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To cite this version:

Olivier Maurice. Elements for modeling transmission between persons, inspired from circuits modeling. 2020. hal-02530419

HAL Id: hal-02530419

https://hal.science/hal-02530419v1

Preprint submitted on 2 Apr 2020

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Elements for modeling transmission between persons, inspired from circuits modeling.

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April 2, 2020

1 Introduction

The purpose of this paper is to exchange techniques whose allow to model transmissions between elementary networks. At this time where virus behavior modeling becomes essential, it seems interesting to explore perhaps some different approaches in this thematic. I present here some ideas for modeling agent propagation using the tensorial analysis of networks enriched with the gamma matrices technique. Working not in the biology perimeter, my proposals are perhaps interestless or already aborded, specialist readers will be able to judge.

2 Major principles

One person is associated with a little network made of two meshes. A first mesh represents the near outer world of this person and a second mesh represents the person activity and his healthy. Persons properties are organized as the diagonal elements of a fundamental tensor. Their activities are similar to current vector and the energy or agent are the source covector. When a person is moving and goes near another, it is translated through the adding of a coupling function, unidirectional, representing the interaction between the two persons. The simulation output is all the second current of all the networks (persons) involved in the computation. The lower this current, the lower the person health.

3 One person model

The advantage of using two meshes is to translate the fact to wear the agent on one side, and to be or not to be affected by the agent on another side. The external properties of the person i are associated with an operator E_i while the internal properties are represented by an operator I_i. The person activity is represented by a 2-vector $\left(x^i, a^i\right)$. The component x^i is an intermediate variable without any intererest. Our observable is a^i. The source covector has two components: the first component is the agent energy m (seen as a marker). The second component A_i is the nominal person energy. A healthy person is symbolized by the system $T_q = \zeta_{qk} a^k$ or by developing:

$$\begin{bmatrix} 0 \\ \\ A_q \end{bmatrix} = \begin{bmatrix} E_{qk} & 0 \\ \\ 0 & I_{qk} \end{bmatrix} \begin{bmatrix} x^k \\ \\ a^k \end{bmatrix} \tag{1}$$

The same person being affected by the agent is associated to the equation:

$$\begin{bmatrix} m \\ \\ A_q \end{bmatrix} = \begin{bmatrix} E_{qk} & 0 \\ \\ 0 & I_{qk} \end{bmatrix} \begin{bmatrix} x^k \\ \\ a^k \end{bmatrix} \tag{2}$$

Before looking to the agent impact, let's understanding how the unitary model works. As we said before, our observable is a^i. For the first person, this observable is obtained through:

$$a^1 = \frac{A_1}{I_{11}} \tag{3}$$

When the person wears the agent, the intermediate variable x^1 is defined by:

$$x^1 = \frac{m}{E_{11}} \tag{4}$$

As long as the "external state" E_{ii} and the "internal state" I_{ii} are not coupled, the a^1 value doesn't depend on x^1 value. We add a coupling function β and study its effect. The "person properties tensor" ζ is now defined by:

$$\zeta = \begin{bmatrix} E_{ii} & 0 \\ \\ \beta & I_{ii} \end{bmatrix} \tag{5}$$

In presence of the agent, a^1 is obtained by:

$$a^1 = \frac{1}{E_{11} I_{11}} \left(A_1 E_{11} - \beta m \right) \tag{6}$$

To take into account rise time and fall time in the person health, the coupling function can be completed by time functions as:

$$\beta \to \beta \left(e^{-t/\tau_1} - e^{-t/\tau_2} \right) \tag{7}$$

τ_1 and τ_2 being values similar to development and recovery durations of the maladie. If we set $E_{ii} = E$, $I_{ii} = I$ and $A_i = 1$ we obtain:

$$a^1 = \frac{1}{EI} \left(1 - \beta \left(e^{-t/\tau_1} - e^{-t/\tau_2} \right) \right) \tag{8}$$

For one person, if for $t > \tau_1$, $a^1 < s$ where s is a threshold, then $A = 0$ and the person is dead.

Comparing (6) and (7) we can define m by:

$$m = \left(e^{-t/\tau_1} - e^{-t/\tau_2} \right) \tag{9}$$

and if m acts at a given period t_c we write:

$$m = \left(e^{-(t-t_c)/\tau_1} - e^{-(t-t_c)/\tau_2} \right) \tag{10}$$

4 Two persons mechanism: gamma matrices

T, ζ being the mathematical representation of a person, if we want to take into account two persons we make $\zeta(2) = \zeta \otimes \zeta$. We obtain, using the previous assumptions:

$$\zeta(2) = \begin{bmatrix} E & 0 & 0 & 0 \\ \beta_1 & I & 0 & 0 \\ 0 & 0 & E & 0 \\ 0 & 0 & \beta_2 & I \end{bmatrix} \tag{11}$$

The source covector of this system is:

$$T = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix} \tag{12}$$

If the first person is infected, the source covector becomes:

$$T = \begin{bmatrix} m & 1 & 0 & 1 \end{bmatrix} \tag{13}$$

When this person moves and encounter the second person, the marker m must be transmitted as the third component of T. This is realized by a matrix called "gamma matrix". In our case of two persons, this matrix is:

$$\gamma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

The operation γT^T gives the new source covector at the time t_c:

$$T(t_c) = \begin{bmatrix} m & 1 & m & 1 \end{bmatrix} \tag{15}$$

γ translates the fact that the second person is infected. Depending on β_2, this person will remain healthy or not. As T is defined for $t = t_c$ we have :

$$m = \left(e^{-(t-t_c)/\tau_1} - e^{-(t-t_c)/\tau_2}\right) \tag{16}$$

γ can be constructed as the sum of two matrices. A first matrix is the person normalized energy. This matrix is purely diagonal and all its components of even number are equal to 1, while all its components of odd number are equal to 0. A second matrix wears the agent diffusion. This matrix is a column matrix where only the first column has some components different to zero.

$$\gamma = \gamma_1 + \gamma_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

The first matrix evolves only when a person dies. In this case, the corresponding source becomes null. The second matrix changes each time the agent propagates. As the value of m becomes automatically null after the time $\tau_1 + \tau_2$, the matrix γ_2 can only be continuously enriched, translating by that the agent diffusion. The order in which the matrix is filled can be determined probabilistically and based on neighboor relations and professional trips or for tourism. Each period nt_c we solve the equation :

$$\gamma(nt_c)T_k(nt_c) = \gamma(nt_c)\dots\gamma(0)T_k(0) = \zeta_{k\eta}a^{\eta} \tag{18}$$

5 Organizing the persons depending on the neighborhood

Based on the direct sum of tensors, the construction of the operator *zeta* can be made increasing the scale following the diagonal of the *zeta* matrix. In fact the distance on the diagonal between two persons follows the physical distance between the same persons. On this principle, the maximum distance between two persons is in relation with trips by flights. The smallest distance is at the family scale.

The organisation chosen for ζ remains anyway valid for a limited time, as persons moves. That's the gamma matrix which translate first of all the punctual neighborhood between persons. But for defining at the beginning the ζ matrix a choice must be done and starting by the home and family proximities seems reasonable. This choice makes analysis and construction of matrices easier.

6 Agent diffusion

The figure 1 shows how the γ_1-matrix filled depending on the transmissions between persons. The γ-matrix includes all the transfers at the first step. Each time it is multiplied by the previous covector T value, it gives the next T value.

The figure 2 shows how evolves the agent part of the covector T depending on the successive products by γ_1. It shows that the γ-matrix can be filled previously to its application, and using a probabilistic method.

A flag is necessary to memorize the current period. It gives the t_c value to fix the start of the transmission evolving in time. A simplest solution consists in using m rather than 1 in γ and to set m to the current t_c value.

7 Increasing the model complexity

The cœfficient β can be a complex operator transmitting an agent more or less fast depending on the person health. This value can even be determined by a bayesian process involving various influent factors.

8 Conclusion

The equation (18) can be seen as a generic equation in order to simulate propagation process in a population where each person behavior can be modeled

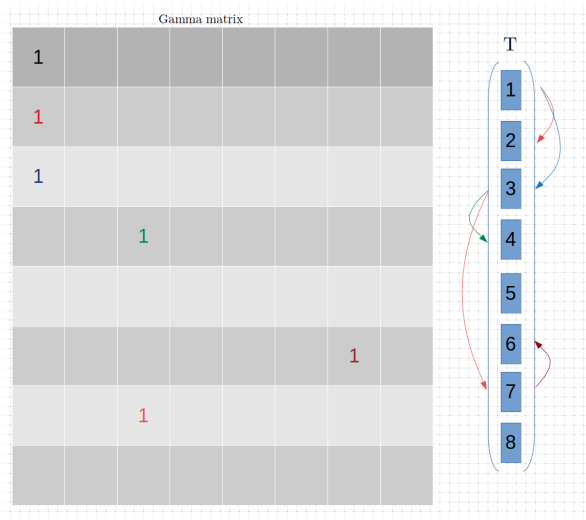

Figure 1: γ_1 agent diffusion process

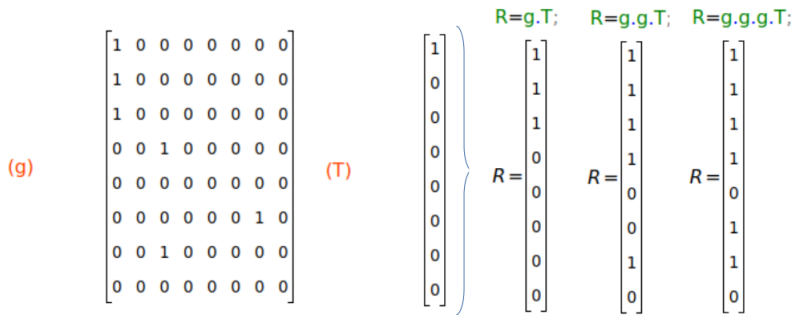

Figure 2: Product $\gamma_1^n T$

through a network of dimension two. These two dimensions allow to take into account all mechanism where one frontier determine the transmission between an outworld and an inworld. Bayesian probabilitites can be used to define the propagation matrix γ either the properties matrix ζ, giving all operators advances behaviors able to take into account real evolving. And the structure of each of these matrices remains simple, allowing computations with millions of persons with computing times still reasonables.