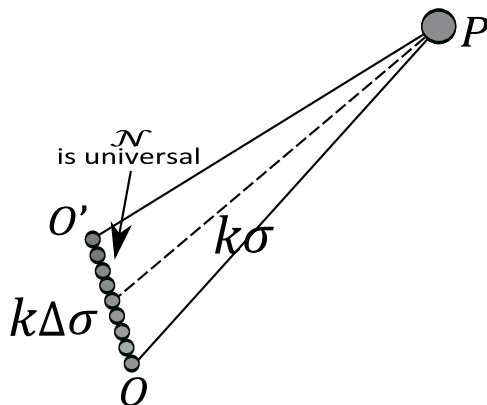


Figure 2

### – The asymmetry of distances

Let two observers  $A, B$ . The vector  $\sigma_{AB}$  satisfies the relation

$$\sigma_{AB} = -\sigma_{BA}$$

and the distances constructed on  $\Sigma$  are symmetric,  $\|\sigma_{AB}\| = \|\sigma_{BA}\|$ . On the other hand the action of  $k$  (linked to an observer) makes these distances asymmetrical,  $k_A\|\sigma_{AB}\| \neq k_B\|\sigma_{BA}\|$ .

Universes with asymmetric distances are predicted in Mathematics (Gromov, 2007; Mennucci, 2013), but unfortunately this concept is not fully exploited in physics.

### – The absolute luminosity of a star

An observer  $E$  measures the absolute luminosity  $L_*$  of a star  $S$  starting from the apparent brightness  $\varphi_*$ . He uses the relationship  $\varphi_* = \frac{L_*}{4\pi(k_E\sigma_{ES})^2}$ . In this

sense the distance  $k_E\sigma_{ES}$  is a "true" distance since this data alone (known in advance by trigonometry) allows to deduce the absolute luminosity of the star (neglecting the absorption by dust on the travel). The temperature  $T_*$  of the star is also left unchanged and the Hertzsprung-Russel diagram is not modified.

– **The variability of the absolute luminosity of a star**

For variable stars (for instance the Cepheids) the pulsation period is left unchanged by the new paradigm. However if a sudden phenomenon is produced (for instance a supernova explosion of a massive star) then the delay of reception of the signal (propagating with the speed  $c$ ), depends on the so-called laser distance  $d_E^{laser}(S, E) = \int_E^S k.d\sigma$  and not from the distance  $d_E(E, S) = k_E\sigma_{ES}$ . Obviously, a test for the validity (or not) of the present paradigm on the relativity of interstellar distances would simply be to measure this delay. It is indeed a practical test, but both our fixed position in space and the absence of a network of observers unfortunately forbid its use at this time.

## 2.4 The function $k$

In reality the function  $k : \mathbb{R}^3 \rightarrow \mathbb{R}_*^+$  defining the  $\Sigma$ -structure does not need to be smooth. As  $\mathbb{R}$  is equipped with a Euclidean structure it is also equipped with a topology then we can consider on  $\mathbb{R}^3$  a structure of measured space  $(\mathbb{R}^3, \mathcal{A}, \mu)$  where  $\mathcal{A}$  is the Borel algebra and  $\mu$  the Lebesgue measure. We will consider functions  $k$  differentiable almost everywhere with null differential where the differential exists. Such functions exists, if  $A$  is a borelian set such that  $\mu(\partial A) = 0$ , then its characteristic function ( $\mathbb{I}_A(x) = 1$  on  $A$  and 0 on its complementary) is differentiable everywhere but on  $\partial A$  with a null differential. Of course, all linear combination  $k$  of such functions fullfills the condition that  $k$  is almost everywhere differentiable with null differential so locally constant at each point where it is differentiable.

There are also examples of continuous non constant functions almost everywhere differentiable with null differential : the Cantor functions.

## 3 The Fermat principle and the propagation of light

The function  $k$  varies continuously as a function of  $\sigma$ . However we can assume that this function is not absolutely continuous (we can imagine that the repartition of matter in a galaxy is fractal and that this can imply a fractal variation of  $k$ ). As a first approximation we can then represent the  $k$ -function as a Cantor function (Bass, 2013).

Let two flat steps on this curve separated by a jump. Let on each step an inertial observer, resp.  $L$  and  $L'$ . We have by invariance of ( $l_H \equiv r_H$ )

$$l_H = k_L\sigma_H = l_{H'} = k_{L'}\sigma_{H'}$$

and for the speed of light ( $c_l = c$ )

$$c_l = \frac{l_H}{\Delta t} = c_{l'} = \frac{l_{H'}}{\Delta t}$$

When a beam of light moves from a step to another one its speed does not change. There is no refraction following the Fermat principle. On the other hand, for a remote observer  $O$ , the apparent speed of light makes a sudden jump from  $c_{app} = \frac{k_O}{k_L}c$  to  $c'_{app} = \frac{k_O}{k_{L'}}c$ . Then the observer  $O$  sees the beam of light to accelerate or to decelerate according to respectively  $k_L > k_{L'}$  or  $k_L < k_{L'}$ , while otherwise the beam propagates in straight line for him. This result is especially important for explaining the superluminal jets observed in some quasars as we will see a little later.

Likewise a free particle maintains the same velocity when moving from a step to another one. This particle does not feel the discontinuity (but the apparent velocity suddenly changes for a remote observer).

In the following the function  $k$  is chosen absolutely continuous. A fixed observer  $O$  has a global view on the ray path, but for him the speed of light is variable (even though this effect is not real but apparent). Let  $\mathbf{r} = k_O\boldsymbol{\sigma}$  and  $\frac{d\mathbf{r}}{ds} = k_O\frac{d\boldsymbol{\sigma}}{ds}$  ( $ds = \|d\mathbf{r}\|$ ). We put  $k_O = 1$  with no loss of generality. Our aim is to obtain an equation expressing the light propagation and also another one for a massive particle. These motions are perceived by  $O$  and for him they are apparent. Must an apparent motion be constrained by a variational principle? In fact no, but we can always try to do that! As usually we must minimize an action integral

$$S = \int_A^B \mathcal{L} ds$$

We express the lagrangian  $\mathcal{L}$  by using the couple of independent variables ( $\mathbf{R} = k\boldsymbol{\sigma}$ ,  $\frac{d\mathbf{R}}{ds} = k\frac{d\boldsymbol{\sigma}}{ds}$ )<sup>3</sup>. We consider an arbitrary variation ( $\mathbf{R} \rightarrow \mathbf{R} + \delta\mathbf{R}$ ,  $\frac{d\mathbf{R}}{ds} \rightarrow \frac{d\mathbf{R}}{ds} + \delta(\frac{d\mathbf{R}}{ds})$ )<sup>4</sup>. The lagrangian is varied as

$$\partial\mathcal{L} = \frac{\partial\mathcal{L}}{\partial(\frac{d\mathbf{R}}{ds})}\delta(\frac{d\mathbf{R}}{ds}) + \frac{\partial\mathcal{L}}{\partial\mathbf{R}}\delta\mathbf{R}$$

We impose the condition  $\frac{d}{ds}(\delta\mathbf{R}) = \delta(\frac{d\mathbf{R}}{ds})$ , that is the parallelogram formed by the four vectors  $\delta\mathbf{R}$ ,  $d\mathbf{R}$ ,  $d\mathbf{R} + \delta(d\mathbf{R})$ ,  $\delta\mathbf{R} + d(\delta\mathbf{R})$  is assumed to be closed and  $\delta(d\mathbf{R}) = d(\delta\mathbf{R})$ . Admitting that  $\delta\mathbf{R}$  is taken equal to 0 the extremities of the integral  $A$  and  $B$ , this leads to the usual Euler-Lagrange equations

<sup>3</sup> a. The vector  $\mathbf{R} = k\boldsymbol{\sigma}$  is the same vector as  $\mathbf{r} = k_O\boldsymbol{\sigma}$ , but its length is measured by the local observer  $L$  located near the current point.

b. In spite of the notation, the displacement  $d\mathbf{R}$  is not the differential of  $\mathbf{R}$ , but it is a differential form. For two non collinear real displacements  $d\mathbf{R}$  and  $d'\mathbf{R}$  both issued from  $\mathbf{R}$ , we have

$$\begin{aligned} d(d'\mathbf{R}) - d'(d\mathbf{R}) &= \partial_i k d\sigma^i d'\sigma^j e_j - \partial_i k d'\sigma^i d\sigma^j e_j = (\partial_i k e_j - \partial_j k e_i) d\sigma^i d'\sigma^j \\ &= (\nabla k \cdot d\boldsymbol{\sigma}) d'\boldsymbol{\sigma} - (\nabla k \cdot d'\boldsymbol{\sigma}) d\boldsymbol{\sigma} \neq \mathbf{0} \end{aligned}$$

The integral of  $d\mathbf{R}$  on a closed contour over  $\Sigma$  is not necessarily null.

<sup>4</sup>The arbitrary displacement issued from  $\mathbf{R}$ ,  $\delta\mathbf{R}$ , has not the same signification as  $d\mathbf{R}$ , the latter one being real and disposed along the trajectory of the particle. We have more freedom with  $\delta\mathbf{R}$ .

$$\frac{d\left(\frac{\partial\mathcal{L}}{\partial\left(\frac{d\mathbf{R}}{ds}\right)}\right)}{ds} - \frac{\partial\mathcal{L}}{\partial\mathbf{R}} = \mathbf{0}$$

with the lagrangian  $\mathcal{L} = \sqrt{\left(\frac{d\mathbf{R}}{ds}\right)^2}$ , we obtain the corresponding eikonal equation

$$\frac{d}{ds} \left( \frac{d\mathbf{R}}{ds} \right) = \frac{d}{ds} \left( k \frac{d\mathbf{r}}{ds} \right) = \mathbf{0} \implies \frac{d^2\mathbf{r}}{ds^2} + \left( \frac{d}{ds} \ln k \right) \frac{d\mathbf{r}}{ds} = \mathbf{0} \quad (4)$$

### – The motion equation of a massive particle

For a particle of mass  $m$  submitted to a potential  $\phi$ , the lagrangian is

$$\mathcal{L} = \frac{1}{2}m\left(\frac{d\mathbf{R}}{dt}\right)^2 - \phi(\mathbf{R})$$

The motion equations seen by  $O$  are

$$m\frac{d}{dt}\left(k\frac{d\mathbf{r}}{dt}\right) + \frac{\partial\phi}{\partial(k\mathbf{r})} = \mathbf{0} \quad (5)$$

Let us note that the motion seen by  $O$  is not real but apparent (relative to him). With no force applied to the particle, the observer  $O$  yet sees the particle accelerates or decelerates. The principle of impulsion conservation seems to be violated in the motion, but it is an apparent motion and that this principle be checked is not necessarily required (however the principle of impulsion conservation must be checked for a local observer  $L$ , and indeed it is :  $\frac{d}{dt}(m\frac{d\mathbf{R}}{dt}) = \mathbf{0}$ ).

## 4 The relativistic generalisation

The relativistic generalisation naturally follows. We start from the action

$$S = \int_{t_1}^{t_2} mc\sqrt{1 - \frac{1}{c^2}\left(\frac{d\mathbf{R}}{dt}\right)^2} dt$$

We obtain the relativistic motion equation of a particle of mass  $m$

$$\frac{d}{dt} \left( \frac{m}{\sqrt{1 - \frac{1}{c^2}\left(\frac{kd\mathbf{r}}{dt}\right)^2}} k \frac{d\mathbf{r}}{dt} \right) = \mathbf{0}$$

Likewise from a geometric point of view, we take now the action

$$S = \int \eta_{ij} \frac{dX^i}{ds} \frac{dX^j}{ds} ds$$

with  $\eta_{ij} = (1, -1, -1, -1)$ ,  $ds^2 = c^2 dt^2 - d\boldsymbol{\sigma}^2$ ,  $i, j = 0, 1, 2, 3$ ,  $dX^0 = cdt$ ,  $dX^\alpha = kd\sigma^\alpha$  ( $\alpha = 1, 2, 3$ ). We deduce the equation of the motion projected on the  $\Sigma$  space

$$\frac{d}{ds}\left(\frac{dX^\alpha}{ds}\right) = 0 \Rightarrow \frac{d^2x^\alpha}{ds^2} + \left(\frac{d \ln k}{ds}\right) \frac{dx^\alpha}{ds} = 0$$

or

$$\frac{d^2x^\alpha}{ds^2} + \frac{\partial \ln k}{\partial x^\beta} \frac{dx^\beta}{ds} \frac{dx^\alpha}{ds} = 0$$

This equation can be rewritten in the form

$$\frac{d^2x^\alpha}{ds^2} + \Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{ds} \frac{dx^\gamma}{ds} = 0$$

We find by identification the connexion symbols (let us note that these coefficients are not symmetric)

$$\Gamma^\alpha_{\beta\gamma} = \frac{\partial \ln k}{\partial x^\beta} \delta_\gamma^\alpha$$

The Ricci tensor is

$$R_{\alpha\beta} = \partial_\mu \Gamma^\mu_{\beta\alpha} - \partial_\beta \Gamma^\mu_{\mu\alpha} + \Gamma^\mu_{\mu\lambda} \Gamma^\lambda_{\beta\alpha} - \Gamma^\mu_{\beta\lambda} \Gamma^\lambda_{\mu\alpha}$$

and by simple calculations

$$R_{\alpha\beta} = \left(\frac{\partial \ln k}{\partial x^\mu \partial x^\beta} - \frac{\partial \ln k}{\partial x^\beta \partial x^\mu}\right) \delta_\alpha^\mu + \left(\frac{\partial \ln k}{\partial x^\mu} \frac{\partial \ln k}{\partial x^\beta} - \frac{\partial \ln k}{\partial x^\beta} \frac{\partial \ln k}{\partial x^\mu}\right) \delta_\lambda^\mu \delta_\alpha^\lambda = 0$$

The metric for any observer  $O$  is Ricci-flat.

Taking account now the gravity, the usual Einstein equations for a observer  $O$  are left unchanged, but with  $\chi_{app} = \frac{k_E}{k_L} \chi$  ( $k_L$  local value of  $k$ ).

## 5 Application

A local observer  $L$  applies the gravitational Newtonian law in situ where the gravitational force acts, i.e. on him or on a mass  $m$  located near him (the label-vector  $\sigma$  goes from the mass  $M$  to the mass  $m$ ). We assume that the attractive mass,  $M$ , is concentrated in a central zone, i.e. the bulge of the galaxy (it is a drastic simplification but sufficient here for an illustrative purpose). The dimensionality of the constant  $G$  will be adequately redefined in order to respect the homogeneity. For instance  $G \rightarrow \bar{G} = \frac{G}{r_g^3}$  where  $r_g$  is the scale factor for a galaxy. This reference length is taken equal to the distance galactic center-Sun,  $\sim 8 \text{ kpc}$ . It is the mean radius measured by any observer located inside it. This radius is the apparent distance separating him from the galactic center (and measured by him), which is the same measured by all observers located in the outskirts of a galaxy (the habitable zone). First we must derive determined quantities. For any local observer we assume the usual law :

$$\frac{d}{dt}\left(m \frac{d\mathbf{R}}{dt}\right) = -GMm \frac{\mathbf{R}}{\|\mathbf{R}\|^3} \quad (6)$$

from which it follows that

$$\underbrace{mr_g \frac{d}{dt} \left( k \cdot \frac{d\boldsymbol{\sigma}}{dt} \right)}_{\text{local observer}} = \underbrace{-\bar{G}Mmr_g \frac{k \cdot \boldsymbol{\sigma}}{(k \cdot \|\boldsymbol{\sigma}\|)^3}}_{\text{Influence of the attractive centre, the vector } \boldsymbol{\sigma} \text{ points toward the local observer}} \quad (7)$$

*Influence of the attractive  
centre, the vector  $\boldsymbol{\sigma}$  points  
toward the local observer*

with  $\mathbf{R} = r_g k \boldsymbol{\sigma}$  and  $d\mathbf{R} = r_g k d\boldsymbol{\sigma}$ <sup>5</sup>,

The gravitational force (right side) can be derived from a potential<sup>6</sup>

The expression (eq. 3) is still copied from the classic law with the change  $\mathbf{r} \rightarrow \mathbf{R} = k \cdot \boldsymbol{\sigma}$  and  $d\mathbf{r} \rightarrow d\mathbf{R} = k \cdot d\boldsymbol{\sigma}$ . Newton's laws do not work at the

<sup>5</sup>here  $\sigma$  is dimensionless. Cf. note 1 for the signification of the "d" in  $d\mathbf{R}$ .

<sup>6</sup>We can cut the bulge of mass  $M$  in a series of spherical concentric shells. For a spherical shell of mass  $dM$  with its center located at the galactic center, we have

$$d^3\Phi = -\frac{\bar{G}dm}{k(\|\boldsymbol{\sigma} - \boldsymbol{\sigma}'\|)}$$

where

$$dm = m_H n'(k'\sigma)/(k'\sigma')^2 d(k'\sigma')/\sin\theta d\theta d\varphi$$

denotes the mass element surrounding the point  $A'$  ( $n' \equiv n'(k'\sigma')$  is the particle density seen by the current observer located at  $A'$ ). On the other hand the angles  $\theta$ ,  $\varphi$ , their differentials and  $dm$  are invariant).

By an usual integration over  $\theta$  and  $\varphi$ , we obtain

$$d\Phi^f = -\frac{\bar{G}dM}{k\sigma}$$

The effect of a spherical shell is the same as a point mass  $dM$  placed at the galactic center.

$$\Phi = -\frac{\bar{G}M}{k\sigma}$$

by application of the operator  $\nabla_{k\boldsymbol{\sigma}}$ .

The  $k$ -factor in  $k\|\boldsymbol{\sigma} - \boldsymbol{\sigma}'\|$  is  $k$  and not  $k'$ . This can be still derived from the analogy with the electrostatic interaction in a medium with a variable permittivity (see for instance Landau and Lifshitz, 1984).

Let a free charge  $Q$  placed at the centre of a dielectric ball of permittivity  $\epsilon(r)$ . The displacement vector  $\mathbf{D}$  is

$$\text{div}\mathbf{D} = Q\delta(\mathbf{r}) \quad \text{rot}\mathbf{D} = \mathbf{0}$$

Combining the spherical symmetry with the Gauss law immediately gives the solution  $D = \frac{Q}{4\pi} \frac{r}{r^3}$ . Admitting now the constitutive relation  $\mathbf{D} = \epsilon(r)\mathbf{E}$  (see Landau and Lifshitz, 1984, eq. 7.1), we deduce the electric field

$$\mathbf{E}(r) = \frac{Q}{4\pi} \frac{\mathbf{r}}{\epsilon r^3}$$

The force acting on a trial particle of charge  $q$  located at a distance  $r$  from the centre is

$$\mathbf{F}(r) = \frac{Qq}{4\pi} \frac{\mathbf{r}}{\epsilon r^3}$$

Then by analogy using the transform  $Q, q \Rightarrow M, m, r \Rightarrow \sigma, \epsilon(r) \Rightarrow k^2(\sigma)$

$$\mathbf{F}(r) = -GMm \frac{k\boldsymbol{\sigma}}{(k(\sigma)\sigma)^3}$$

Let us however notice that even though such an analogy gives the same result that using the right procedure, i.e. by passing from eq. 6 to eq. 7, it should not taken at face value.

nanoscopic scale of atoms and molecules and we need to use quantum mechanics. At a very large scale, i.e. in the galaxy realm, we assume that the newtonian gravitational law is still valid even though now expressed as above (eq. 7).

The motion of a star is grossly circular in a galaxy, then ( $k$  is assumed to be a function of the radial distance  $\sigma$ , then for a circular motion  $\frac{dk}{dt} = \frac{dk}{d\sigma} \frac{d\sigma}{dt} = 0$ ). The first member simplifies as

$$\frac{d}{dt}(k \cdot \frac{d\sigma}{dt}) = \frac{dk}{dt} \frac{d\sigma}{dt} + k \frac{d^2\sigma}{dt^2} \simeq k \frac{d^2\sigma}{dt^2}$$

If we assume that all observers in the outskirts of a galaxy are equivalent, the quantity  $k \cdot \sigma$  is the same everywhere in this region. The basis of this assumption is obviously reflected in the galactic flat curve for the velocities. Subsequently any observer in a galaxy supplies the same estimation of the distance for the galactic centre, after comparison with its proper local reference (but for each of them this distance is a true distance). Unfortunately at the present time this assumption is impossible to check ! The quantity  $k \cdot \sigma$  can be put equal to 1 in the outskirts of a galaxy. Then we obtain :

$$r_g k \cdot \ddot{\sigma} = -\bar{G}M r_g k \cdot \sigma$$

On the other hand (the index  $O$  designating an outer observer) :

$$r_g k_0 \cdot \ddot{\sigma} + \bar{G}M r_g k_0 \sigma = 0$$

(the dot notation above a quantity is used here to represent the time derivative). Eventually putting  $\mathbf{r} = r_g k_0 \sigma$  :

$$\ddot{\mathbf{r}} + \bar{G}M \mathbf{r} = 0$$

We recognize the equation of the harmonic oscillator. For a circular motion  $r\dot{\theta} \propto r$  (we recall that the angle and its time derivative are defined quantities and the same for all observers). This relationship expresses a quasi-rigid body-like rotation and any spiral structure, once built-up, is conservative (without outer ingredients artificially added to the theory such as, for instance, the density wave hypothesis needed to maintain this structure). However the velocity  $v$  measured locally is  $r_g(k \cdot \sigma)\dot{\theta}$ , and this is the same value measured by any inertial observer located along the galactic radius (this so-called inertial observer is a fictive observer who does not participate to the rotation of the galaxy). Evaluating the frequency ratio we find  $\frac{\Delta\nu}{\nu} = \frac{v}{c} = \text{Constant}$ . Any outer observer measuring this ratio (this is the same ratio because the speed of light and the frequency are universal quantities and  $\frac{v}{c} = \frac{v_{app}}{c_{app}}$ ) immediately deduces an apparent flat rotation curve as attested by spectroscopic measurements (Metzger, Calwell, Schechter, 1998).

The spiral structure and the flat galactic curve are no longer independent but are now conjugate phenomena by an "universal law"  $k \cdot \sigma = 1$  in the outskirts of a galaxy. In other words a spiral substructure can be permanent, compatible with a flat rotation curve (as observed spectroscopically), but with no winding problem.

The figure 3 describes the typical variation of  $k$  over a large spectrum of distances, starting from the singularity (a sleeping black hole), assumed to be



present at the center of a "normal" galaxy, up to the size of a galaxy cluster. The mean curve, which aims to be universal, is parametrized by  $r_{ref}$  which could be linked to the mass of the central black hole  $M_{BH}$  (maybe this link could be established from straightforward numerical simulations but which remain to be done). If we report us to the paragraph 5.3 on the AGNs (assumed to be equipped of a supermassive active black hole), replacing the bulge by the accretion disk surrounding the black hole, a law  $r_{ref} \sim M_{BH}^{-\frac{1}{2}}$  could be suggested<sup>7</sup>. We can also propose a simple relationship between  $k$  and the density  $\rho$  (in the case of the Milky Way  $r_{ref} \simeq 8 \text{ kpc}$ )

$$e^{-[(\frac{1}{k-0.16})-1.19]} = \frac{\rho}{\rho_{ref}} \quad (8)$$

This law is assumed to be universal and applicable to other spiral galaxies (but with another couple  $r_{ref}, \rho_{ref}$ ). For  $0.5r_{ref} < r < 3r_{ref}$ , we have

$$e^{-[(\frac{1}{k-0.16})-1.19]} = \frac{\rho}{\rho_{ref}} \sim e^{-\frac{r}{r_{ref}}} \Rightarrow k \sim \frac{r_{ref}}{r} \quad (9)$$

The figure 4 reproduces the corresponding curve for the velocities (radius of the bulge  $\sim 3 \text{ kpc}$  and mean density  $\rho_b \sim 0.18 M_{\odot} \text{pc}^{-3}$ ; Galactic disk treated as a thin disk (thickness  $\sim 300 \text{ pc}$ ) with an exponential decreasing surface density (stars + gas), taking  $\rho_D = 0.14 M_{\odot} \text{pc}^{-3}$  at  $r_{ref} = 8 \text{ kpc}$ ). We can see an oscillating series of pieces of pseudo keplerian curves superimposed to a mean flat plateau. This oscillation is characteristic of most observational velocity profiles supplied for the Milky Way (see the figure 10 of Chemin, Renaud and Soubiran (2015) and the figure 2 of Morz et al (2019)). We can note that the pseudo-keplerian behavior in the solar neighborhood perturbing the mean flat plateau can lead to conflicting measurements (to compare Bovy and Tremaine, 2012 and Moni Bidin et al, 2014).

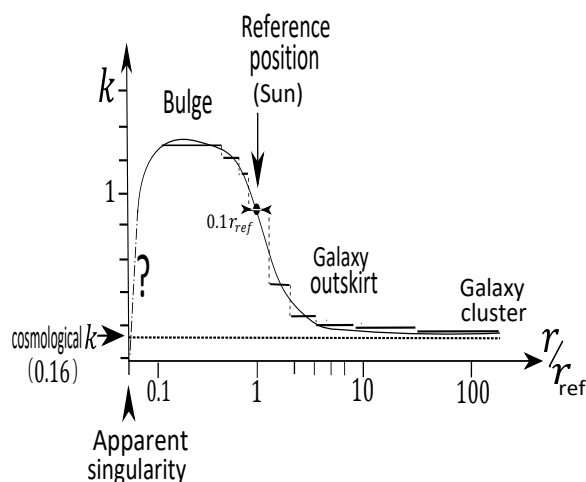


Figure 3

<sup>7</sup>Even though in a different context, a connection between a "dark matter effect" (whose extension zone is linked to  $r_{ref}$ ) with the galaxy center and the supermassive black hole has been suggested (Bogdán and Goulding, 2015).

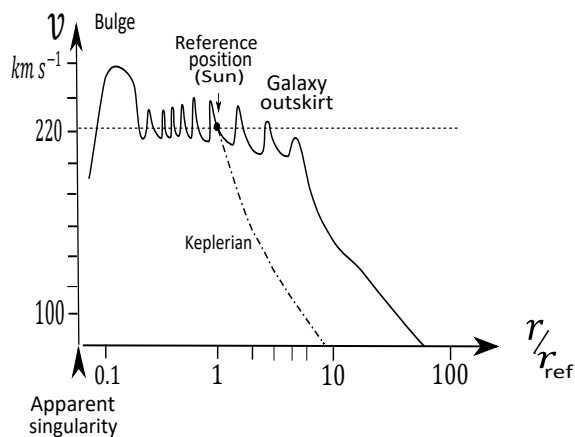


Figure 4

### – The case of the elliptical, dwarfs and large ultradiffuse galaxies

An elliptical galaxy can approximately be represented by a triaxial spheroid. Both radial velocity measurements (Méndez et al, 2001, Romanowsky et al., 2003; Lane, Salinas and Richtler, 2014) and estimations of mass-to-light ratios in elliptical galaxies deduced from gravitational lensing (Treu, 2010), seem to suggest that the "dark matter effect" is weak. Transposed in the context of the new paradigm this could signify that the mean value of  $k$  is much higher in these objects than in the outskirts of the spiral galaxies (grossly an elliptical galaxy is a kind of "big bulge" with high  $k$ ). However, in any case, these observational data are still disputed and there exists no consensus on the rate of dark matter in the elliptical galaxies. Consequently, it is difficult to fix  $k$  in these objects, but we can suggest  $k \sim Const \simeq 1$  (no "dark matter effect").

At the opposite side the dwarf galaxies or the large ultradiffuse galaxies are dominated by a strong "dark matter effect" (Kormendy and Freeman 2016) and the  $k$  is weak ( $1 > k > 0.16$ ).

### – Self-gravity in a galaxy

Let a cluster composed of  $N$  stars in a galaxy. The stars interact between them. The effect of the star  $j$  on the star  $i$  is expressed by ( $i \neq j$ ).

$$m_i r_g \frac{d}{dt} \left( k_i \cdot \frac{d\sigma_i}{dt} \right) = - \sum_{j=1}^N \frac{\bar{G} m_i m_j}{(k_i \cdot \|\sigma_i - \sigma_j\|)^3} r_g k_i \cdot (\sigma_i - \sigma_j) \quad (10)$$

and with  $k_i = \frac{k_O}{k_O} k_O$  (and assuming  $|\frac{1}{k_i} \frac{dk_i}{dt}| \ll |\frac{\ddot{\sigma}_i}{\sigma_i}|$  for a cluster whose radius is  $\ll r_g$ ):

$$r_g k_O \cdot \ddot{\sigma}_i = - \sum_{j=1}^N \left( \frac{k_O}{k_i} \right)^3 \frac{\bar{G} m_j}{(k_O \cdot \|\sigma_i - \sigma_j\|)^3} r_g k_O \cdot (\sigma_i - \sigma_j)$$

Putting  $\mathbf{r}_{i,j} = r_g k_O \cdot \sigma_{i,j}$  we obtain (following the global point of view of an outer observer  $O$ )

$$\frac{d^2 \mathbf{r}_i}{dt^2} = - \sum_{j=1}^N \left( \frac{k_O}{k_i} \right)^3 \frac{G m_j}{\|\mathbf{r}_i - \mathbf{r}_j\|^3} (\mathbf{r}_i - \mathbf{r}_j) \quad (11)$$

This is the usual equation but with a factor  $\left(\frac{k_O}{k_i}\right)^3$  which in appearance strongly amplifies the phenomenon of self-interaction in the outer region of the galaxy (with  $k_i < k_O$ ). Let us notice that we obtain the same equation by replacing in the classical equation the gravitational constant  $G$  by the apparent constant  $G_{app} = \left(\frac{k_O}{k_i}\right)^3 G$ . We could thus mimic the effect of dark matter by a "variable" gravitational constant as some authors have made (Moffat, 2006) (with the sole, but though important, difference that here the variability is not real but apparent (measured locally the gravitational constant is  $G$ )).

$$\frac{d^2 \mathbf{r}_i}{dt^2} = - \sum_{j=1}^N \frac{G_{app} m_j}{\|\mathbf{r}_i - \mathbf{r}_j\|^3} (\mathbf{r}_i - \mathbf{r}_j) \quad (12)$$

However for an inertial observer placed at  $i$ , putting  $\mathbf{R}_{i,j} = r_g k_i \cdot \boldsymbol{\sigma}_{i,j}$  in eq. (5) gives

$$\frac{d^2 \mathbf{R}_i}{dt^2} = - \sum_{\substack{j=1 \\ i \neq j}}^N \frac{G m_j}{\|\mathbf{R}_i - \mathbf{R}_j\|^3} (\mathbf{R}_i - \mathbf{R}_j)$$

i.e. the standard form. Thus a local measurement (within the extrasolar solar system under consideration) would provide  $G$ .

For an outer observer,  $G$  varies as  $r^3$  in the outskirts of a galaxy. But beyond in the intergalactic medium following this rule, the  $G_{app}$  coefficient would diverge which is not observed. We must thus assume that there certainly exists a cut-off value, for  $r$  of the order of a few  $r_g$ , which limits this effect. Fixing this cut-off value (or in other words the width of the bump of the function  $k$ ) is difficult because the galactic velocity curve shows no evidence of a decline at some distance.

For the Milky Way a refinement of the function  $k$  needs a deep knowledge of the galactic rotation curve. Unfortunately the observational data diverge according to the authors. For instance Gnaniński (2018) finds that the Milky Way rotation curve has its shape between keplerian and flat in the outskirts while, on the contrary, Sofue, Honma and Omodaka (2009) or Russeil et al (2017) find in some cases (following the parametrization) a rising rotation curve.

### – The central black hole

At the opposite side for  $r \rightarrow 0$ ,  $G_{app} \rightarrow 0$  and again it is not a realistic situation. A natural manner to remedy to that is to imagine that  $G_{app}$  tends toward a finite limit  $\neq 0$ . However may be can we also imagine that  $G_{app}$  strongly increases toward the infinite near the centre and mimics a galactic black hole.

It is well admitted that a supermassive black hole (SMBH) resides at the center of any galaxy (including ours) with a mass varying from  $10^6$  to  $10^{10} M_\odot$  (Marziani and Sulentic, 2011, Fig. 9). It is a real singularity of the space-time of which it is hard to get rid off (a black hole is a zero-volume object into which the entire mass is concentrated, a concept where modern physics breaks down). The paradigm with  $G_{app} \rightarrow \infty$  when  $r \rightarrow 0$  reproduces the same phenomena

(the observation of very high apparent velocities around the galactic center), but the great advantage now is that the singularity is no longer real but is just an apparent effect from the point of view of a distant observer. An apparent infinite appears much less severe than an actual infinite. Limiting us just for a moment to a continuous physics (without mathematical infinite), if an observer goes to the center of the Galaxy and measures the gravitational constant, he obtains the usual value of  $G$ . Moreover nothing is abnormal for him because the singularity has disappeared. However today there exists a large consensus that black holes seem to be present in most galaxies (indifferently "active" or "normal"). In this case we must necessarily keep the idea that a (sleeping) black hole is existing at the center of a "normal" galaxy as ours, but that its mass is lower than expected by some factor that is to be determined (taken into account of the broader array of values of the masses of galactic black holes (Marziani and Sulentic, 2011), their estimate could possibly be subject to a large uncertainty). By contrast we must specify that to attribute a very large mass to supermassive black holes, living at the center of the active galaxies, appear inevitable (these objects are actively accreting black holes). The reason is that the radiations emitted by these objects are indeed very energetic (high frequencies and very high luminosities), while in the framework of the present paradigm the frequencies  $\nu$  are left unchanged ( $\nu = \frac{c}{\lambda}$  (the apparent speed of light  $c$  and the apparent wavelength  $\lambda$  vary in the same way, the ratio is left invariant)).

On the other hand the fact that the quasars were much more numerous at high redshift than today seems to suggest that sleeping SMBHs are maybe hidden in many nearby "normal" galaxies. The frontier between "active" and "normal" galaxies can be a bit fuzzy.

#### – The galactic bulge

Otherwise let us note now from the figure 3, that an extrasolar system located in the bulge of the Galaxy must appear weakly bounded from the point of view of a terrestrial observer. Let  $v_b$  the velocity measured locally by an observer located in the bulge and  $v_{appE}$  the same velocity (in fact more precisely the motion projected on the sky plane) measured from the Earth. We have

$$\frac{k_b}{k_E} \sim 0.78 \implies \frac{v_{appE}}{v_b} \sim 0.78$$

That is for a Sun (mass  $M$ )-Earth (mass  $m$ ) system transported in the bulge  $b$  of the Galaxy : i.e.  $(S, E) \rightarrow (Eb, Sb)$ , we have  $v_{appE} \sim 23 \text{ km s}^{-1}$ . Obviously the local observer at  $b$  does not remark nothing of special. For him  $v_b$  is still equal to  $30 \text{ km s}^{-1}$  (the mean velocity of the Earth around the Sun). We can also study the phenomenon starting from the Newton's gravitational equation. In the bulge

$$mr_g k_b \cdot \ddot{\sigma}_{Eb} = - \frac{\bar{G} M m}{(k_b \cdot \|\sigma_{Eb} - \sigma_{Sb}\|)^3} r_g k_b \cdot (\sigma_{Eb} - \sigma_{Sb})$$

Or, putting  $\mathbf{R}_E = r_g k_b \cdot \sigma_{Eb}$  and  $\mathbf{R}_S = r_g k_b \cdot \sigma_{Sb}$ , we find the usual equation with the same  $G$  as those measured on Earth

$$\frac{d^2 \mathbf{R}_E}{dt^2} = -\frac{GM}{\|\mathbf{R}_E - \mathbf{R}_S\|^3} (\mathbf{R}_E - \mathbf{R}_S)$$

On the other hand from the point of view of a terrestrial observer

$$mr_g k_E \cdot \ddot{\boldsymbol{\sigma}}_{Eb} = -\left(\frac{k_E}{k_b}\right)^3 \frac{\bar{G} M m}{(k_E \cdot \|\boldsymbol{\sigma}_E - \boldsymbol{\sigma}_S\|)^3} r_g k_E \cdot (\boldsymbol{\sigma}_{Eb} - \boldsymbol{\sigma}_{Sb})$$

Putting now  $\mathbf{r}_E = r_g k_E \cdot \boldsymbol{\sigma}_{Eb}$  and  $\mathbf{r}_S = k_E \cdot \boldsymbol{\sigma}_{Sb}$  leads to the equation with apparent  $G_{app}$  :

$$\frac{d^2 \mathbf{r}_E}{dt^2} = -G_{app} \frac{M}{\|\mathbf{r}_E - \mathbf{r}_S\|^3} (\mathbf{r}_E - \mathbf{r}_S)$$

For  $G_{app} \sim 0.48 G$  we see that the apparent acceleration is reduced by a factor 2.

### – The vicinity of the black hole

A contrario, near the galactic center, the motions (seen from the Earth) appear amplified by a very high  $G_{app}$  near this centre, even if the central black hole has a mass weaker than admitted (i.e.  $\sim 4 \cdot 10^6 M_\odot$ ). The noticeable fact with the empirical curve reported in fig. 3 is that now both the dark matter and black hole effects, even though very different in nature, are treated on the same plane through an unique paradigm (an apparent  $G$ ). Another interest is that the black hole mass can appear strongly magnified from the point of view of a distant observer. In the "normal" galaxies as ours the central black hole does not appear energetically very active; for instance we do not observe the spectacular jets or strong emissions of gamma rays or X-rays which characterize the active galactic nuclei such as quasars and blazars. The sole noticeable effect produced by the center of the Galaxy is essentially kinematic with an apparent amplification of velocities of nearest objects. Let  $C$  and  $E$  the indexes which respectively label the velocities measured near the core, and on Earth. We have for the tangential components

$$v_{t_{appE}} = \left(\frac{k_E}{k_C}\right) v_{tC}$$

and for the radial components

$$v_{rE} = v_{rC} = v_{tC}$$

(assuming a velocity inclined to  $45^\circ$  on the line of sight for the second equality). We do not label "app" the radial velocities because these ones are true velocities (measured by spectroscopy and the ratio  $\frac{\Delta\nu}{\nu}$  is universal).

$$\frac{v_{rE}}{v_{t_{appE}}} = \frac{v_{tC}}{v_{t_{appE}}} = \left(\frac{k_C}{k_E}\right)$$

If  $k_C \ll k_E$  then  $v_{rE} \ll v_{tE}$ . This result is a criterion of falsifiability of the model. The orbits appear squeezed on the sky plane (perpendicular to the line of sight).

A possible conclusion is that the black holes residing at the center of "normal" (not active) galaxies, as in the case of the Milky Way, could be smaller than usually admitted (by a factor which needs to be determined), hence their "lack of vigour". On the contrary the black holes assumed to be at the center of active galaxies (Seyfert and quasars) can obviously be very massive.

### 5.1 The consideration of the galactic translation

An observer  $O$  located outside a galaxy measures the velocity of an object  $P$  assumed to be located inside this galaxy. The point of view of the observer  $O$  differs from that of a series of local inertial observers  $L, L', \dots$  attending to the translation of the galaxy as a whole (but not to its rotation). However we admit the relation of velocity composition

$$\mathbf{v}(P/O) = \frac{k_O}{k_L} \mathbf{v}(P/L) + \mathbf{v}(L/O)$$

with

$$\mathbf{v}(P/O) = k_O \dot{\boldsymbol{\sigma}}(P/O) \quad \mathbf{v}(P/L) = k_L \dot{\boldsymbol{\sigma}}(P/L)$$

and

$$\mathbf{v}(L/O) = k_O \dot{\boldsymbol{\sigma}}(L/O) \text{ independent of } L$$

For two distinct inertial observers  $L, L'$ , we have

$$\begin{aligned} \mathbf{v}(L/O) &= \mathbf{v}(L'/O) \\ \mathbf{v}(L'/L) &= k_L \dot{\boldsymbol{\sigma}}(L'/L) = k_L (\dot{\boldsymbol{\sigma}}(L'/O) - \dot{\boldsymbol{\sigma}}(L/O)) = \mathbf{0} \end{aligned}$$

as appropriate.

#### – Point of view of an inner (local and inertial) observer $L$

The inertial observer  $L$ , accompanying the translation of the galaxy as a whole, is assumed to be placed on the line  $PO$  near  $P$ . The unit vector of this line is labelled by  $\mathbf{n}$  and any vector perpendicular to it is denoted by  $\mathbf{t}$ . We recall that the directions are univocally defined by all observers (especially here  $L$  and  $O$ ). Both orthoradial and radial motions respectively following  $\mathbf{n}$  and  $\mathbf{t}$  (measurements made by the inner observer  $L$ ) can be composed in a natural manner to give the vector relation

$$\mathbf{v}(P/L) = v_n(P/L) \mathbf{n} + v_t(P/L) \mathbf{t}$$

From spectroscopic measurements we have ( $r$  for radial)

$$\frac{\Delta\nu}{\nu}(P/L) = \frac{v_n(P/L)}{c} \equiv \frac{v_r(P/L)}{c}$$

#### – Point of view of an outer observer $O$

A important point is that the tangential and radial motions are not treated in the same manner by an outer observer. The tangential motions are obtained by

proper motion measurements and the radial motions by spectroscopic measurements.

**Tangential motion** (perpendicular to the line of sight with unit vector  $\mathbf{t}$ ), issued from proper motion measurements made by the observer  $O$  located outside the galaxy

$$v_t(P/O) = \mathbf{v}(P/O) \cdot \mathbf{t}$$

**Radial motion** (along the line of sight with unit vector  $\mathbf{n}$ ) issued from spectroscopic measurements made by this observer :

$$\frac{\Delta\nu}{\nu}(P/O) = \frac{vr(P/O)}{c} = \frac{v_r(P/L) + v_r(L/O)}{c}$$

Subtracting the constant term  $v_r(L/O)$ , the  $\frac{\Delta\nu}{\nu}$  is the same seen by  $O$  and  $L$ .

## 5.2 The spiral galaxies

There exists a current consensus that the disk-like galaxies (e.g. the Milky Way) gradually form from a slow accretion of gas (Mo, Van Den Bosch and White, 2010; Naab and Ostriker, 2017).

In an obvious scenario the formation of the spiral substructure can be produced by the encounter of two galaxies with a statistically non-null impact parameter, but it is a rare phenomenon. Another more general way to form a spiral morphology is still to consider it as an instability which spontaneously develops in a self-gravitating homogeneous disk with a bar-shaped concentration of mass in the central regions (on the other hand it is well known that computer simulations spontaneously develop disks with transient spiral like morphologies, see for instance the figure 2 of Naab and Ostriker, 2017). The difficulty is therefore not in the build-up of a spiral substructure, which seems to be an inevitable process, but rather in its conservation over times larger than 1 – 2 billions of years. Regardless of the mode of formation of the spiral substructure this one can be maintained for a very long time (of the order of the Universe age) in view of the paradigm described in the present paper.

The equation of an ideal rotating-in-block logarithmic spiral (representing a spiral arm) with rotational velocity  $\Omega$  is given by (we denote by  $ob$  any object in the galaxy)<sup>8</sup>

$$Z_{ob}(t) = a_{ob}e^{(\theta_{ob}+i\Omega t)}$$

We introduce a bit more of physics if we admit that the spiral arm possesses a thickness and that the motion of various objects which compose it is not perfectly circular. Then we can now associate to each type of objects  $ob$  a complex number

$$Z_{ob}(t) = [a_{ob} + b_{ob}e^{i(\omega t + \eta_{ob})}]e^{(\theta_{ob}+i\Omega t)}$$

---

<sup>8</sup>We must note that the global solid body rotation of any distant galaxy cannot be perceived by a terrestrial observer. Rather spectroscopic measurements provide indeed a flat rotation curve (the proper motions are very small and not measurable in a distant galaxy).

where  $b_{ob} \ll a_{ob}e^{\theta_{ob}}$ ,  $\eta_{ob}$  is an arbitrary phase and  $\omega$  is an epicyclic frequency. Then, at this highly idealized stage, the spiral substructure can be seen as a series of stars or patches of gas which oscillate around a mean position (the spiral substructure can periodically distort over time : to pulsate in block or to vary its thickness).

Eventually "rotating in block" does not mean that everything is immutable or just periodic in a disk-like galaxy or still that the latter one is a solid structure! The reason is that the various objects such as stars, HII regions, giant molecular clouds, etc, possess random velocities. A rapid examination of the stellar kinematics near the Sun suffices to convince ourself that the peculiar velocities can be high and strongly depending on the type (the mass, the age, the metallicity, etc) of stars (old metal-poor red dwarfs, K Giants or young metal-rich A dwarfs, etc).

Moreover once formed the spiral substructure acts as a potential well which attracts molecular clouds and these ones collide with one another. This phenomenon drives shock waves through the gas triggering the production of new *HII* regions and *OB* stars which, in turn, stochastically underlines the spiral substructure.

Evidence against long-lived spiral arms has been suggested on the basis that, in the wave density model, the star formation sequence (SF) should then show a spatial ordering (dense HI  $\Rightarrow$  molecular gas  $\Rightarrow$  stars dust-enshrouded  $\Rightarrow$  young stars  $\Rightarrow$  unobscured young stars  $\Rightarrow$  clusters), a process which is not observed (Foyle et al, 2010). A contrario, in the framework of the present paradigm, it is clear that a stationary dynamical spiral pattern of well-defined speed (but it is not here a wave density) can coexist with no spatial ordering for the formation sequence.

There exists another source which also alters the body solid rotation of the galaxies. The function  $r \rightarrow k$  is represented in fig. 3 by a staircase function surimposed to a mean continuous curve. Only a few steps are reproduced, even though this structure is likely fractal (a Cantor function). For the Galaxy, the rotation curve exhibits a clear wavy behavior (see the figure 10 of Chemin, Renaud and Soubiran (2015) and the figure 2 of Morz et al (2019)). These oscillations are attributed to the presence of local rings or arm pieces, and here this behavior is mimicked by the fractal nature of  $k$  (however at a large scale and at a lower resolution, for instance for a remote galaxy, the function  $k$  would appear continuous). We can fix the step of the function  $k$  to approximately 1 *kpc* wide near the Sun (i.e. grossly the thickness of the Orion arm). Obviously on each step determined by  $k = Const$ , the matter cannot rotate as a rigid ring and the rotation becomes naturally differential (as this is the case in the vicinity of the Sun as attested by proper motions measurements, Olling and Dehnen, 2003; Bovy, 2017). Then how to conciliate this differential rotation with a sustainable spiral substructure (i.e. a global solid body rotation)? The figure 4 (not to scale for the distances and the velocities)<sup>9</sup> shows that such a situation can easily be realized if we assume that streams of stars (and of other objects) in the form of vortices of all sizes are disposed along the spiral substructure. These vortices

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<sup>9</sup>The black arrows indicate the rigid motion of a piece of the long-lived spiral arm and the triple arrows indicate the differential motions within this arm.





(on a step of the function  $k$ ,  $\Omega$  varies in a keplerian manner, i.e. as  $R^{-\frac{3}{2}}$ ). The parameters  $\alpha$  and  $\beta$  respectively represent the minor and major axes of the vortex, assumed to be elliptical, and  $\gamma$  is the orientation of the major axis related to  $\mathbf{e}_x$ ). We can put  $\alpha + \beta = 1$ .

The comparison with the expression (2) of Olling and Dehnen (2003) immediately gives

$$\begin{pmatrix} \omega_v \beta \sin \gamma & -\frac{1}{2} \Omega_0 + \omega_v \alpha \cos \gamma \\ -\Omega_0 - \omega_v \beta \cos \gamma & \omega_v \alpha \sin \gamma \end{pmatrix} = \begin{pmatrix} K + C & A - B \\ A + B & K - C \end{pmatrix}$$

From Bovy (2017), we have (unit expressed in  $km s^{-1} kpc^{-1}$ )

$$\begin{pmatrix} K + C & A - B \\ A + B & K - C \end{pmatrix} = \begin{pmatrix} -6.5 & 27.2 \\ 3.4 & -0.1 \end{pmatrix} \quad (13)$$

The fit  $\Omega_0 = -27.5$ ,  $\omega_v = 43$ ,  $\alpha = \frac{1}{3}$ ,  $\gamma = -13^\circ$  provides the matrix coefficients

$$\begin{pmatrix} -7.3 & 26.3 \\ 2.1 & -3.6 \end{pmatrix} \quad (14)$$

The comparison between (10 and (11) shows that the paradigm proposed in the present paper gives an acceptable representation for the solar environment.

### 5.3 Relativistic jets in AGNs

Close to the core of active galaxies (AGNs), some collimated relativistic jets of matter are observed with superluminal expansion (Thomson, MacKay and Wright, 1993; Biretta, Sparks, Macchetto, 1999; Blandford, Meier, Readhead, 2019). The usual explanation is that the jet is directed toward us and that the illusion of a superluminal velocity results from a relativistic effect. The interpretation proposed in the present context is not antithetical with this idea but rather intervenes as a complementary aspect<sup>11</sup>. In these jets, the apparent speed of light seems to increase from low values  $\sim 0.01 c$  to high values  $2-3 c$  and sometimes more, over a distance of a few parsecs. For a quasar we can still report us to the figure 3, but with the change  $r_{ref} = r_g \sim 8 kpc \rightarrow 10^2 r_S \sim 100 pc$  ( $r_S$  denotes the Schwarzschild radius of the central black hole) (this meant shrinking the curve following the abscissa but with no change of the ordinate) (fig.6). Let a jet with an apparent velocity  $v_{app}$ . Seen from the Earth this jet will appear such as  $v_{app} > c$  ( $c$  being the speed of light measured in the laboratory). However we shall have  $v_{app} < c_{app}$ . The apparent speed of a photon estimated by a terrestrial observer can then be larger than  $c$ , whereas its true speed estimated locally would be always  $c$ . For an inertial observer placed near the jet the velocity of a blob of gas is always subluminal and smaller than  $c$  (respecting the

<sup>11</sup>It should be made clear that the mechanism responsible of the formation of these relativistic jets is left unchanged, i.e. the presence of a supermassive black hole (SMBH) surrounded by an accretion disk which powered the jet up to velocities with a very high Lorentz factor.

causality principle). Contrarily to the orthodox interpretation, the interest is, here, that the jet has no preferential orientation. Another interesting point is that from the Earth the jets are seen accelerating whereas, locally, the velocity is constant. This acceleration is apparent and no mechanism such as for instance a MHD-powered one as early proposed (see for a short review Asada et al, 2014) is needed.

Let  $J$  a particle in the jet,  $L$  an local observer at rest relatively to  $O$ . We have

$$\mathbf{v}(J/O) = \frac{k_O}{k_L} \mathbf{v}(J/L) + \mathbf{v}(L/O) = \frac{k_O}{k_L} \mathbf{v}(J/L)$$

For  $v(J/L) \sim c$  and  $\frac{k_O}{k_L} \sim 6$ , we obtain  $\mathbf{v}(J/O) \lesssim 6 c$ .

The usual relativistic mechanism proposed by Rees (Rees, 1966) can also occur together with the one that we propose in this paper, especially in the rare cases when the speed of the jet is  $\geq 6 c$  (with a record  $\sim 50 c$  measured in PKS 0805-07, Blandford, 2019).

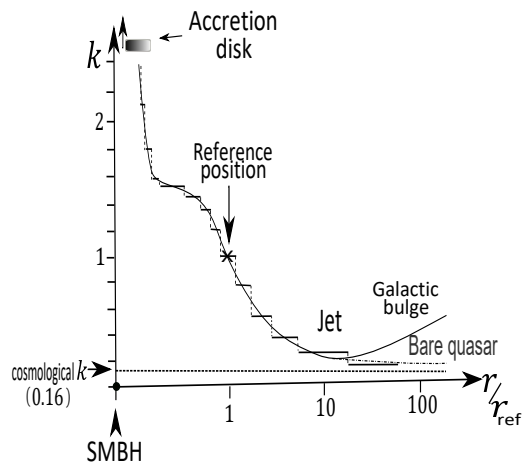


Figure 6

The figure 7 provides the result of a depending on time numerical simulation of a relativistic jet, emitted from the outer rim of an accretion disk, surrounding a SMBH of mass  $M \sim 10^9 M_\odot$ <sup>12</sup>. The density plot  $\rho$  is given in gray scale (we must note that the very weak nebulosity is not visible, only the very luminous central point (the quasar) and the jet can appear on a snapshot, however see the figure at a very large scale of Blandford, Meier, and Readhead, 2019). The jet is perpendicular to the line of view. The calculations have been made with the PLUTO code which targets high Mach number flows in astrophysical fluid dynamics (Mignone et al, 2015). The RMHD (relativistic MHD equations) module expressed in spherical coordinates has been used throughout with the ideal equation of state.

<sup>12</sup>We assume here a more simplified model than the usual accretion-ejection mechanism in SMBHs (see Asada et al, 2008, for a short review). A similar accretion-ejection mechanism (but with a more weaker energetics) is also acting in TT-Tauri stars (Ferreira, J., Pelletier, G. and Appl, S, 2000). Let us note that in the present model the matter is not emitted along the pole axis but is issued from the outskirts of the disk. A weak magnetized nebulosity is formed with an empty large cavity above the accretion disc. The jet is produced by a pinch effect

The initial conditions are the following : an homogeneous magnetized torus of major radius equal to  $r_M = 10 r_S \sim 10 pc$  (or 1/10 of the reference radius  $r_{ref}$ ) and minor radius equal to  $5 r_S$  with an uniform density of particle  $\sim 10^6 cm^{-3}$  and temperature  $\sim 10^6 K$ . The magnetic field in the torus is fully azimuthal and of the order of  $0.3 Gs$ . The velocity of the barycenter of a section is assumed be of the order of  $0.5c$ . This initial velocity is communicated to the torus by a MHD process which is not questioned here. Let us specify that the model exposed above is far removed from the standard model of AGNs, but it is presented as just an illustrative way for a simple comparison of the kinematics (true or apparent) of the jet.

After expansion fig. 7 shows that the velocity of the gas decreases as a function of the distance  $r$  and the velocity of the jet goes from  $0.5c$  at  $r = r_M$  to  $0.3c$  at  $r = 10 r_M$ . The jet<sup>13</sup> develops by magnetic striction and the part which is represented is of the order of  $1 kpc$  long (but it can be much longer before decollimation). In spite of this decrease a remote observer will see the velocity of the jet increases from  $0.2c$  to  $1.8c$ , fictitiously suggesting to this observer that a driving mechanism is really at work. The jet has been taken perpendicular to the line of view and the quasars is obscured by the dusty torus surrounding it (and therefore is not visible). Of course the model also indifferently works if the jet is inclined on the line of view at any angle, and the quasar made visible (for a study on torus-obscured and unobscured quasars, see for instance DiPompeo et al, 2016).

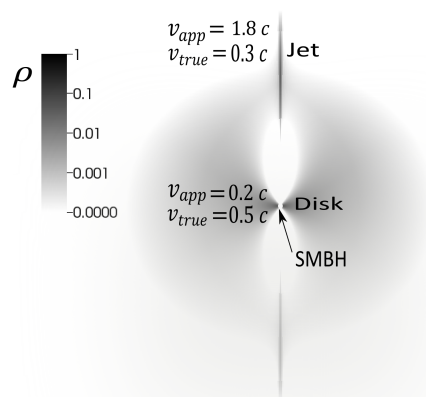


Figure 5

Figure 7

The explanation is the following. Let  $v_{true}$  the true velocity (locally measured) and  $v_{app}$  the apparent velocity (measured by a remote observer  $O$ ). We have the relationship

<sup>13</sup>A counter-jet is existing but it is weaker. The toroidal magnetic field has not been taken symmetric with respect to the mean plane of the equatorial disk (there exists absolutely no reason why such a symmetry should be realized (Sparks et al, 1992)).

$$v_{app} = \frac{k_O}{k} v_{true}$$

Correspondingly the link between the true and apparent accelerations (denoted by a point) is

$$\dot{v}_{app} = \frac{k_O}{k} \left( -\frac{\dot{k}}{k} v_{true} + \dot{v}_{true} \right) \quad \left( \frac{\dot{k}}{k} < 0 \right)$$

We can distinguish four cases

- i.  $v_{true} < 0$    a.  $\left( \frac{|\dot{k}|}{k} > \frac{|\dot{v}_{true}|}{v_{true}} \Rightarrow v_{app} > 0 \right)$    b.  $\left( \frac{|\dot{k}|}{k} < \frac{|\dot{v}_{true}|}{v_{true}} \Rightarrow v_{app} < 0 \right)$
- ii.  $v_{true} = 0 \Rightarrow v_{app} > 0$
- iii.  $v_{true} > 0 \Rightarrow v_{app} > 0$

The figure 5 corresponds to the case i.a, but the AGNs can certainly produce all possible situations described above.

#### 5.4 The gravitational lensing

Especially for the deviation angle  $\alpha$  of light beams emitted from a very distant object and deflected by a close cluster of galaxies, we have

$$\alpha = \frac{G_{app}}{G} \left( \frac{c}{c_{app}} \right)^2 \frac{2GM}{c^2 b} = \left( \frac{c_{app}}{c} \right) \frac{2GM}{c^2 b}$$

where  $b$  is the impact parameter measured by the terrestrial observer  $E$ . With  $c_{app} = 6 c$  in a galaxy cluster we can deduce that the deviation angle is  $\sim 6$  times the value calculated with just the visible matter and without dark matter. Starting from the equation (7), an usual calculation gives

$$r_c k \cdot \ddot{\sigma} = -\frac{GM_{vis}}{(r_c k \cdot \sigma)^3} r_c k \cdot \sigma$$

( $r_c$  designs the mean galaxy cluster radius and the origin of  $\sigma$  is taken at the center of the galaxy cluster, assumed to be spherically symmetric) and

$$r_c k \cdot \frac{\dot{\sigma}^2}{\sigma} = -\left( \frac{GM_{vis}}{r_c k \cdot \sigma} \right)^2$$

The velocity measured by spectroscopy (i. e. the radial velocity,  $\gamma$  is the angle between the velocity vector and the line of sight) is

$$v_r = \left( \frac{GM}{r_c k \cdot \sigma} \right)^{\frac{1}{2}} \cos \gamma = \left( \frac{k}{k_E} \right)^{\frac{1}{2}} \left( \frac{GM_{vis}}{r_c k_E \cdot \sigma} \right)^{\frac{1}{2}} \cos \gamma$$

Once again the amplification factor  $\left( \frac{k}{k_E} \right)^{\frac{1}{2}}$  mimics a dark matter effect. With  $\frac{k}{k_E} \simeq 6$ , we find  $M_{vis} \simeq 0.17(M_{DM} + M_{vis})$  (for Abell 2744 the observational data supplies a slightly higher ratio  $\simeq 0.25$  corresponding to an amplification factor  $\frac{k}{k_E} \simeq 4$ ).

## 6 The standard model of cosmology

The Friedmann equations is

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_{vis} + \rho_{DM}) - \frac{\kappa c^2}{a^2} + \frac{\Lambda c^2}{3} \quad (15)$$

where  $H$  is the Hubble parameter,  $a$  the dimensionless scale factor,  $\kappa$  the curvature,  $\Lambda$  the cosmological constant,  $\rho_{vis}$  the density of visible matter,  $\rho_{DM}$  the dark matter density. The new paradigm leaves unchanged all the conclusions of the standard model of cosmology but without dark matter. Within the present formalism, it would *a priori* seem that we can directly rewrite the Friedmann equation as

$$\left(\frac{k_E \dot{\sigma}}{k_E \sigma}\right)^2 = \frac{8\pi G_{app}}{3} \rho_{vis} - \frac{\kappa c_{app}^2}{(k_E \sigma)^2} + \frac{\Lambda c_{app}^2}{3}$$

However the velocity which is measured is not  $k_E \dot{\sigma}$  but  $k \dot{\sigma}$  (we recall that the radial velocity is measured by spectroscopy). With the adequate change we obtain the equation

$$H^2 = \left(\frac{k \dot{\sigma}}{k_E \sigma}\right)^2 = \frac{8\pi \left(\frac{k}{k_E}\right)^2 G_{app}}{3} \rho_{vis} - \frac{\kappa \left(\frac{k}{k_E}\right)^2 c_{app}^2}{(k_E \sigma)^2} + \frac{\Lambda \left(\frac{k}{k_E}\right)^2 c_{app}^2}{3}$$

Eventually

$$H^2 = \left(\frac{k \dot{\sigma}}{k_E \sigma}\right)^2 = \frac{8\pi \frac{k_E}{k} G}{3} \rho_{vis} - \frac{\kappa c^2}{(k_E \sigma)^2} + \frac{\Lambda c^2}{3} \quad (16)$$

An estimation of the amplification factor  $\frac{k_E}{k}$  is supplied by the Planck data (2018)

$$\frac{k_E}{k} = \frac{\rho_{vis} + \rho_{DM}}{\rho_{vis}} \simeq 6.36$$

## 7 Conclusion

In the framework of the standard astrophysics, we must do appeal for ingredients such as dark matter (not directly observed), wave density (difficult to maintain), or relativistic effects for the quasar jets (but with a strong constraint on the jet direction, which has to be skillfully oriented toward the observer). All these hints are eliminated here. Moreover the physics laws are not modified in their local form.

The fundamental constants  $G, c, \dots$  are universal (the same everywhere), however the measurement process done by an observer makes them appear as pseudovariables  $G_{app}, c_{app}, \dots$  (while the observer does not perceive them as well and rather introduces various artificial effects : dark matter in galaxies and galaxy clusters, density waves in spiral galaxies, or sleeping black holes (assumed to

be located at the center of "normal" galaxies). Even though this new idea still needs a lot more work, the present draft suggests that the representation we have of the Universe could be seen different following we consider the tangential velocities (which are apparent) or the radial velocities (real and measured by spectroscopy). Whereas we can say as in the standard astrophysics that the unit length is the same everywhere (there are not "small" and "large" meters), this is the remote measurement which makes them different. The paradigm exposed will be maybe falsifiable (in the Karl Popper sense) with the next generation of telescopes. A main conclusion is that the measurement of the proper motions in a galaxy or a cluster of galaxies is then expected to show that the mean tangential (apparent) velocities are smaller or larger than the mean radial (true) velocities measured by spectroscopy by a factor  $\sim \frac{k_E}{k}$  comprised between 0.78 (minimum) and  $\simeq 6.36$  (maximum) (following the region which is observed).

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