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Tilings, Tessellations. Learn Geometry by hand.

Marcel Morales

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MARCEL MORALES

MARCEL MORALES,
Tiling, Tessellations.
Learn Geometry by hand
Tessellations

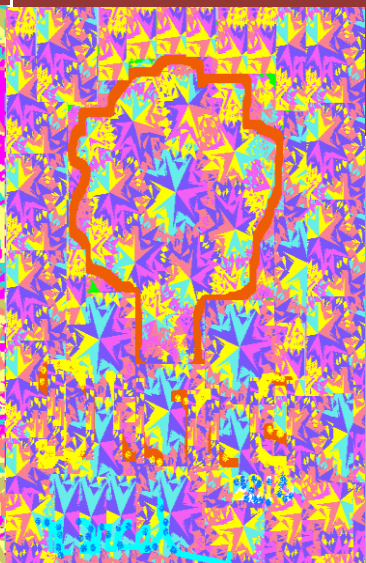
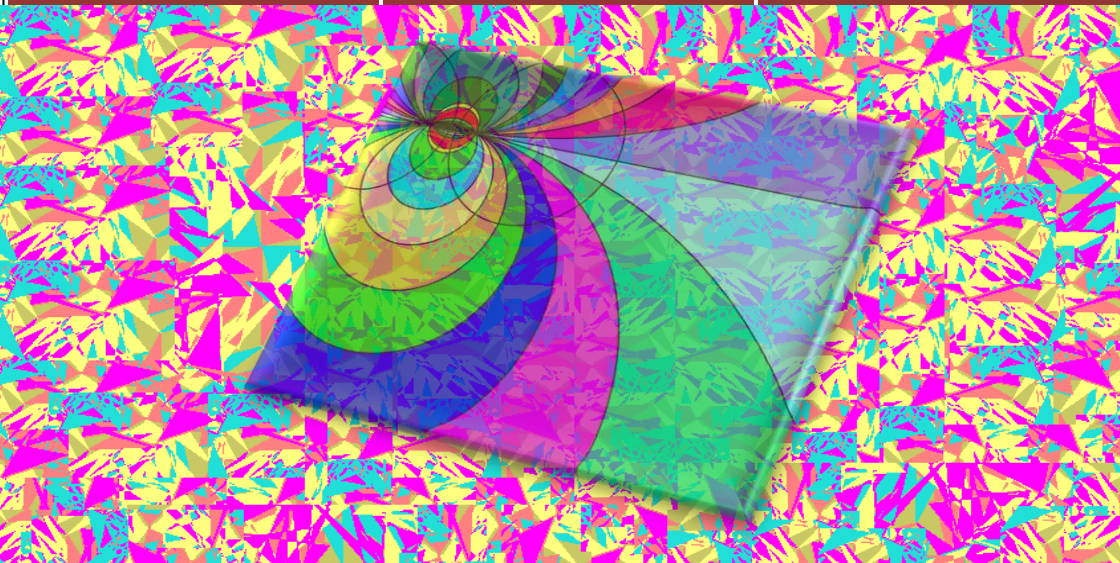
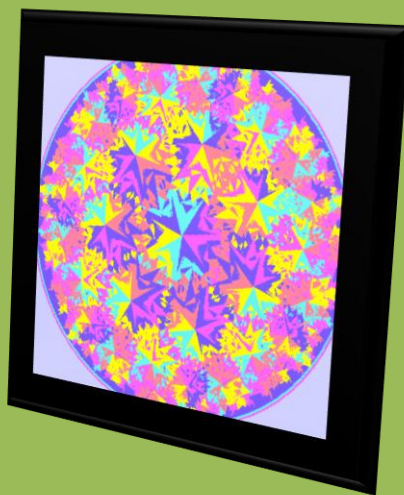


Table des matières

Geometry level One : geometric figures, polygons.....	5
The convex regular Polygons.....	9
The star regular Polygons	10
Grid pattern , square graph paper, triangular graph paper and others.....	13
Puzzles and Tessellations	17
First example :.....	17
Example 2	19
We can also get this animal from two squares :.....	20
The winged Deer and the Elephant	21
Third example:.....	23
Fourth example, Horses:.....	26
Fifth example, Fishes:.....	27
Sixth example with rotations of 60°	28
Seventh example by using symmetries	31
My Software Tessellation.....	32
Tiling p1 or R0	50
Lattice and fundamental region for p1 (R0).....	52
Tiling p2 or R2	54
Lattice and fundamental region for p2 (R2).....	56
Tiling p3 ou R3	58
Lattice and fundamental region for p3 (R3).....	60
Tiling p4 ou R4.....	62
Lattice and fundamental region for p4 (R4).....	64
Tiling p6 ou R6	65
Lattice and fundamental region for p6 (R6).....	67
Tiling pg ou M0	69
Lattice and fundamental region for pg (M0).....	70
Tiling pgg ou M0R2.....	72
Lattice and fundamental region for pgg (M0R2).....	74
Tiling cm ou M1	75
Lattice and fundamental region for cm (M1)	76
Tiling pm ou M1g	77
Lattice and fundamental region for pm (M1g)	80
Tiling pmg ou M1R2	81
Lattice and fundamental region for pgm (M1R2)	83
Tiling pmm ou M2.....	84
Lattice and fundamental region for pmm (M2).....	87
Tiling cmm ou M2R2	88
Lattice and fundamental region for cmm (M2R2)	90
Tiling p4g ou M2R4.....	92

Lattice and fundamental region for p4g (M2R4).....	94
Tiling p4m ou M4.....	95
Lattice and fundamental region for p4m (M4).....	97
Tiling p31m ou M3R3.....	98
Lattice and fundamental region for p31m (M3R3).....	100
Tiling p3m1 or M3.....	101
Lattice and fundamental region for p3m1 (M3).....	103
Tiling p6m ou M6.....	104
Lattice and fundamental region for p6m (M6).....	107
Pattern of a tiling of a cube.....	109
Pattern of a tiling of a dodecahedron.....	109
Paper wall.....	112
The hyperbolic Tilings.....	112

Tiling, tessellation, periodic or non periodic tiling: fulfilling a picture by some figure was used by all the civilizations to decorate walls, carpets, potteries ...

This book can help everybody to produces and drawn his own tiling, without any knowledge in mathematics.

After some introduction to the subject with elementary notions of geometry, we introduce the seventeen groups of tiling of the plane.

I have developed software that helps us to draw a picture for each group of tiling.

Our aim is to introduce people to:

1. Recognize a tiling, and the basic figure,
2. Describe the tiling group, i.e. the transformation used to fulfill the plane, which rotation, symmetries.
3. How realize a tiling by drawing, cutting and gluing without using difficult techniques.
4. For each tiling's group a complete sequence of realization is done, explaining and allowing realizing it quickly.
5. Every tiling group is illustrated by one picture.

The software has been developed by Marcel Morales, but the author has learned a lot from the high school class of Alice Morales. The software has been used by school students during many years, in all the degrees of schools. A joint work with the classroom of Alice Morales has been presented in the international exposition Exposciences International 2001 in Grenoble. The software has been introduced in many scientific expositions in France and outside France: Mexico, Peru, Colombia, Iran, Turkey, and Vietnam.

Moreover, after the expositions and collaborations with high schools, it appears that doing tiling can be a kind of game, but also improves the knowledge in mathematics, without any formal course. We introduce some tiling founded in old civilizations.

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<https://www-fourier.ujf-grenoble.fr/~morales/>
<https://marcel-morales.com/>

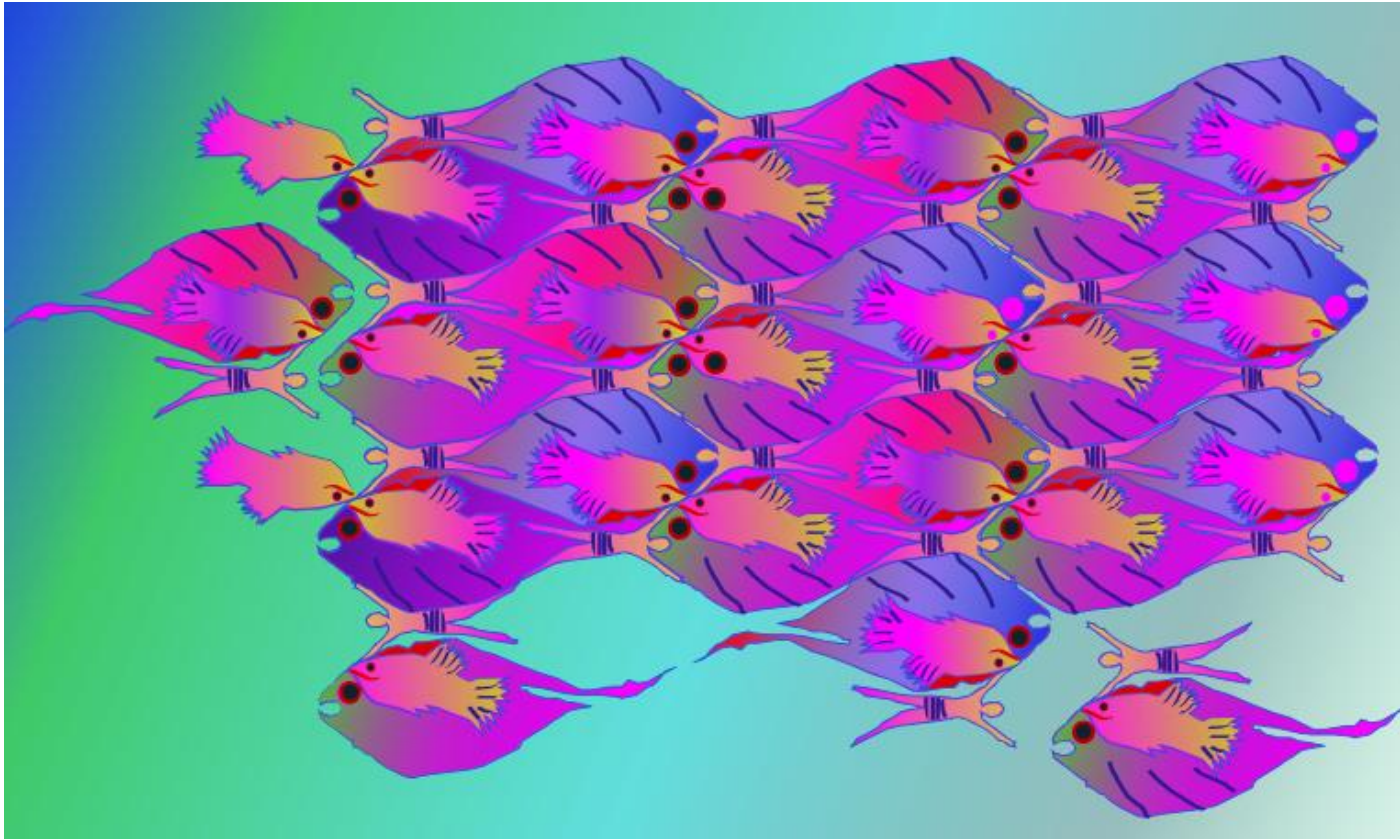
Download my software:

<https://www-fourier.ujf-grenoble.fr/~morales/software-Teselacion-Marcel-Morales.html>

This book is part of my research, the software and the methods developed here are original due to the author.

Tiling or Tessellation consist to cover with a pattern (tile) an entire wall, for this reason it can be called wallpaper. Our aim is to provide a very simple way to discover Tessellations and produce your own Tessellation either by my software or by hand using paper and scissors. My software and this book introduces you to this subject mixing mathematics and art without any background in mathematics.

First we have to learn some words from Geometry. All you need is some transparence paper, a rule, scissors glue and some color pens.

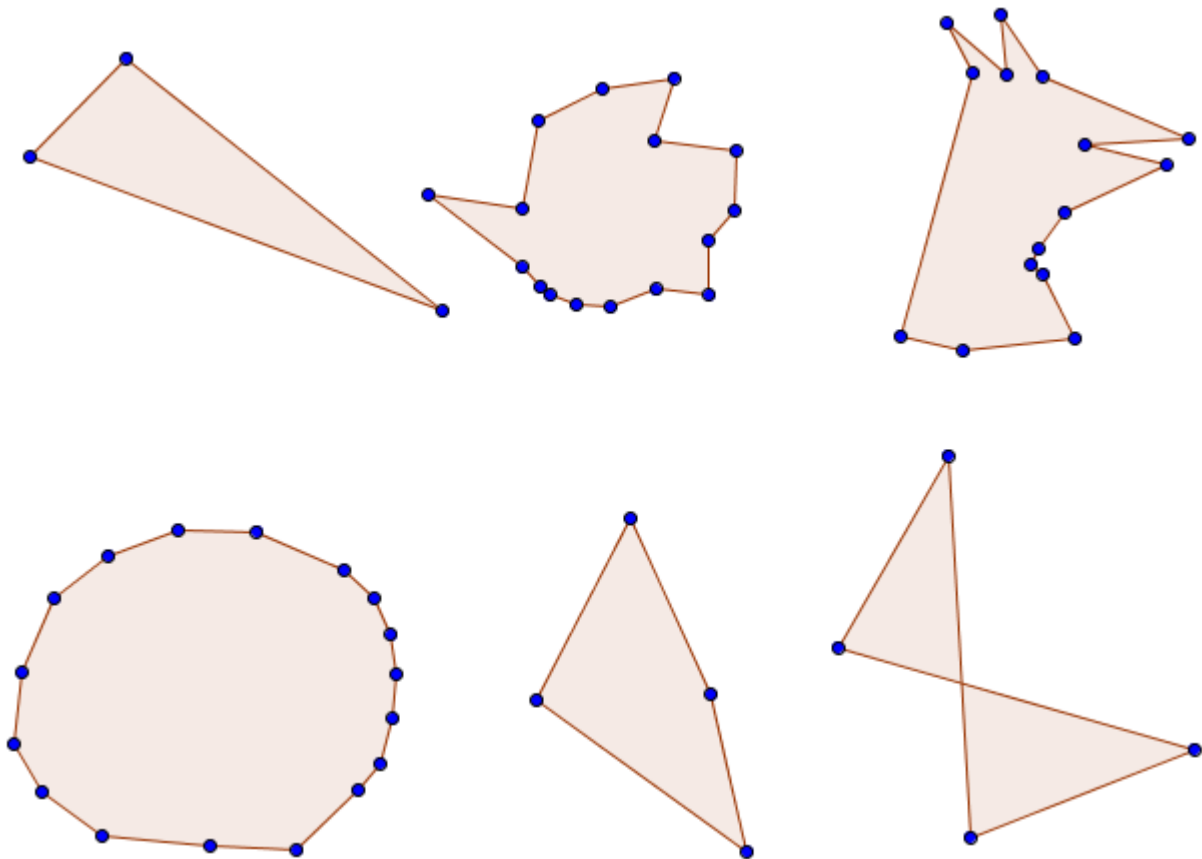


Geometry level One : geometric figures, polygons.

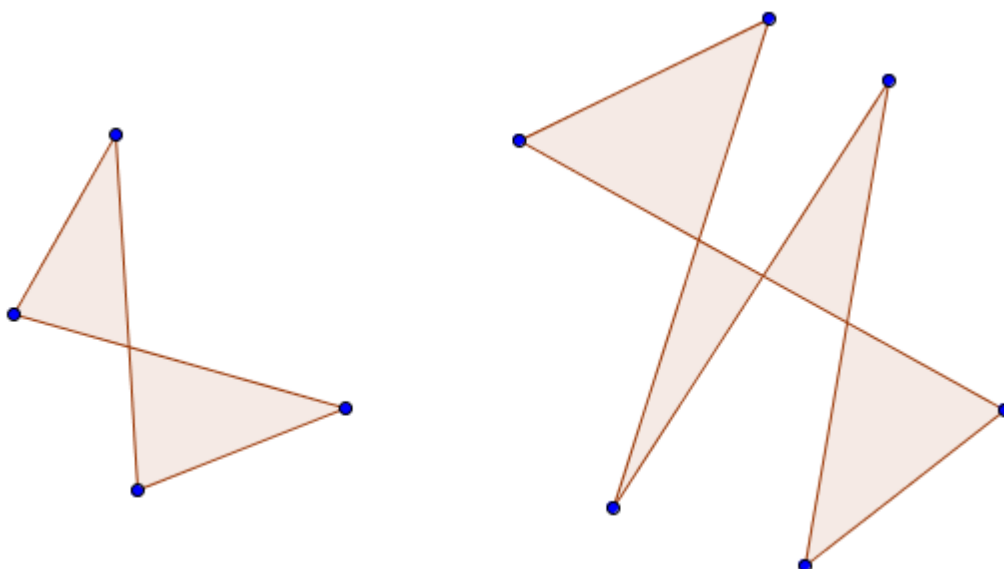
Precise words are very important, so we can speak about the same thing.

A polygon is a plane figure bounded by segments of right lines, each segment is called a side , a common point of two segments is a vertex. We have some examples:

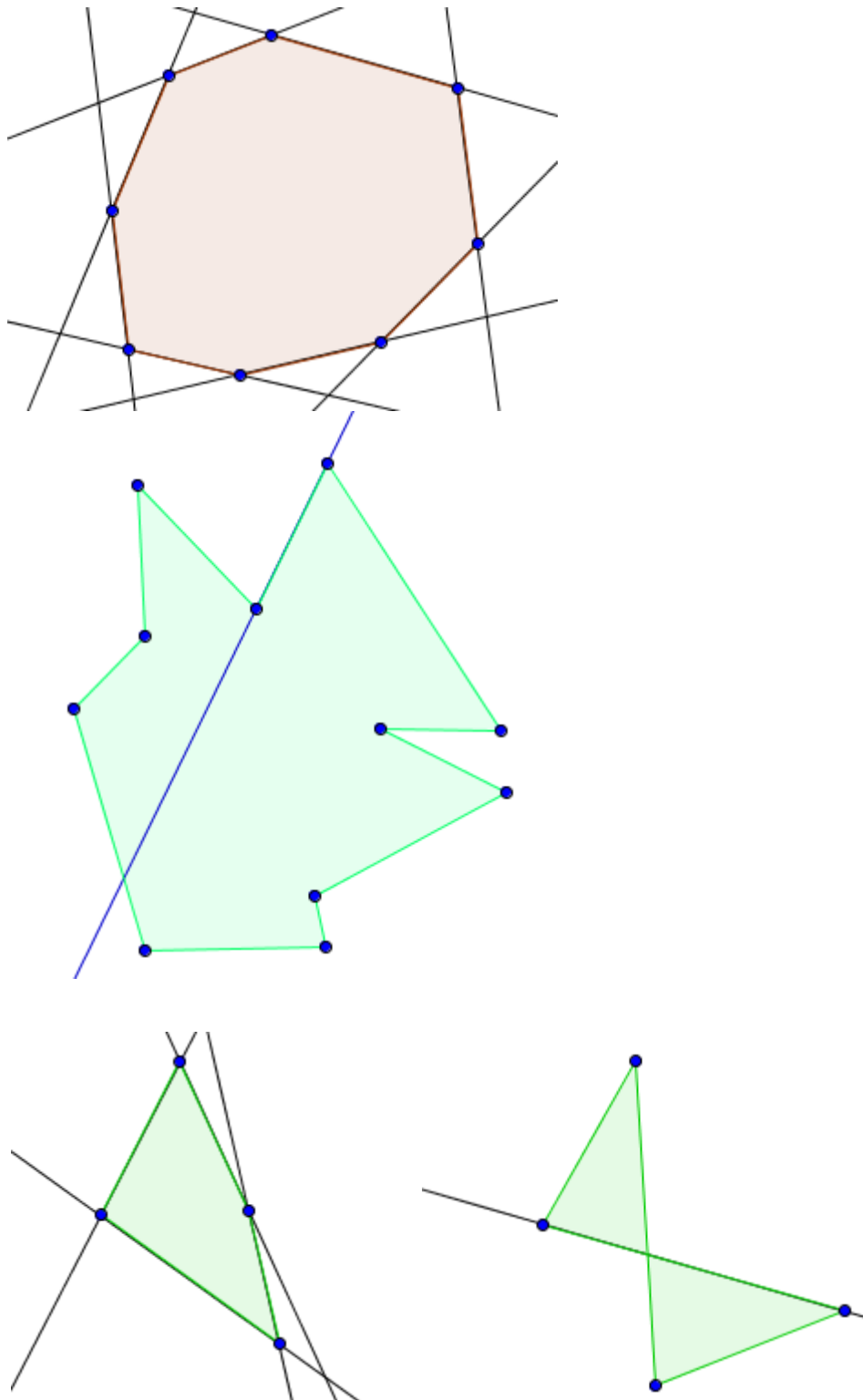




- We can see that there are many differences between these polygons. The number of vertices, of sides and the shape. Let me be more precise.
- The number of sides is very important:
 - • Triangle or Polygon with 3 sides,
 - • Quadrilateral or Polygon with 4 sides,
 - • Pentagon or Polygon with 5 sides,
 - • Hexagon or Polygon with 6 sides,
 - • Heptagon or Polygon with 7 sides,
 - • Octagon or Polygon with 8 sides,
 - • Enneagon or Polygon with 9 sides,
 - • Decagon or Polygon with 10 sides,
 - • Hendecagon or Polygon with 11 sides,
 - • Dodecagon or Polygon with 12 sides,
 - • Icosagon or Polygon with 20 sides.
- The shape is the second point:
- Crossed Polygons or self-intersecting Polygons, two sides intersect outside a vertex.



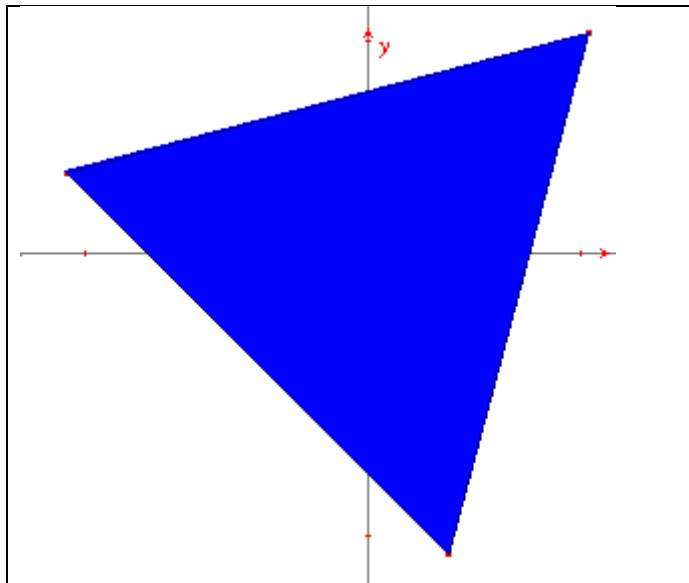
- Convex Polygons and star Polygons: In order to understand convexity we extend each segment to a line, this line cut the plane in two regions. If the Polygon is divided in two regions by some of these extended segments we say that the Polygon is a star. When is not the case the Polygon is called convex. Convexity is illustrated bellow.



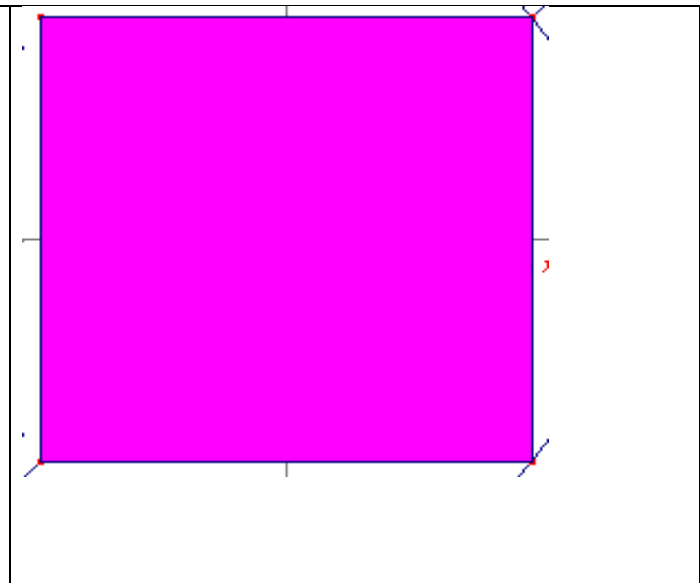
Regular Polygons.

In usual language by regular Polygon we mean a convex Polygon whose angles have the same measure and the sides the same length. In fact we don't need convexity so we speak about Star regular.

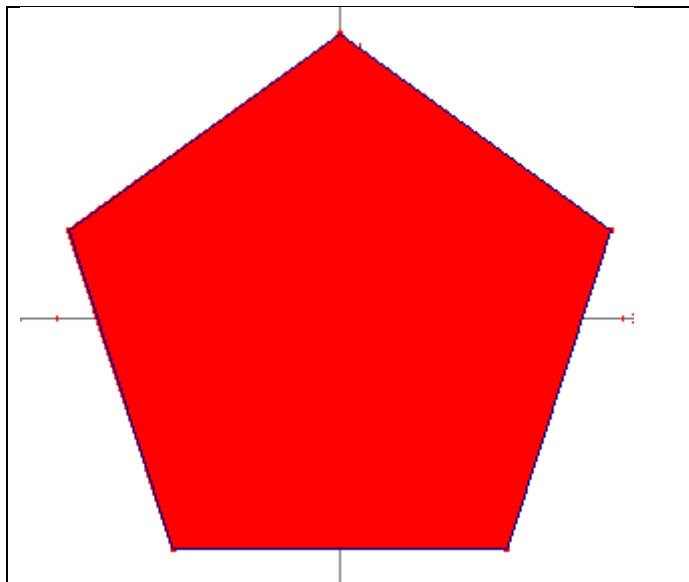
The convex regular Polygons



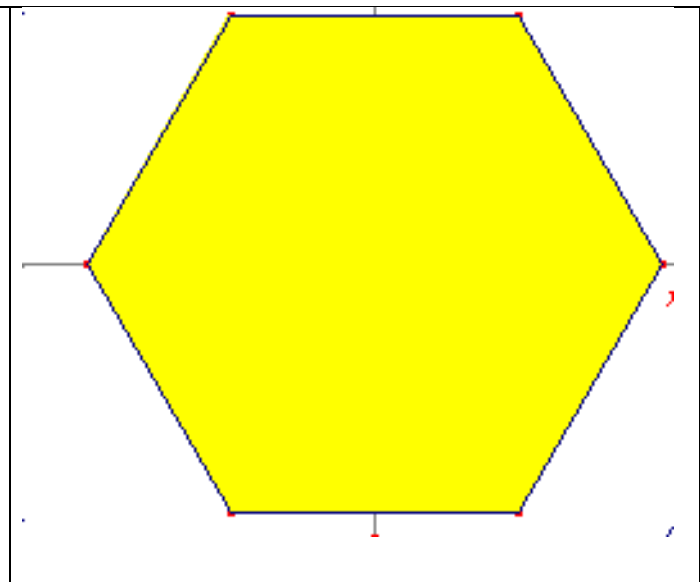
Equilateral Triangle



Square

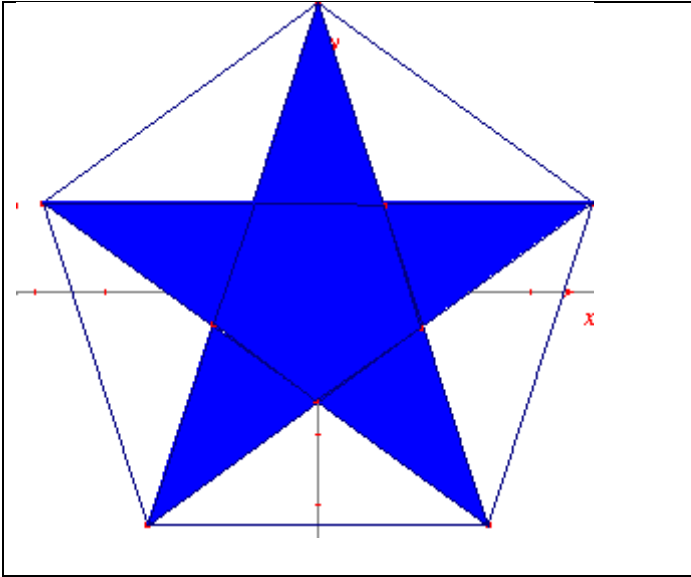
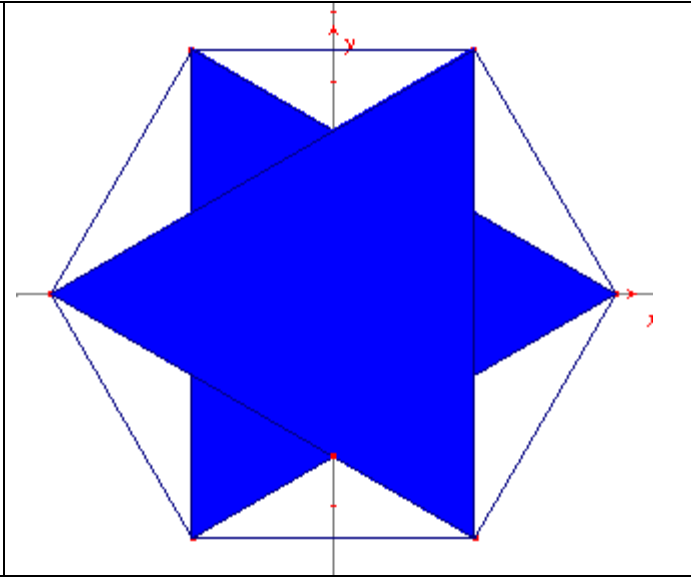
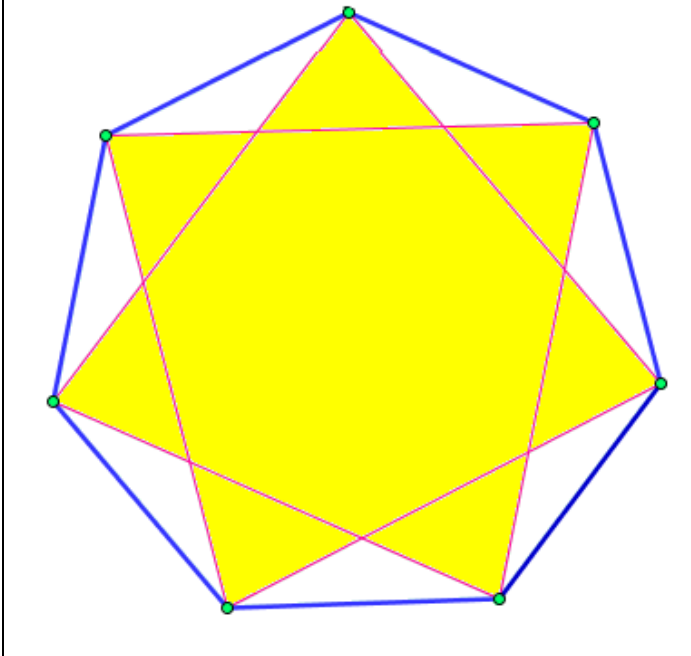
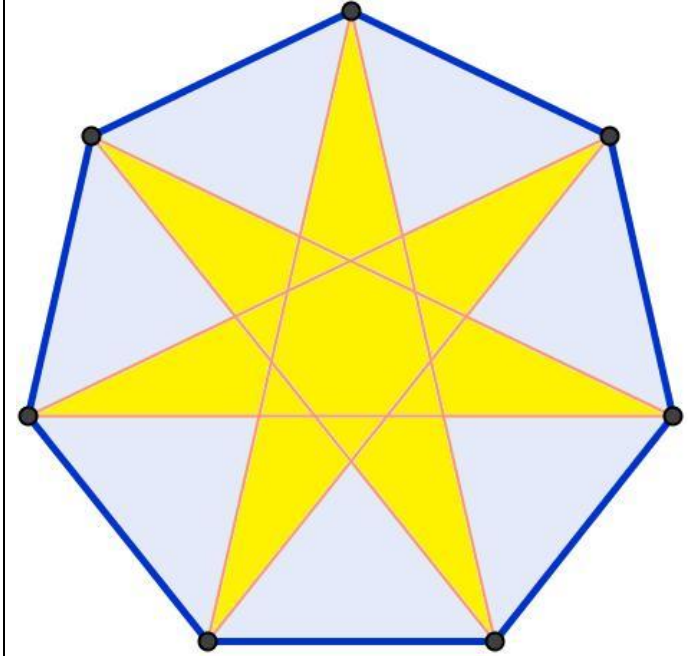


Regular Pentagon



Regular Hexagon

The star regular Polygons

	
<p>Pentagram or star regular pentagon</p>	<p>Star regular Hexagon</p>
	
<p>Star regular Heptagon $\{7/2\}$.</p>	<p>Star regular Heptagon $\{7/3\}$.</p>

The first **Star regular Polygon** is the **star Pentagon** also named pentagram and noted $\{5/2\}$.

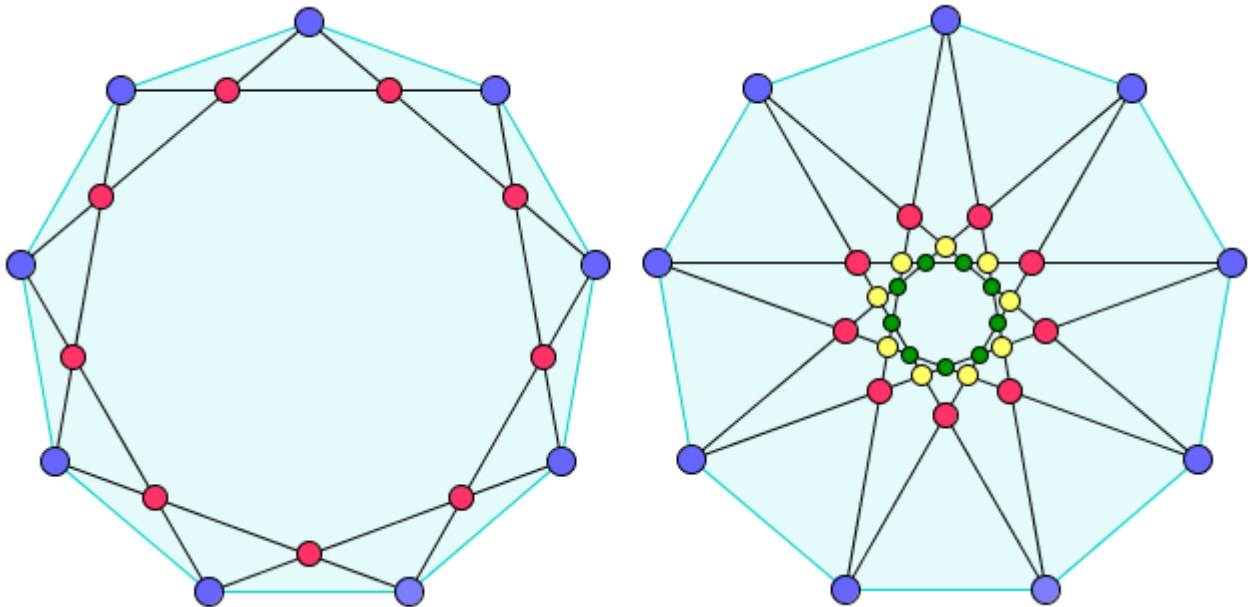
The second **Star regular Polygon** is the **star Hexagon** noted $\{6/2\}$.

The third **Star regular Polygon** is the **star Heptagon**, but there are two **types** :

We draw a regular convex Heptagon. Choose as our start point a vertex (1) draw a segment from the vertex (1) to the vertex (3), then a segment from (3) to (5) we continue in this way until we reach again the vertex (1).

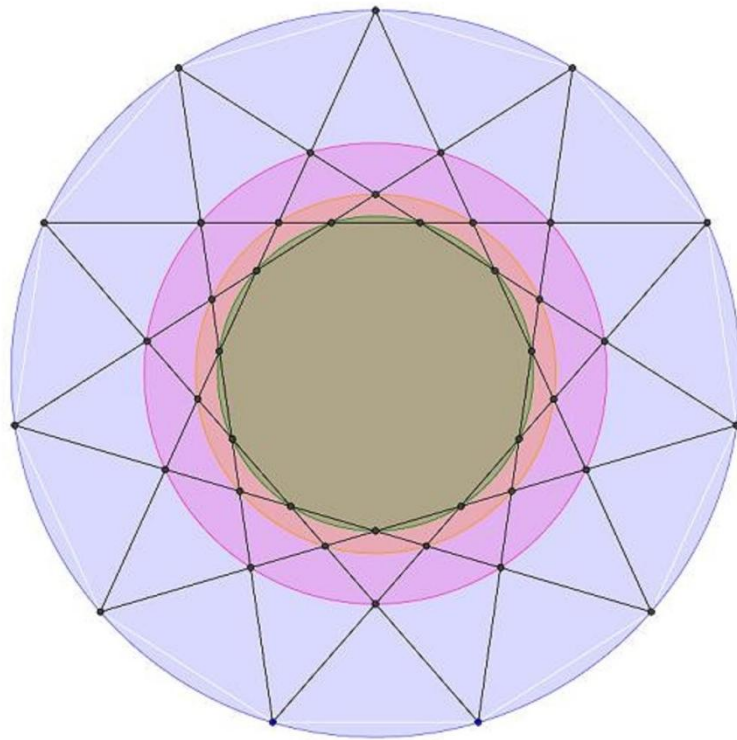
You can see that we have turned twice around the center of the Heptagon. This explain why we denoted $\{7/2\}$.

I hope you understand now why the second Heptagon is noted $\{7/3\}$, explain why.



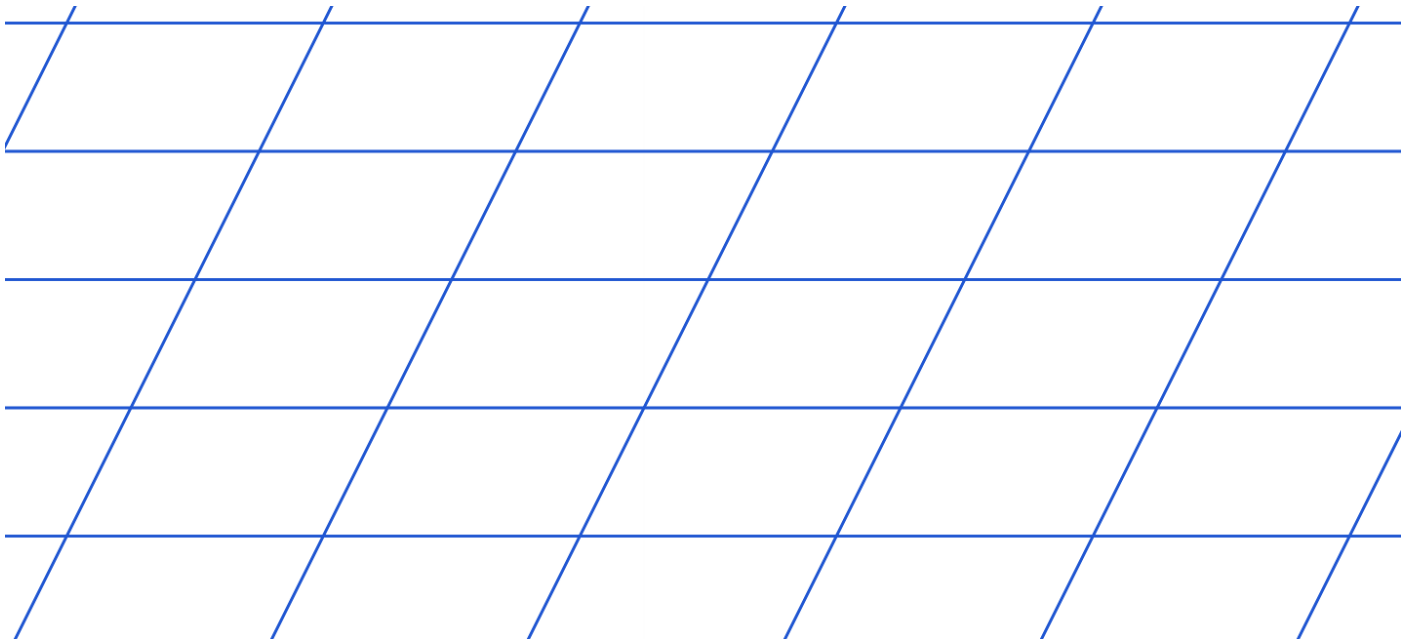
These two figures show two possibilities for a Star regular Enneagon $\{9/2\}$ et $\{9/4\}$. We give an alternative definition of $\{9/4\}$: As we can see we have used four colors to distinguish intersecting points at different.

- 1) In the following picture can you guess the name and notation of this regular star Polygon?
- 2) Using a transparent paper copy the regular Polygons with 11 sides and using a rule draw the star Polygons $\{11/2\}, \{11/3\}$ and so on. Color the points in the same level with the same color.
- 3) Using a transparent paper copy the Polygons with 13 sides and using a rule draw the star Polygons $\{13/2\}, \{13/3\}$ and so on. Color the points in the same level with the same color.

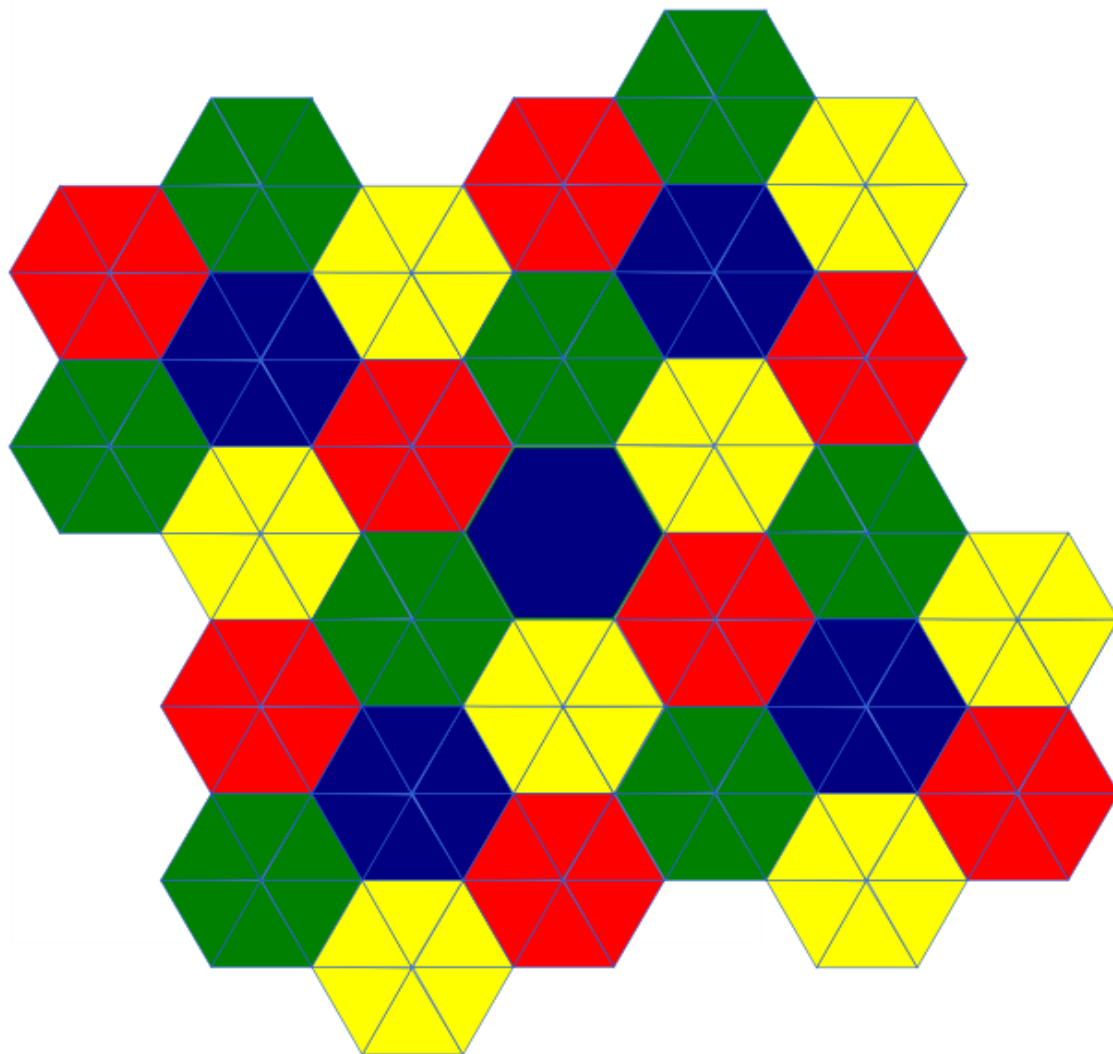


Grid pattern , square graph paper, triangular graph paper and others

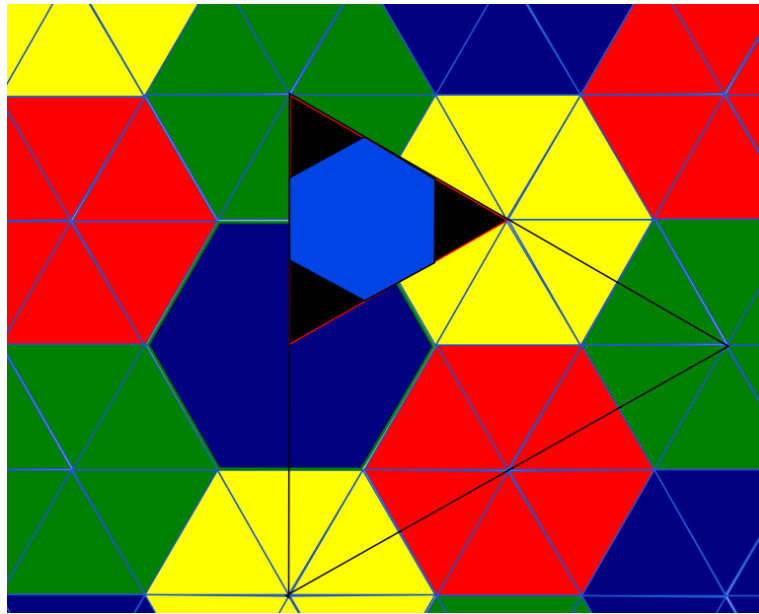
A special grid pattern is given by Parallelograms, Parallelograms are not rectangles but have their opposite sides parallel . Grid Patterns by Parallelograms are very important in Tessellations, of course rectangle grids or square grids are a class of Grid Patterns by Parallelograms.



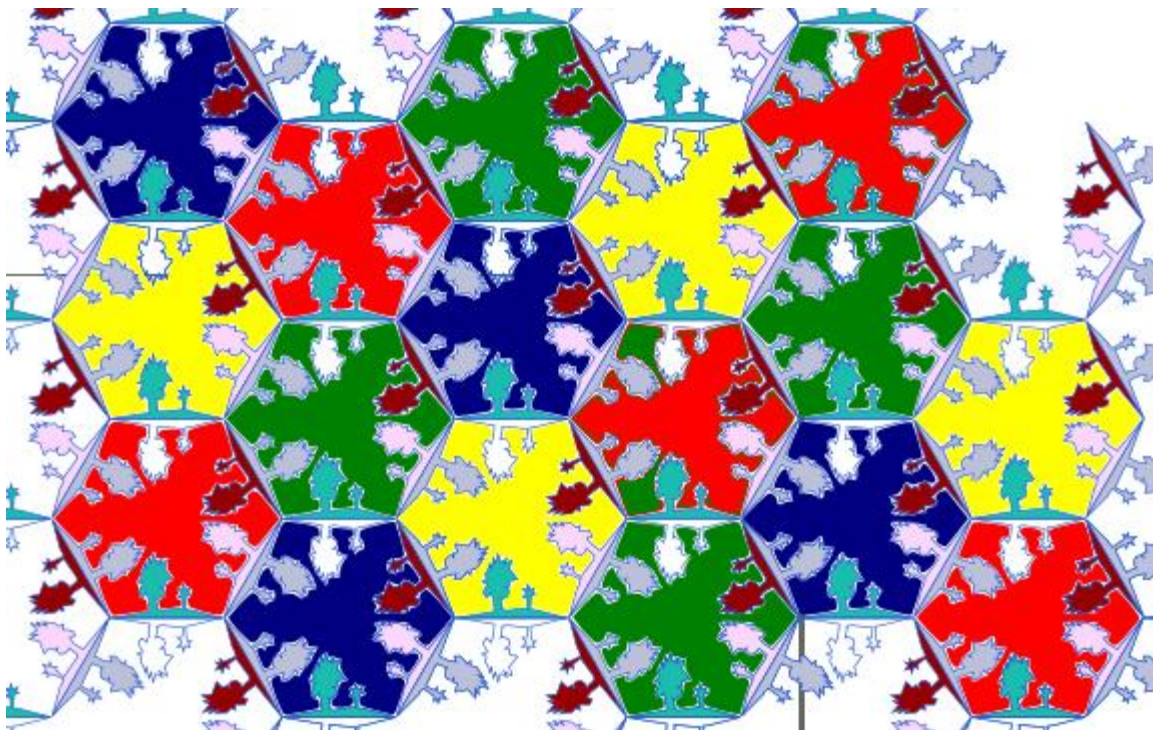
Another grid pattern very useful for Tessellation is given by equilateral triangles. We can see that 6 equilateral triangles give an Hexagon.



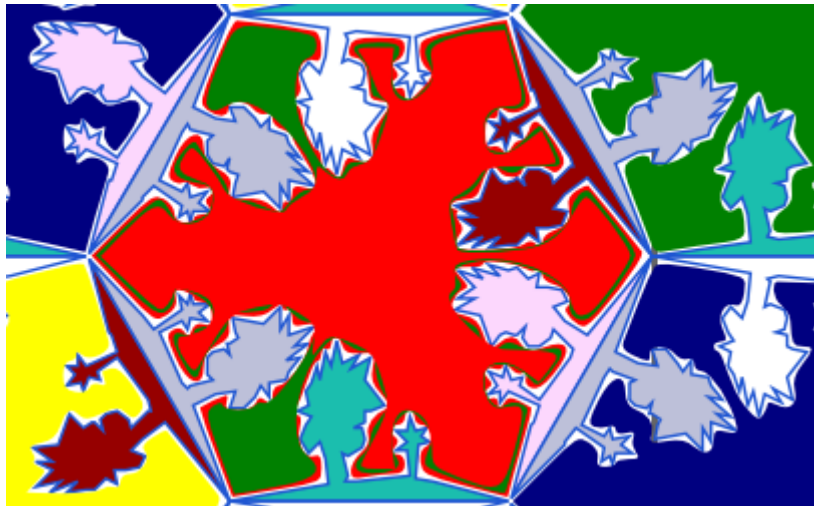
The grids by equilateral triangles produces also a Hexagonal grid. As we will see from this grid by equilateral triangles there are many other interesting triangles and Hexagons: The centers of 3 contiguous Hexagons produce an equilateral triangle. The centers of 6 contiguous triangles produce an Hexagon.



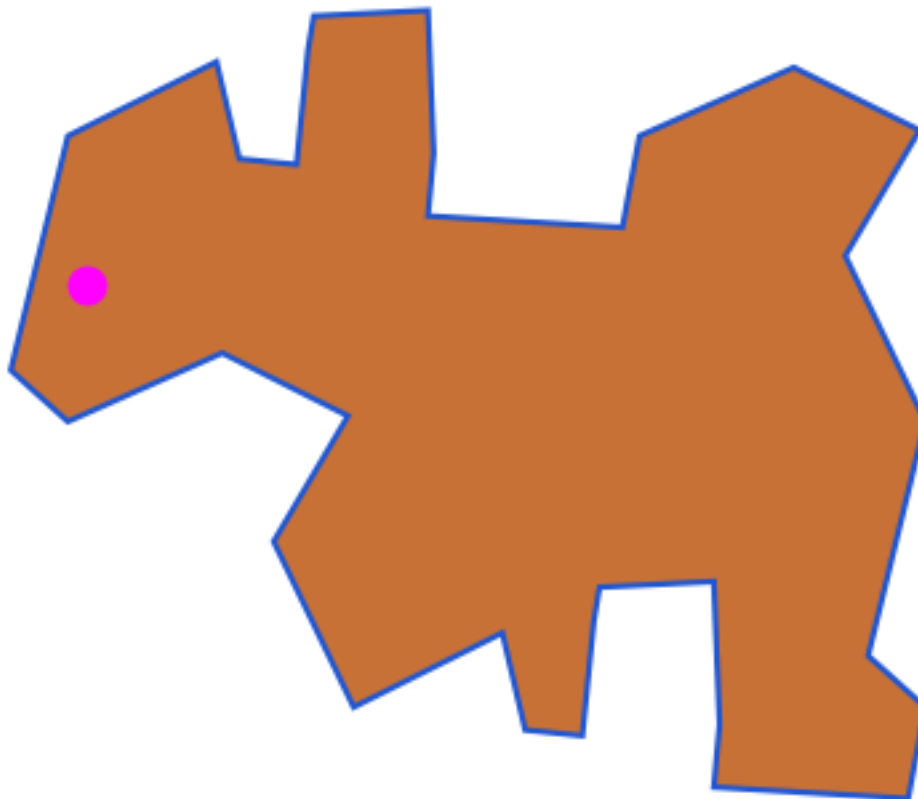
Here we have a Tessellation that use the Hexagonal grid:



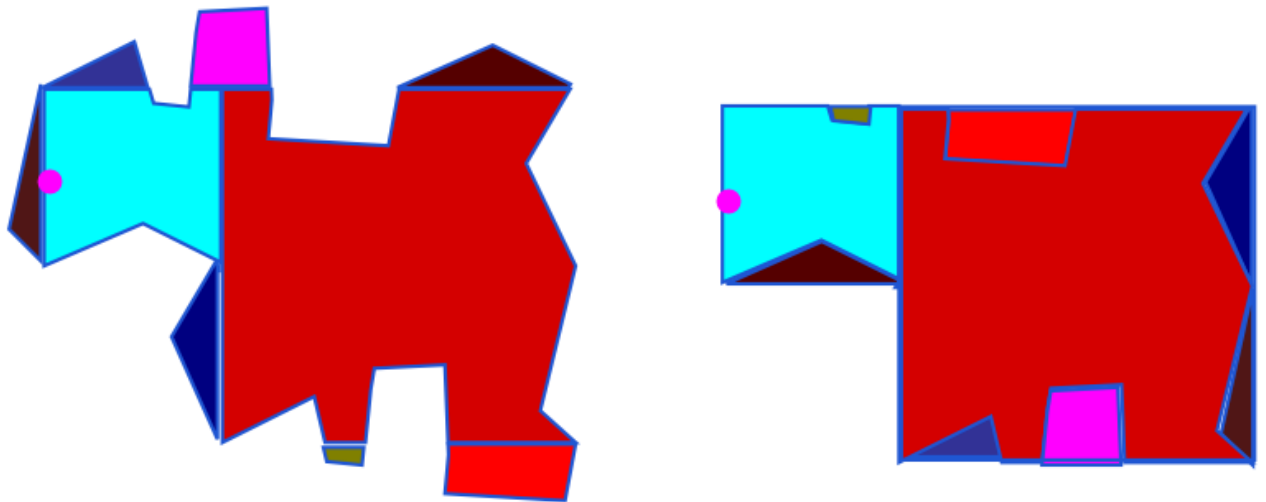
And a zoom of one Hexagon



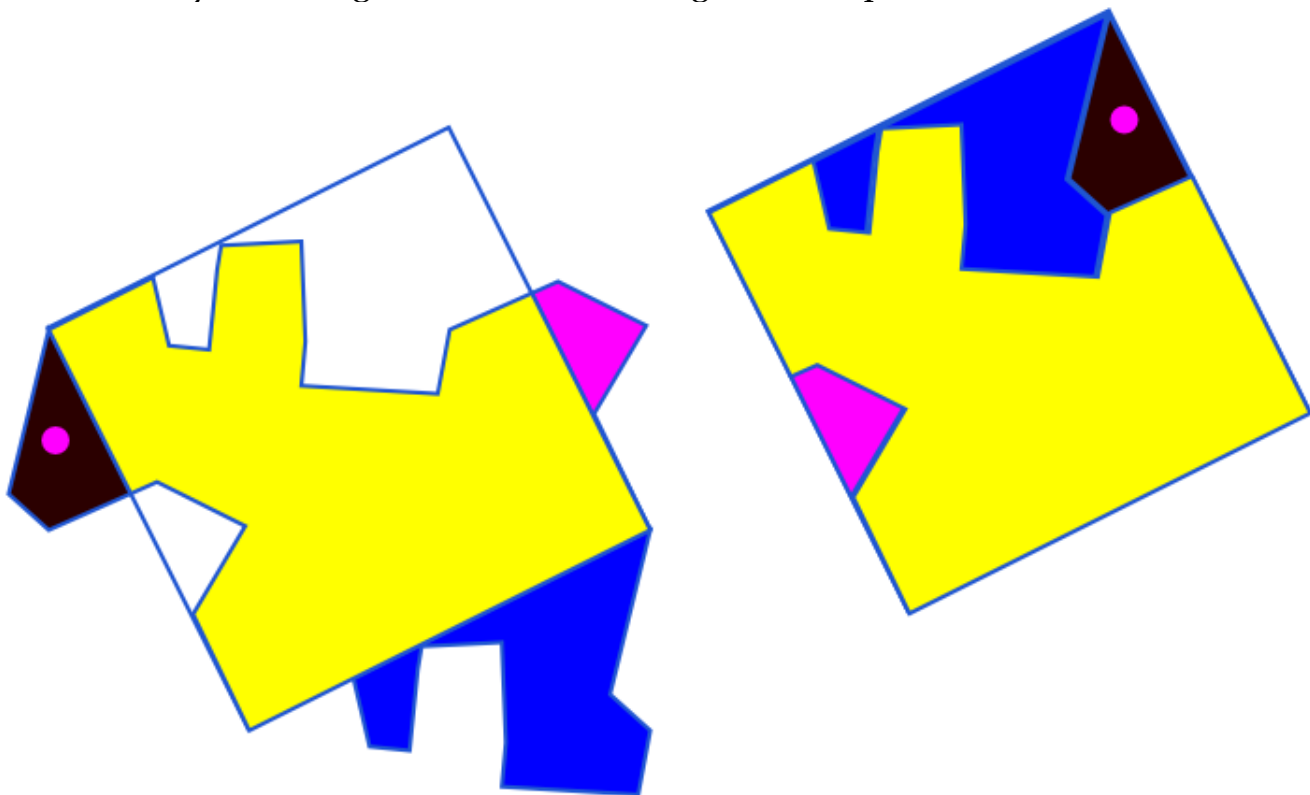
Puzzles and Tessellations
First example :



Make two copies of this figure. We will cut this figure in two ways.
In the first we will transform our figure in two squares;



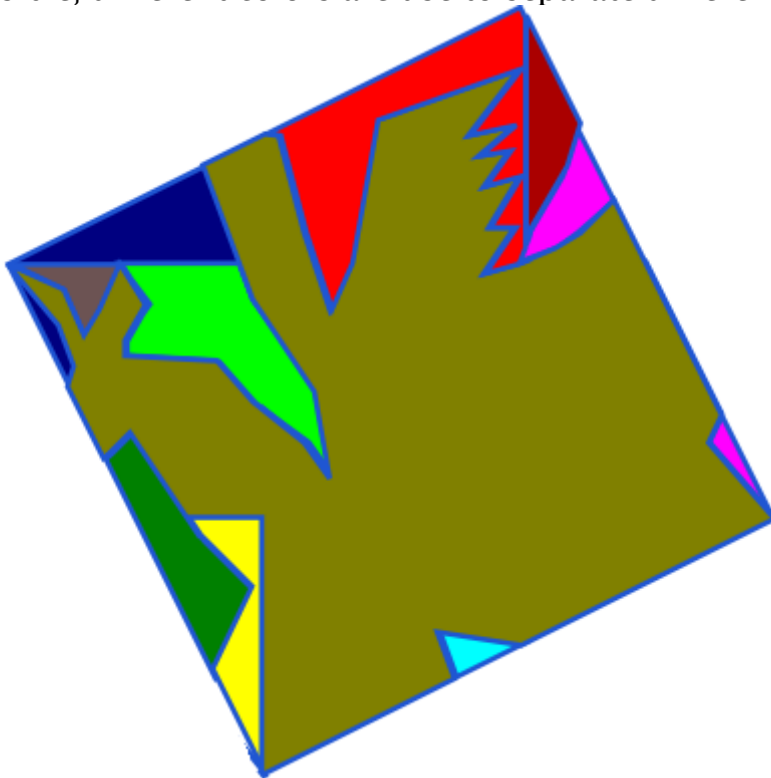
In the second way of cutting we transform the figure in a square.



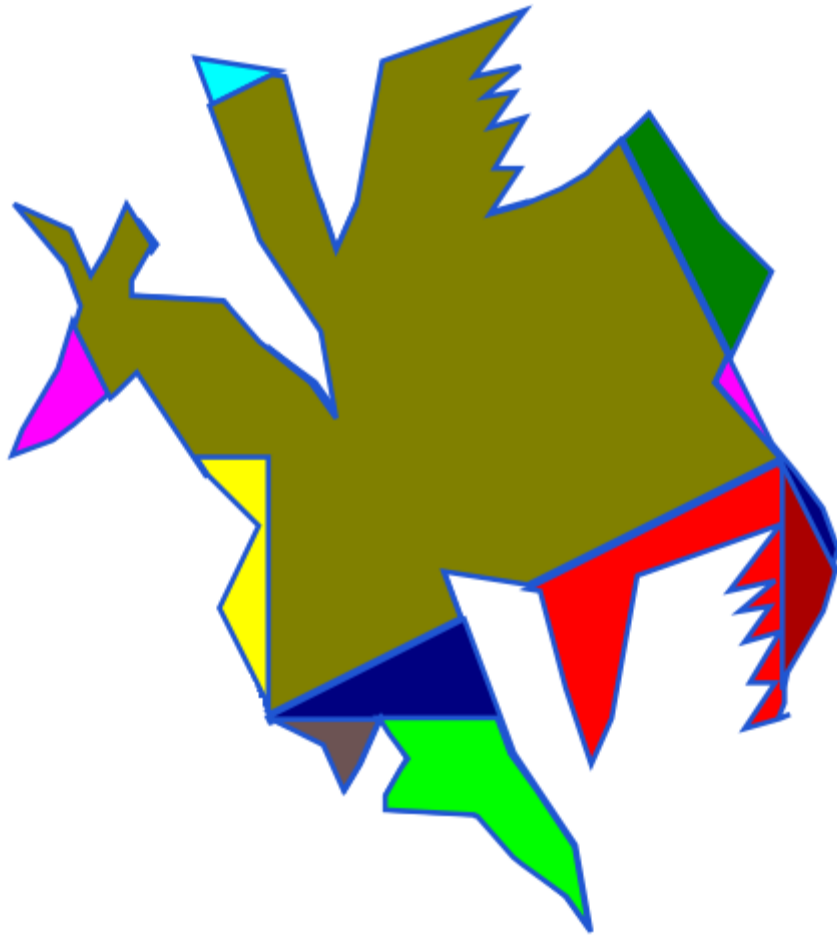
Example 2



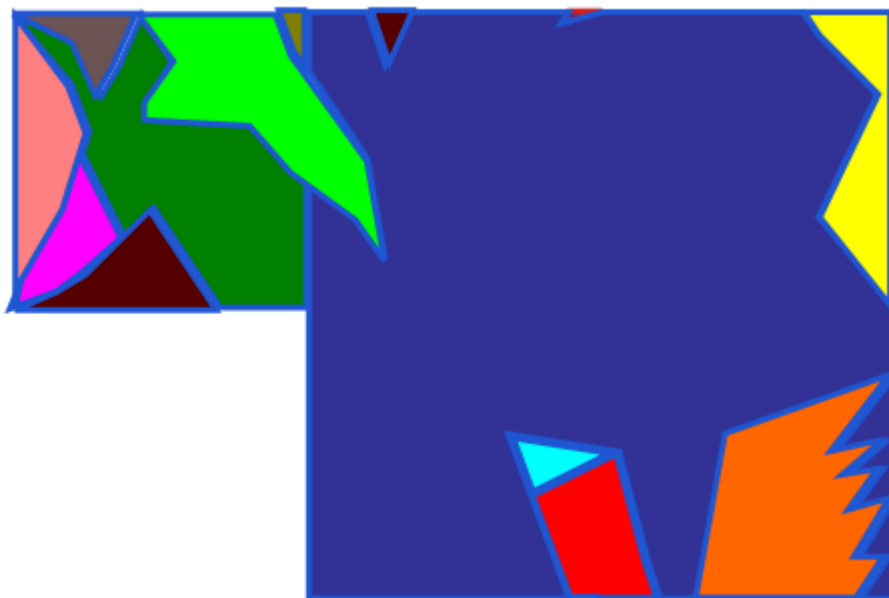
Copy the following square and cut following the segments as exact as possible, different colors are use to separate different regions.



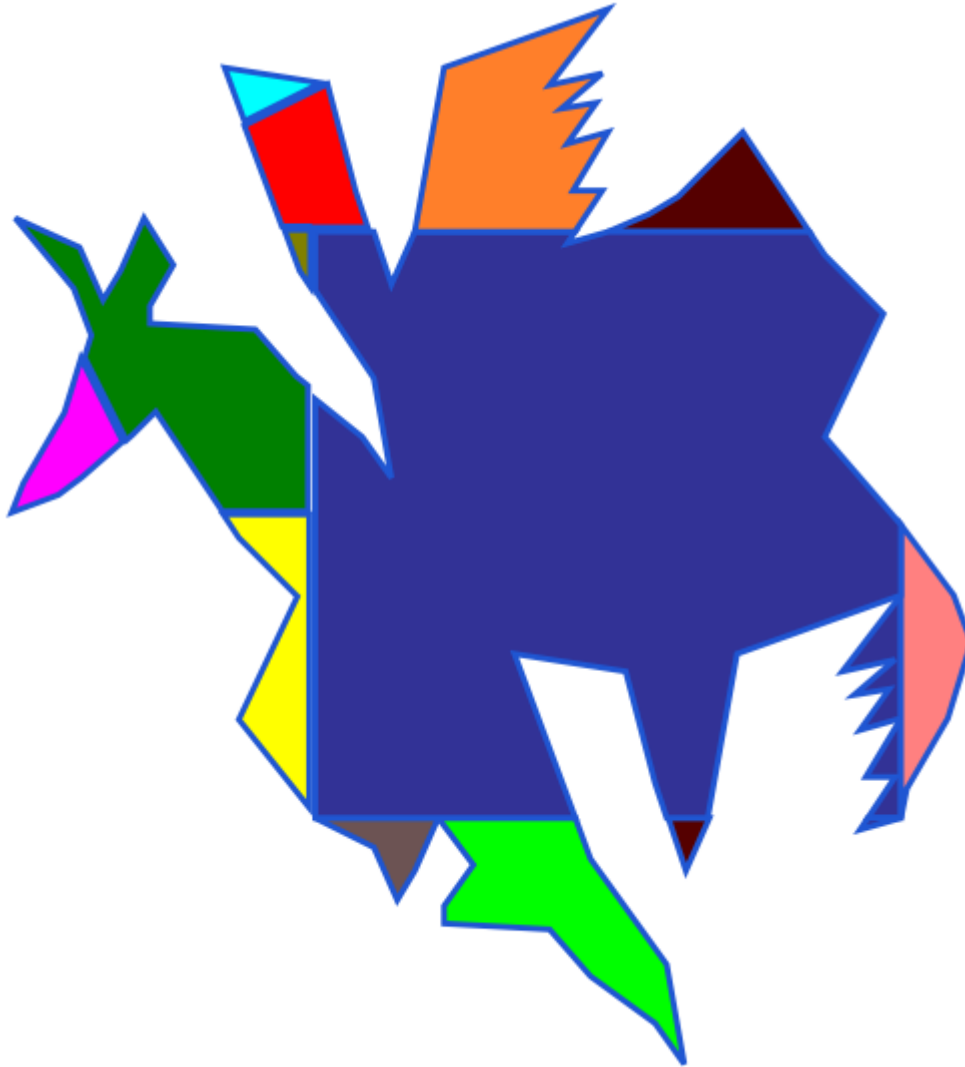
In a sheet of paper glue the different pieces, so we have the following animal.
Can you put a name on this animal ?



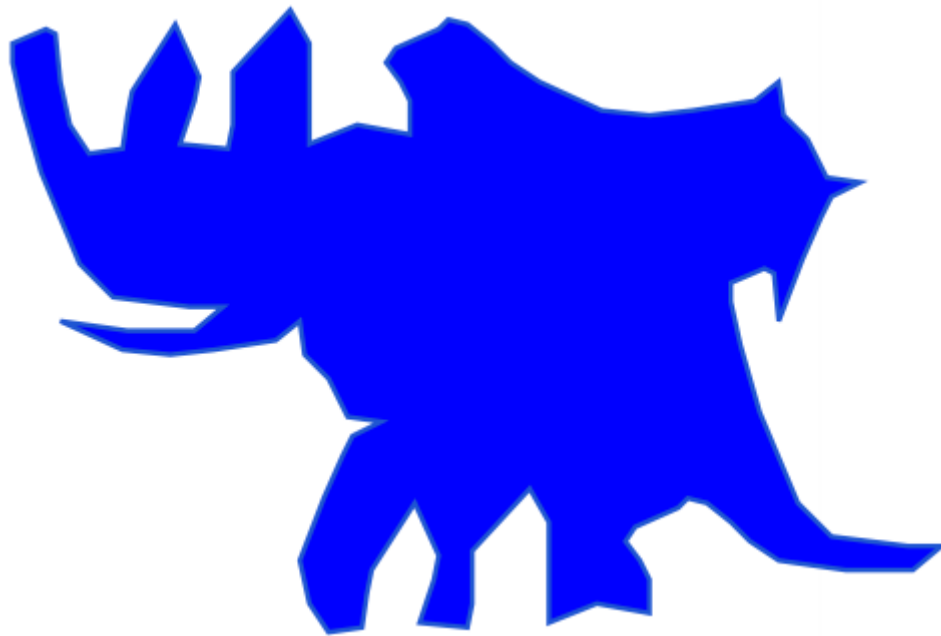
We can also get this animal from two squares :



Do a copy cut all the regions, then glue all in a sheet of paper.



The winged Deer and the Elephant



Do a copy of the next figure, cut the regions of different colors.

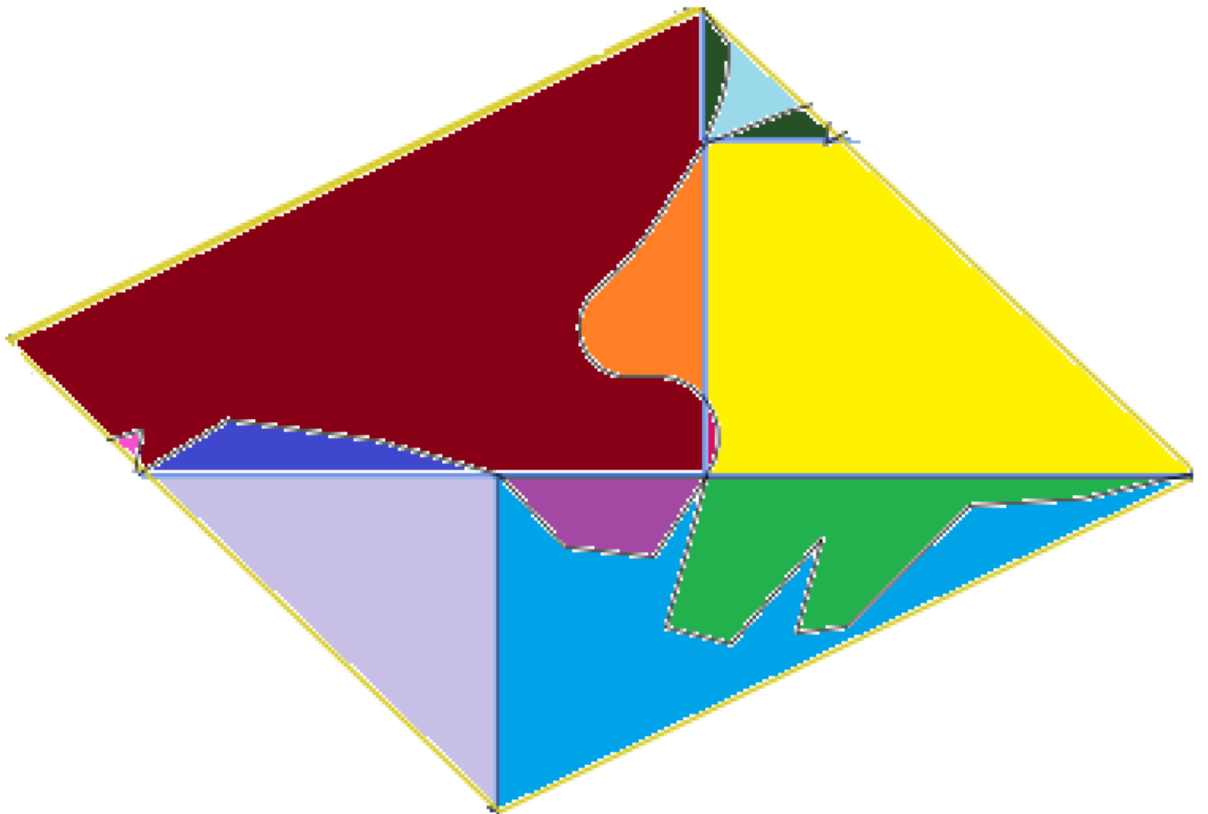


Glue on a sheet of paper the different pieces and get the winged Deer:

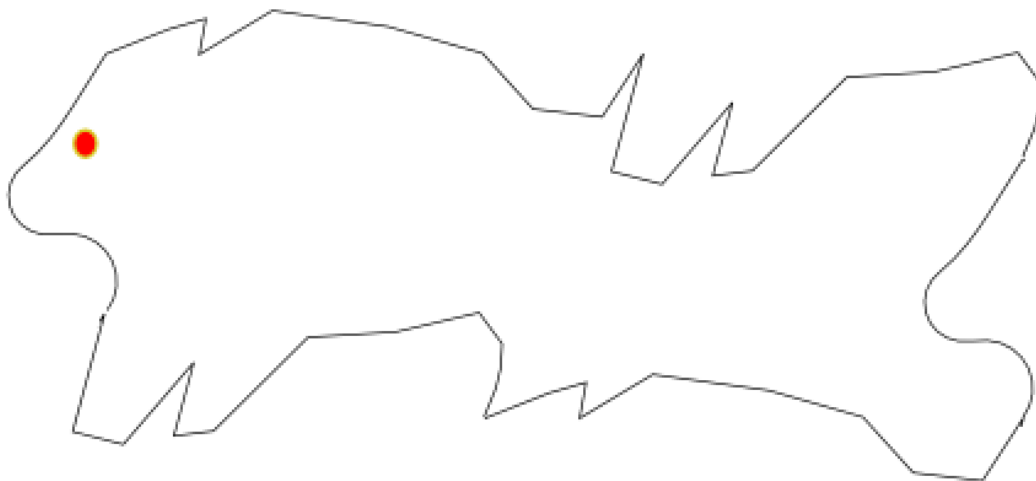


Third example:

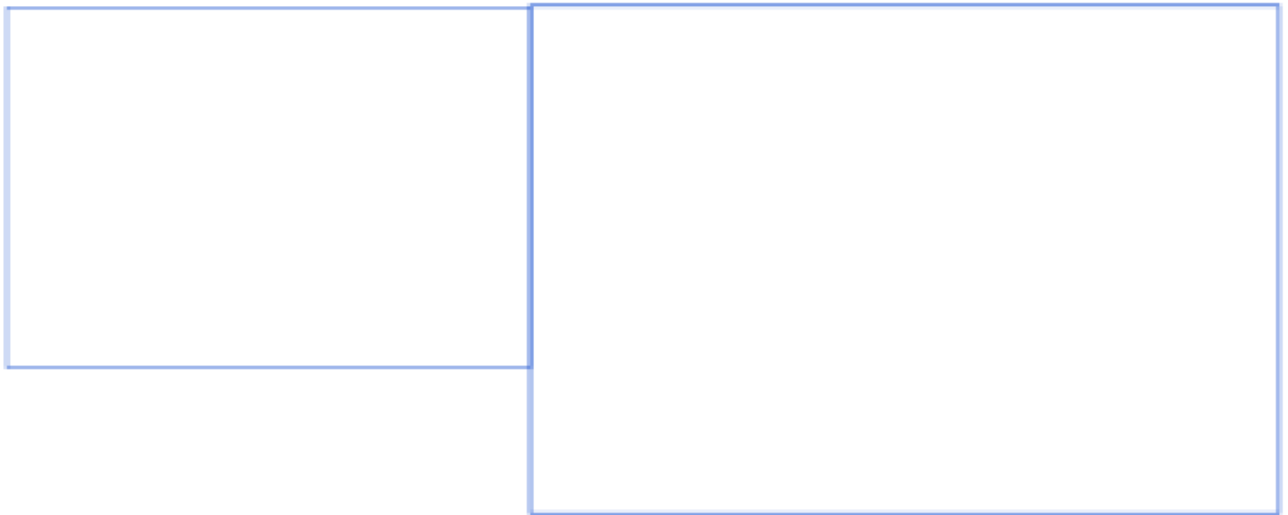
Make two copies of the following figure and cut it in pieces.



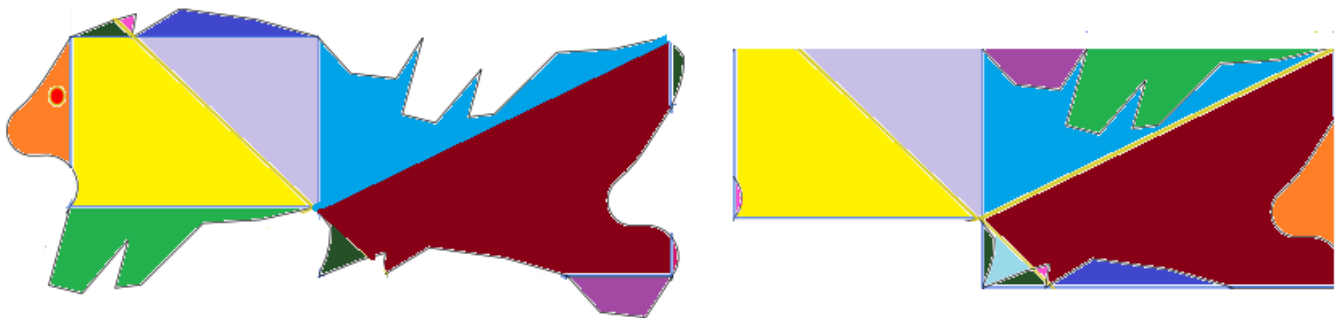
Glue together all the pieces in a sheet of paper, you should get a squirrel :



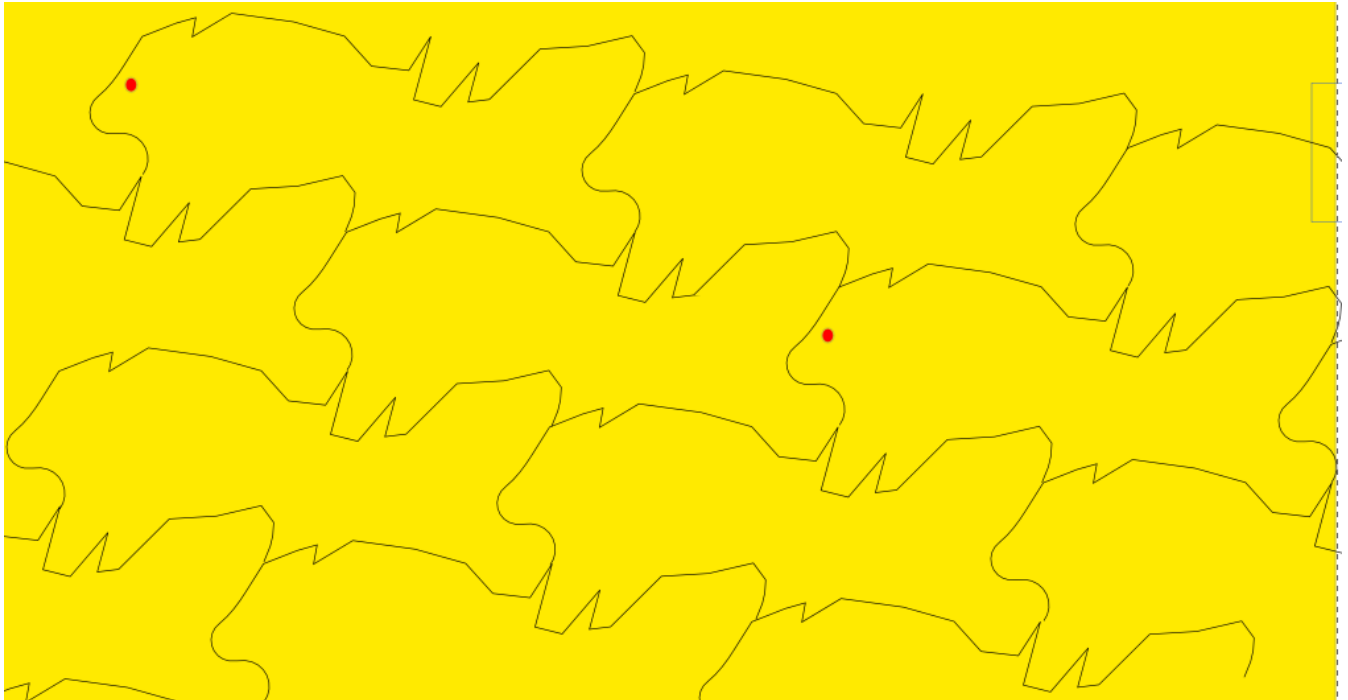
With the pieces of the second copy you should fulfill the following two squares:



Here is the answer:



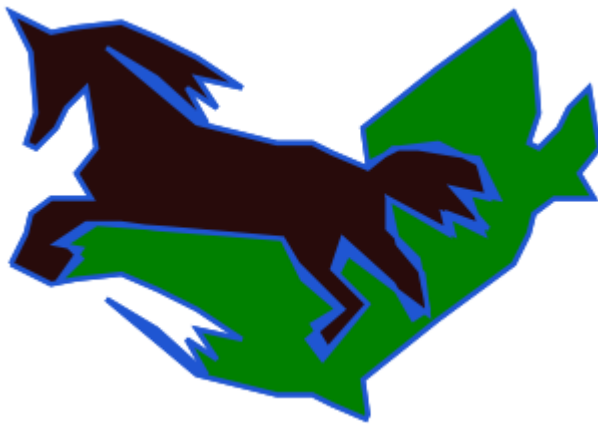
Now we will use the Squirrel to fulfill a sheet of paper. We start with a sheet of paper we put it in landscape, Cut the above squirrel figure, it will be our pattern. Put the pattern on the left up corner of the sheet of paper and with a pen draw the Squirrel on the paper. Then move the pattern to the right taking care of no overlapping the first Squirrel and the border of the new figure coincides with the border of the pattern. Draw with the pen the border of the pattern. You should continue until fulfill the sheet of paper.



Complete the figure on your paper coloring up to you.

Fourth example, Horses:

Make a copy of the following figure composed of a Horse and a small piece of field. This is our Pattern.



Repeat the steps described above to fulfill a sheet of paper with a Squirrel:



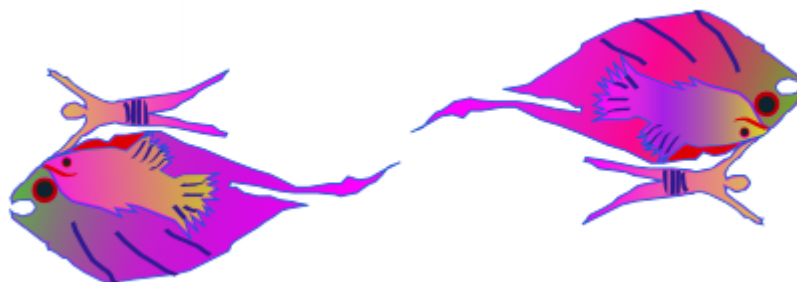
You should customize the figure drawing flowers,...

Fifth example, Fishes:

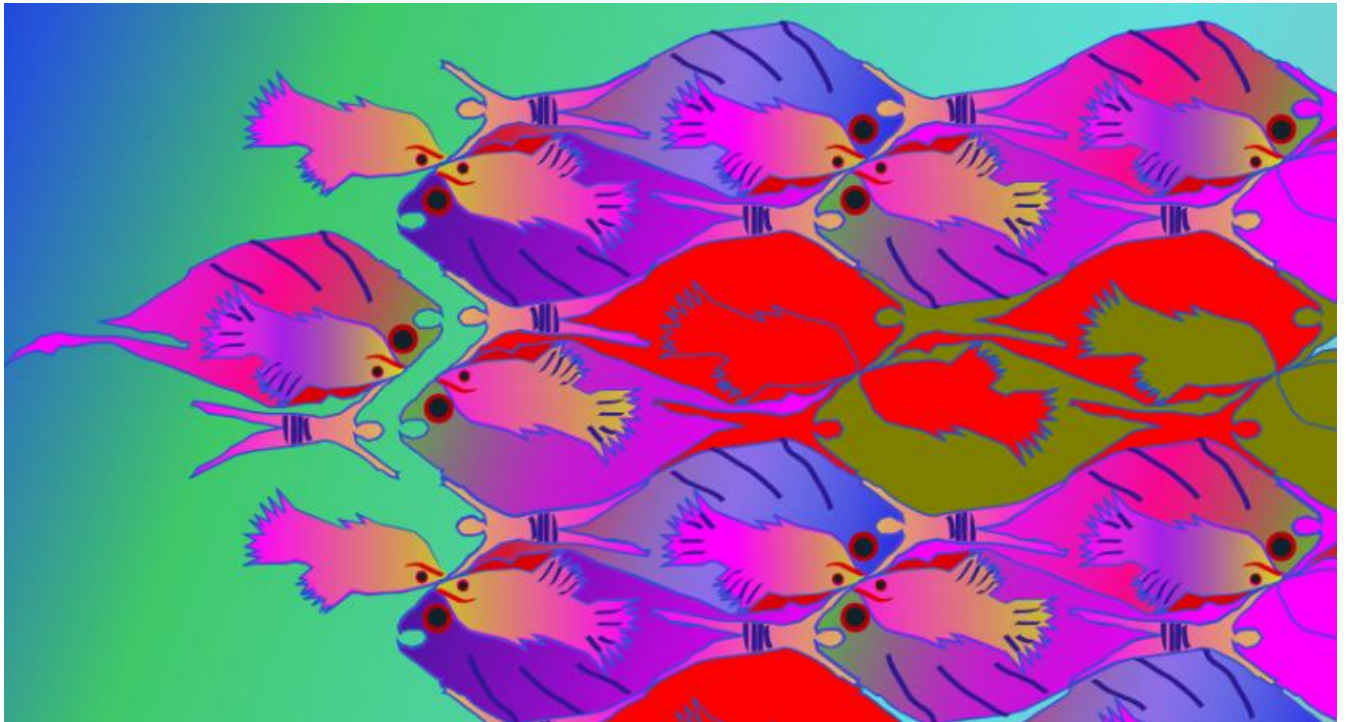
Make a copy of the following pattern composed of a Fish and a swimmer. Cut it very carefully.



The process to fulfill a sheet of paper is similar to the above but after drawing in the paper the first pattern in order to draw the second copy we have to do a U-turn as you can see in the following figure:



Here you can see the outcome after several steps:

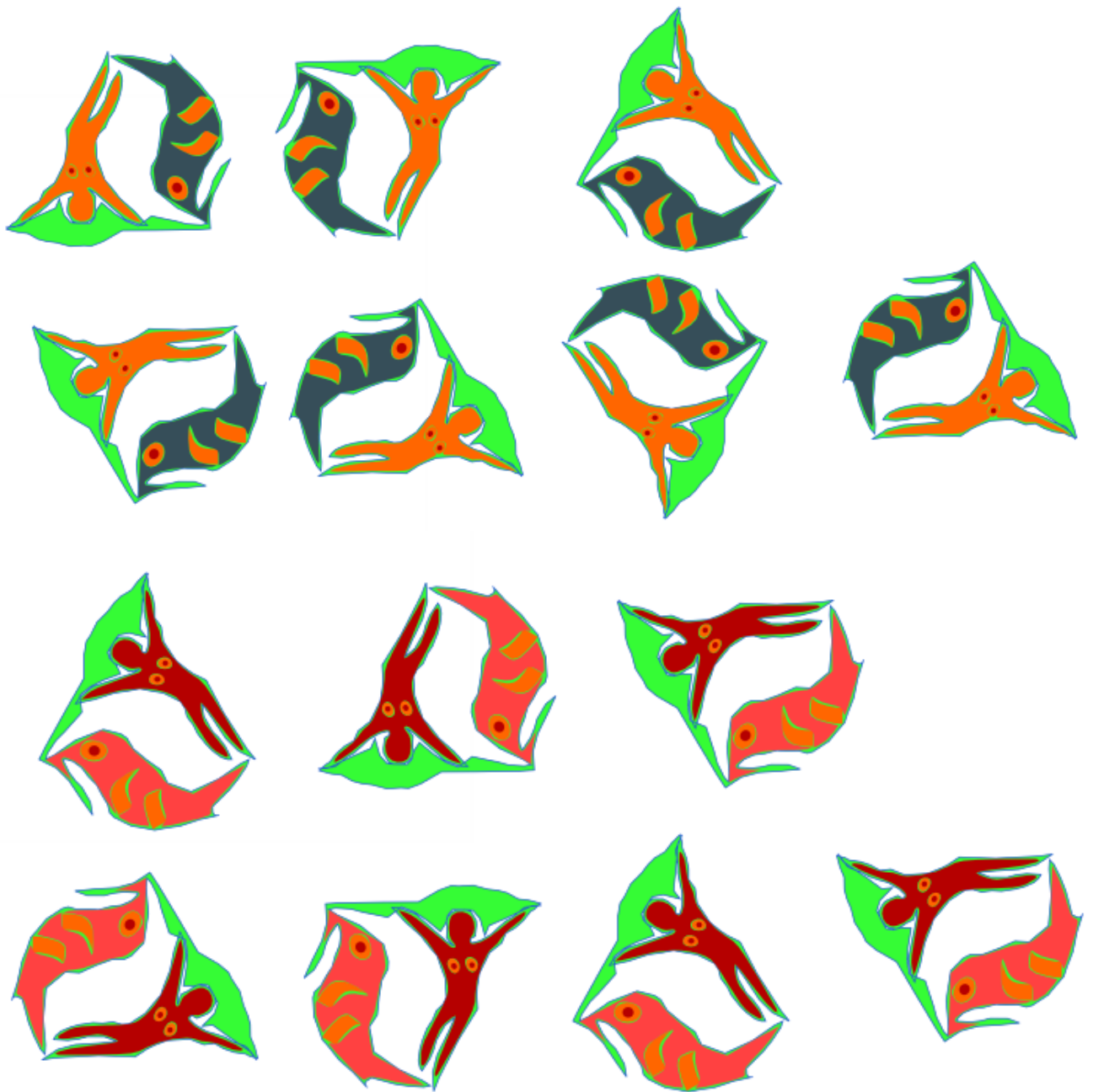


Sixth example with rotations of 60°

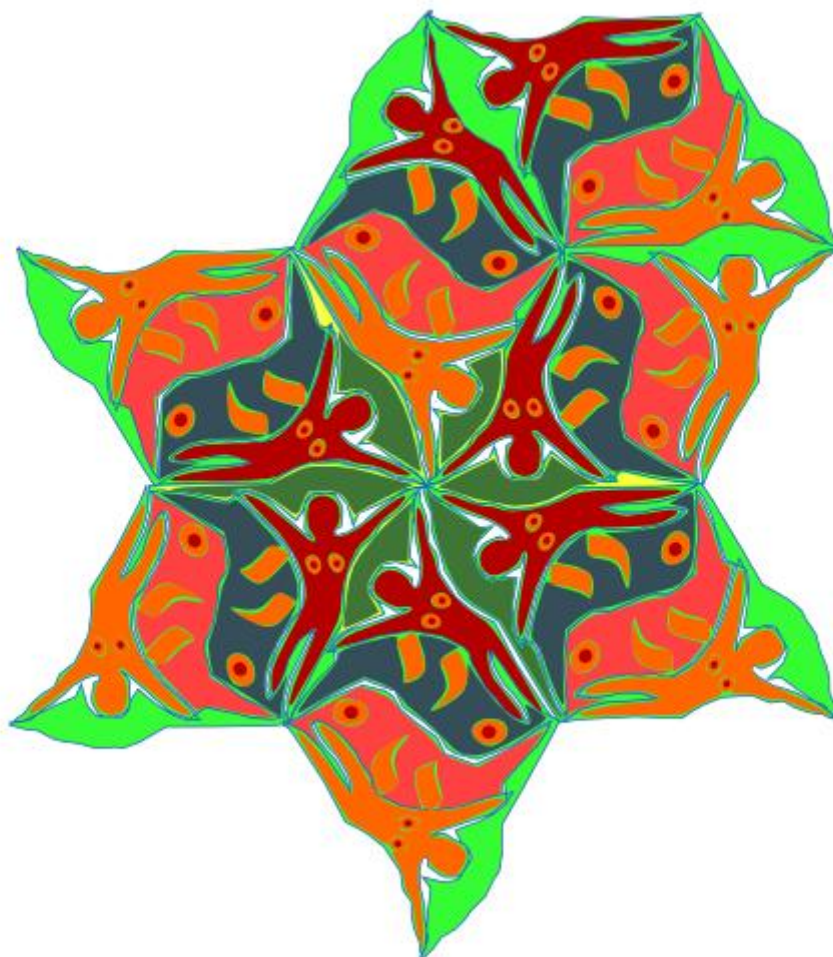
Consider the figure below:



We will rotate this figure with angles $60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ$. Check it.

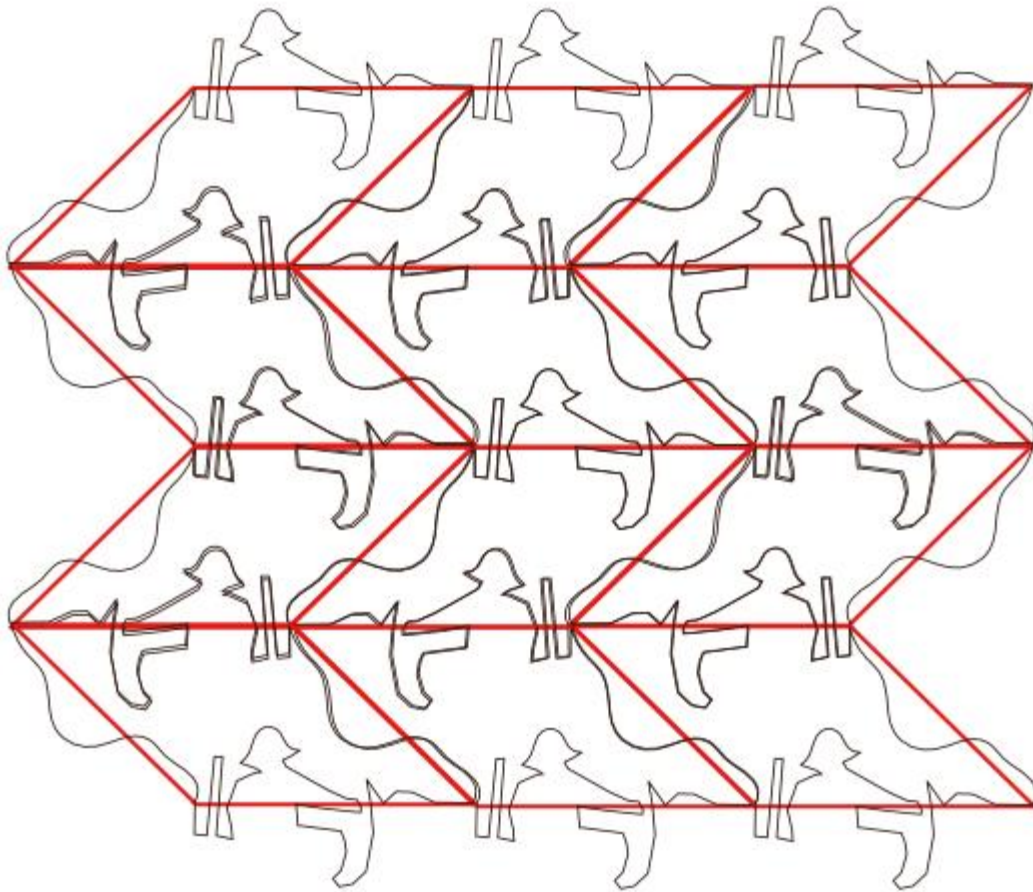


Cut all carefully and put it together as shown in the next picture.



Seventh example by using symmetries





My Software Tessellation

All examples presented above are Tessellations. The figures are due to the author. We explain now how you can learn Tessellations and use my Software or use only scissors.

You should be able :

Recognize if a figure is a Tessellation , that is the kind of rotations or symmetries used.

Do your own Tessellation either using scissors or by a computer with my Software.

Regular Tessellations (or Euclidean Tessellations) are classified by the rotation or symmetries used. All the rotations or symmetries used form a set called a group.

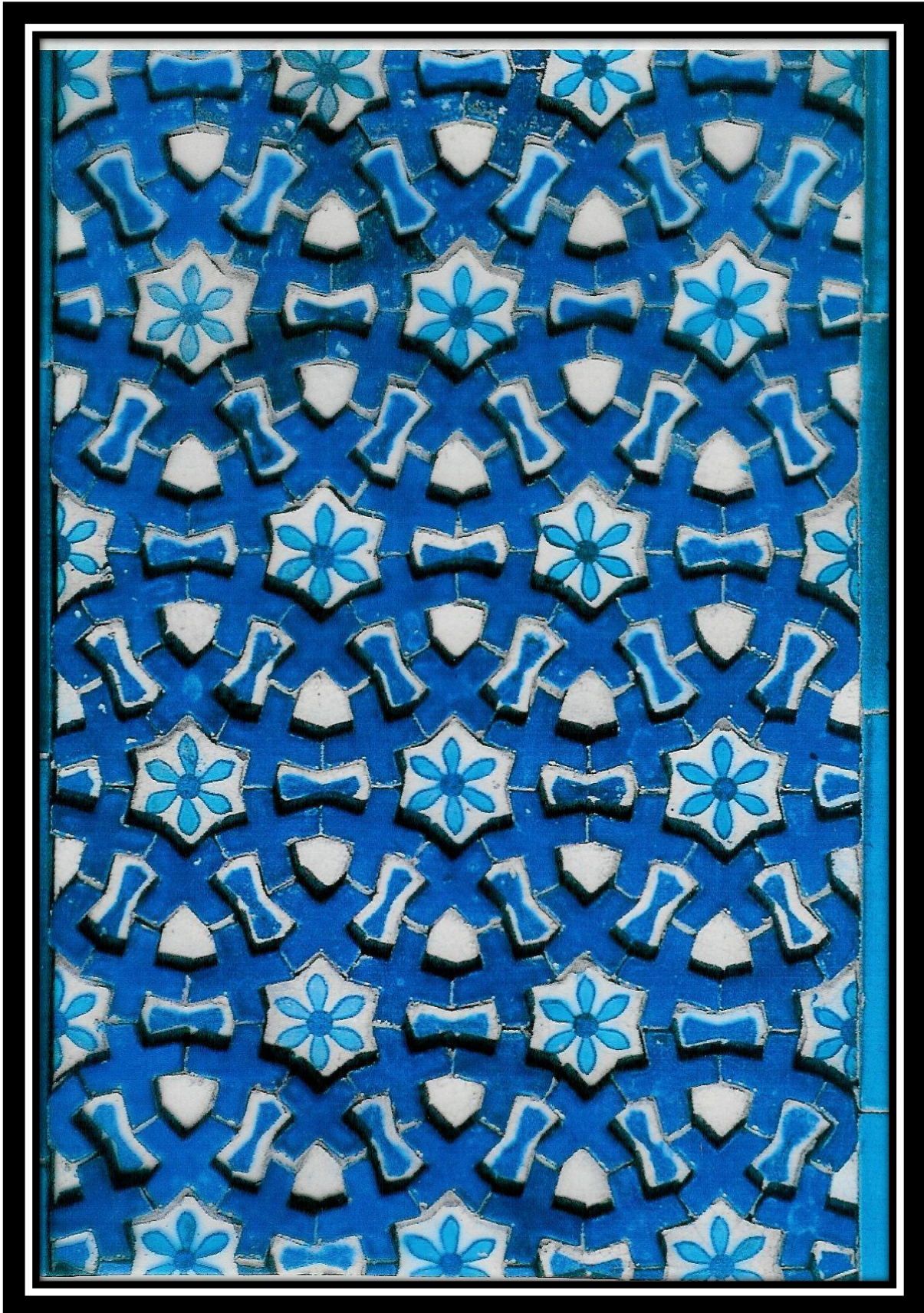
There are exactly 17 groups of Tessellations. For each group we will indicated the way to produce your own Tessellation by using scissors. You can download and use for free my Software. For each of the 17 group a

original process discover by the author is indicated. The process was tested in the Universities of Grenoble and Lyon and also in many classes of high school.

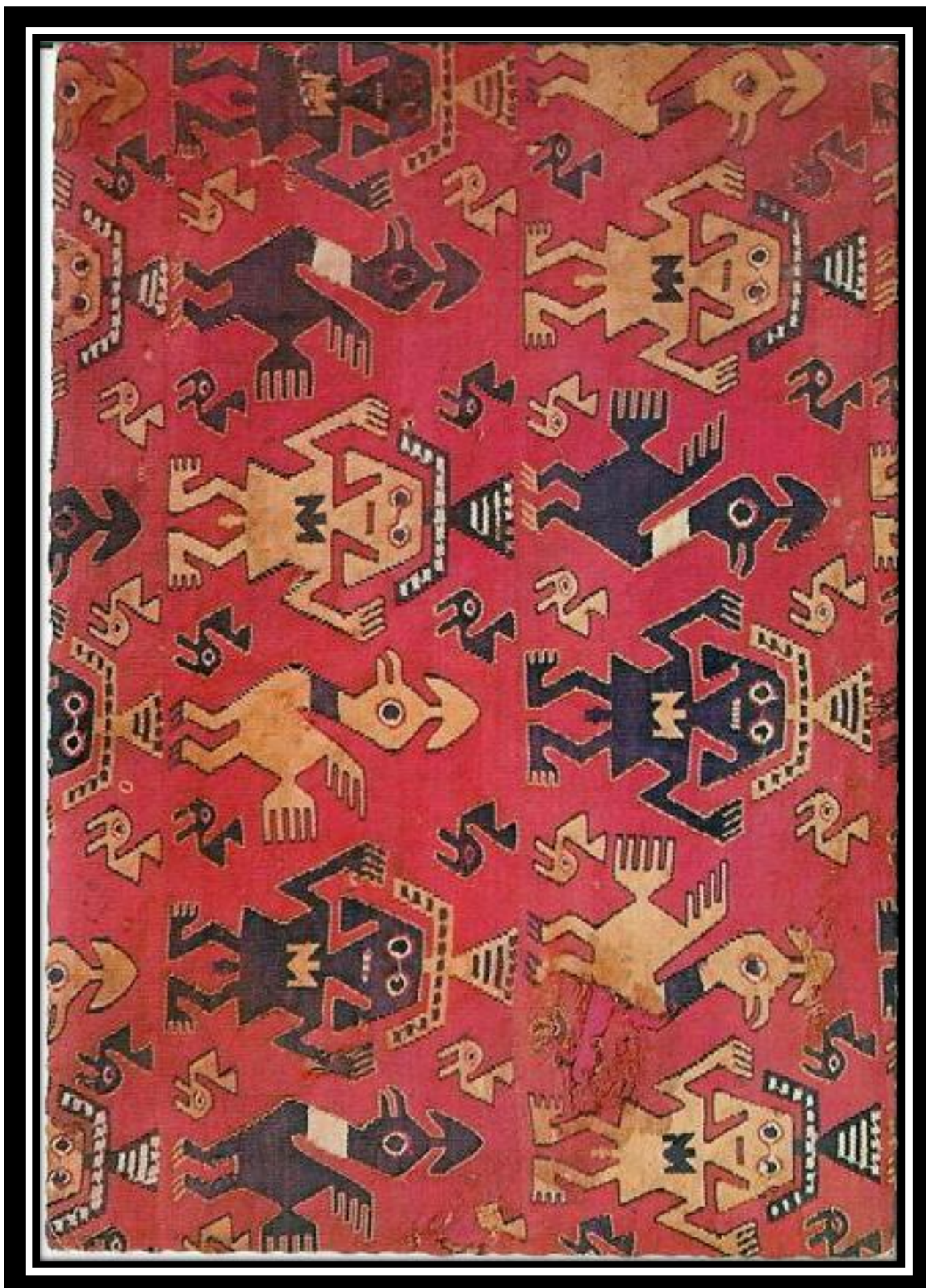
It is the first software developed in Europe. The first version appeared in 1990 during my lessons on Tessellations at the Grenoble University. It was adapted and improved to Young students with the help of Alice Morales. The content of this book was presented in many Universities and high schools in the world: France, México, Peru, Colombia, Iran, Turkey, Vietnam, Spain. A great collaboration with the high school teacher Alice Morales allow us to perform our methods for Young pupils in schools. School classes participated under our guidance to local events as the Sciences festival or international events as Exposciences International 2001 in Grenoble. I was Professor in the University for education of teachers in France (IUFM or ESPE) , part of my lessons consisted to introduces the Tessellations as a part of amusing mathematics but also conceptual to learn mathematics without a formal frame. As a result we can observe that pupils in schools can learn Geometry by intuition, and can describe deep mathematical properties by observation of their practice doing tessellations.

I repeat the goal of this book and my software is to introduce Tessellations to everybody, especially to children and people with very few mathematical background.

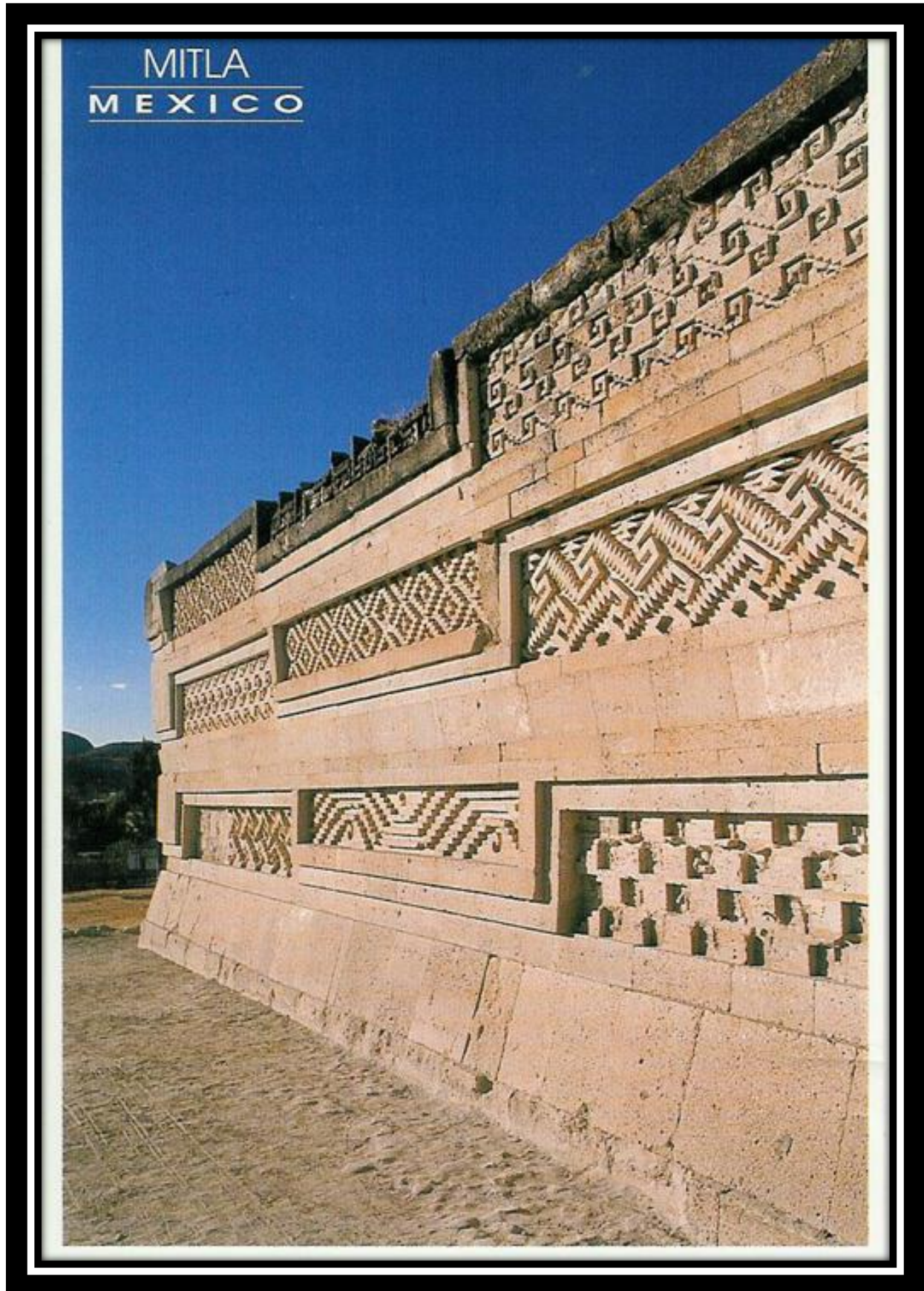
The following pictures show Tessellations produced by different civilizations in several parts of the world, some of them were done two thousand years ago.



Tiling founded on the wall of a mosque.



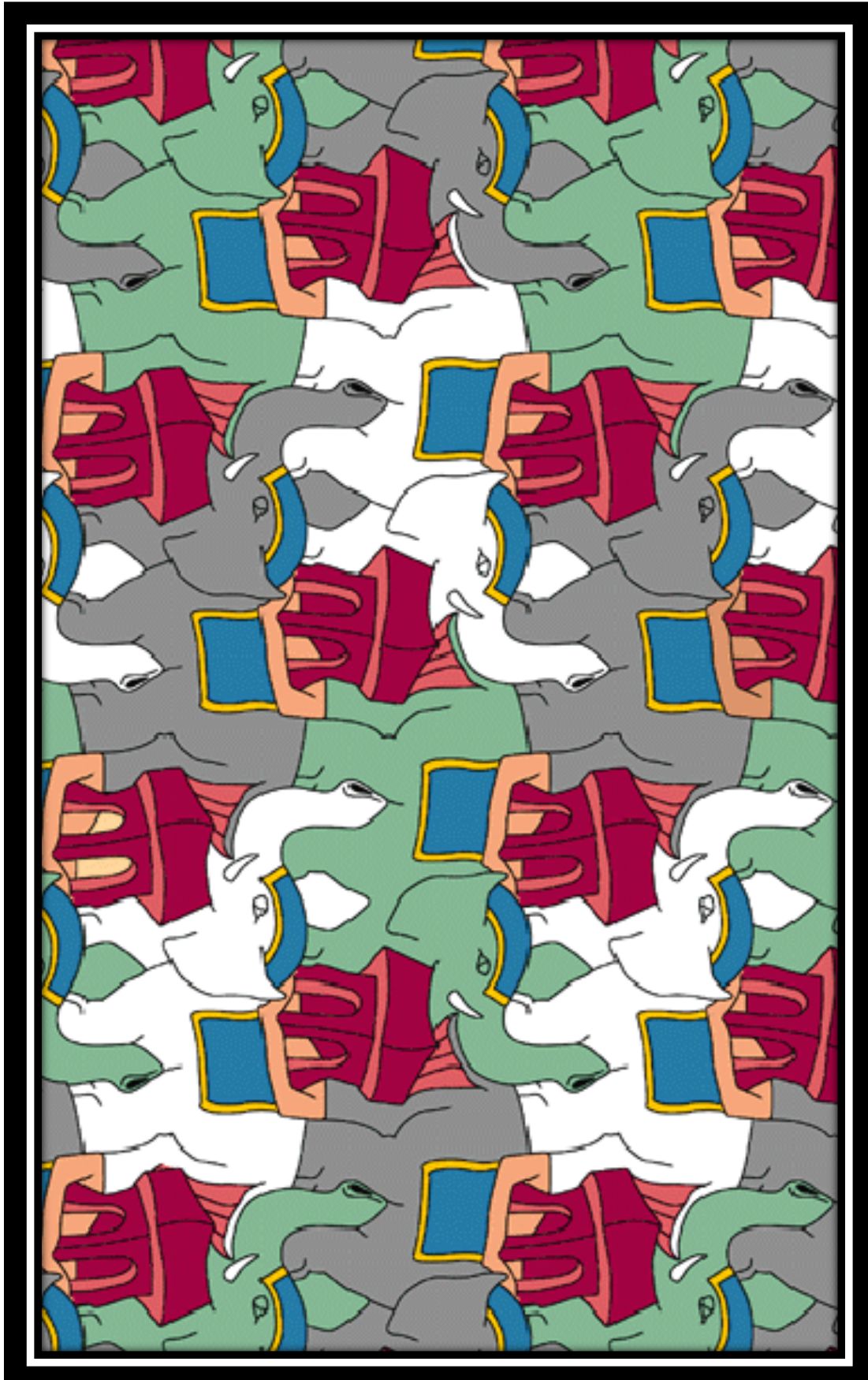
Tiling founded in a Peruvian carpet (Paracas).



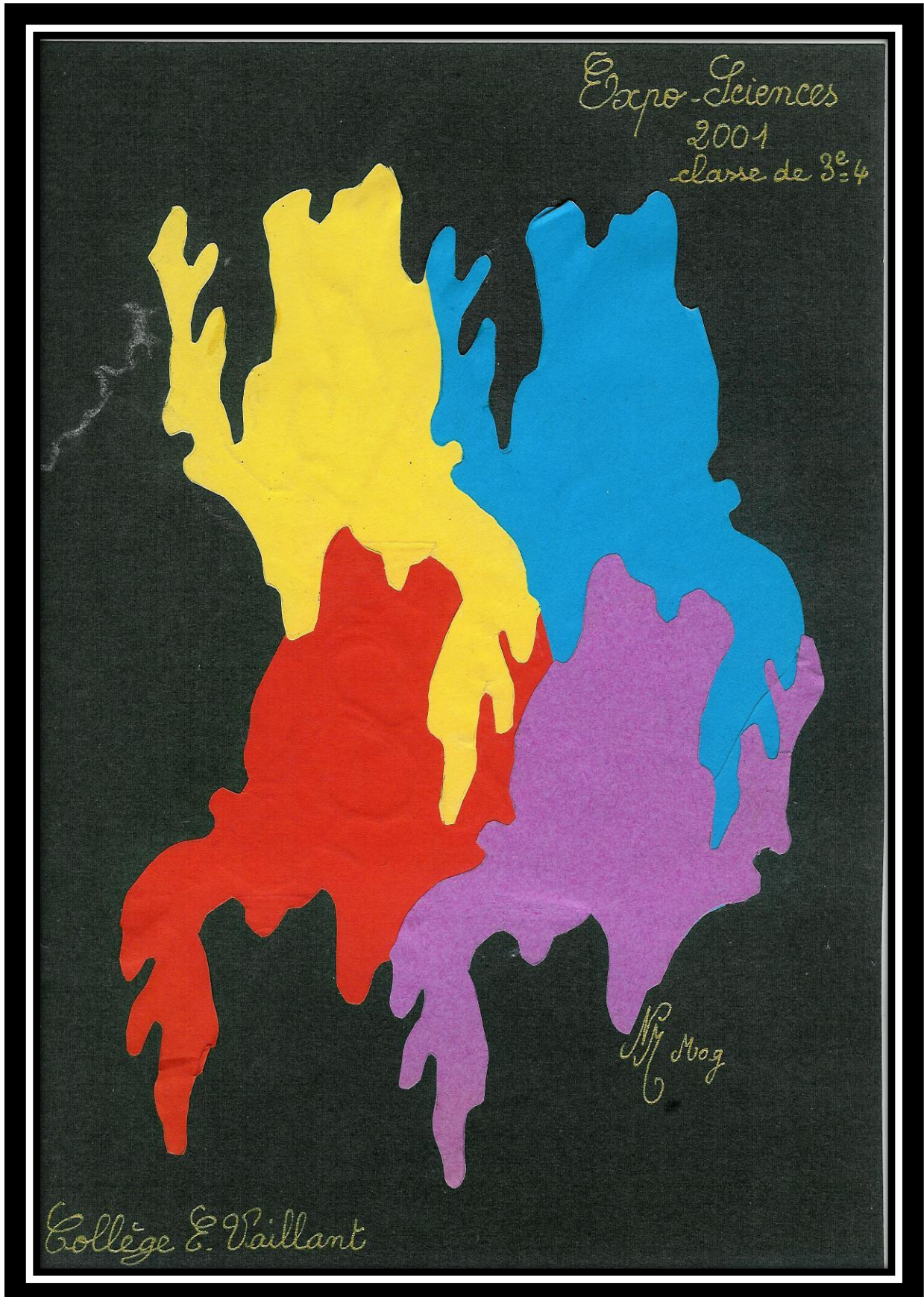
Tiling decoration of a wall in Mitla, Mexico



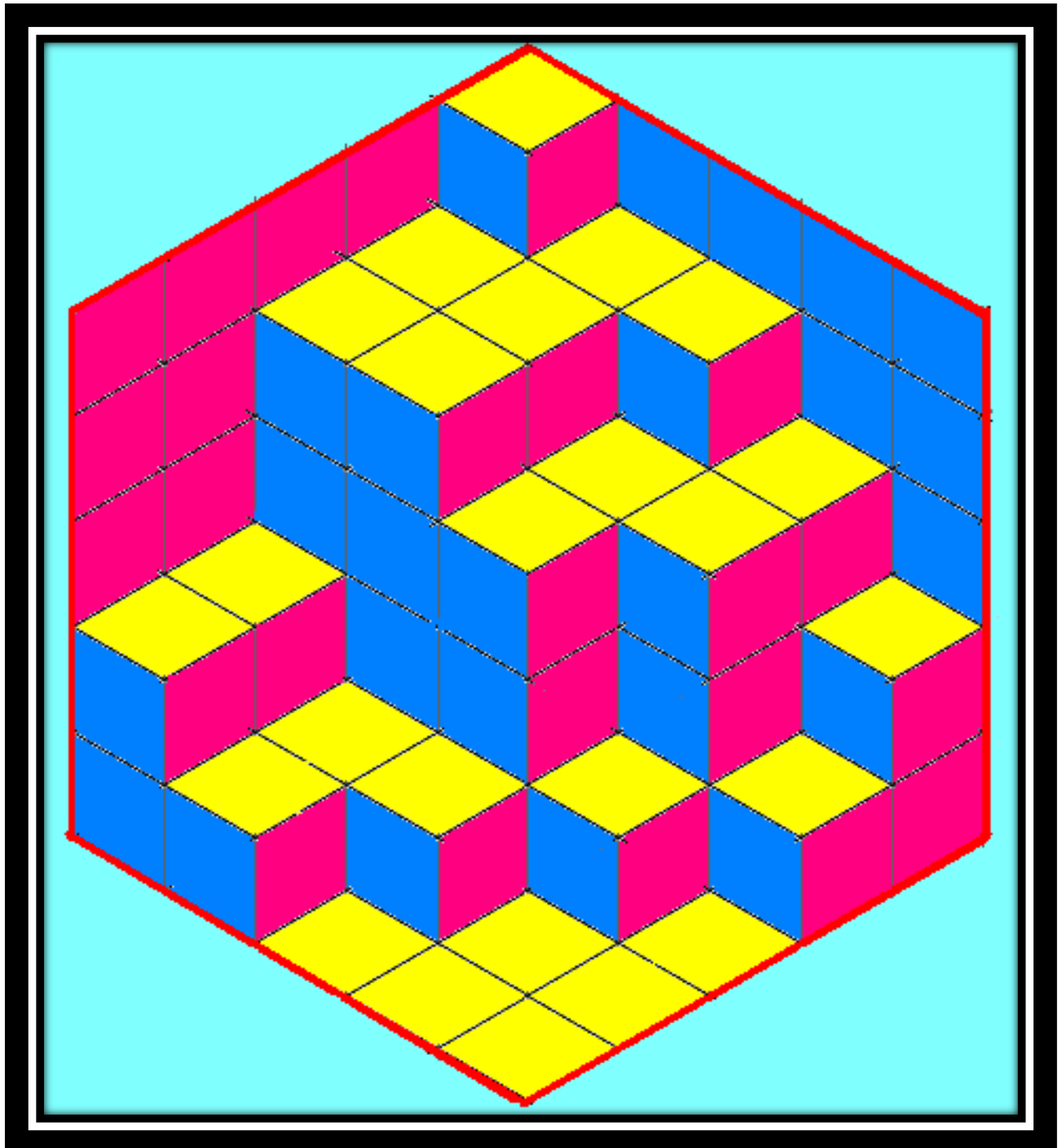
Potteries, Indians from North America.



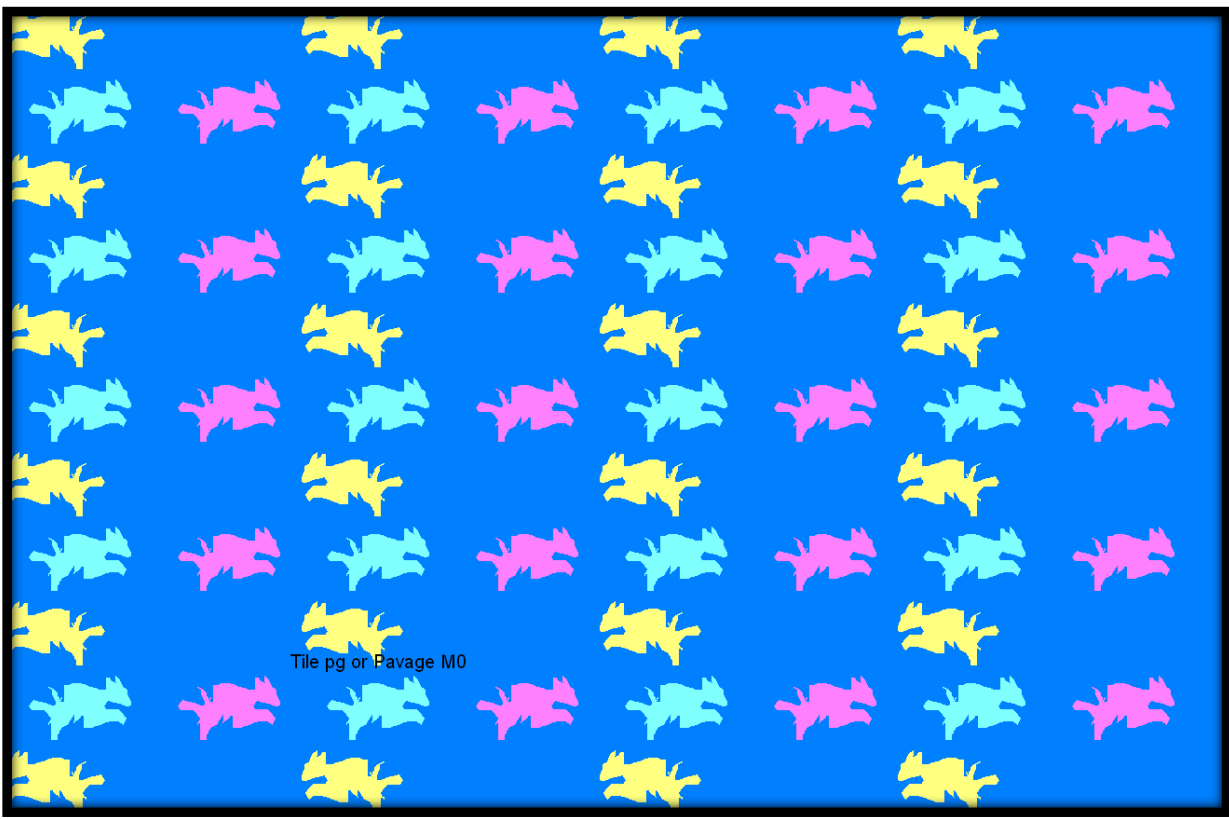
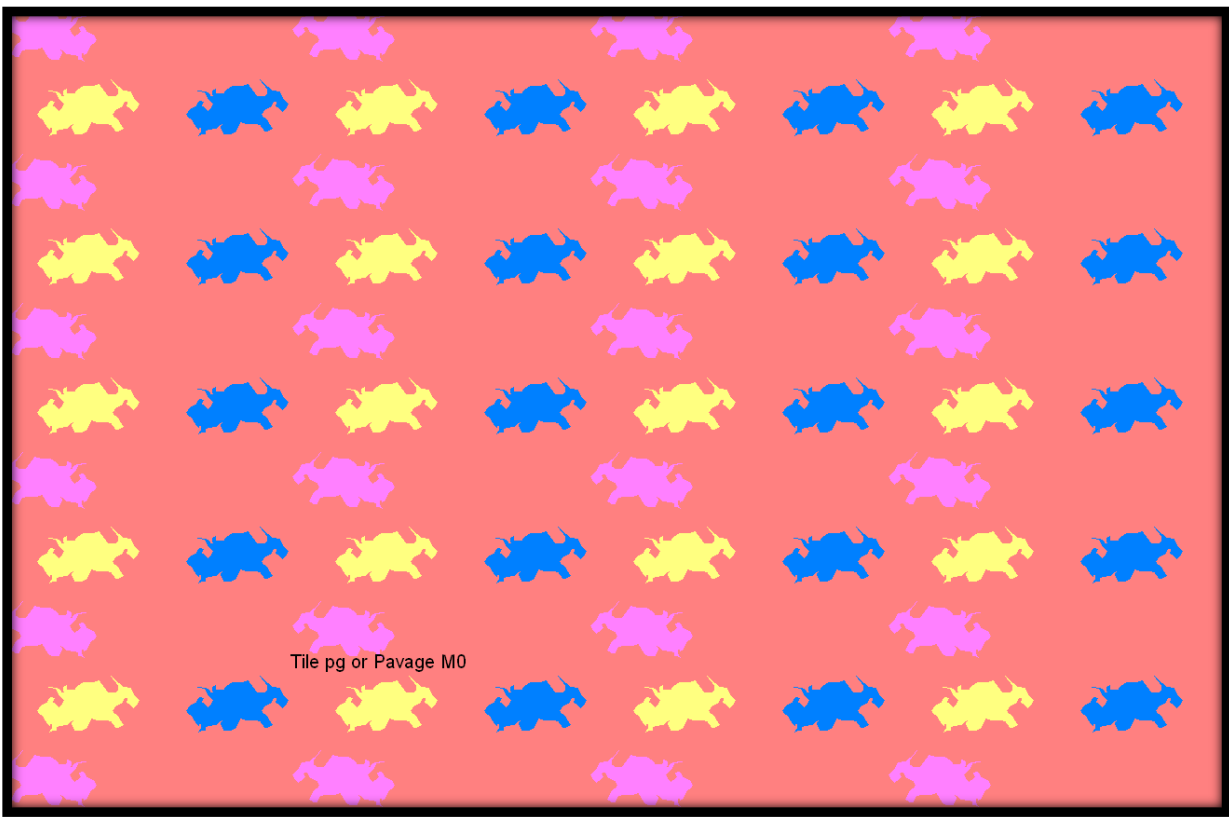
Escher's peint.

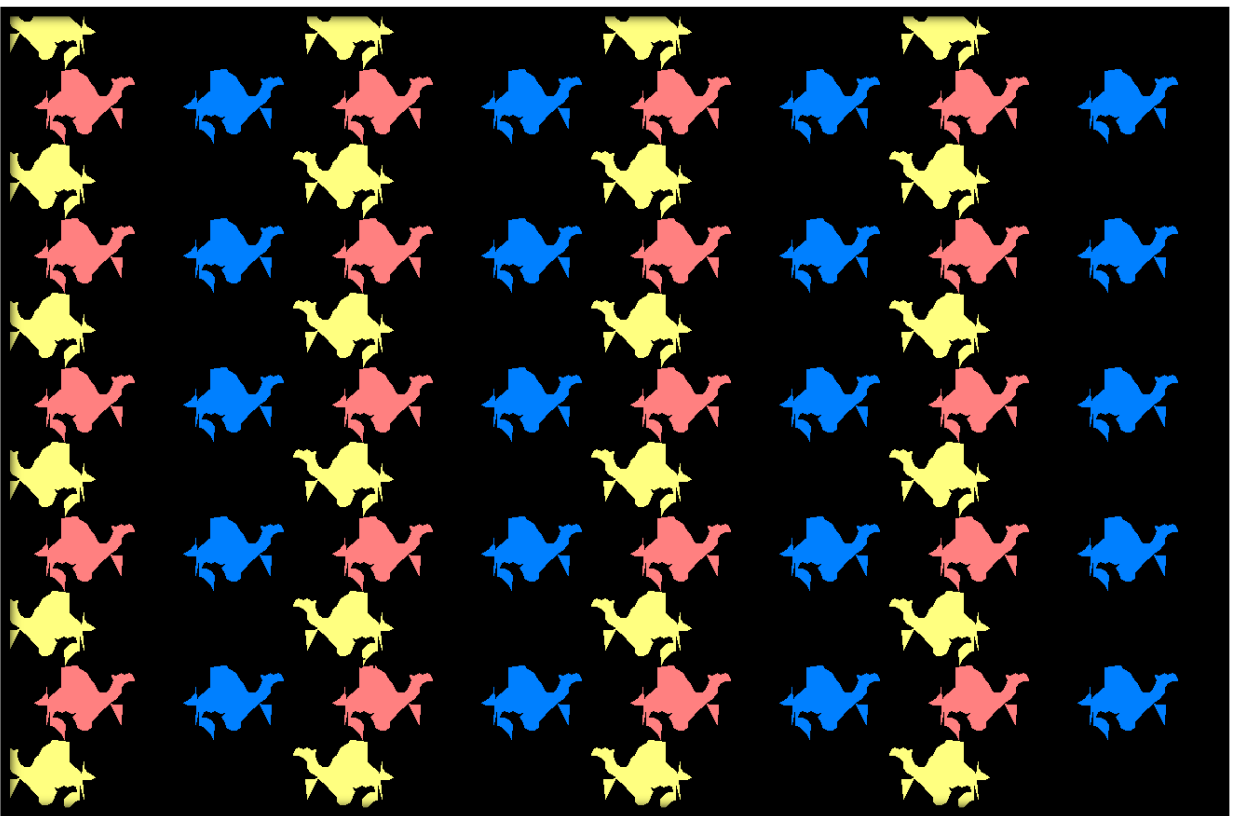
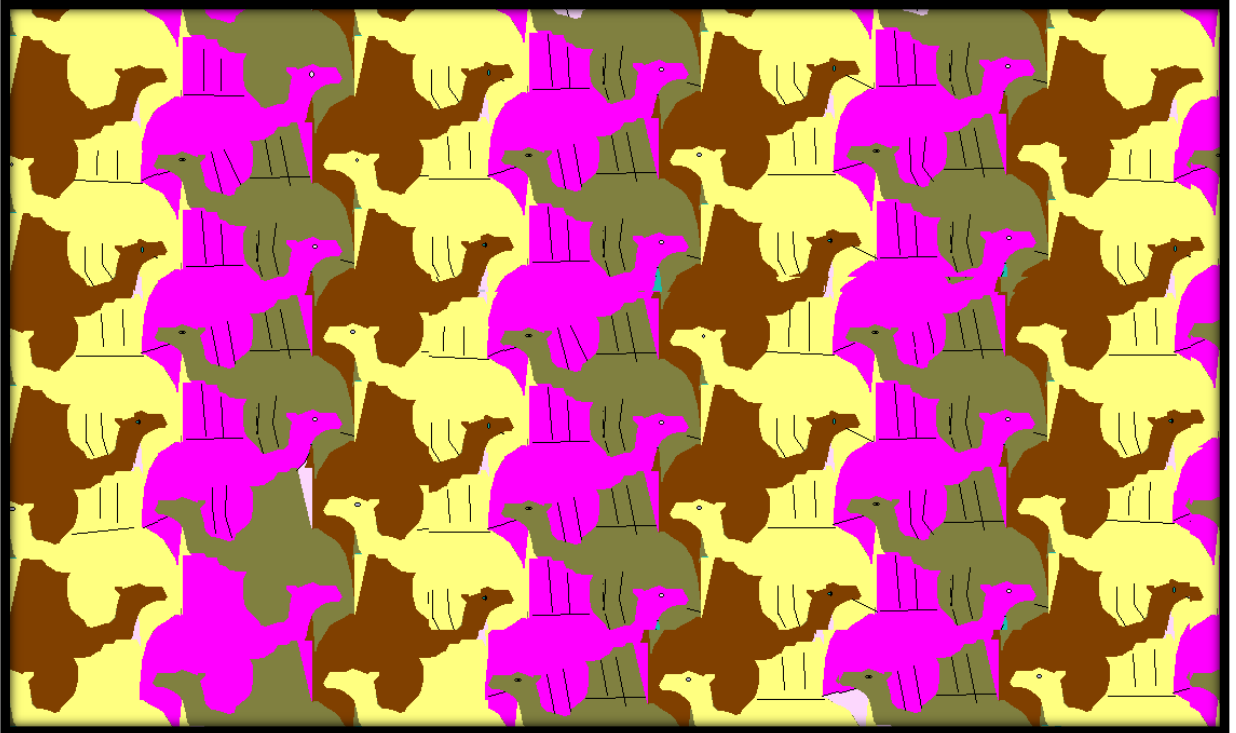


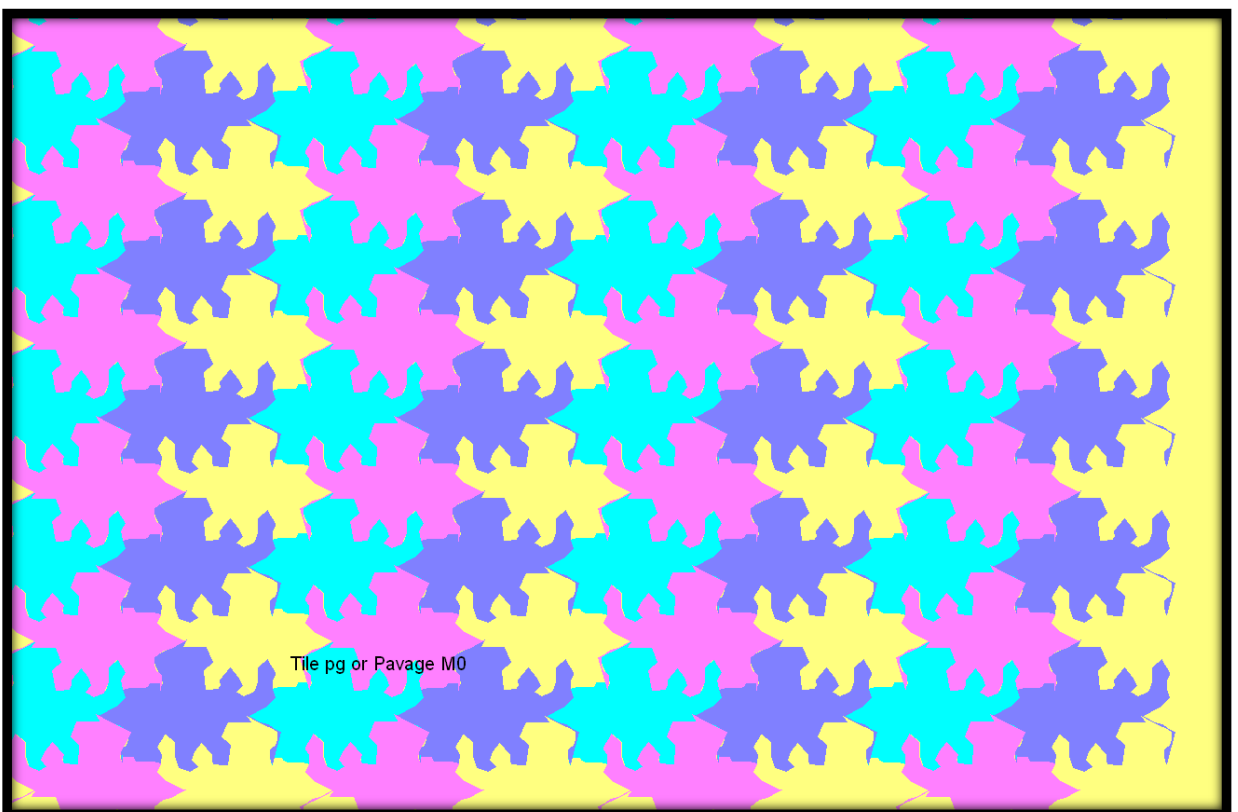
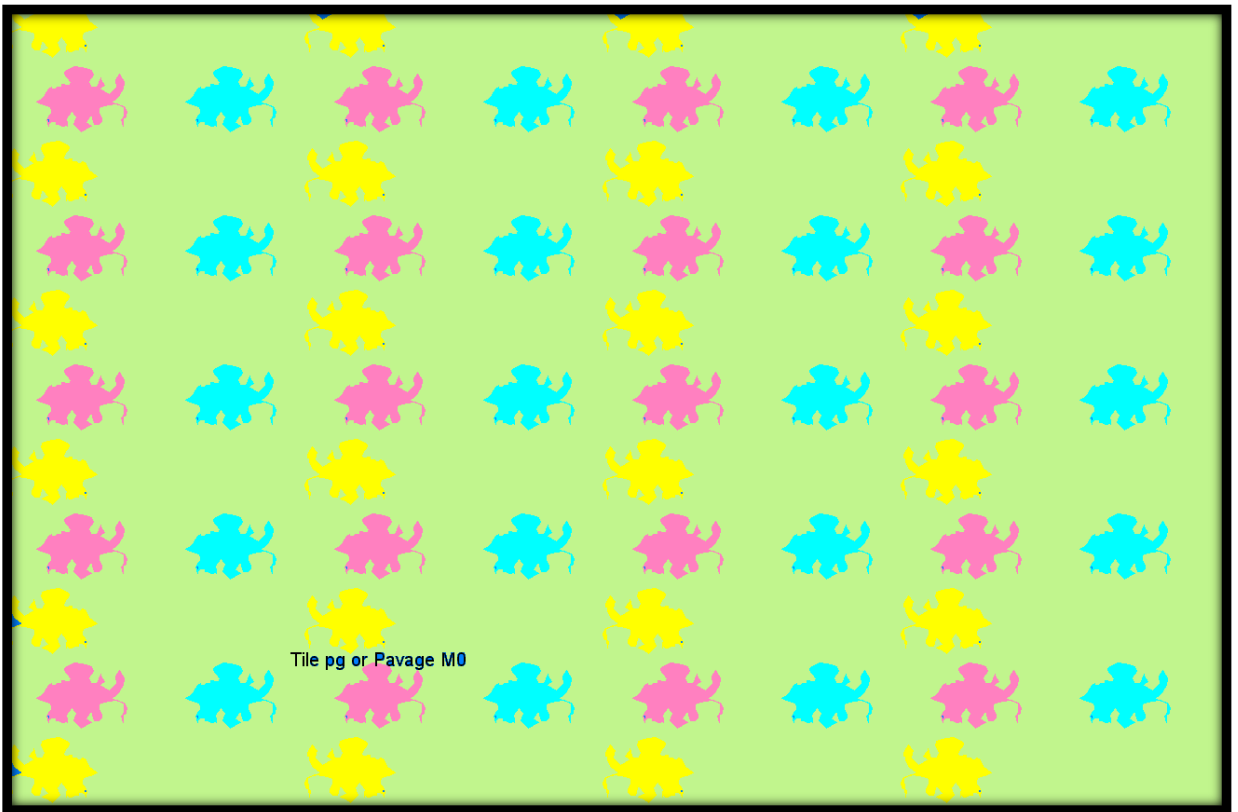
Tiling realized by a pupil in the High school.

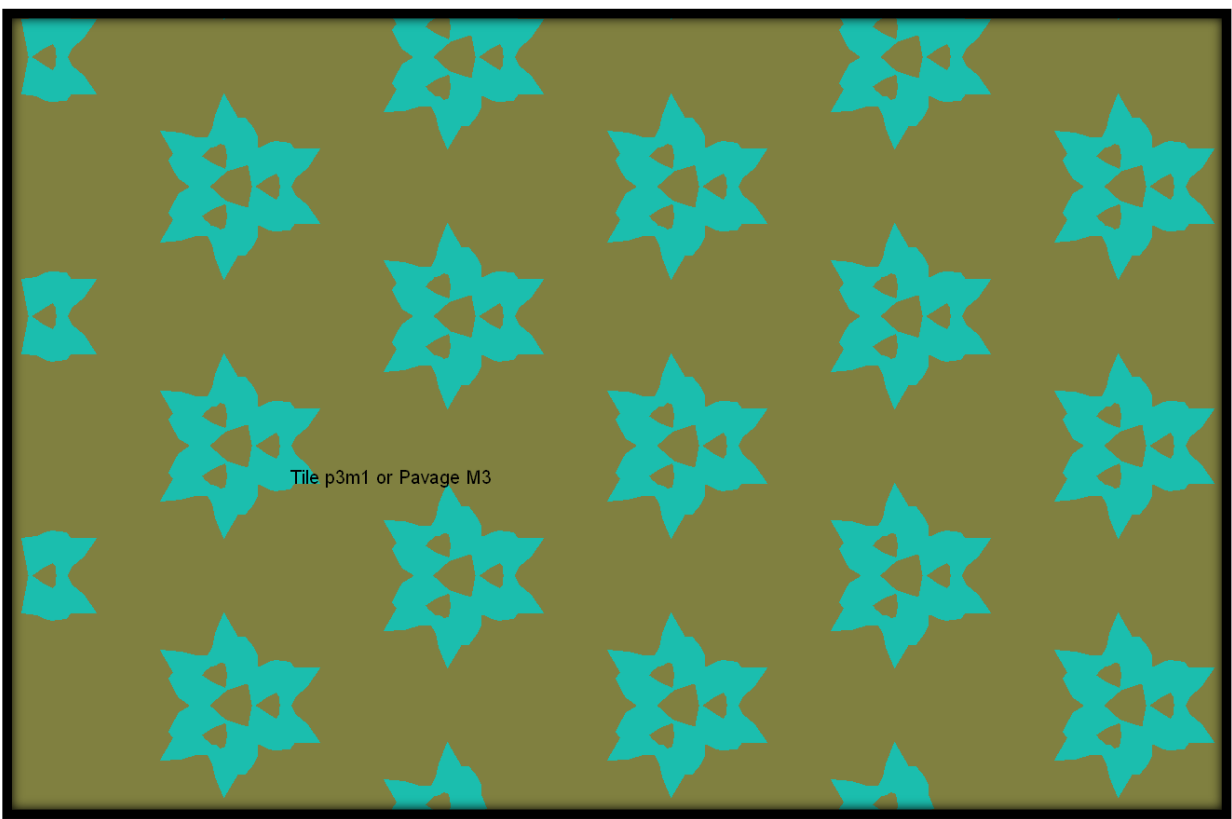
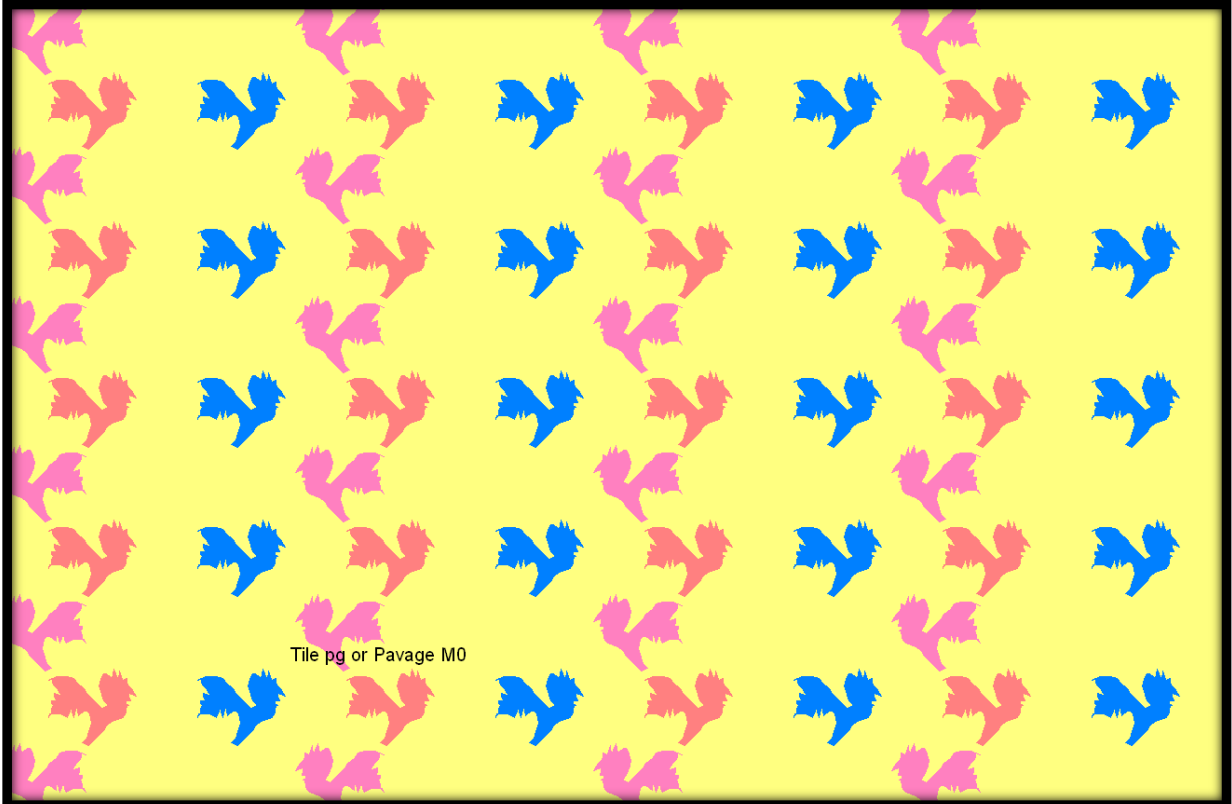


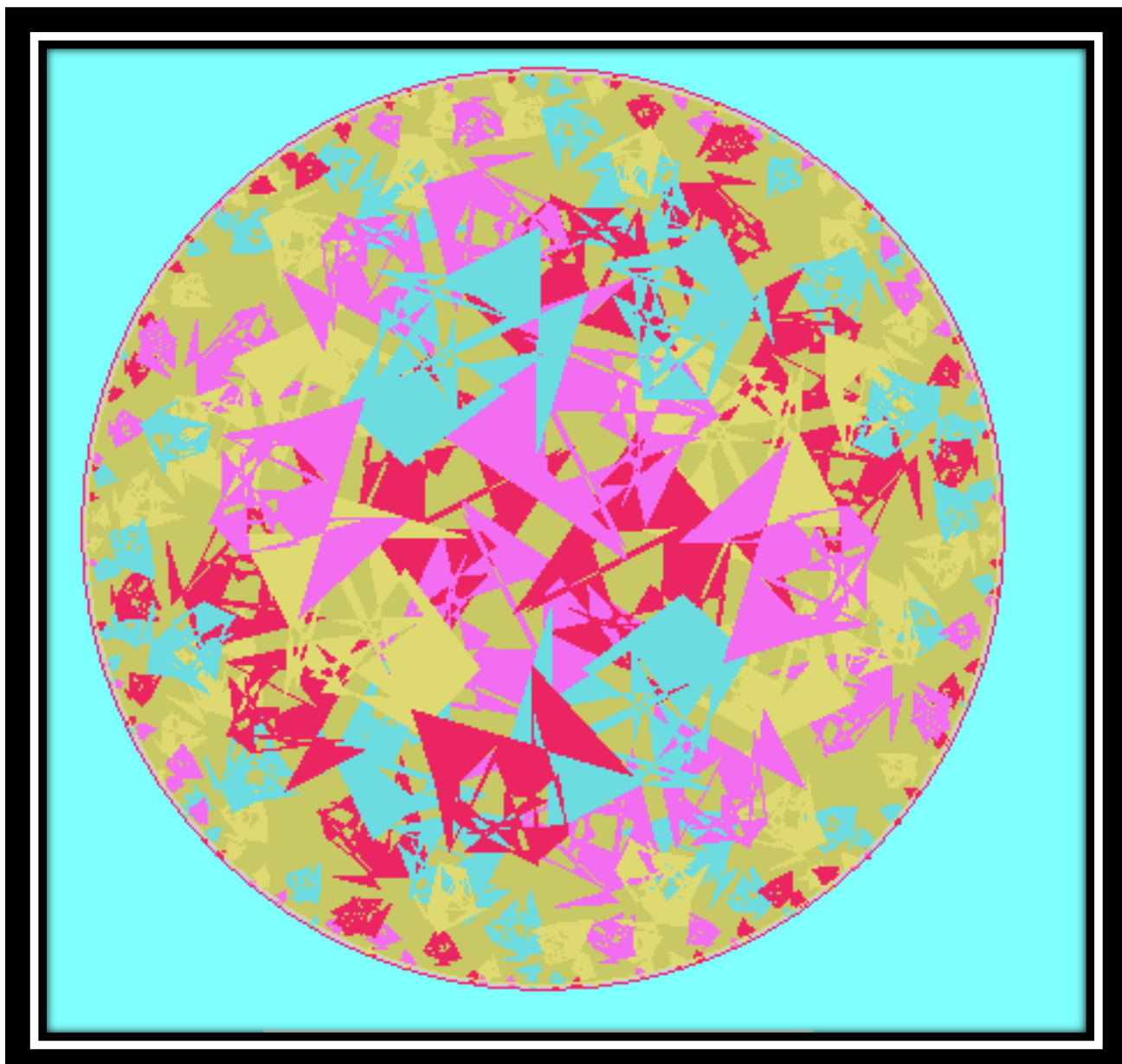
Non periodic tiling of an hexagon, by using a rhomb (colored yellow), transformed by using thee transformations (see the cube up): by translations it remains yellow, by rotation of 120° and translations it is colored red, by rotation of 240° and translations it is colored blue.

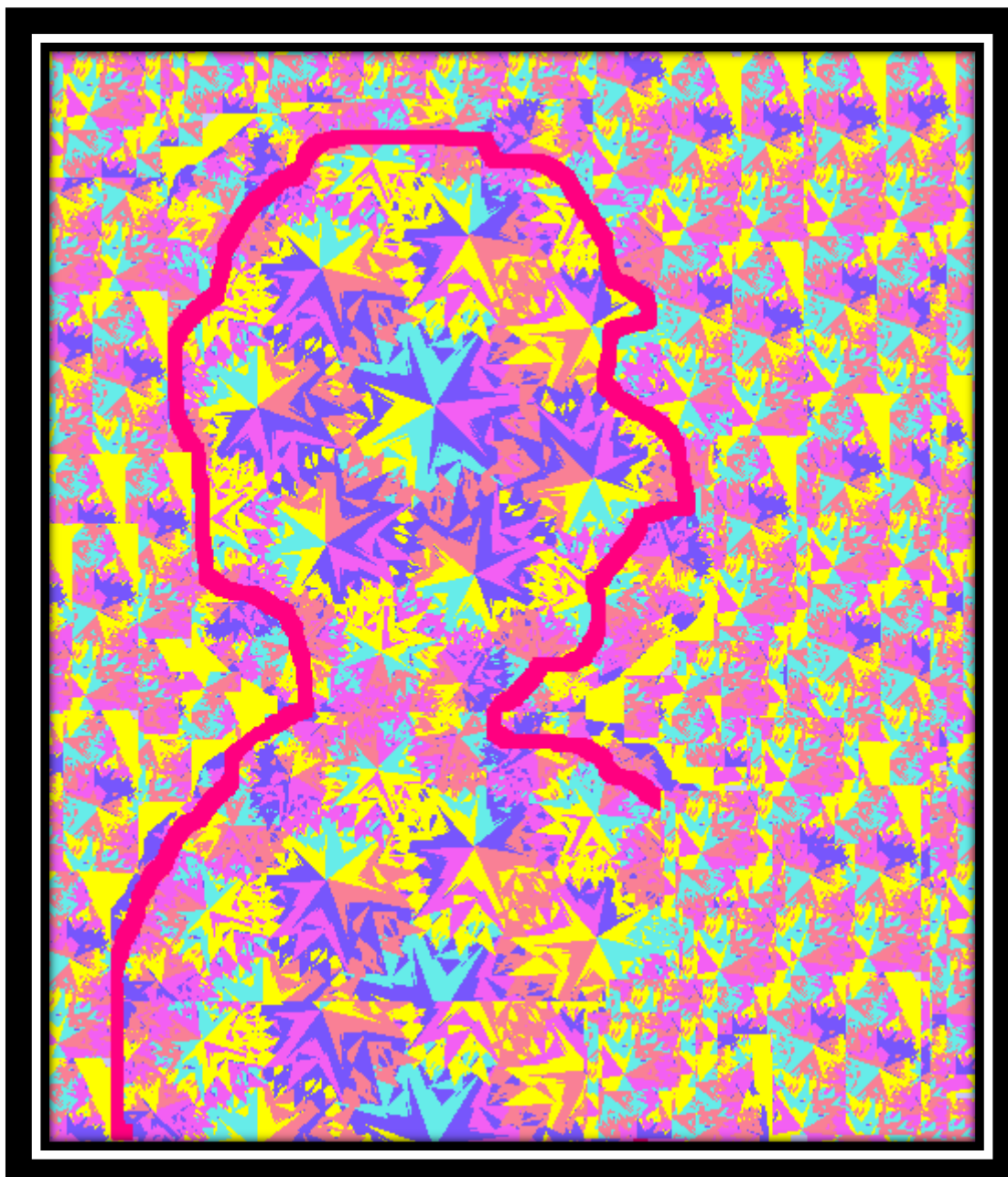


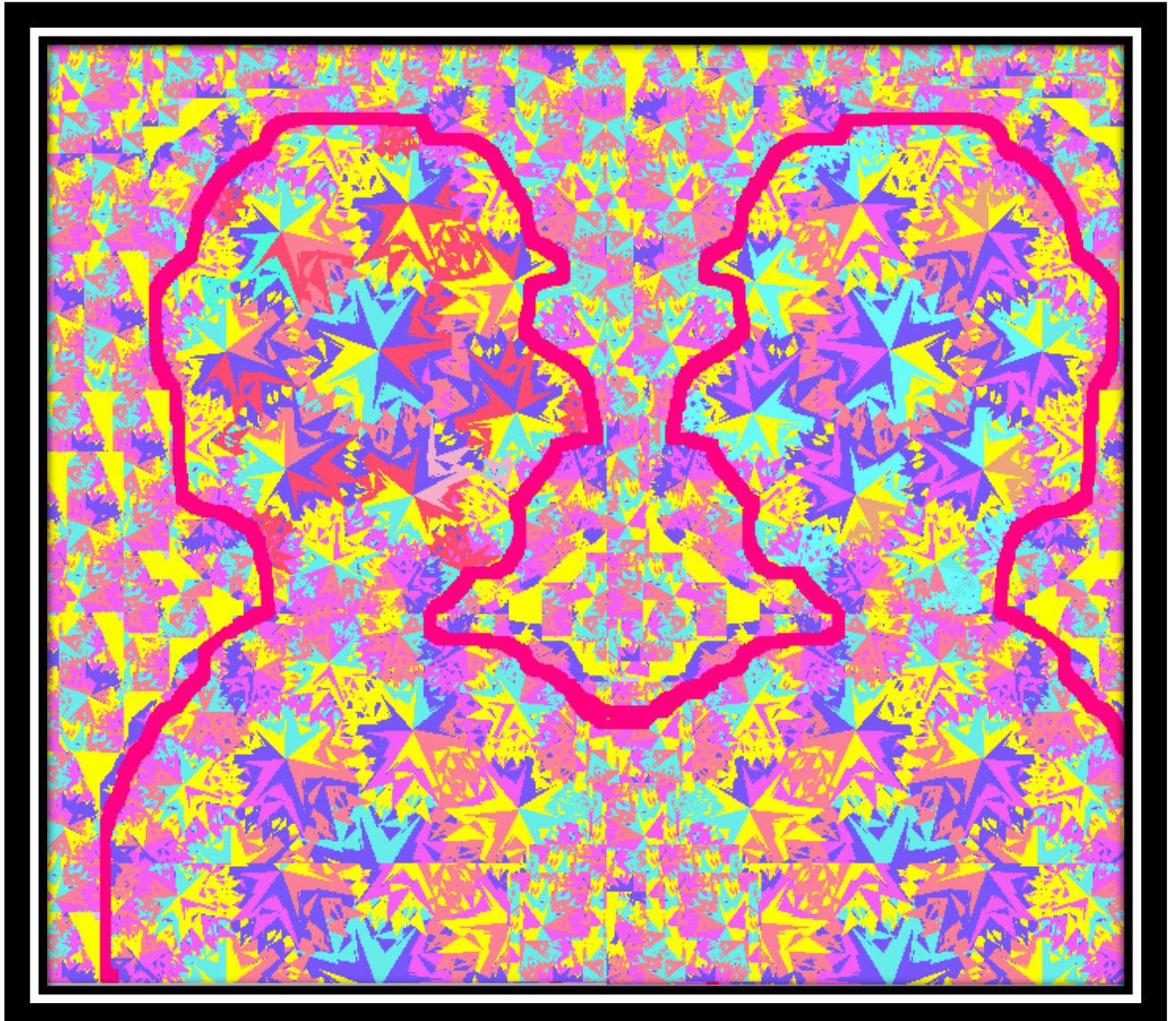


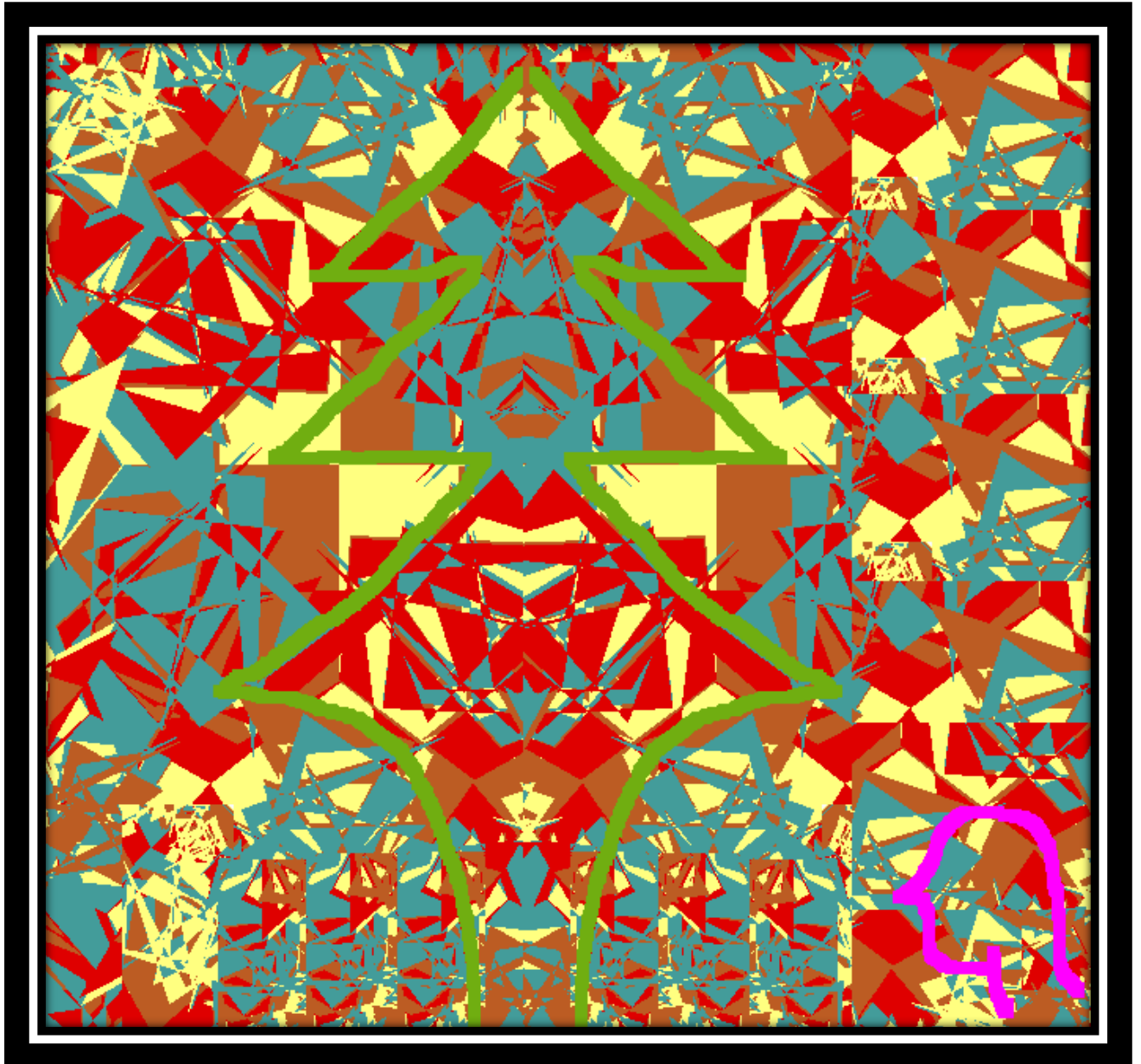








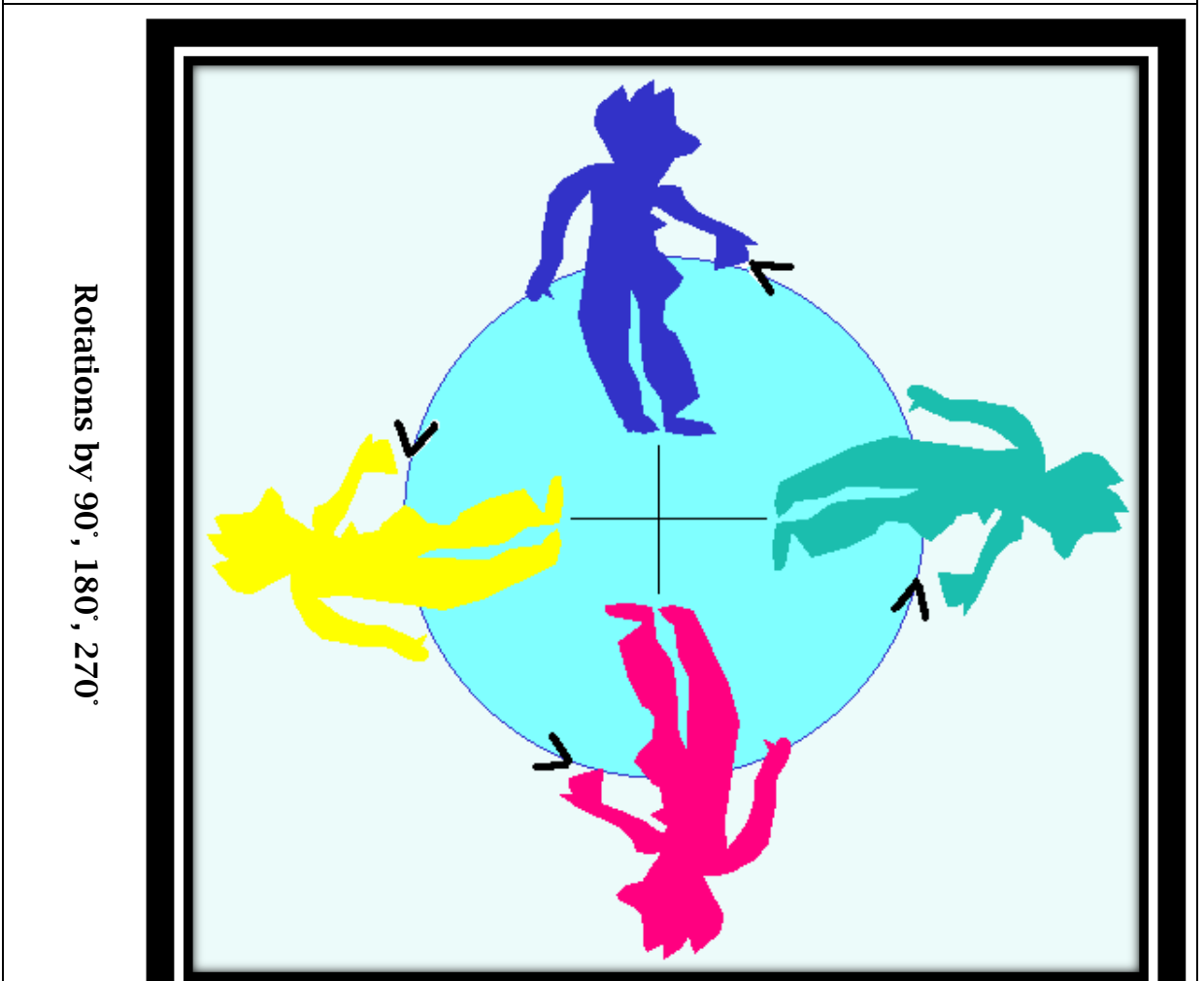




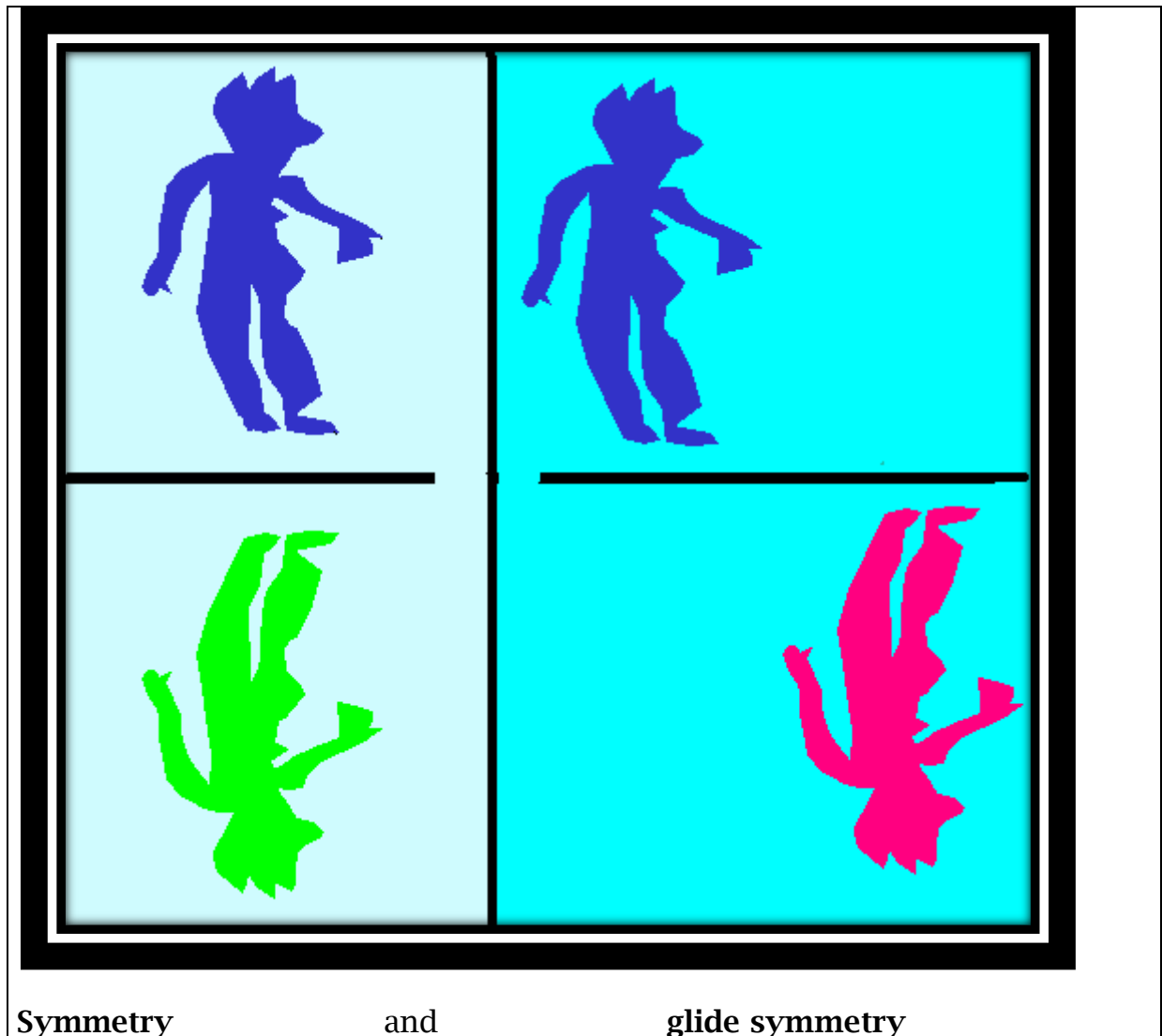
We give now a few on the transformations of the plane in Euclidian Geometry. **Translation, rotations, symmetry, and glide symmetry.**



Translation

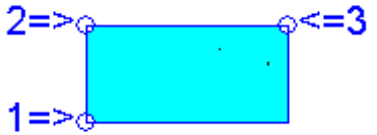


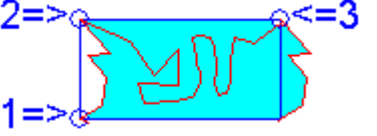
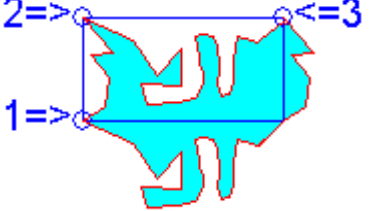
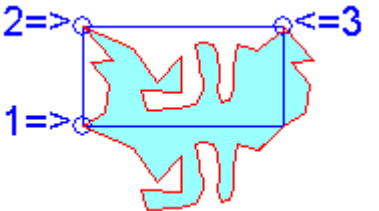


Rotations by 90°, 180°, 270°



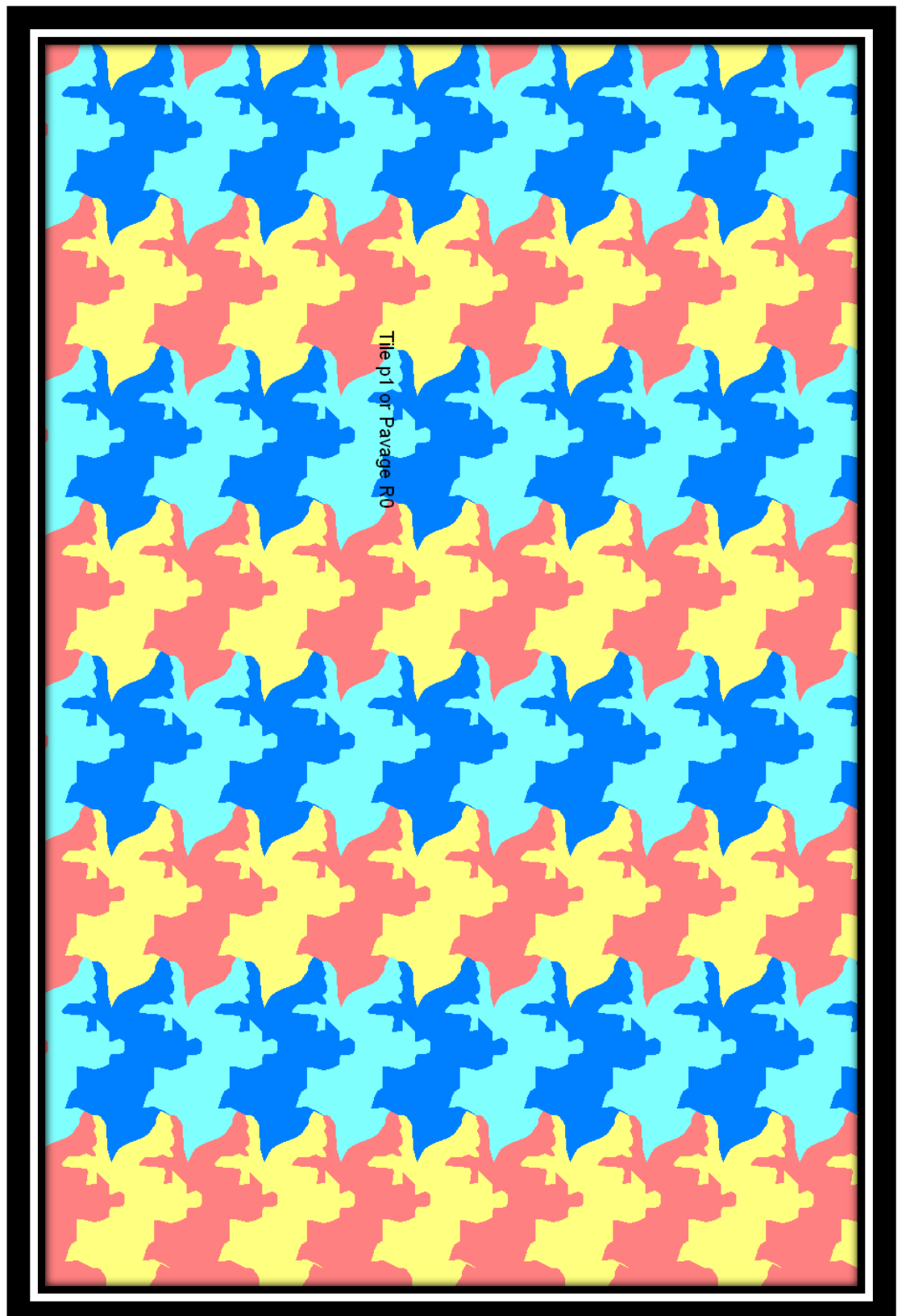
Doing a pattern for tiling the plane by using a sheet of paper and scissors.
We will describe the process for each one of the seventeen groups of tiling.
We will use both notations English and French. In each one we have numbered some special points

Tiling p1or R0

 <p>Take a rectangular piece of paper.</p>	 <p>Draw a simple curve starting in 1 and going to 2</p>	 <p>Cut along the curve and glue it on the right of your rectangle</p>
 <p>Draw a simple curve starting in 2 and going to 3</p>	 <p>Cut along the curve and glue it on the down of your rectangle</p>	 <p>This is your pattern for this tiling group.</p>

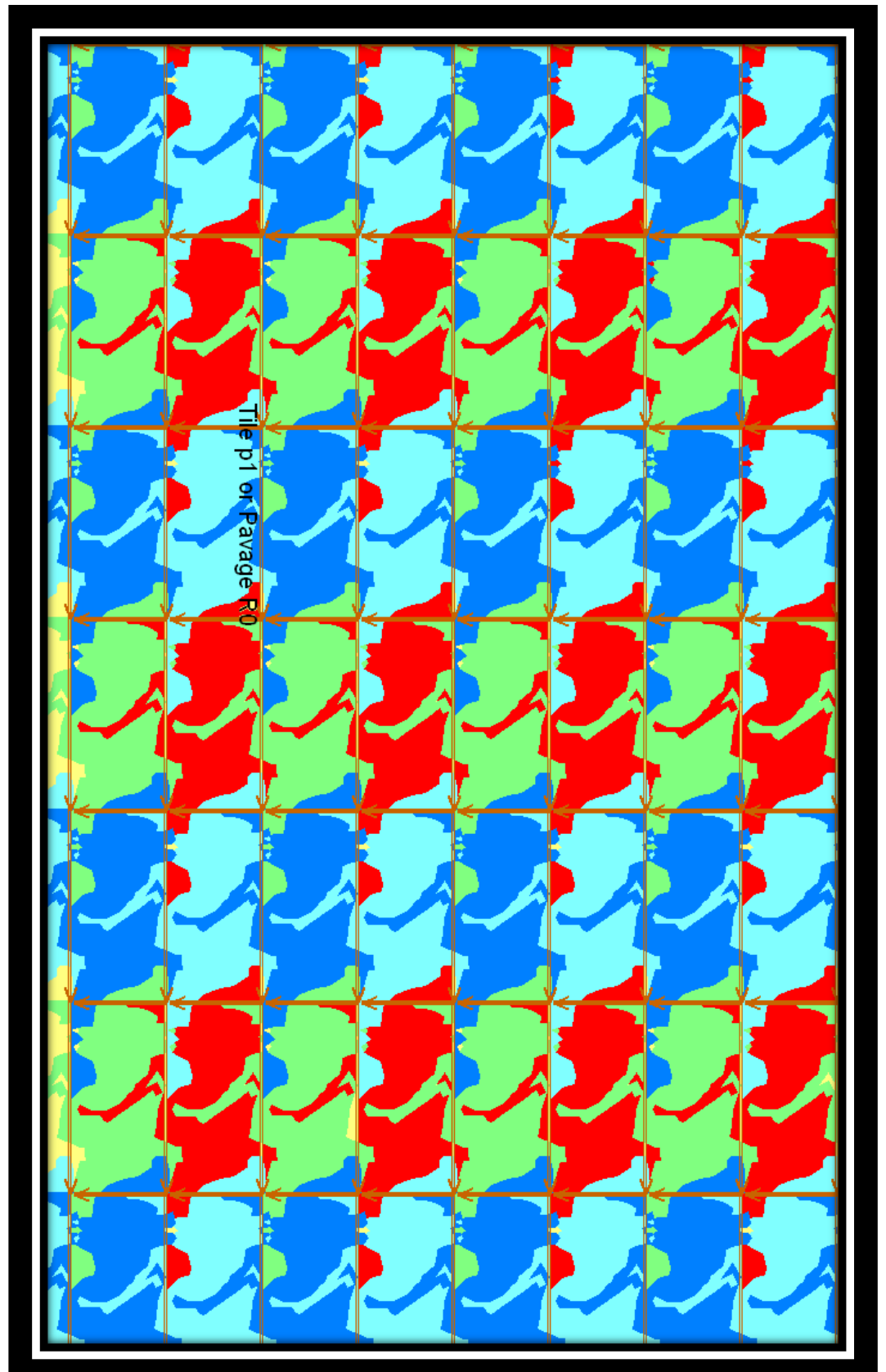
We fulfill the plane by using horizontal and vertical translations; we color it in order to distinguish them:



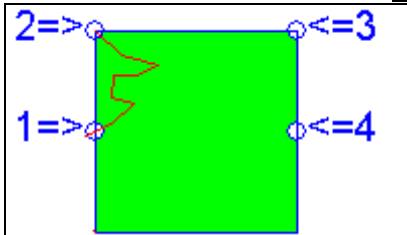
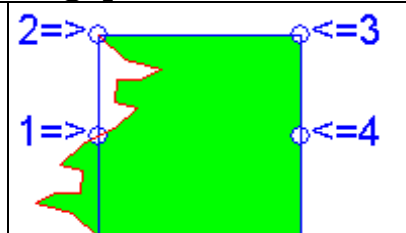
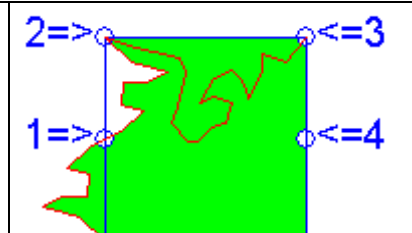
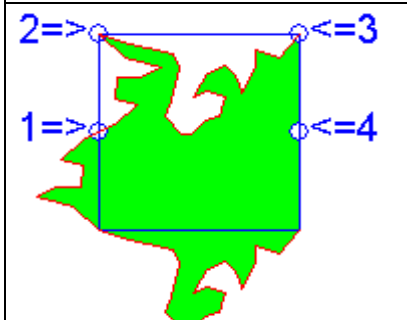
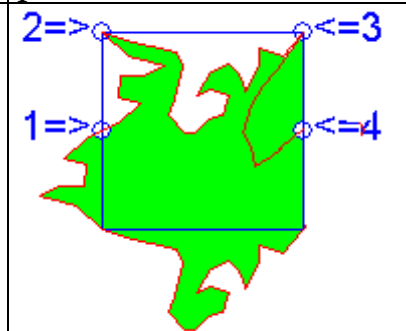
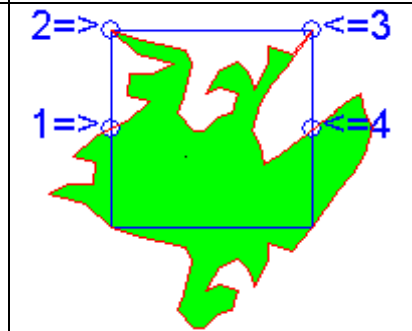


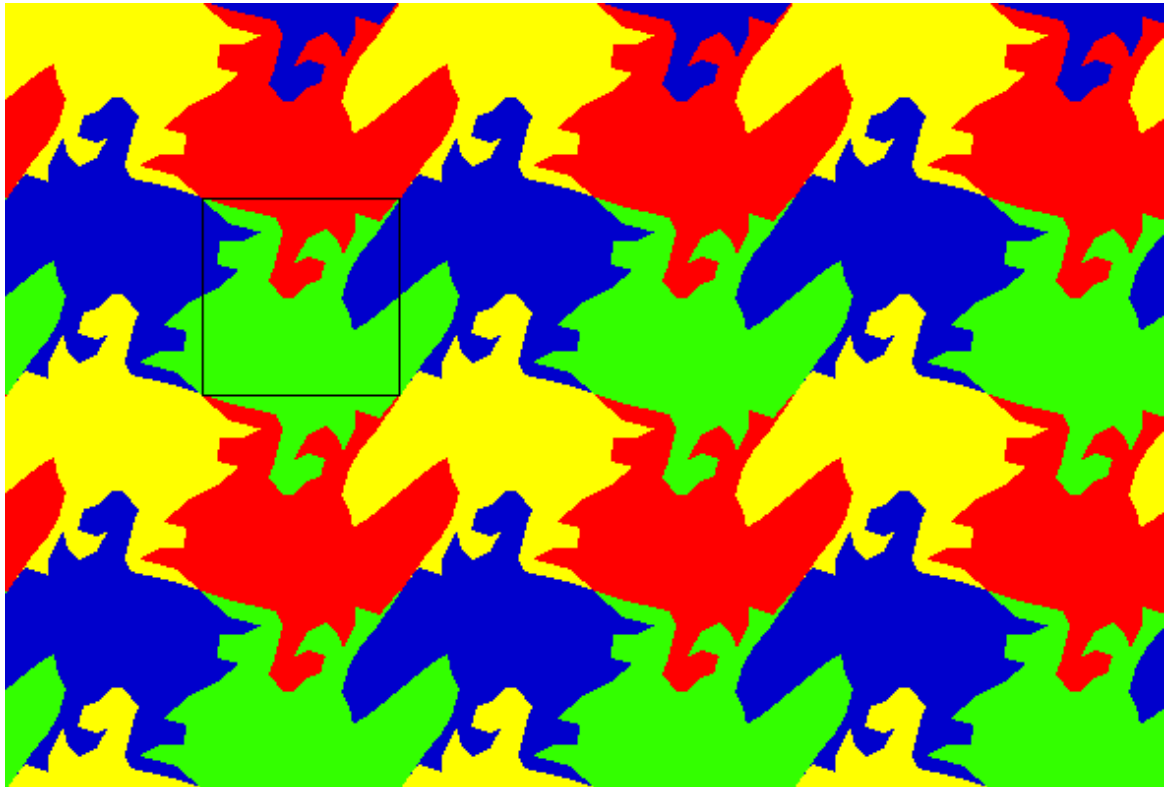
Lattice and fundamental region for p1 (R0)

The following picture is the lattice associated to the Tiling p1 (R0), any one of the rectangles is a fundamental region and as you can check the arrows are translations.



Tiling p2 or R2

 <p>Take a square piece of paper. Draw a simple curve starting in 1 and going to 2.</p>	 <p>Cut along the curve and glue it after doing a half-tour with center in the point 1.</p>	 <p>Draw a simple curve starting in 2 and going to 3</p>
 <p>Cut along the curve, translate it vertically and glue.</p>	 <p>Draw a simple curve starting in 3 and going to 4. Cut along the curve and glue it after doing a half-tour with center in the point 4.</p>	 <p>This is your pattern for this tiling group.</p>



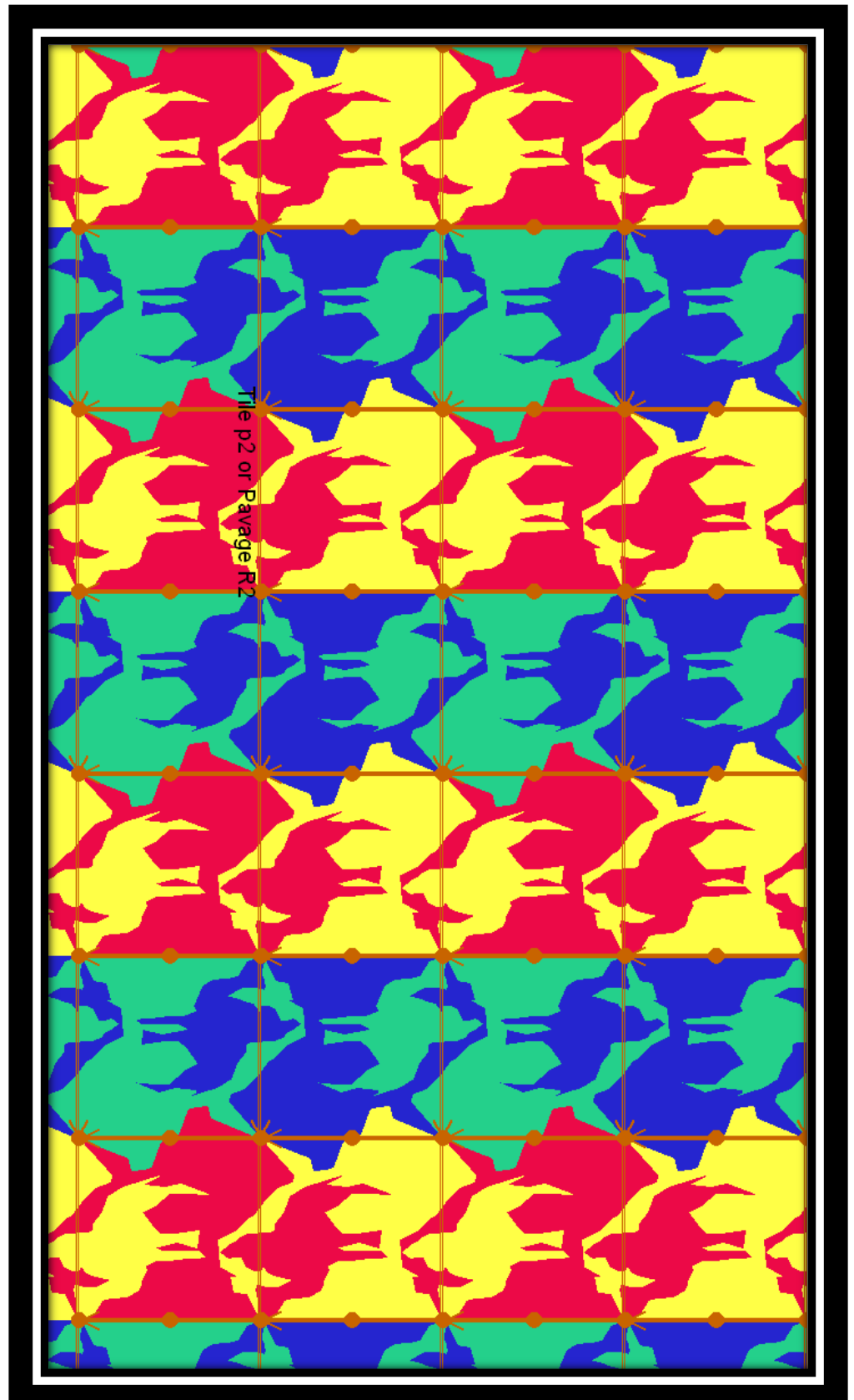
We fulfill the plane by using rotations of angle 180° and translations.



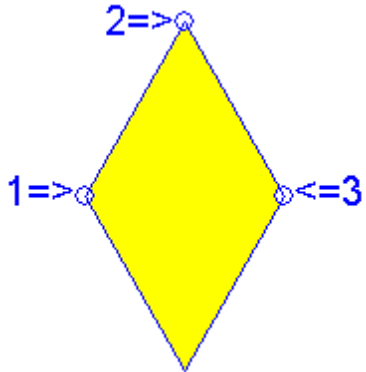
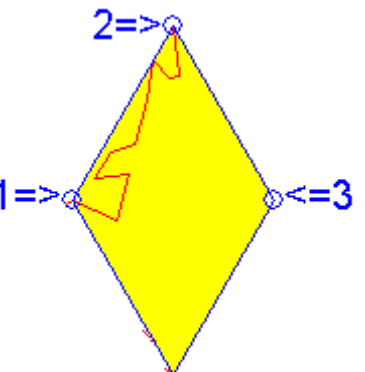
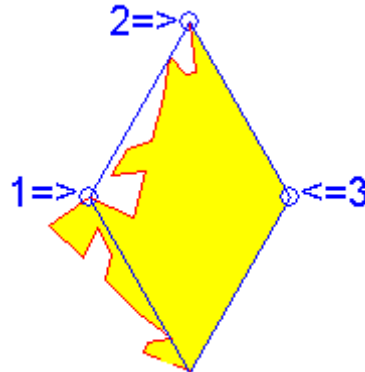
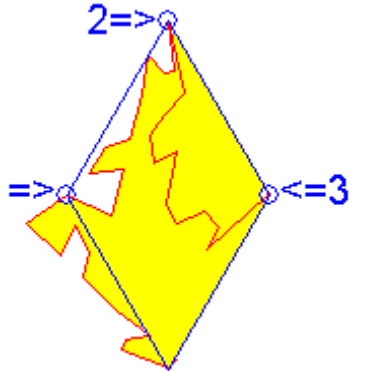
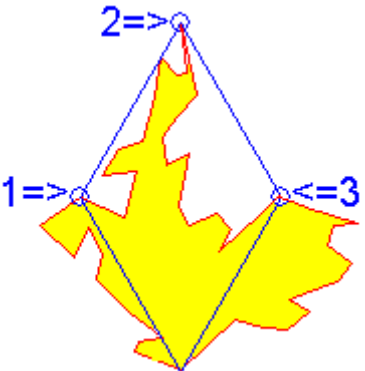
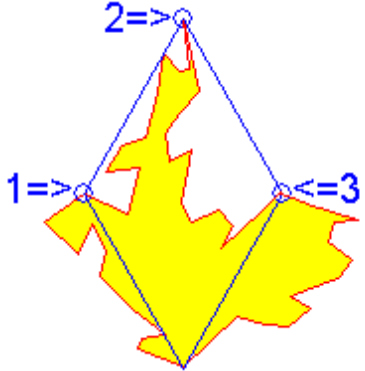
Lattice and fundamental region for p2 (R2)

The following picture is the lattice associated to the Tiling p2 (R2), any one of the squares is a fundamental region and as you can check the arrows are

translations, and the circles are centers of rotation π , also called central symmetry.

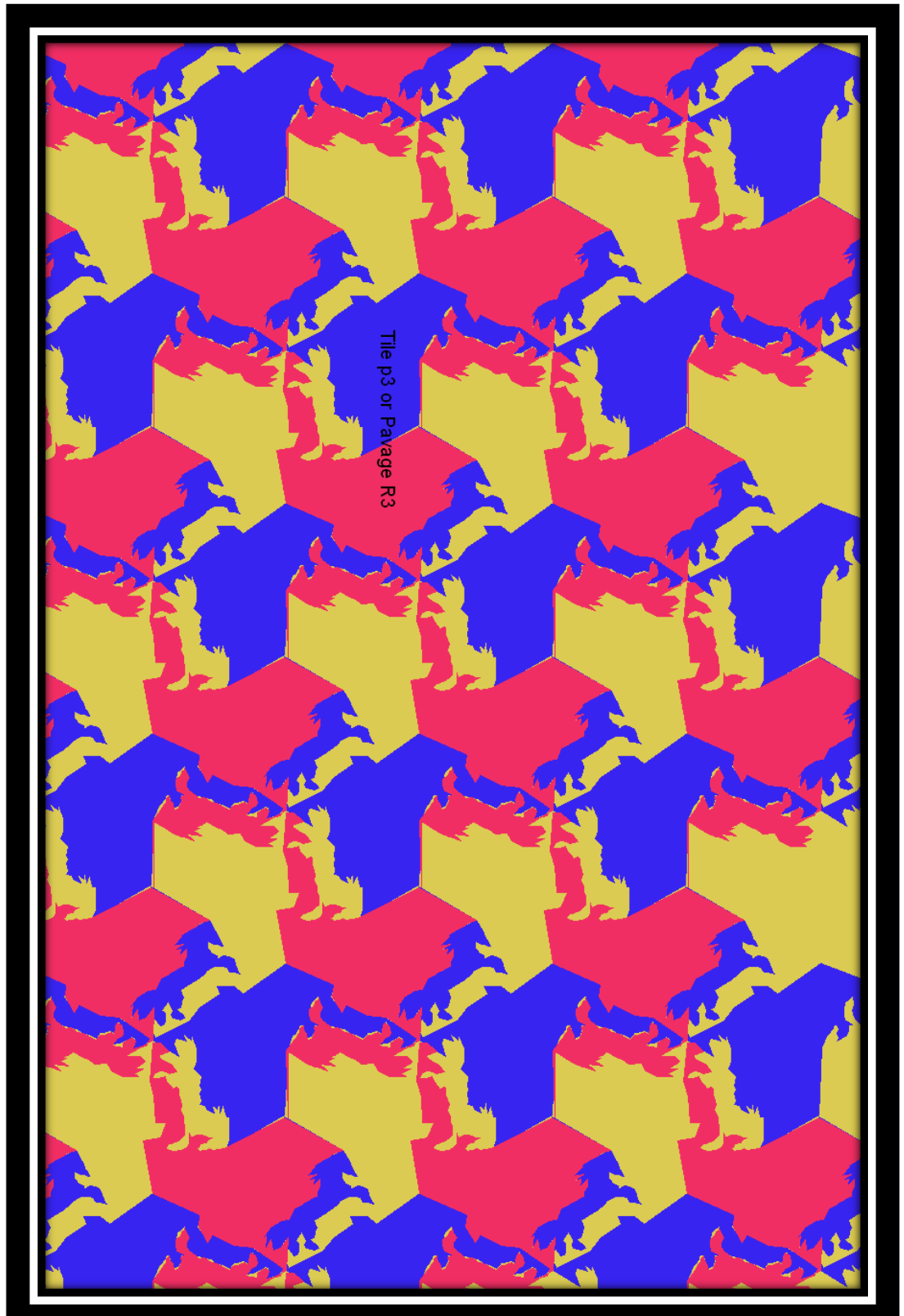


Tiling p3 ou R3

 <p>Take a rhomb piece of paper. (two equilateral triangles)</p>	 <p>Draw a simple curve starting in 1 and going to 2.</p>	 <p>Cut along the curve and glue after a rotation of 120° with center in the point 1.</p>
 <p>Draw a simple curve starting in 2 and going to 3.</p>	 <p>Cut along the curve and glue after a rotation of 120° with center in the point 3.</p>	 <p>This is your pattern for this tiling group.</p>

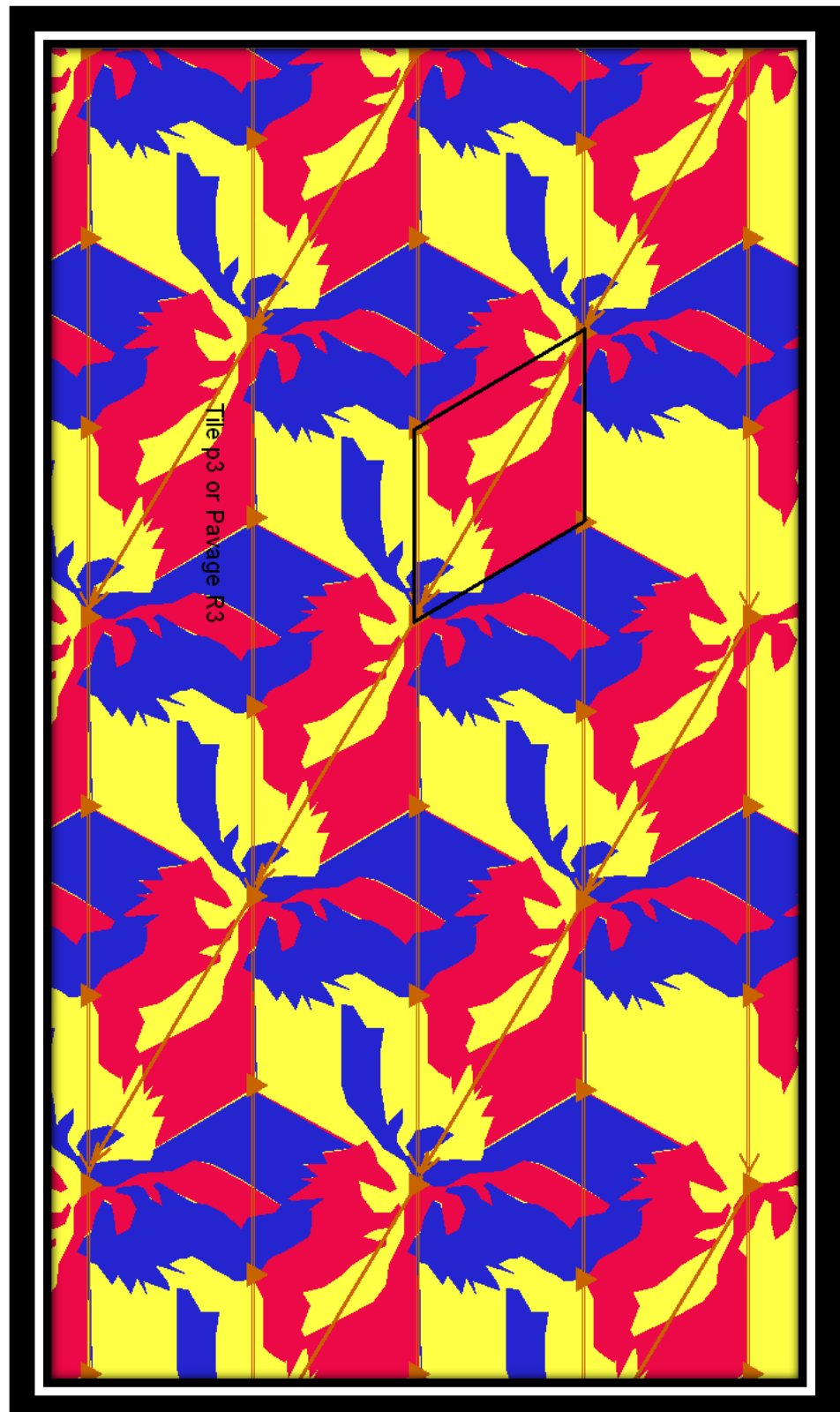
We fulfill the plane by using rotations of angle 120° and translations:



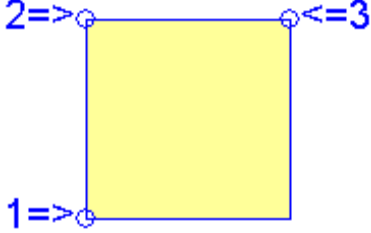
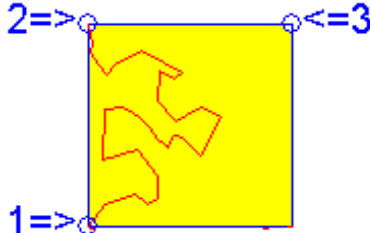
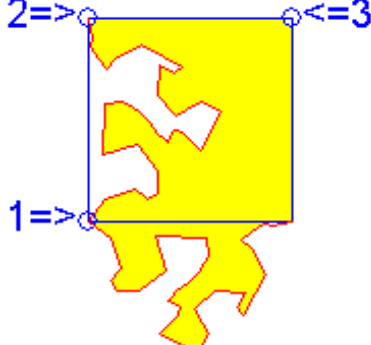
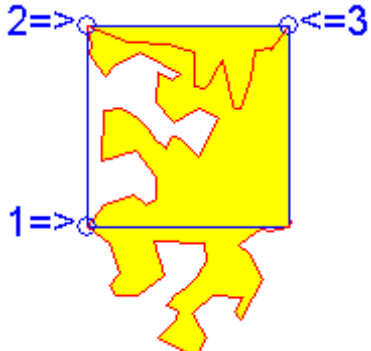
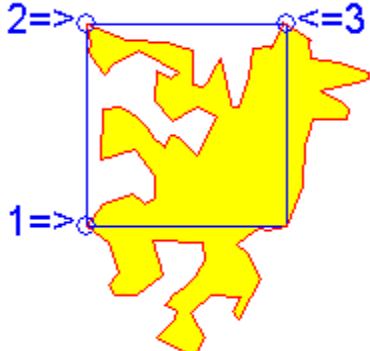
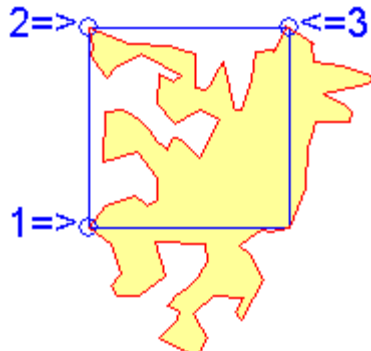


Lattice and fundamental region for p3 (R3)

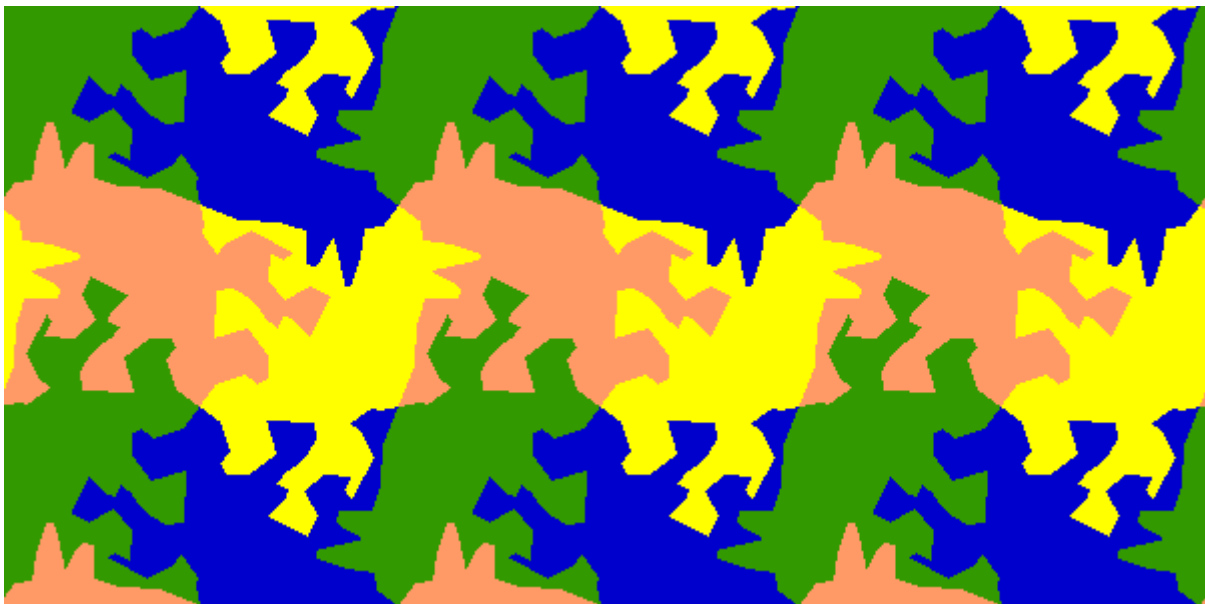
The following picture is the lattice associated to the Tiling p2 (R2), the rhombus drawn is a fundamental region and as you can check the arrows are translations, and the small triangles are centers of rotation $2\pi/3$.



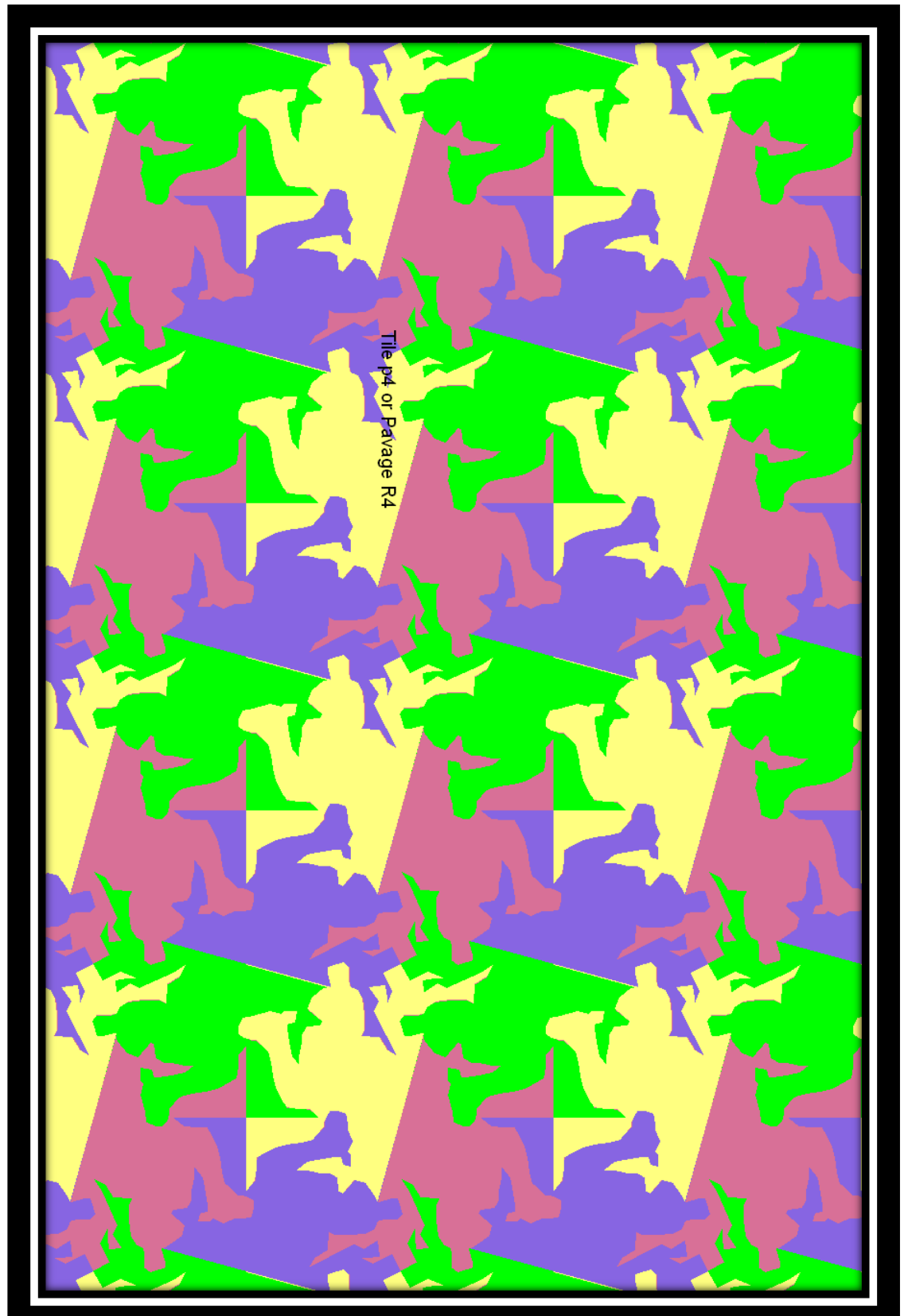
Tiling p4 ou R4

 <p>2=> <=&3</p> <p>1=></p> <p>Take a square piece of paper.</p>	 <p>2=> <=&3</p> <p>1=></p> <p>Draw a simple curve starting in 1 and going to 2.</p>	 <p>2=> <=&3</p> <p>1=></p> <p>Cut along the curve and glue after a rotation of 90° with center in the point 1.</p>
 <p>2=> <=&3</p> <p>1=></p> <p>Draw a simple curve starting in 2 and going to 3.</p>	 <p>2=> <=&3</p> <p>1=></p> <p>Cut along the curve and glue after a rotation of 120° with center in the point 3.</p>	 <p>2=> <=&3</p> <p>1=></p> <p>This is your pattern for this tiling group.</p>

We fulfill the plane by using rotations of angles 90°, 180°, 270° and

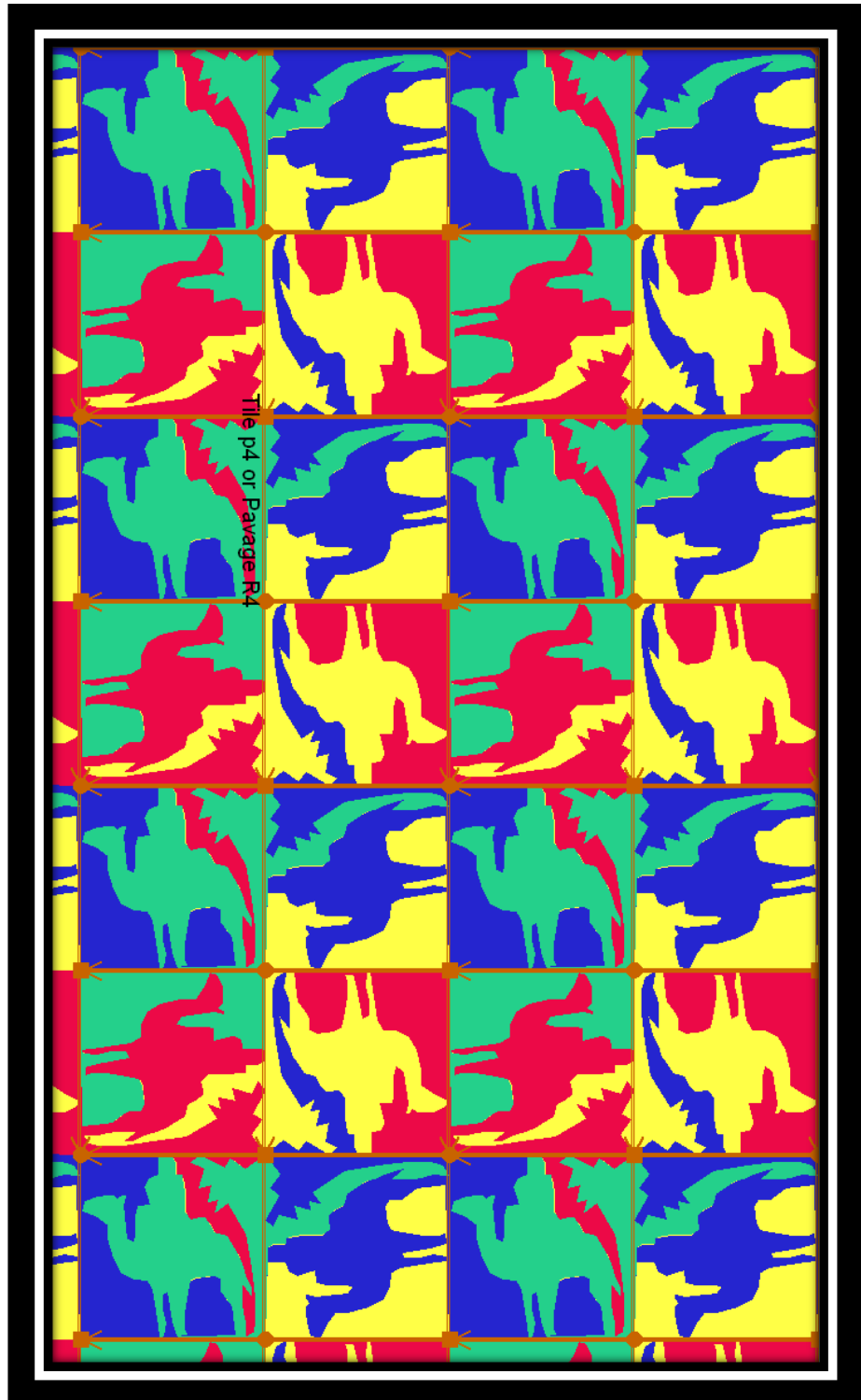


translations:



Lattice and fundamental region for p4 (R4)

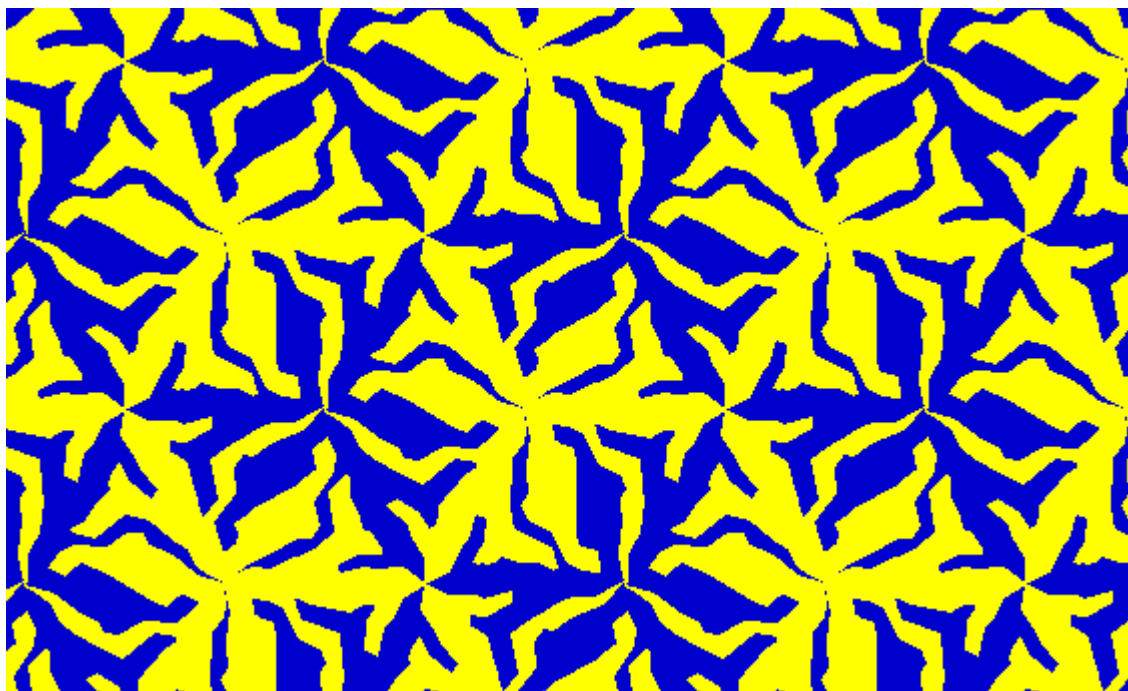
The following picture is the lattice associated to the Tiling p4 (R4), any square drawn is a fundamental region and as you can check the arrows are translations, the circles are centers of rotation π and the small squares are centers of rotation $\frac{\pi}{2}, \pi, \frac{3\pi}{2}$.

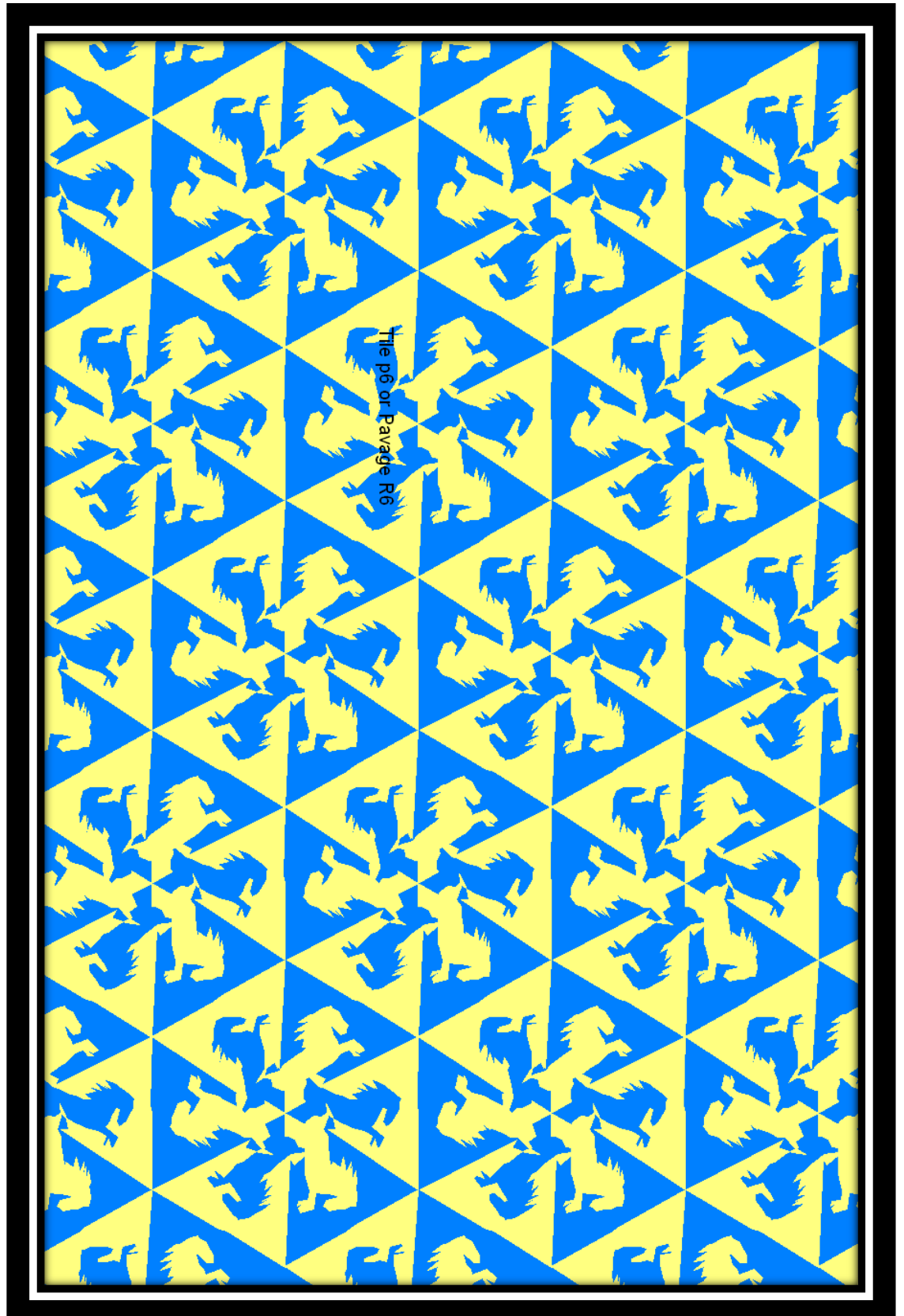


Tiling p6 ou R6

<p>Take an equilateral triangle piece of paper.</p>	<p>Draw a simple curve starting in 1 and going to 2.</p>	<p>Cut along the curve and glue after a rotation of 60° with center in the point 1.</p>
<p>Draw a simple curve starting in 2 and going to 3.</p>	<p>Cut along the curve and glue after a rotation of 180° with center in the point 3.</p>	<p>This is your pattern for this tiling group.</p>

We fulfill the plane by using rotations of angle 60° , 120° , 180° and translations:











Lattice and fundamental region for p6 (R6)

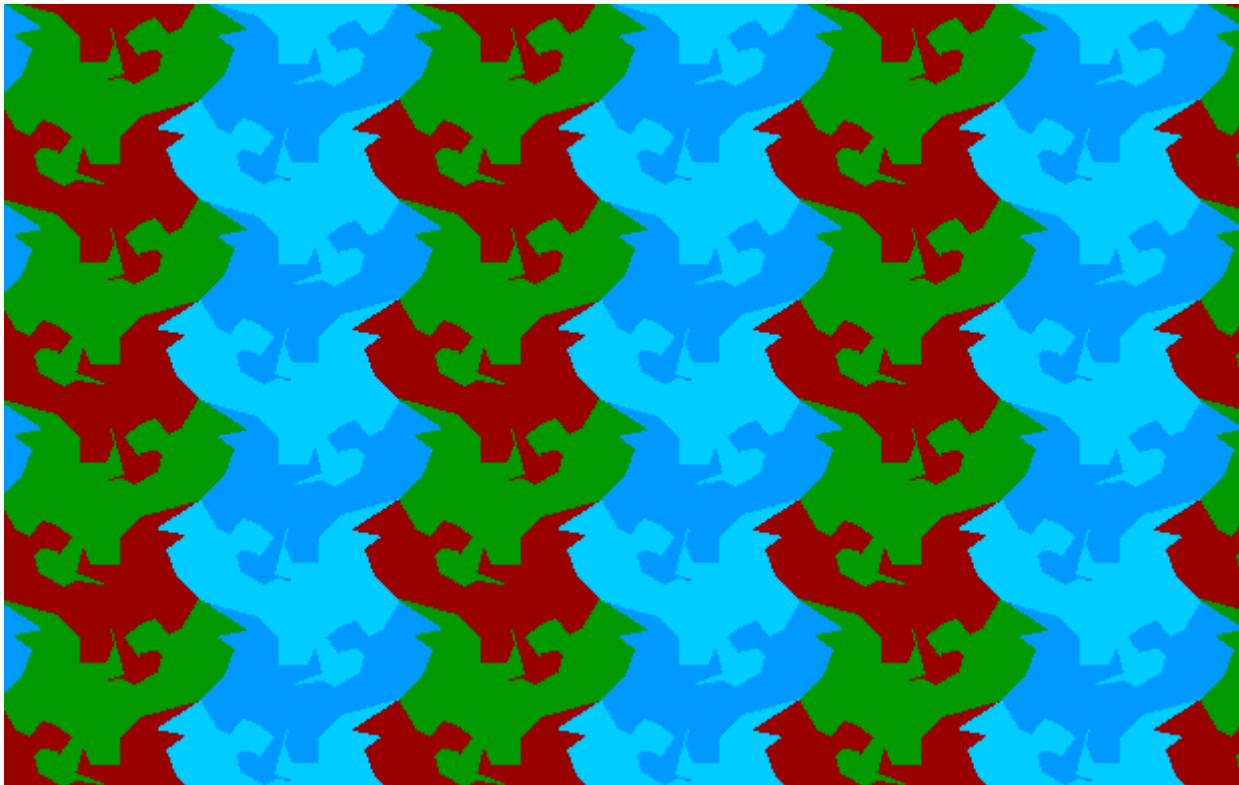
The following picture is the lattice associated to the Tiling p6 (R6), the triangle drawn is a fundamental region and as you can check the arrows are translations, the circles are centers of rotation π , the small hexagons are centers of rotations $\frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$. And the small triangles are centers of rotation $\frac{2\pi}{3}, \frac{4\pi}{3}$.

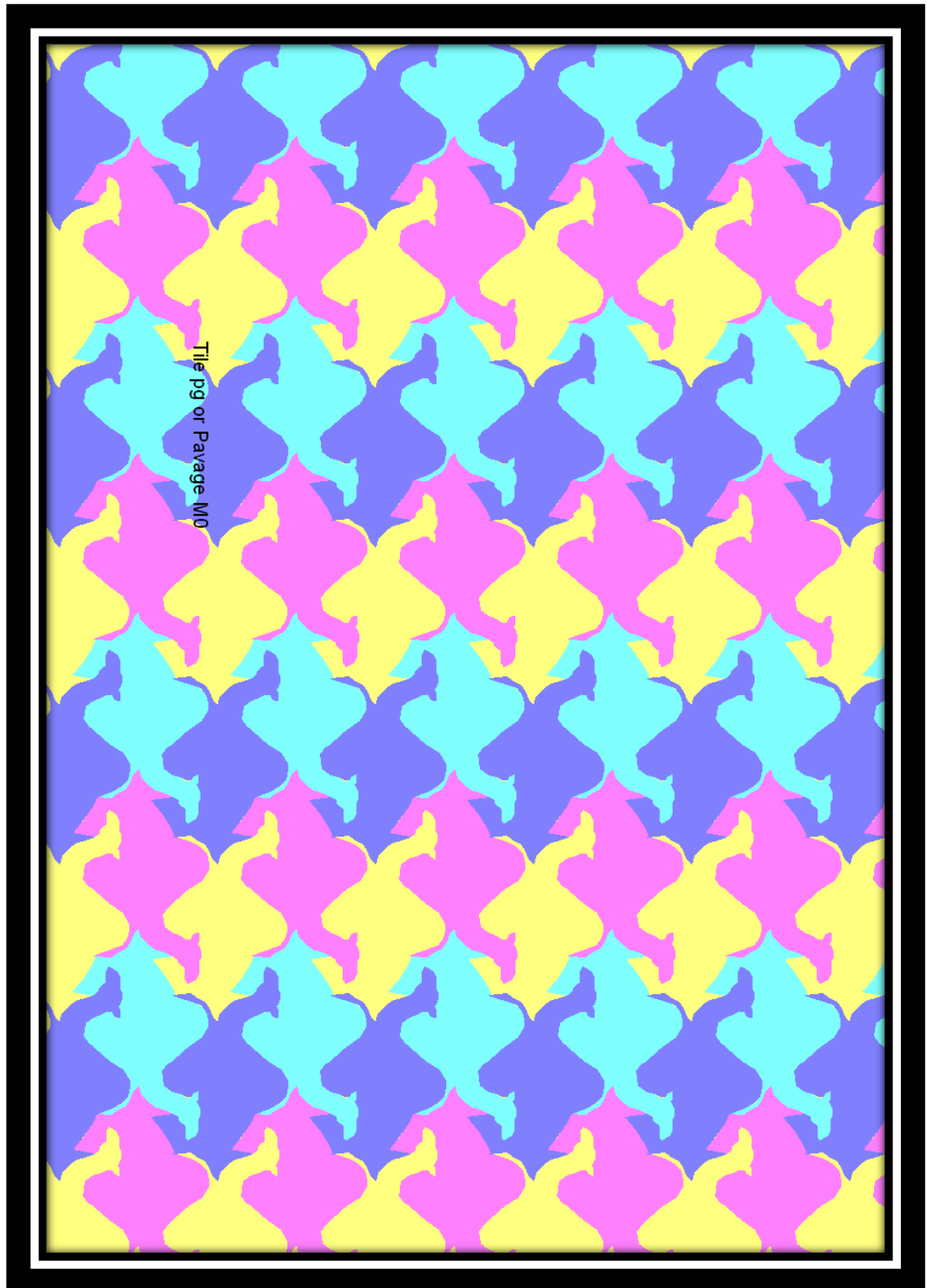


Tiling pg ou M0

 <p>Take a rectangular piece of paper.</p>	 <p>Draw a simple curve starting in 1 and going to 2.</p>	 <p>Cut along the curve and glue it on the right side.</p>
 <p>Draw a simple curve starting in 2 and going to 3.</p>	 <p>Cut along the curve, turn on and glue it on the top side.</p>	 <p>This is your pattern for this tiling group.</p>

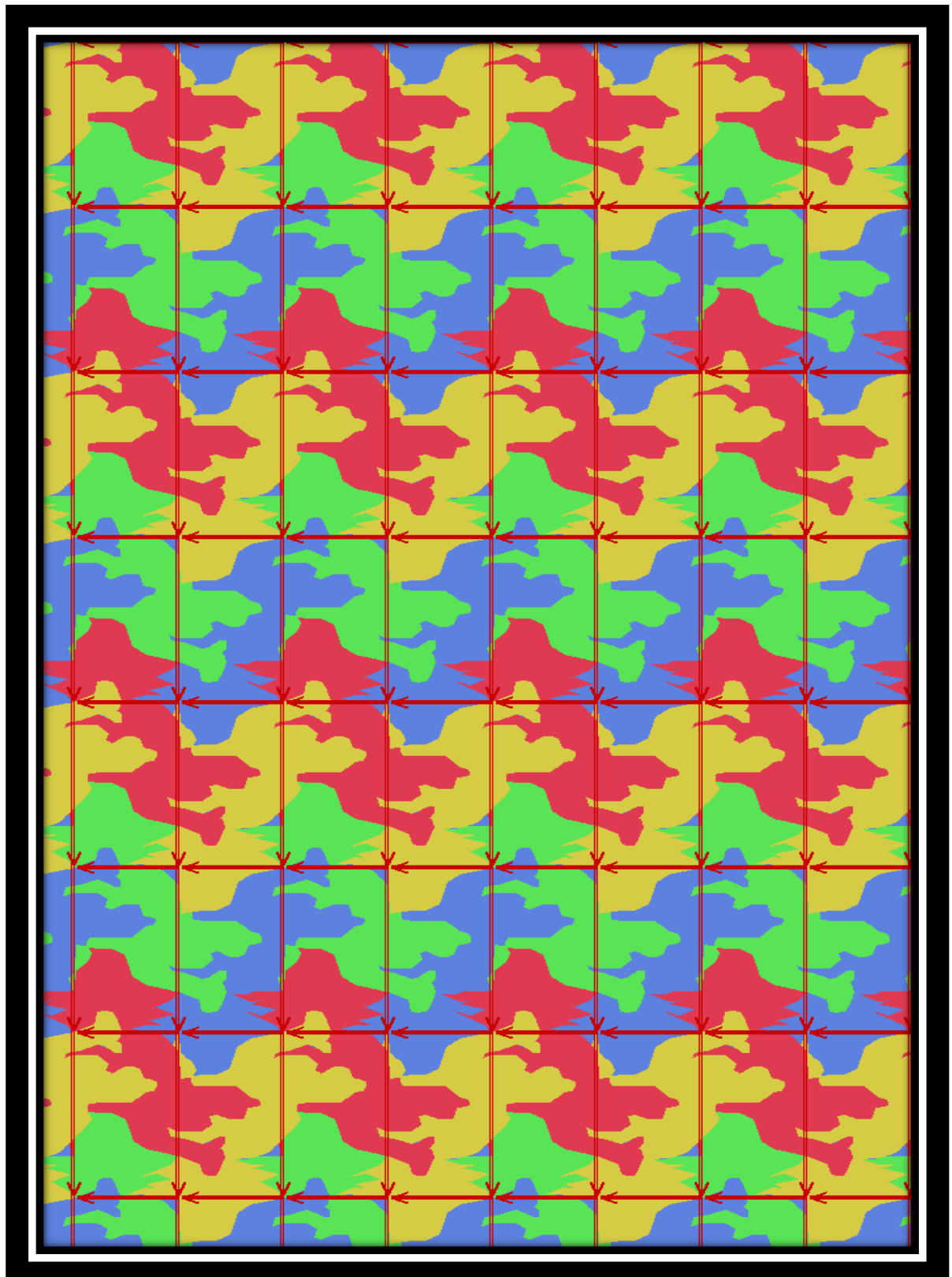
We fulfill the plane by using glides symmetries and translations:




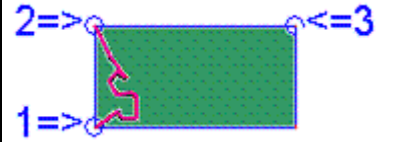






Lattice and fundamental region for pg (M0)

The following picture is the lattice associated to the Tiling pg (M_0), any rectangle is a fundamental region and as you can check the arrows are translations. The lattice is the same as the tiling p_0 (R_0).

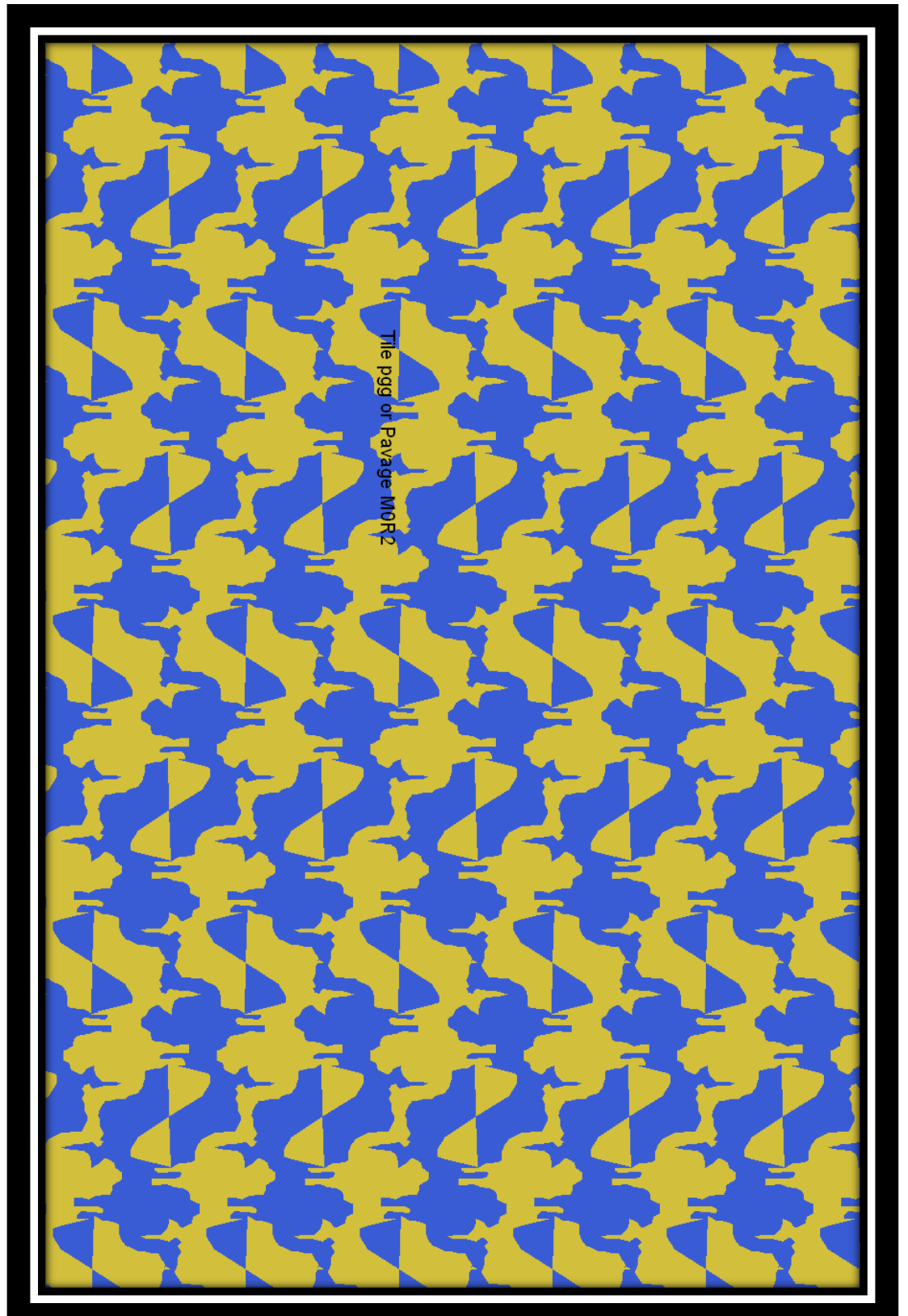


Tiling pgg ou MOR2

 <p>Take a rectangular piece of paper.</p>	 <p>Draw a simple curve starting in 1 and going to 2.</p>	 <p>Cut along the curve, turn on and glue it on the right side.</p>
 <p>Draw a simple curve starting in 2 and going to 3.</p>	 <p>Cut along the curve, turn on and glue it on the top down side.</p>	 <p>This is your pattern for this tiling group.</p>

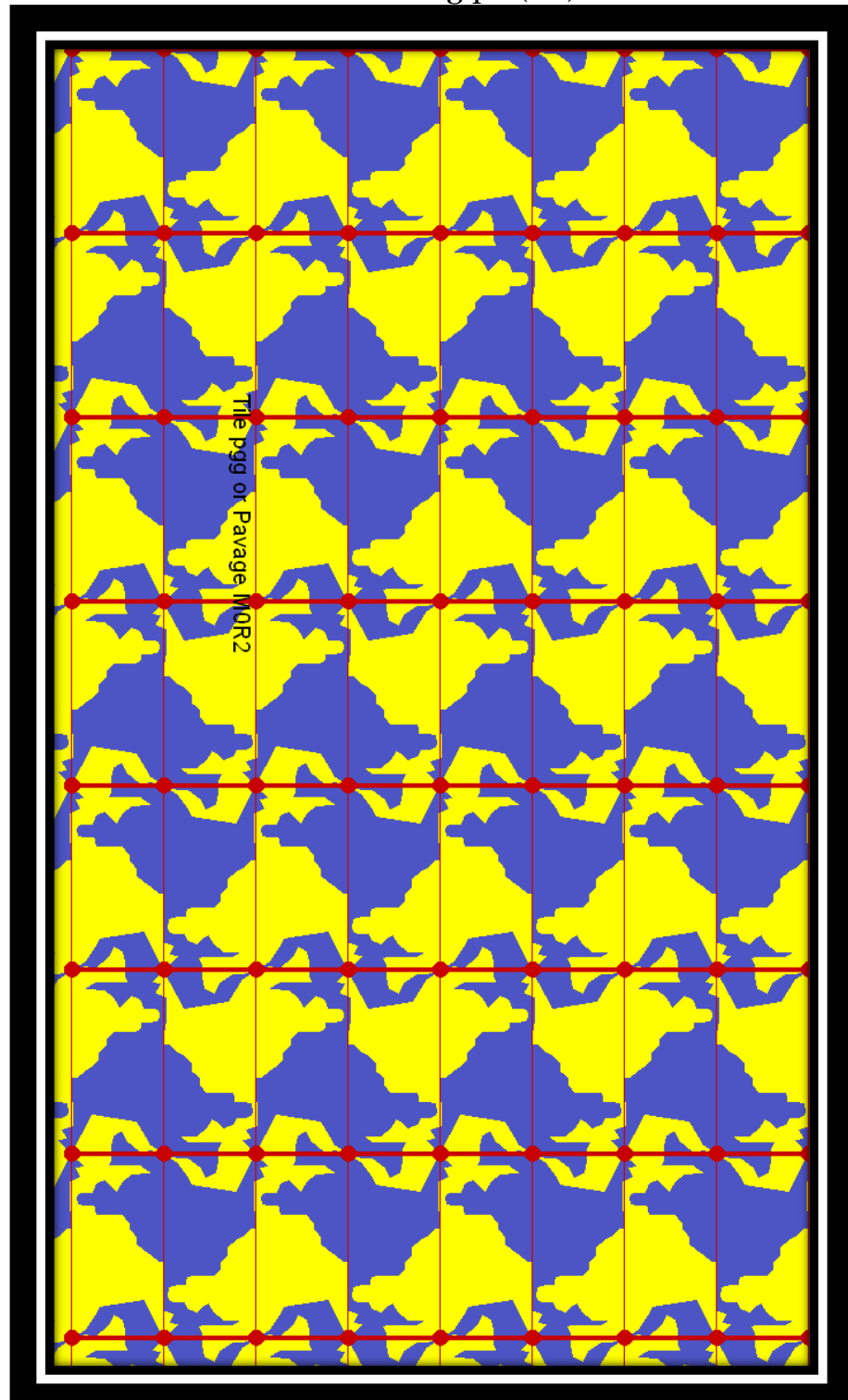
We fulfill the plane by using glides symmetries, rotations of angle 180° and translations:



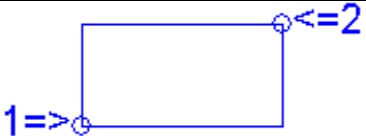

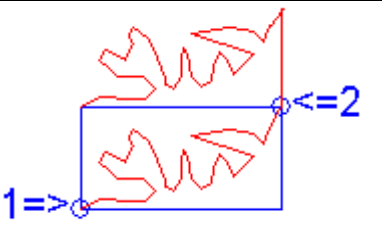
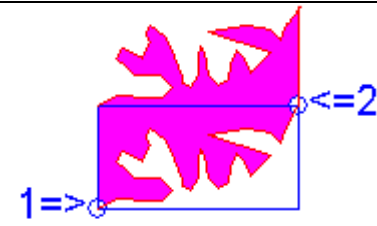


Lattice and fundamental region for pgg (MOR2)

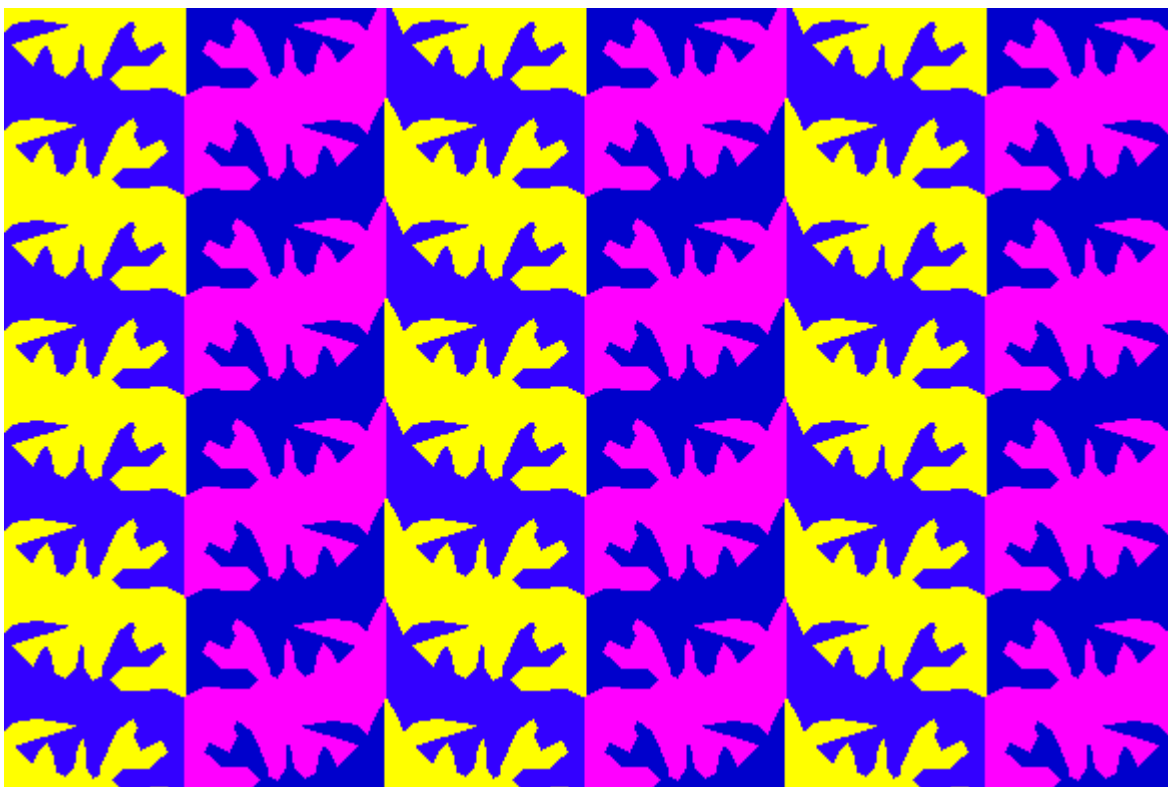
The following picture is the lattice associated to the Tiling pgg (MOR2), any rectangle is a fundamental region and as you can check the arrows are translations. The lattice is the same as the tiling p2 (R2).

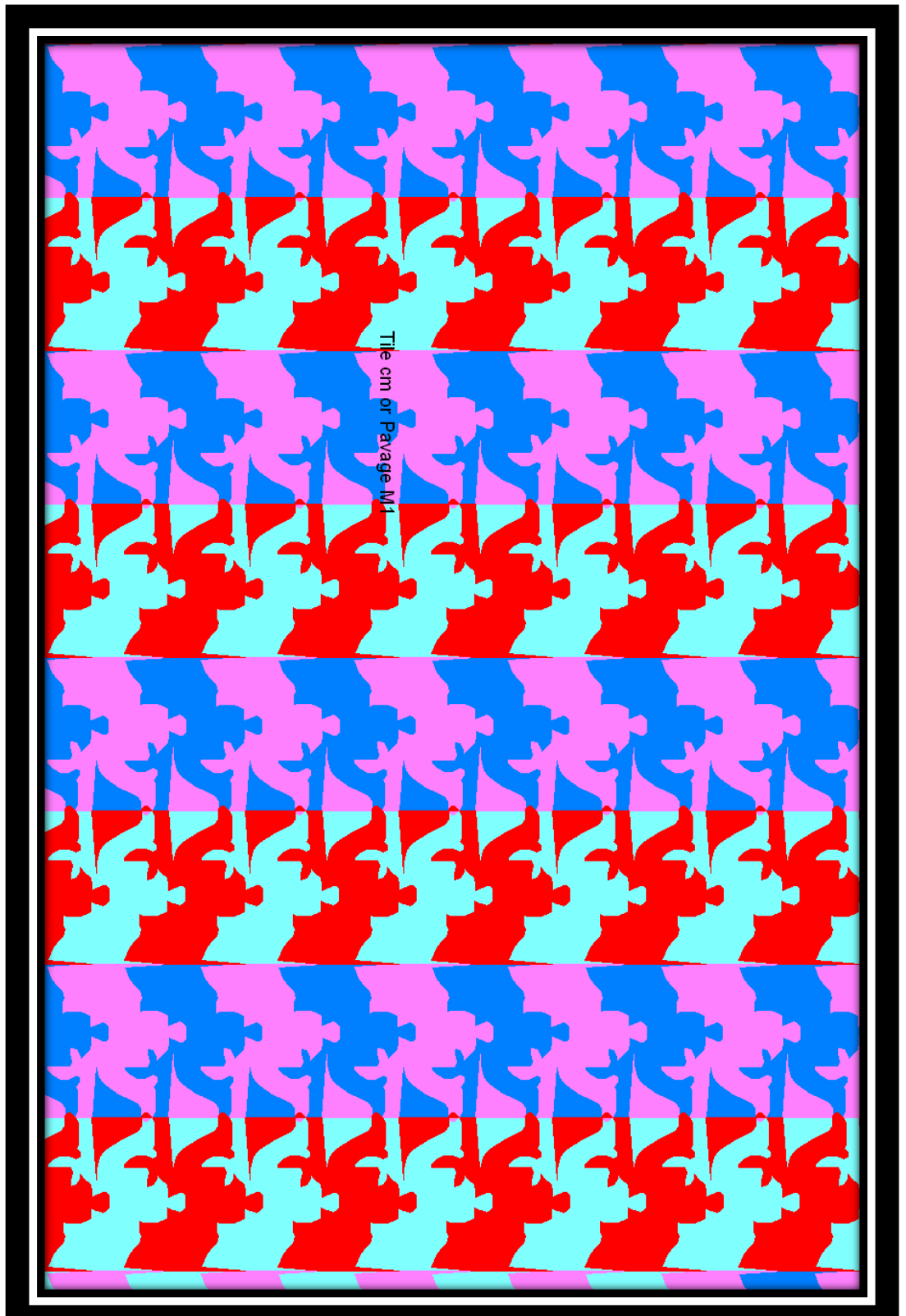


Tiling cm ou M1

 <p>1=> <=2</p> <p>Take a rectangular piece of paper.</p>	 <p>1=> <=2</p> <p>Draw a simple curve starting in 1 and going to 2.</p>	 <p>1=> <=2</p> <p>Cut along the curve and glue it on the top up side.</p>
	 <p>1=> <=2</p> <p>This is your pattern for this tiling group.</p>	

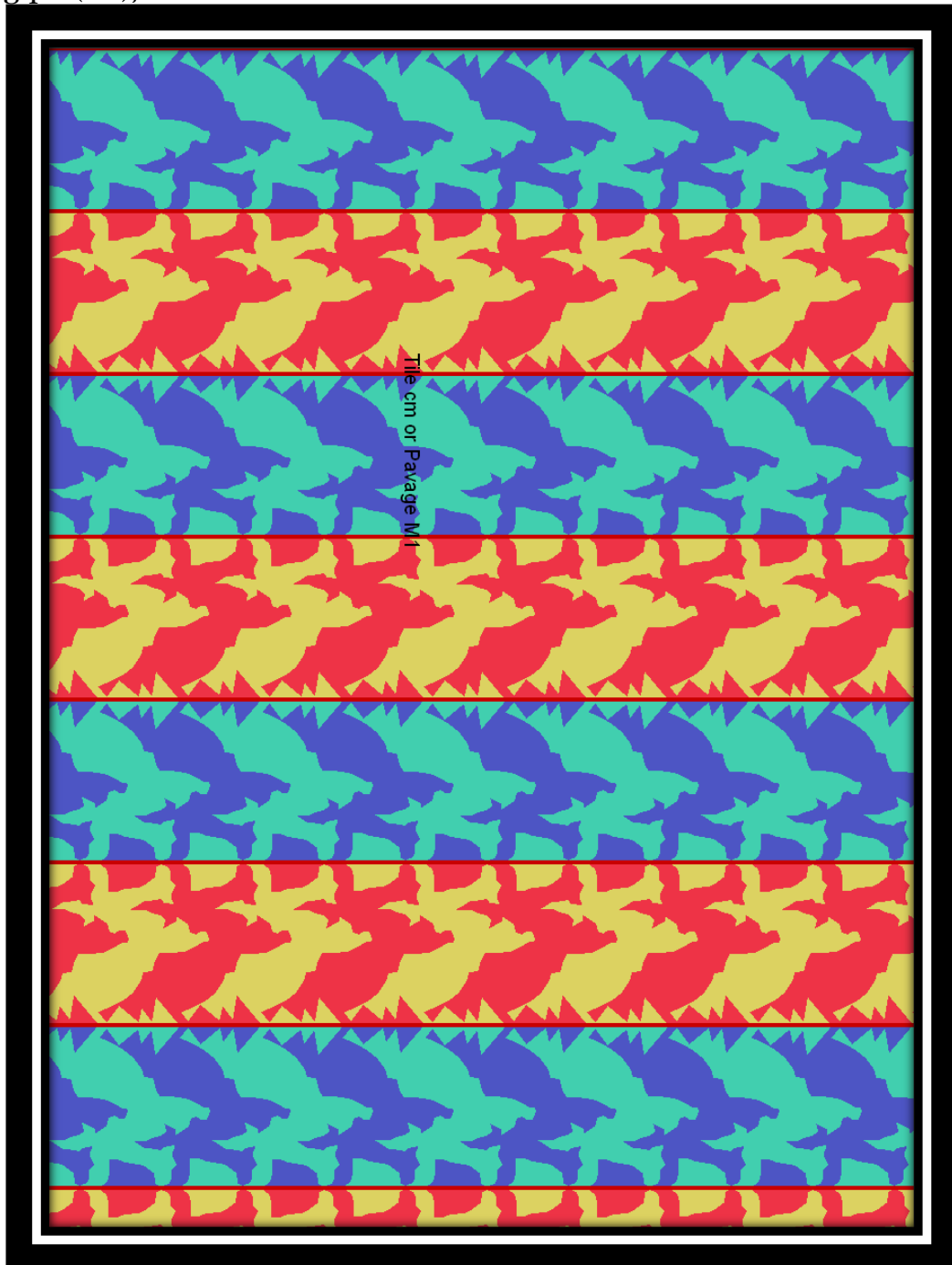
We fulfill the plane by using symmetries and translations:





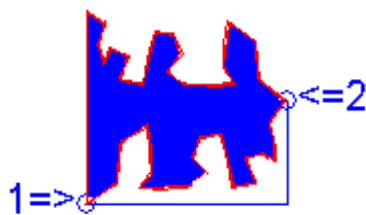
Lattice and fundamental region for cm (M1)

The following picture is the lattice associated to the Tiling cm ($M1$), the fundamental region is rectangle that you are invited to draw, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling $p0$ ($R0$), and we must add the mirror horizontal lines.

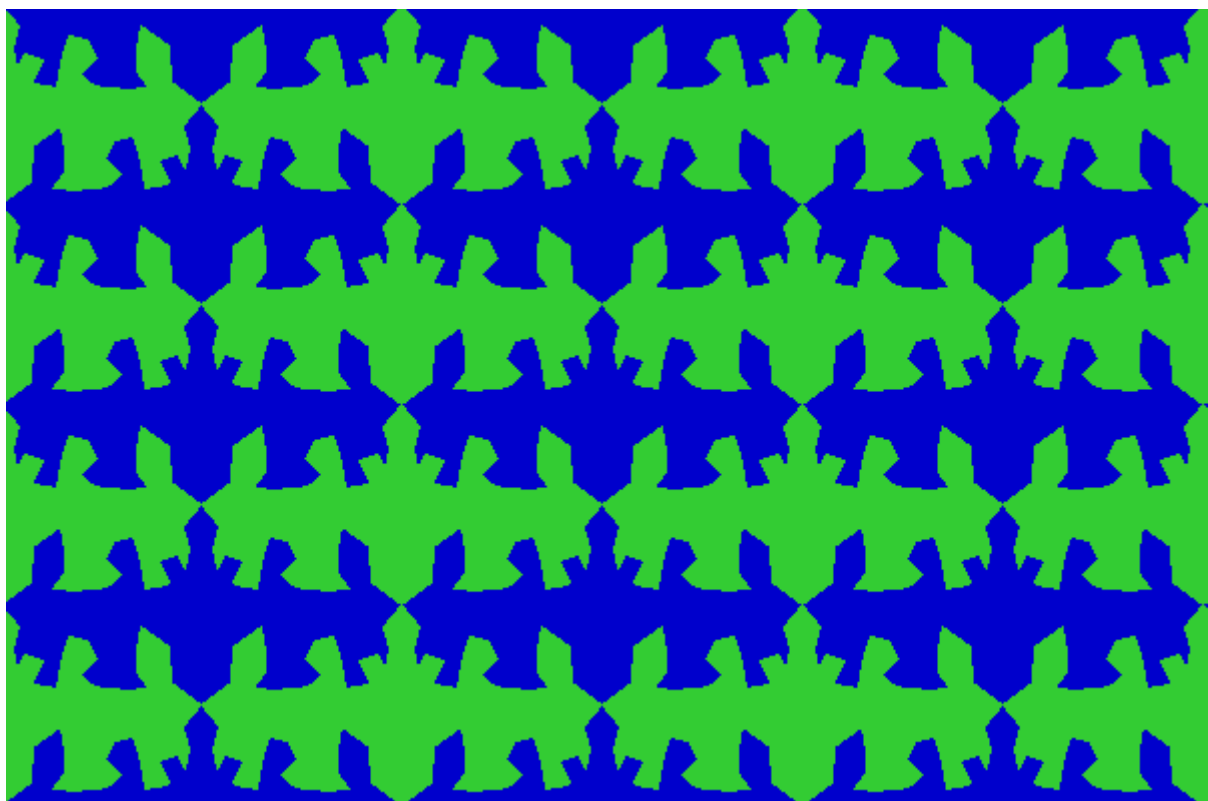


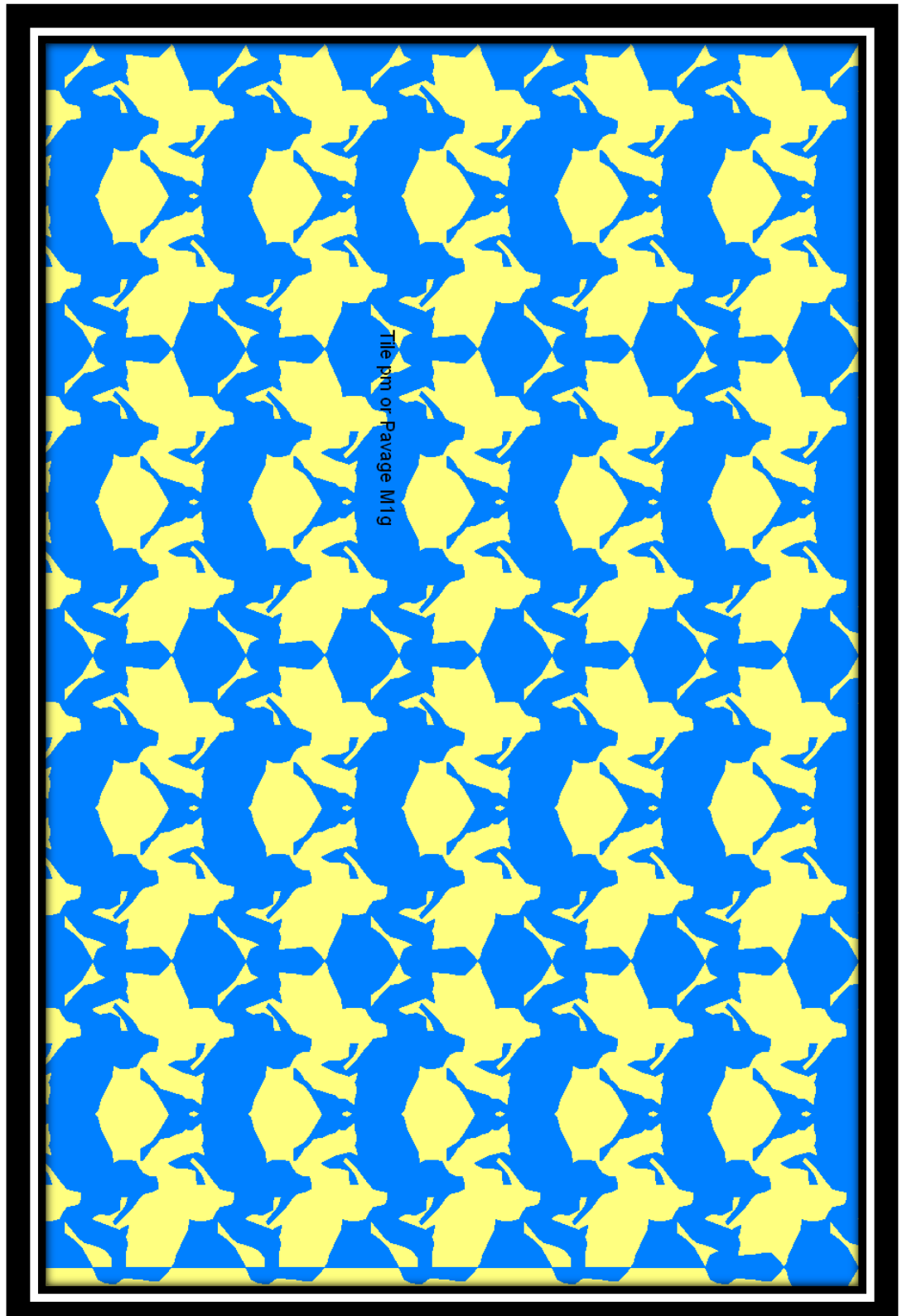
Tiling pm ou M1g

<p>Take a rectangular piece of paper.</p>	<p>Draw a simple curve starting in 1 and going</p>	
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	to 2.	Cut along the curve, turn on and glue it on the top up side.
	 <p>This is your pattern for this tiling group.</p>	

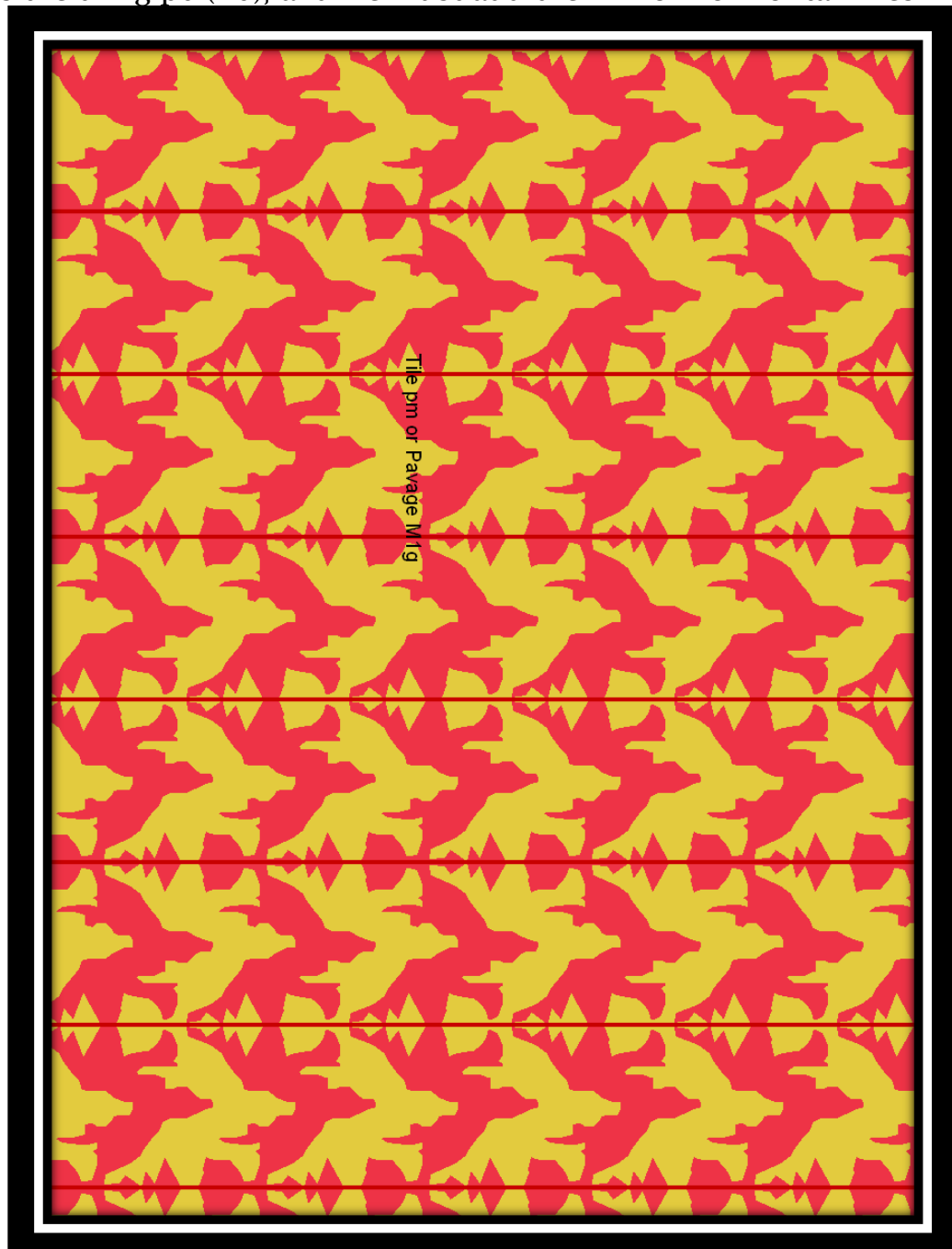
We fulfill the plane by using vertical symmetries and glide symmetries:



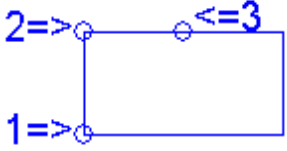
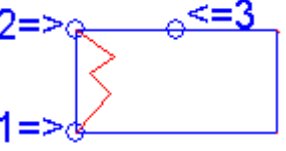
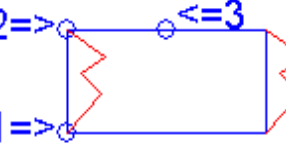


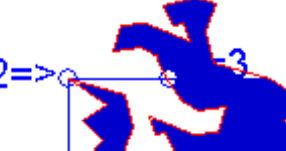


Lattice and fundamental region for pm (M1g)

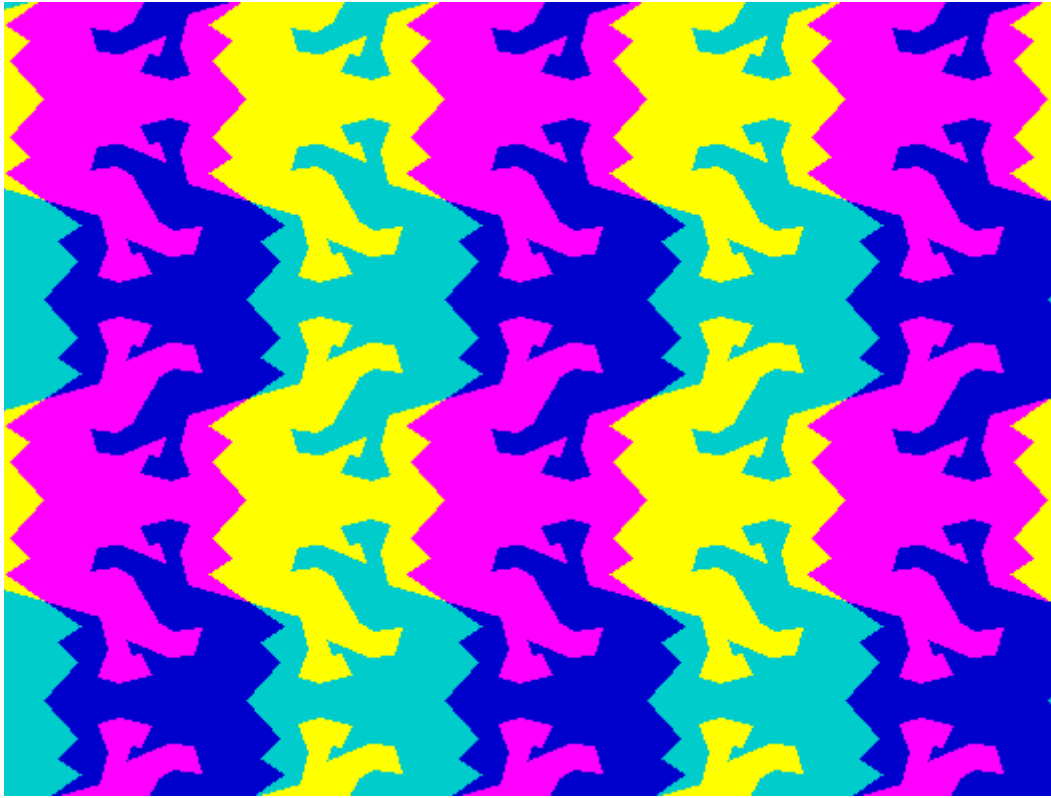
The following picture is the lattice associated to the Tiling pm (M1g), the fundamental region is a rectangle that you are invited to draw, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling p0 (R0), and we must add the mirror horizontal lines.

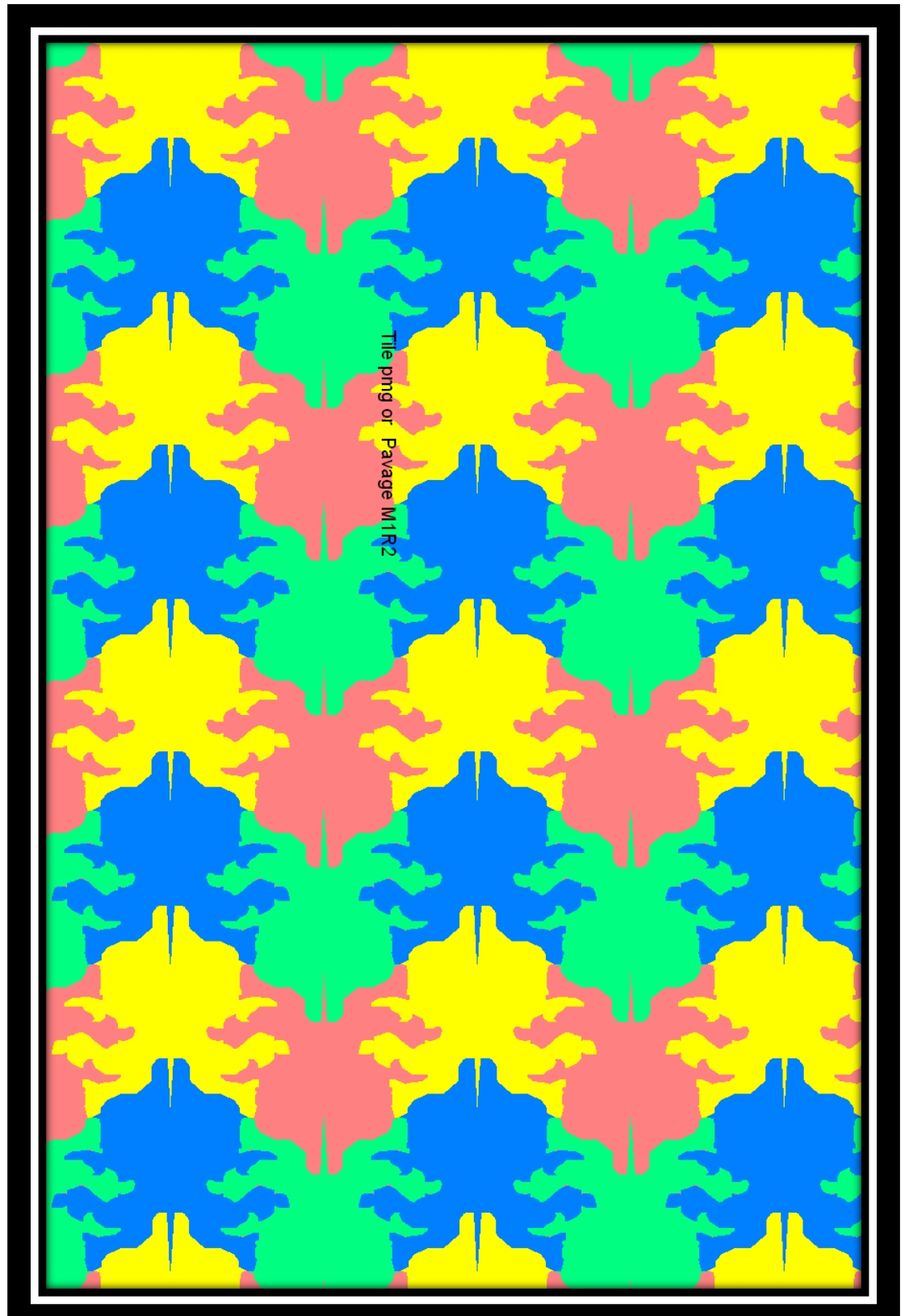


Tiling pmg ou M1R2

 <p>Take a rectangular piece of paper.</p>	 <p>Draw a simple curve starting in 1 and going to 2.</p>	 <p>Cut along the curve and glue it on the right side.</p>
 <p>Draw a simple curve starting in 2 and going to 3.</p>	 <p>Cut along the curve, rotate it of 180° and glue it on the top up side.</p>	 <p>This is your pattern for this tiling group.</p>

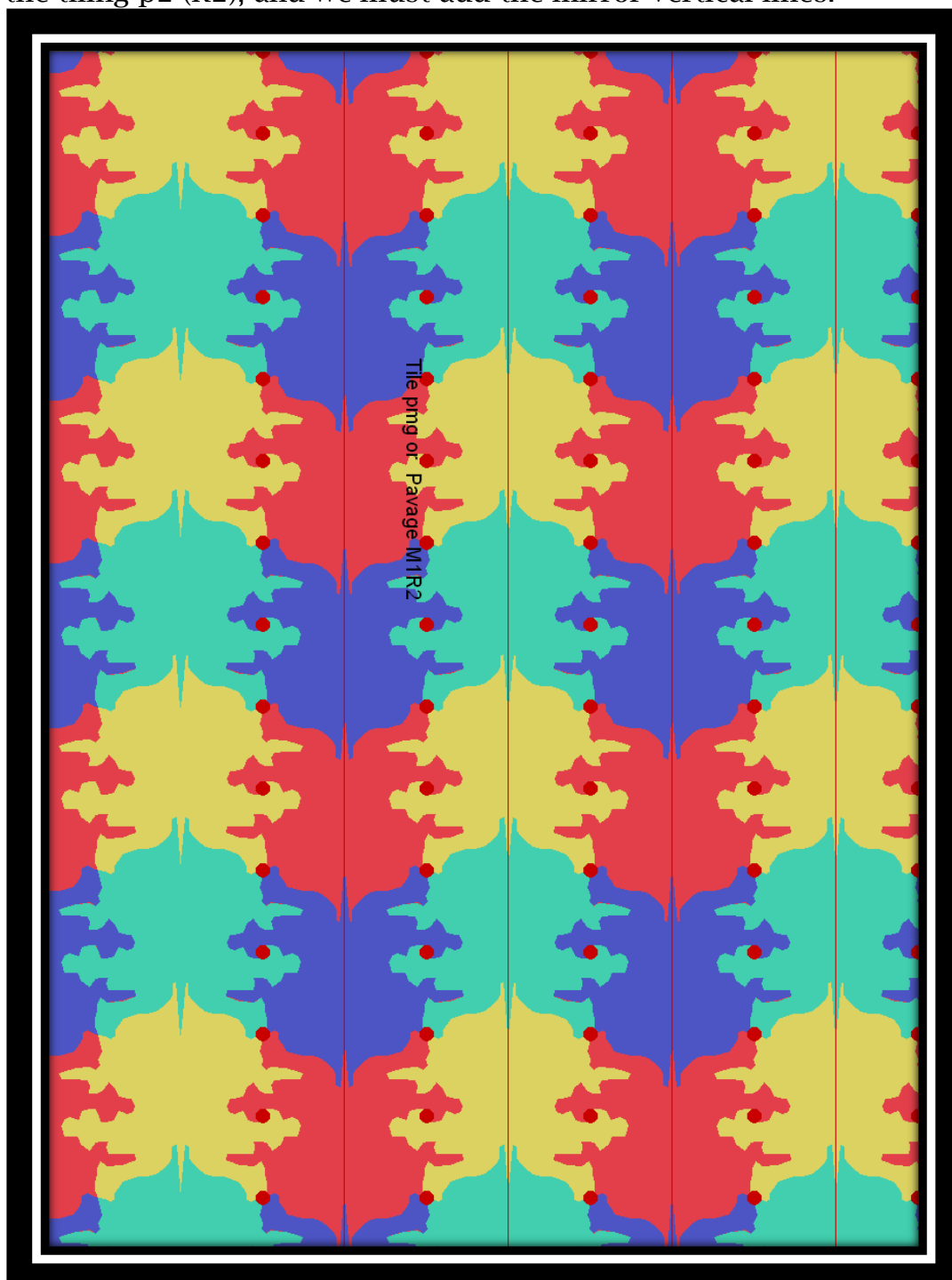
We fulfill the plane by using rotations of angle 180° and horizontal symmetries:



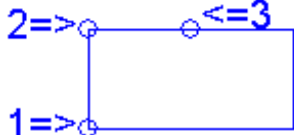
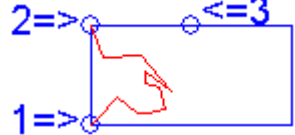




Lattice and fundamental region for pgm (M1R2)

The following picture is the lattice associated to the Tiling pgm ($M1R2$), the fundamental region is a rectangle that you are invited to draw, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling $p2$ ($R2$), and we must add the mirror vertical lines.

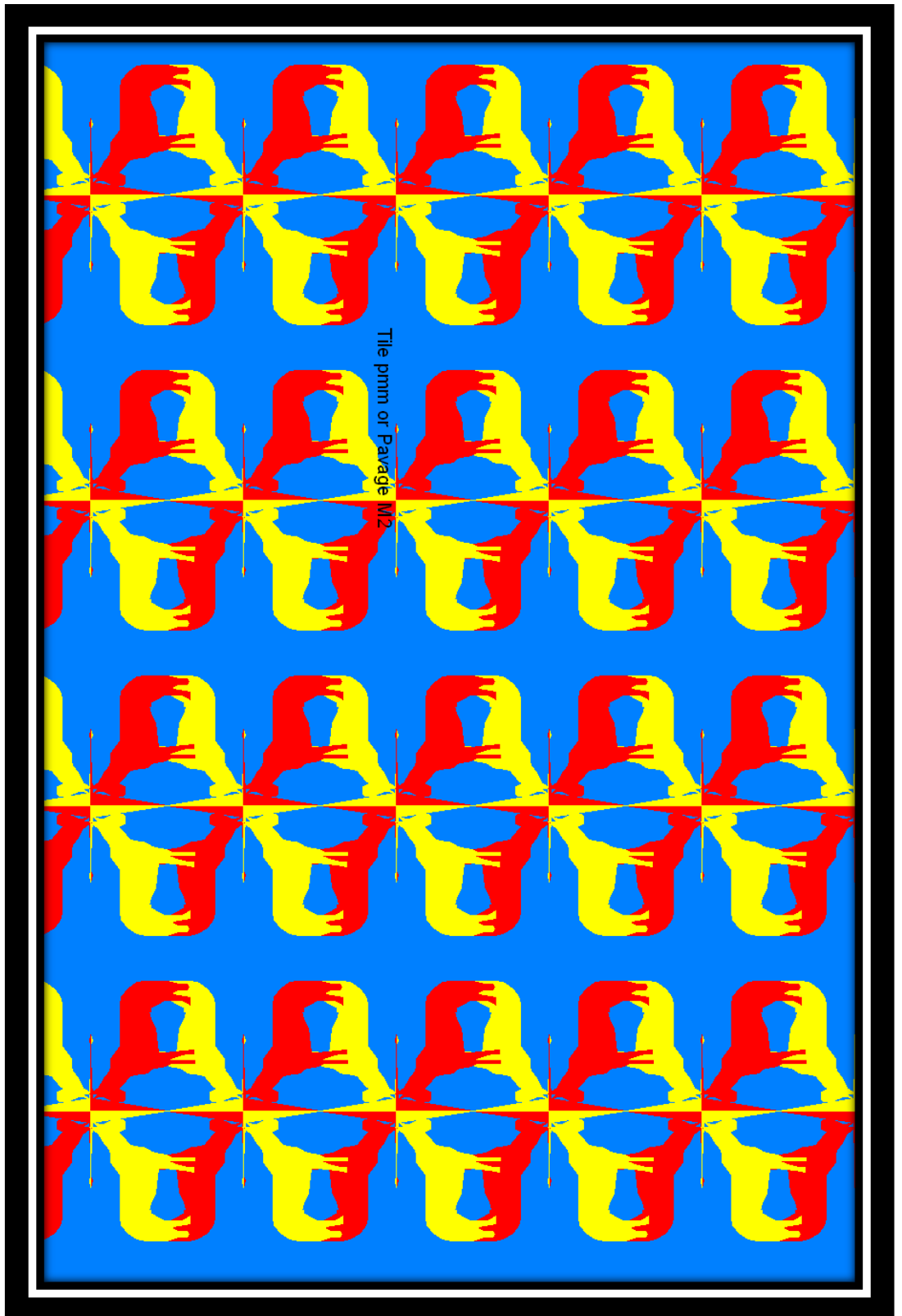


Tiling pmm ou $M2$

 <p>Take a rectangular piece of paper.</p>	 <p>Draw a simple curve starting in 1 and going to 2.</p>	 <p>Draw a simple curve starting in 2 and going to 3.</p>
	 <p>This is your pattern for this tiling group. (a draw rectangle)</p>	

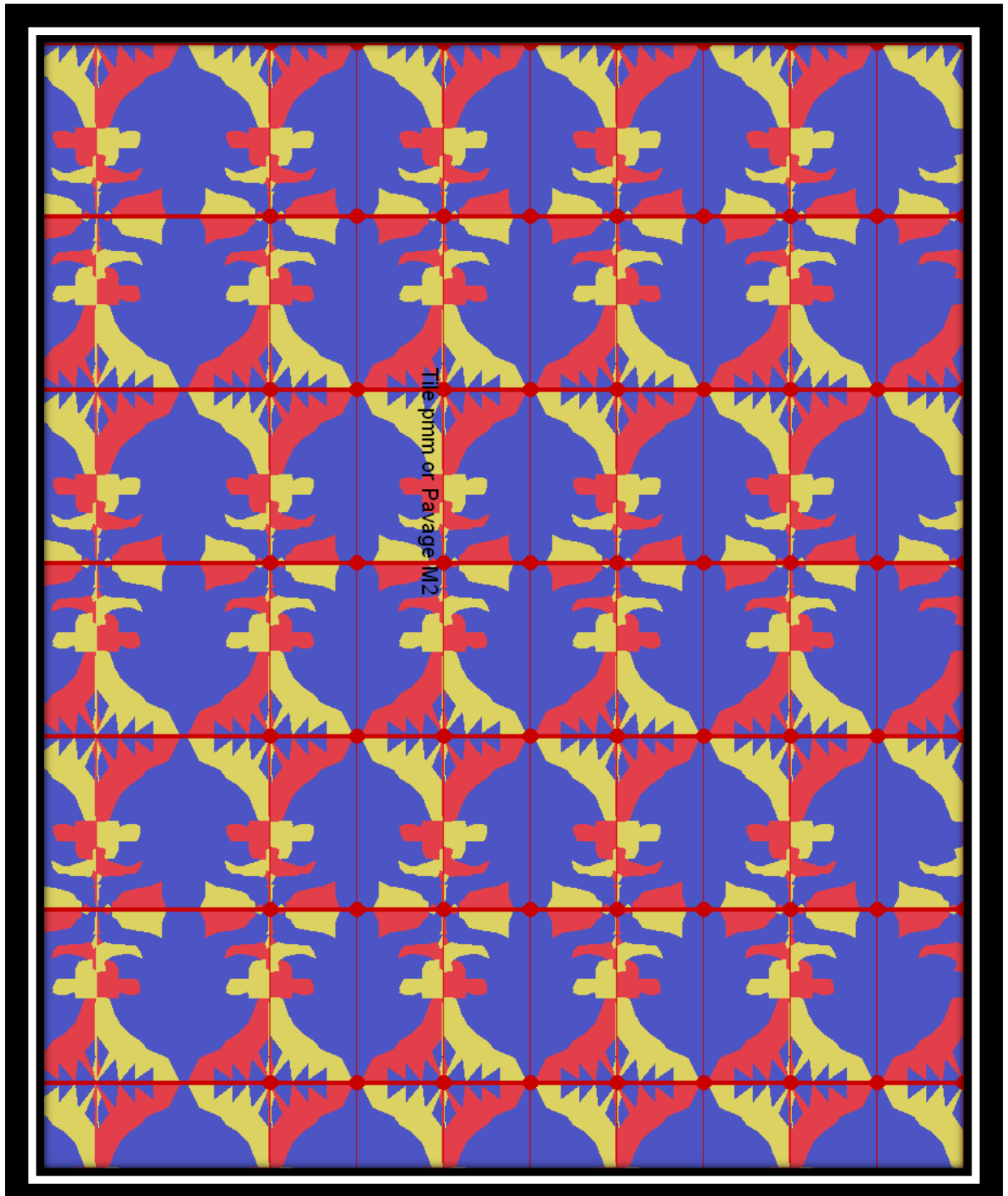
We fulfill the plane by vertical and horizontal symmetries :



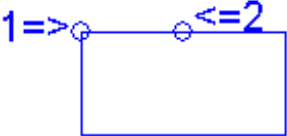
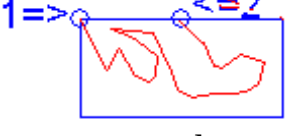
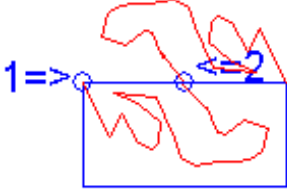



Lattice and fundamental region for pmm (M2)

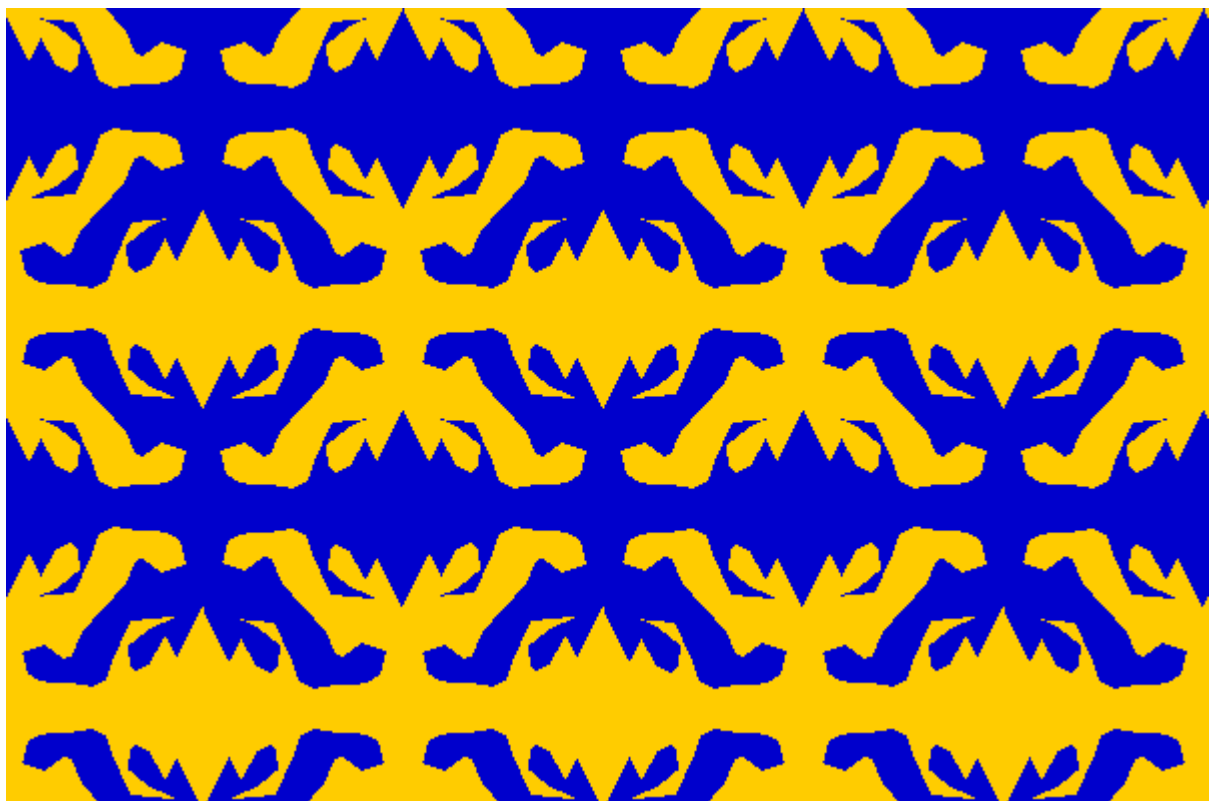
The following picture is the lattice associated to the Tiling cm (M1), the fundamental region is a rectangle, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling p2 (R2), and we must add the mirror horizontal and vertical lines.

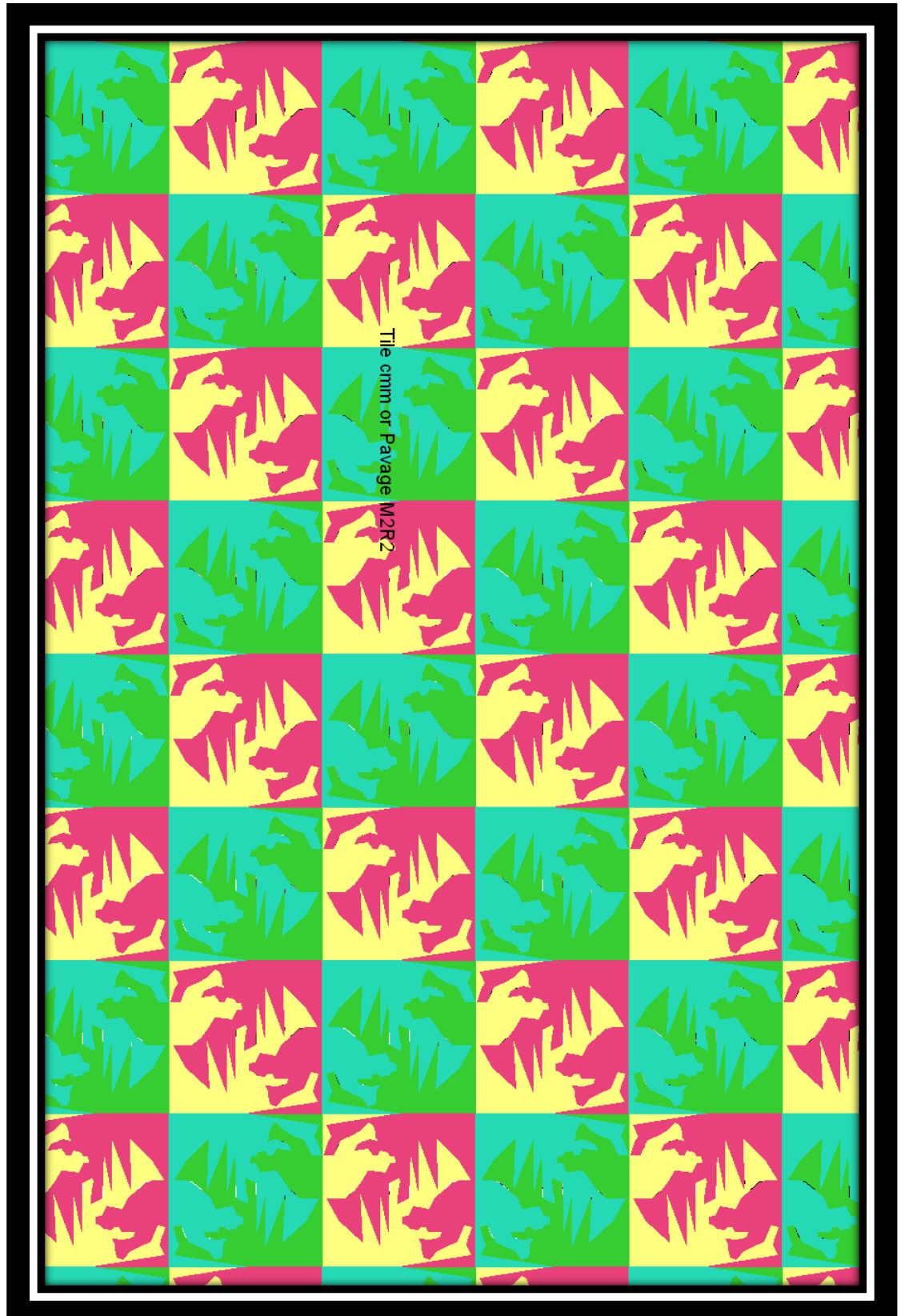


Tiling cmm ou M2R2

 <p>Take a rectangular piece of paper.</p>	 <p>Draw a simple curve starting in 1 and going to 2.</p>	 <p>Cut along the curve, rotate it, and glue it on the top up side.</p>
	 <p>This is your pattern for this tiling group.</p>	

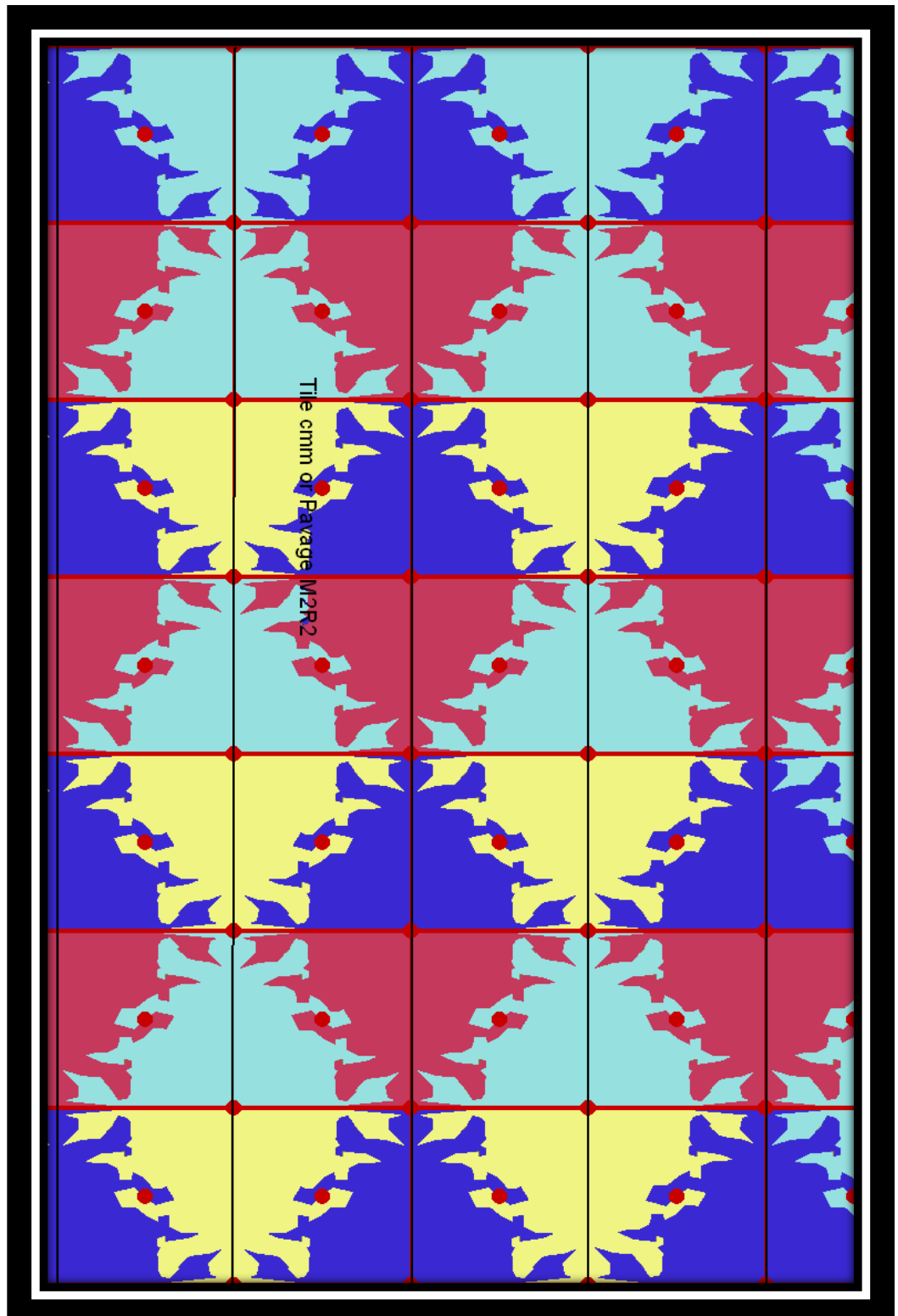
We fulfill the plane by using glides symmetries , rotations of angle 180° and translations:



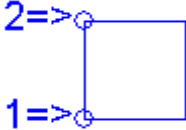
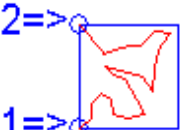
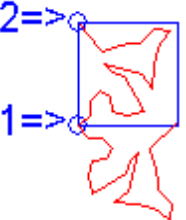
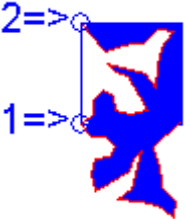


Lattice and fundamental region for cmm (M2R2)

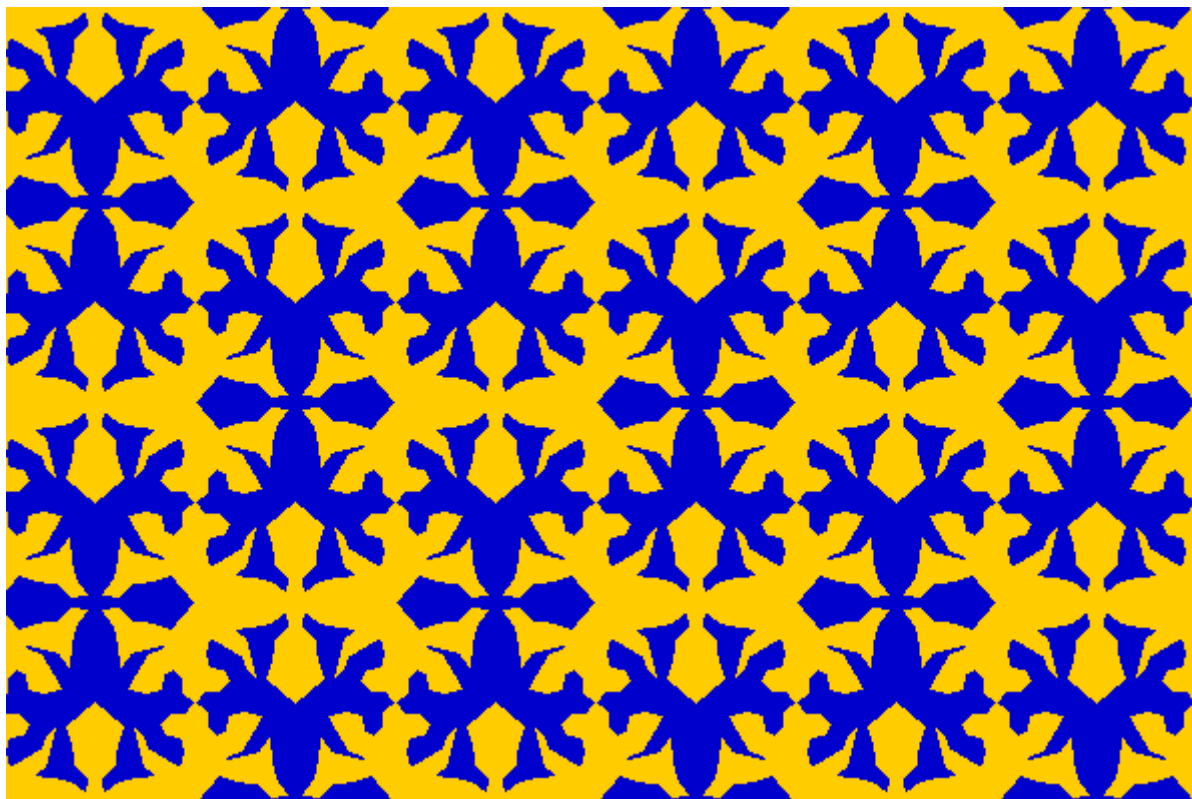
The following picture is the lattice associated to the Tiling cmm (M2R2), the fundamental region is a rectangle, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling p2 (R2), and we must add the mirror horizontal and vertical lines.

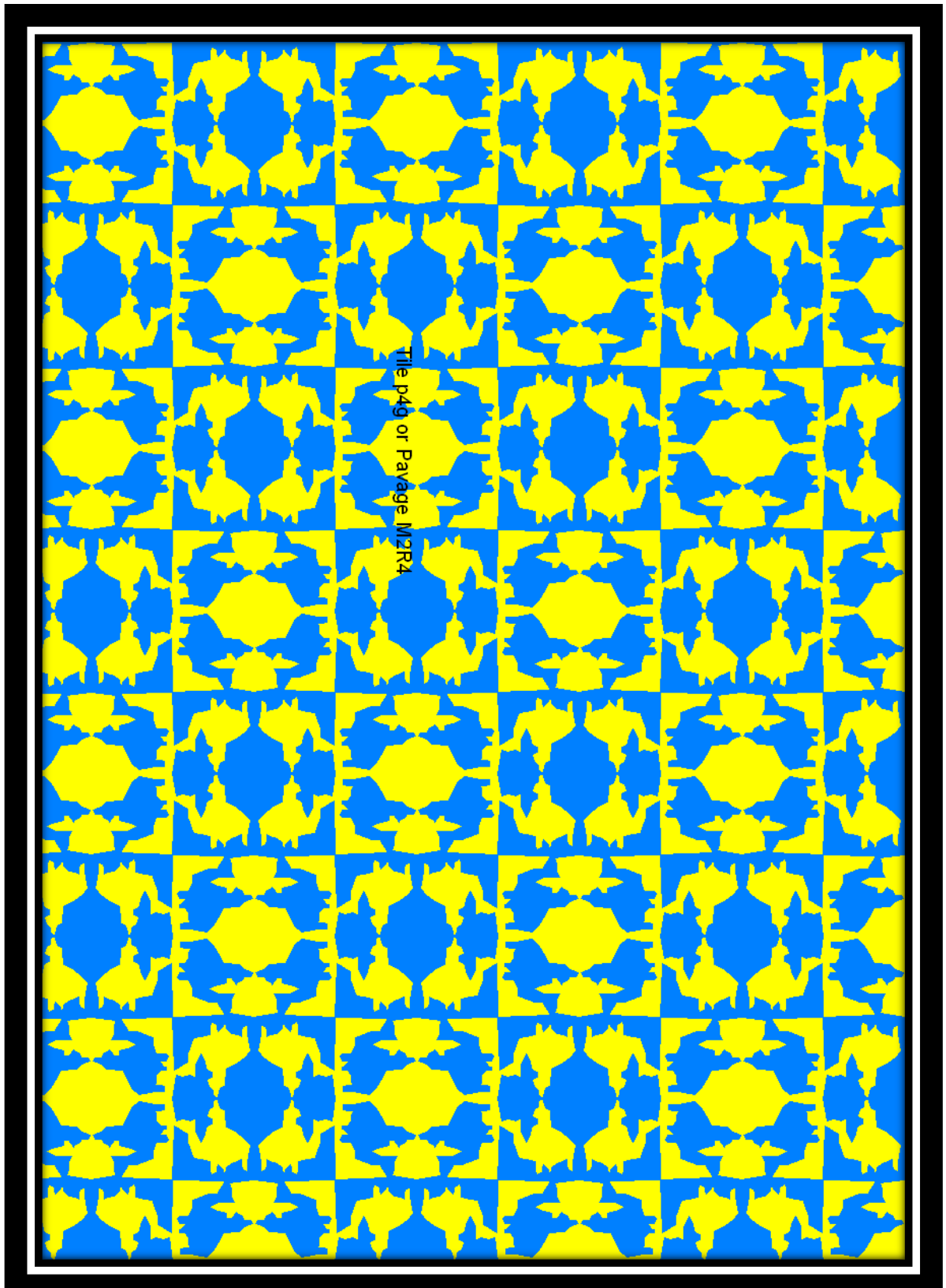


Tiling p4g ou M2R4

 <p>Take a square piece of paper.</p>	 <p>Draw a simple curve starting in 1 and going to 2.</p>	 <p>Cut along the curve, rotate it, and glue it on the top down side.</p>
	 <p>This is your pattern for this tiling group.</p>	

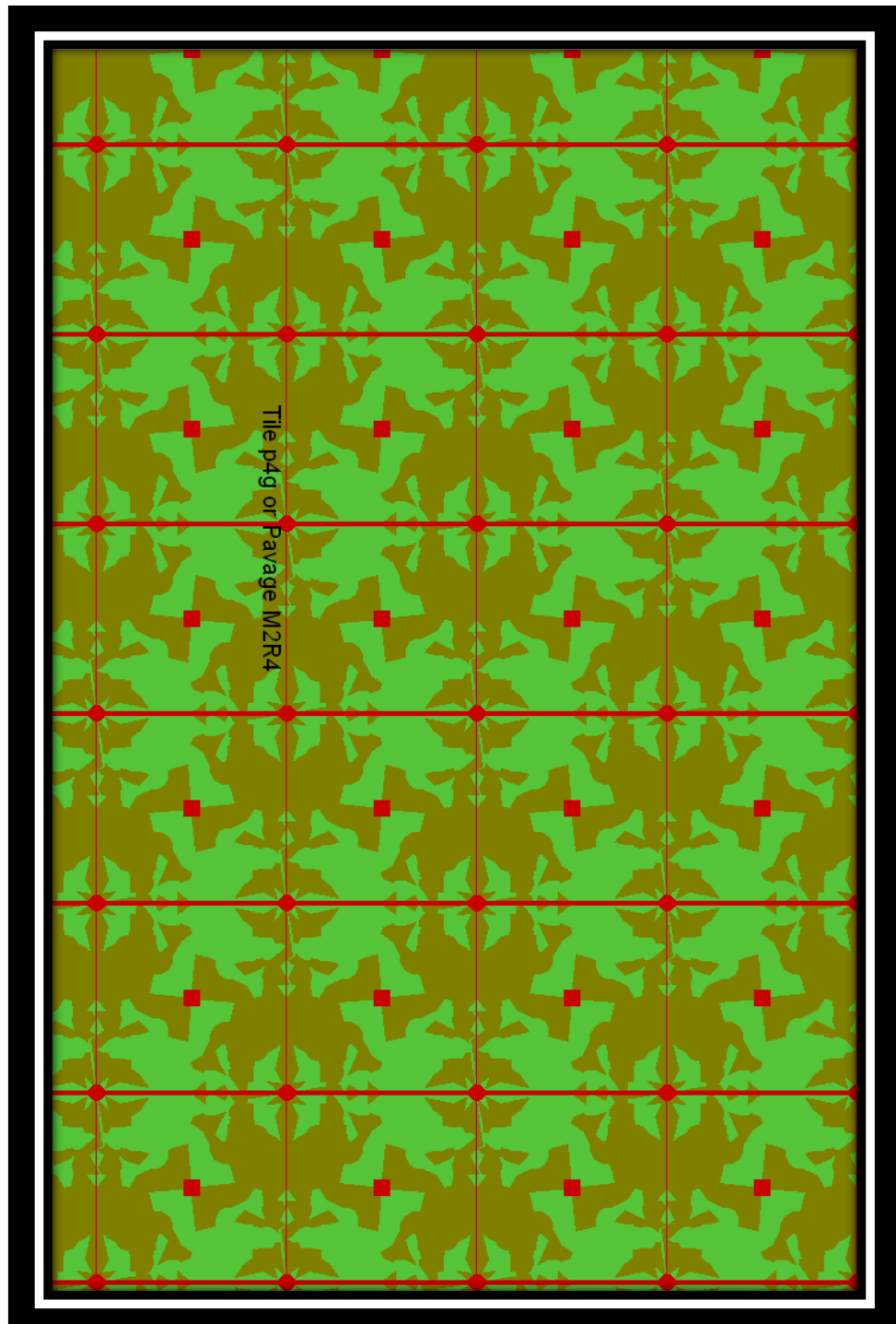
We fulfill the plane by using glides symmetries , rotations of angle 90°, 180°, 270 and translations:



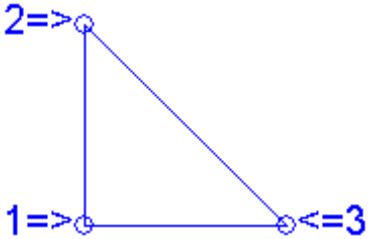
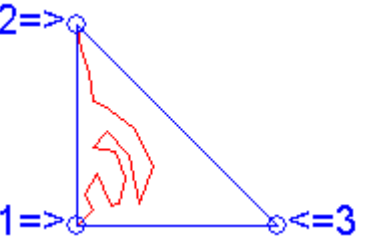
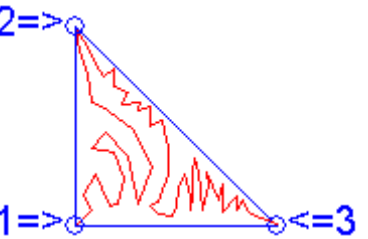
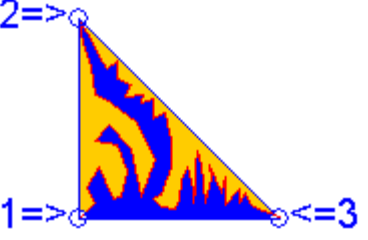


Lattice and fundamental region for p4g (M2R4)

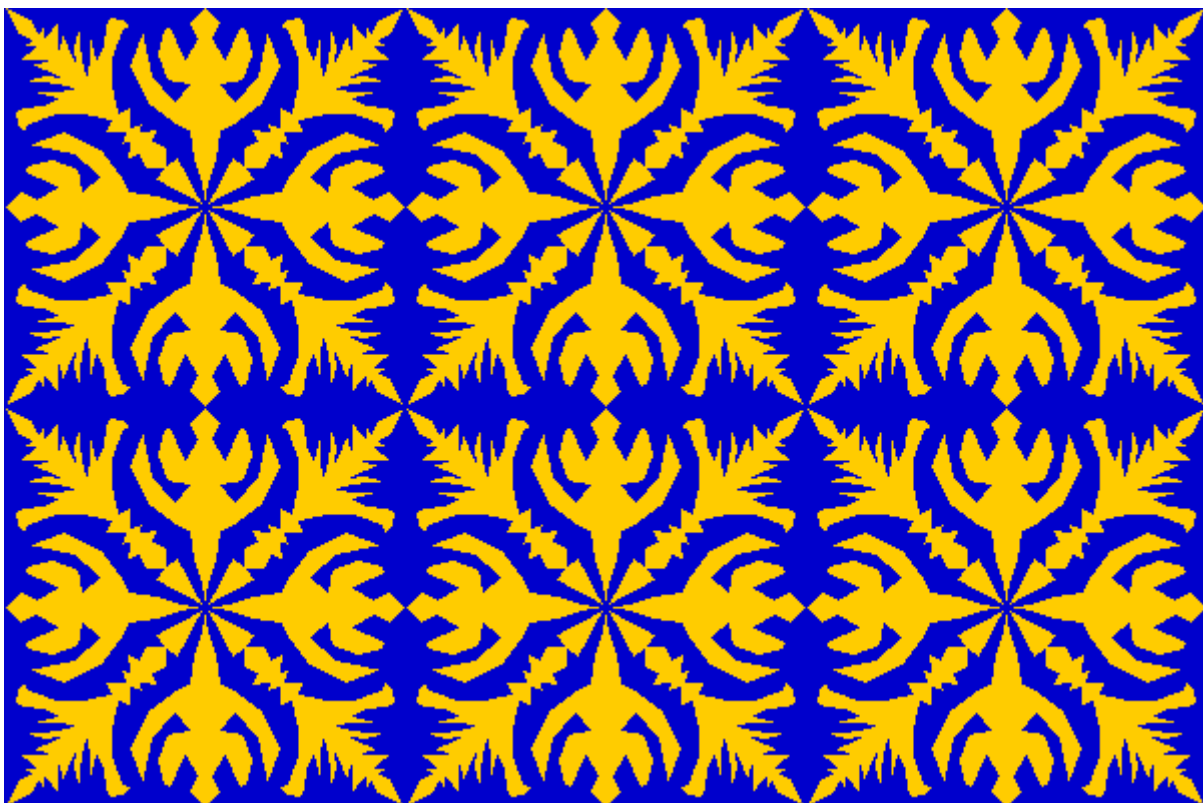
The following picture is the lattice associated to the Tiling p4g (M2R4), the fundamental region is a half of a square, that is a right isosceles triangle, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling p4 (R4), and we must add the mirror horizontal and vertical lines.



Tiling p4m ou M4

 <p>Take a right isosceles triangular piece of paper.</p>	 <p>Draw a simple curve starting in 1 and going to 2.</p>	 <p>Draw a simple curve starting in 2 and going to 3.</p>
	 <p>This is your pattern for this tiling group. (a bicolor triangle)</p>	

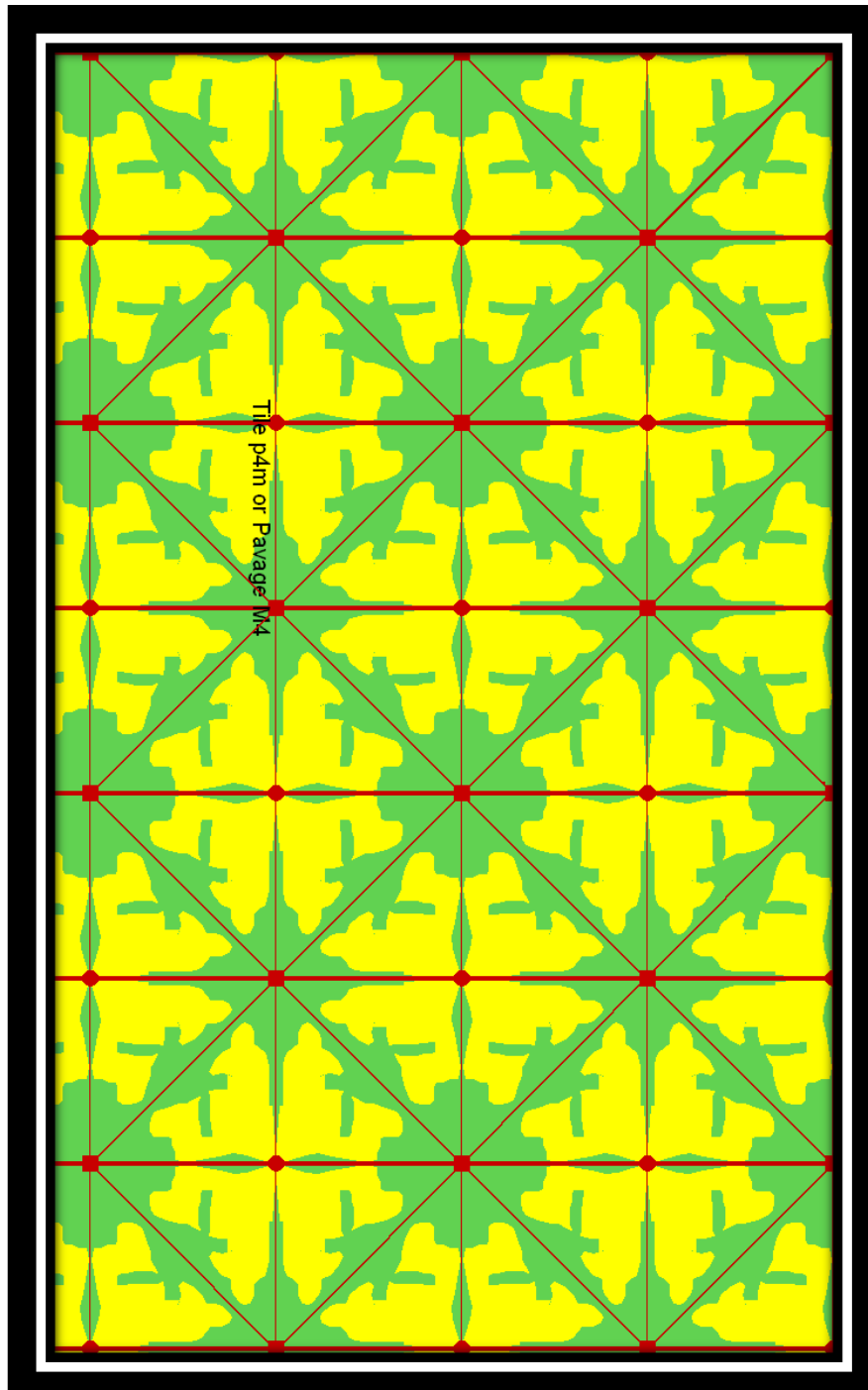
We fulfill the plane by using symmetries with axis the sides of the rectangle triangle:



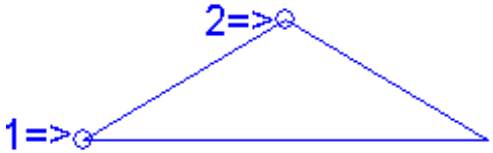
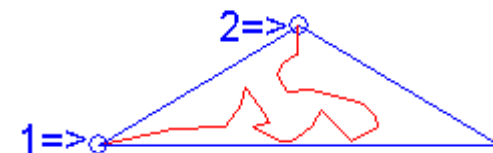
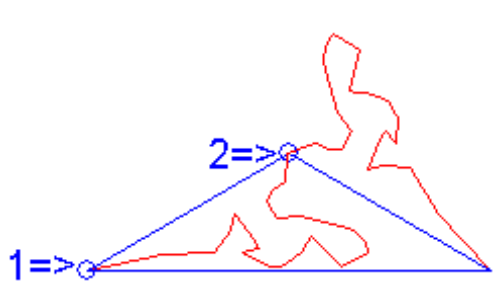
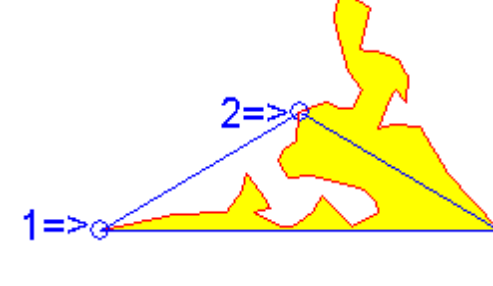


Lattice and fundamental region for p4m (M4)

The following picture is the lattice associated to the Tiling $p4m$ (M4), the fundamental region is a half of a square, that is a right isosceles triangle, and as you can check there are vertical and horizontal translations. The lattice is the same as the tiling $p4$ (R4), and we must add the mirror horizontal and vertical lines.

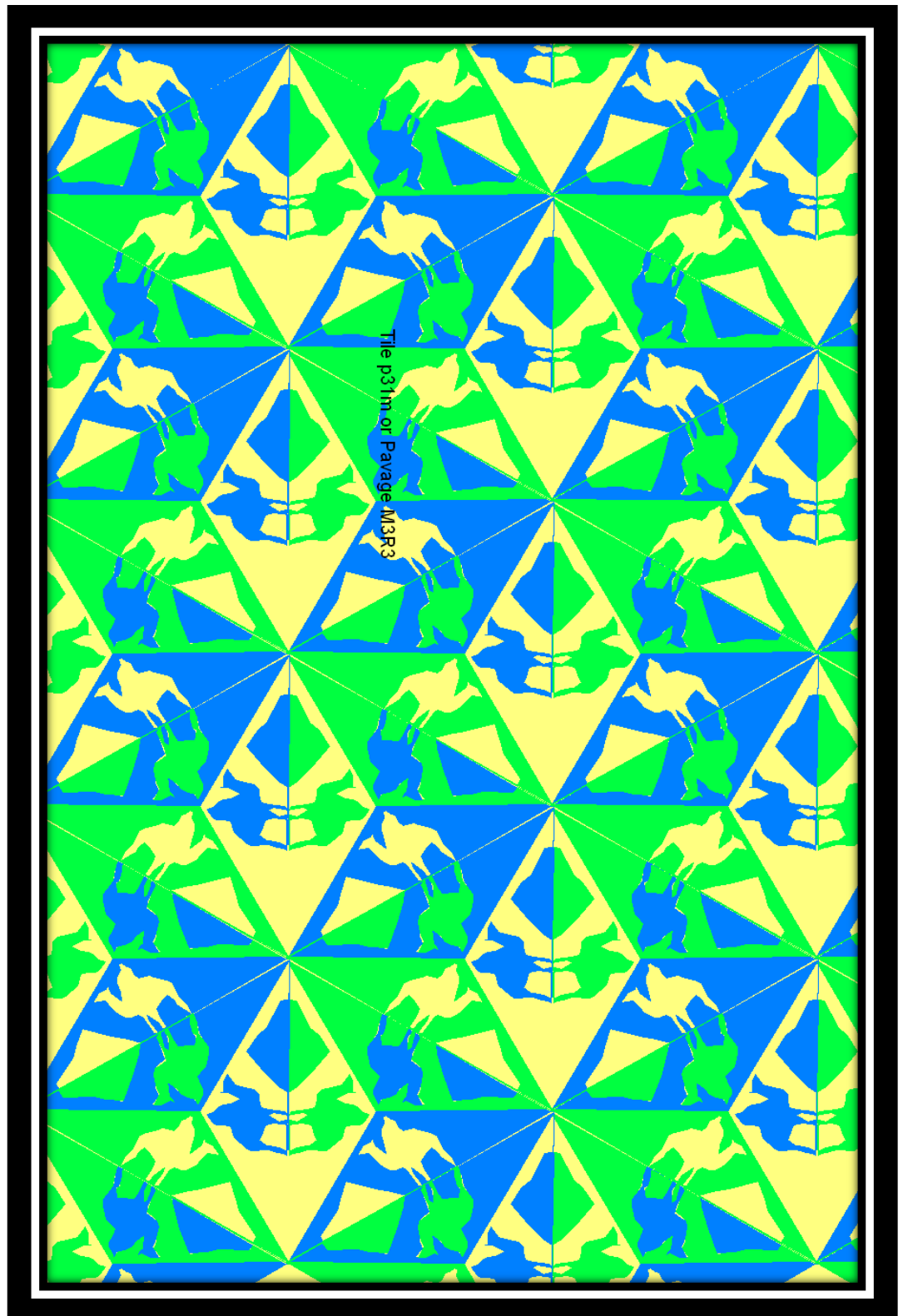


Tiling $p31m$ ou M3R3

 <p>Take a triangular piece of paper. (isosceles having the principal angle 120°)</p>	 <p>Draw a simple curve starting in 1 and going to 2.</p>
 <p>Cut along the curve, rotate it, and glue it on the right side.</p>	 <p>This is your pattern for this tiling group.</p>

We fulfill the plane by using symmetries and rotations of angle 120° :

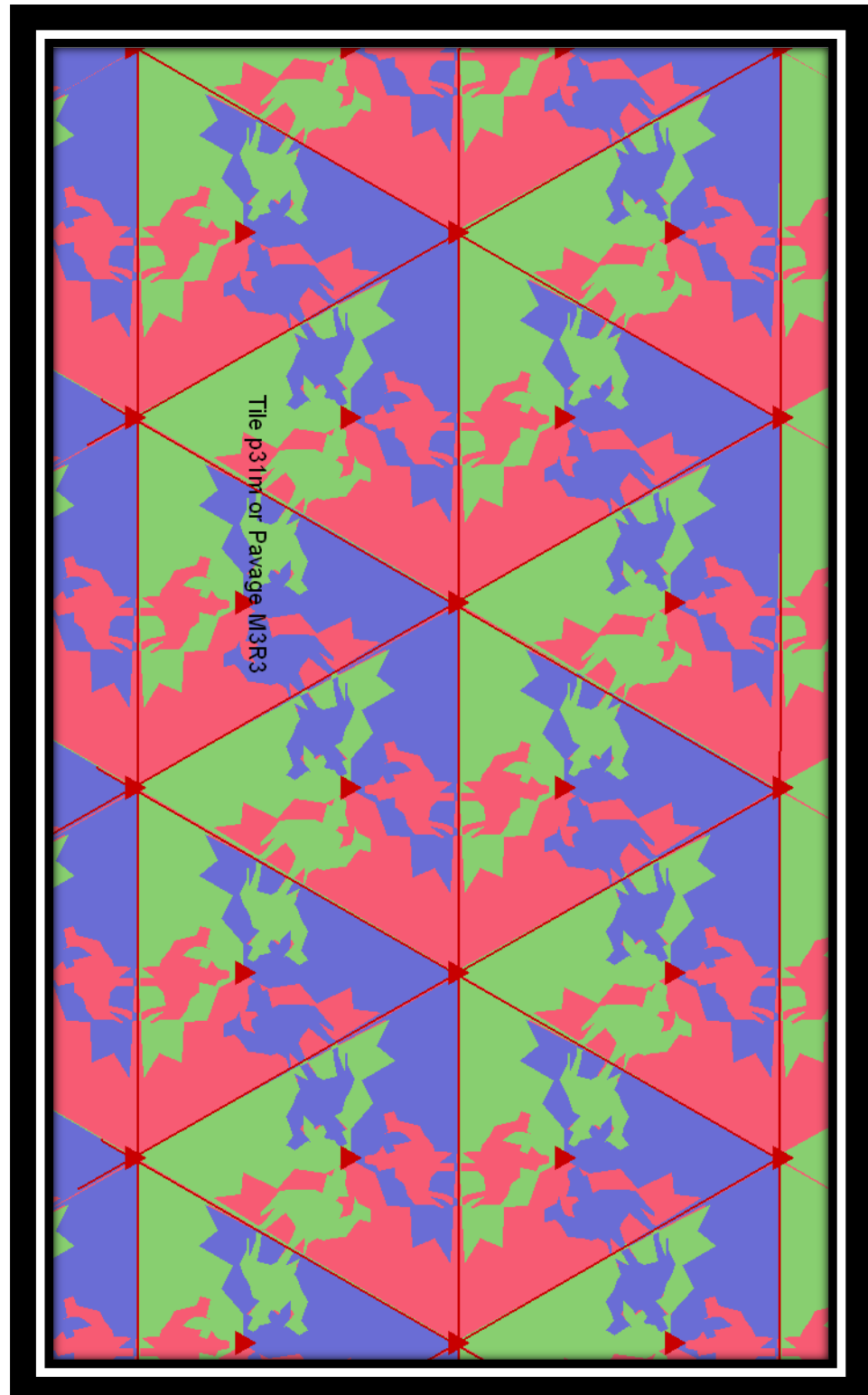




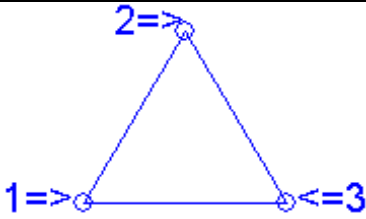
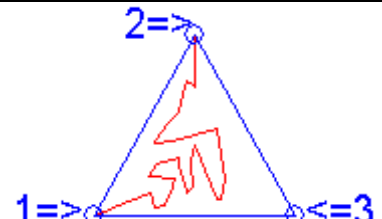
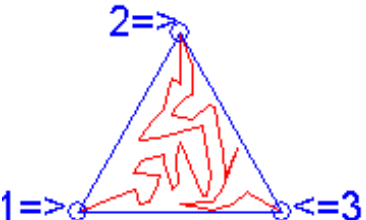

Lattice and fundamental region for p31m (M3R3)

The following picture is the lattice associated to the Tiling p31m (M3R3), the fundamental region is equilateral triangle, and as you can check there are translations following the sides of the triangles. The lattice is the same as the

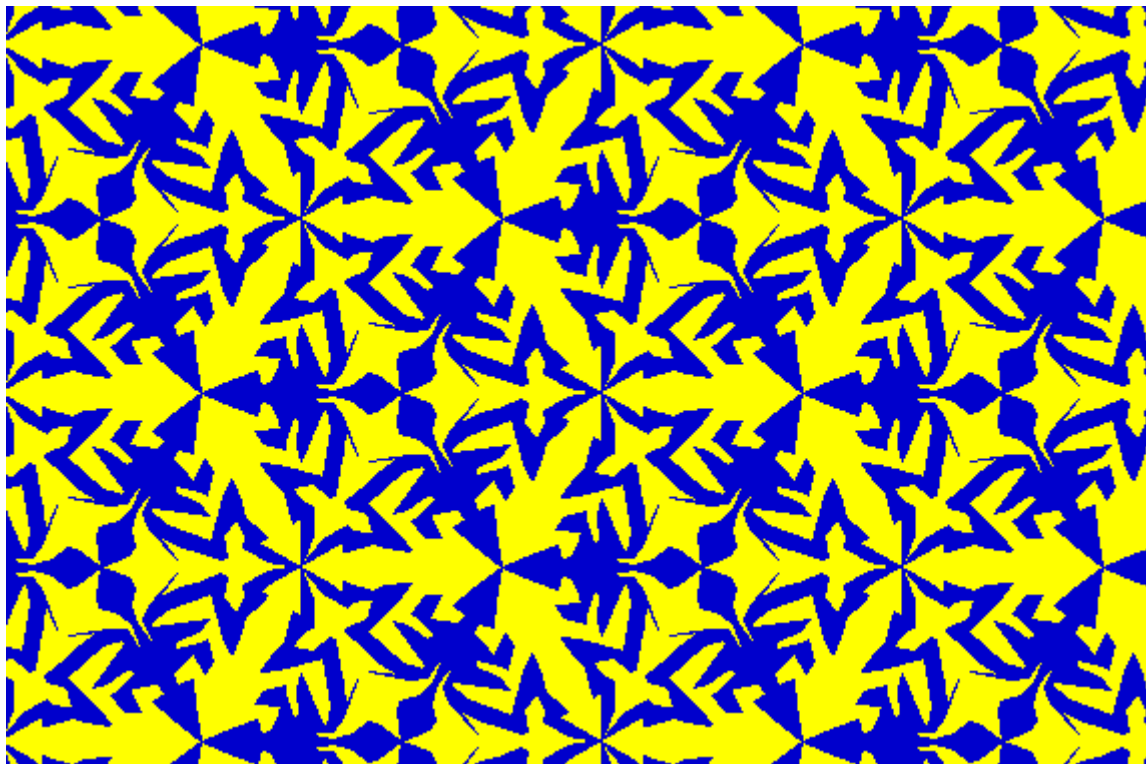
tiling $p3$ ($R3$), and we must add the mirror lines, that is the sides of the triangles.

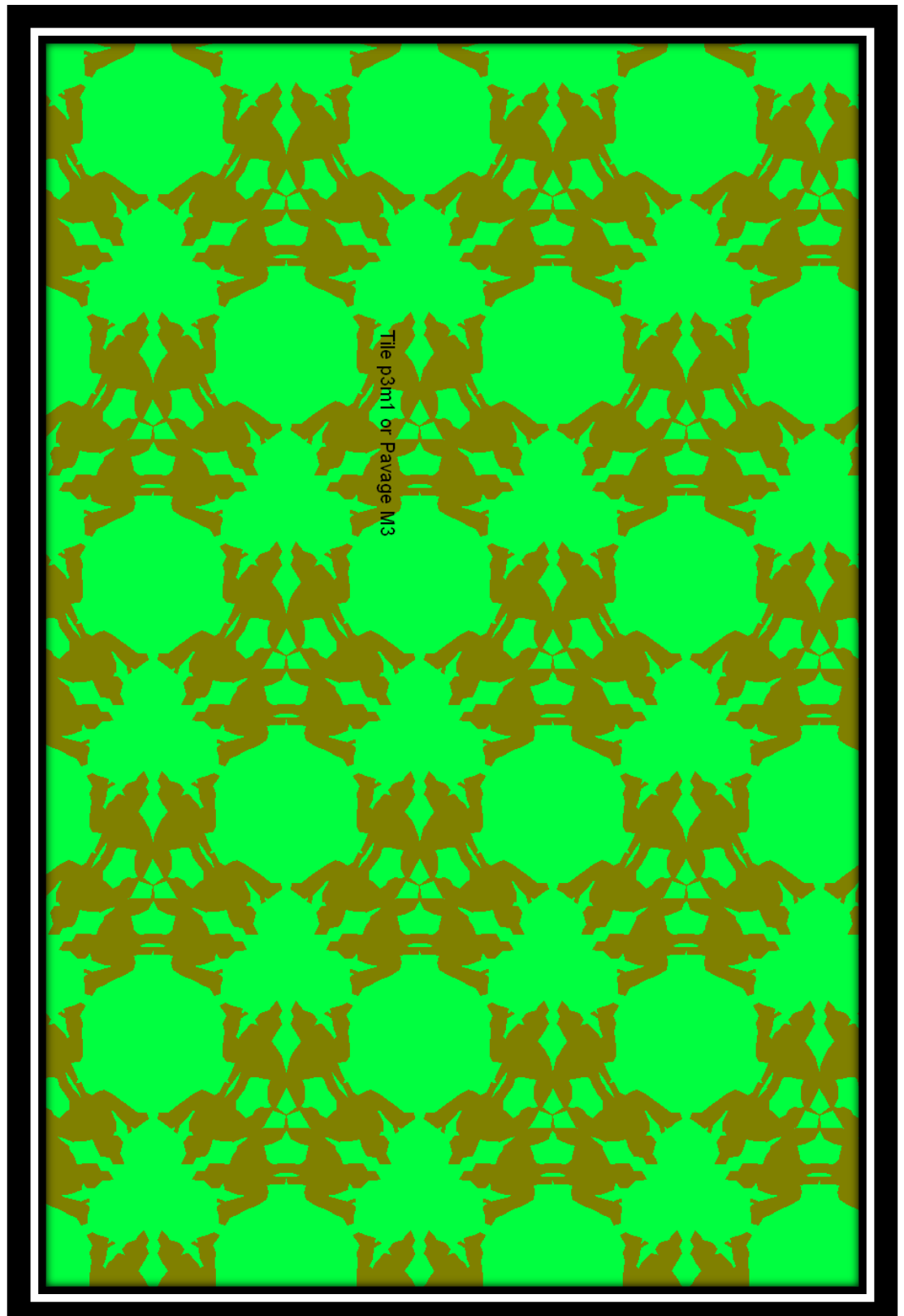


Tiling $p3m1$ or $M3$

 <p>Take a triangular piece of paper (equilateral).</p>	 <p>Draw a simple curve starting in 1 and going to 2.</p>
 <p>Draw a simple curve starting in 2 and going to 3.</p>	 <p>This is your pattern for this tiling group.</p>

We fulfill the plane by using symmetries with axis the sides of the equilateral triangle:

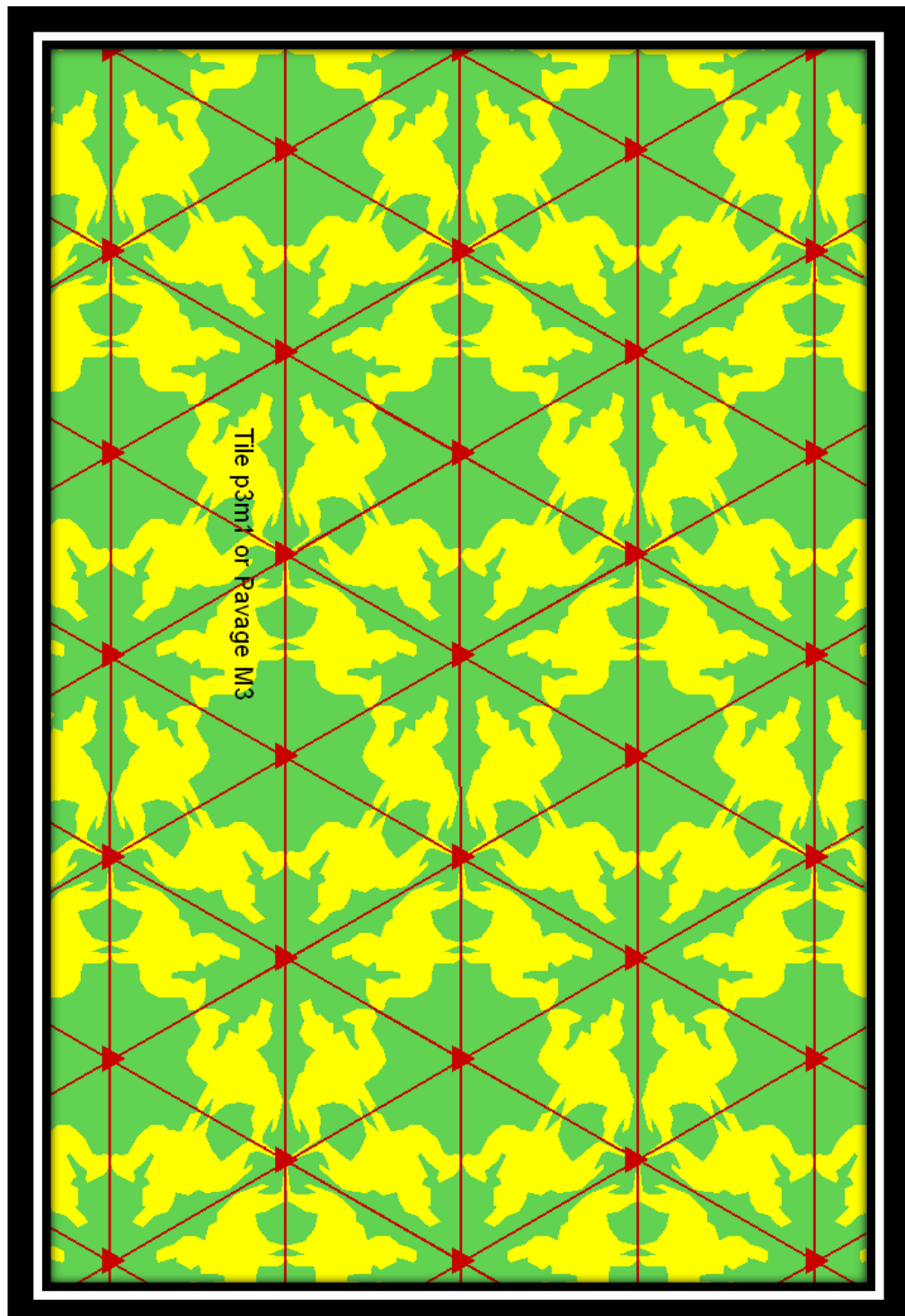




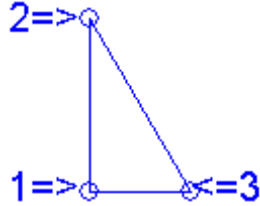
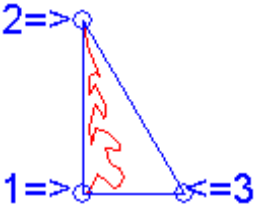
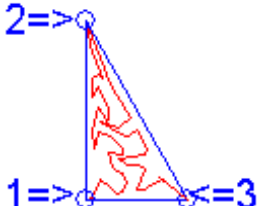
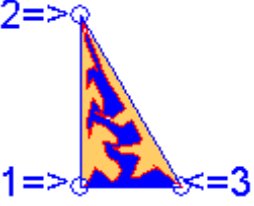
Lattice and fundamental region for p3m1 (M3)

The following picture is the lattice associated to the Tiling p3m1 (M3), the fundamental region is an equilateral triangle, and as you can check there are

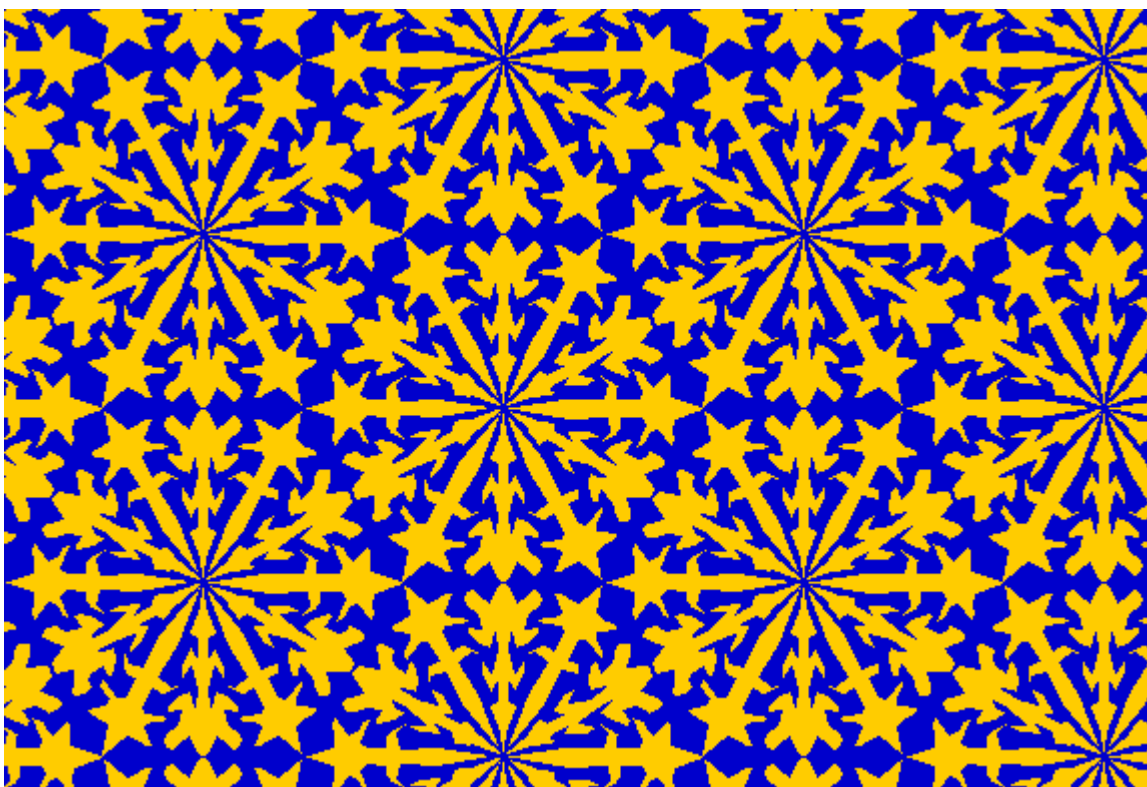
translations following the sides of the triangles. The lattice is the same as the tiling $p3$ ($R3$), and we must add the mirror lines, that is the sides of the triangles.

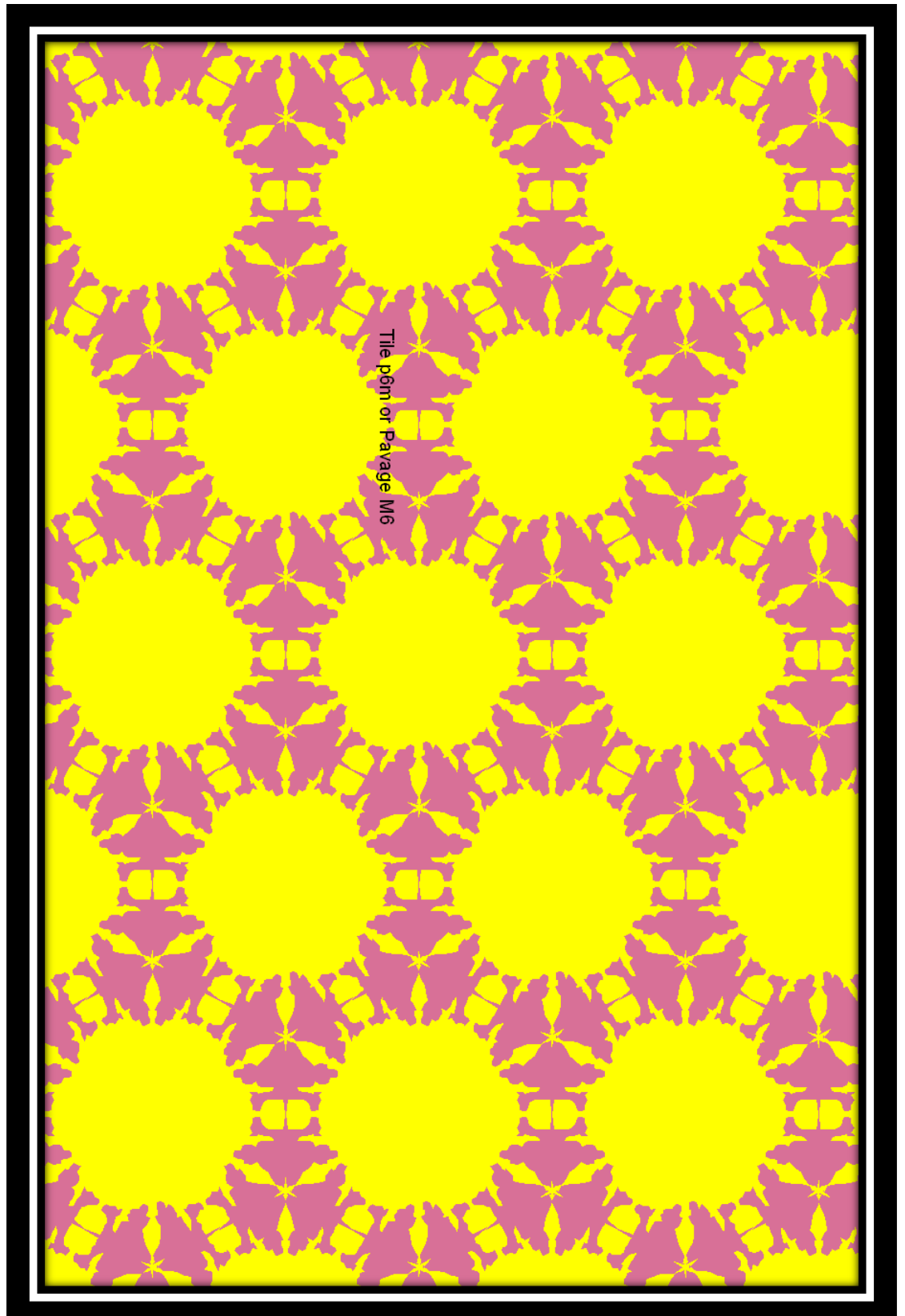


Tiling $p6m$ ou M6

 <p>Take a triangular piece of paper. (The half of an equilateral triangle).</p>	 <p>Draw a simple curve starting in 1 and going to 2.</p>
 <p>Draw a simple curve starting in 2 and going to 3.</p>	 <p>This is your pattern for this tiling group.</p>

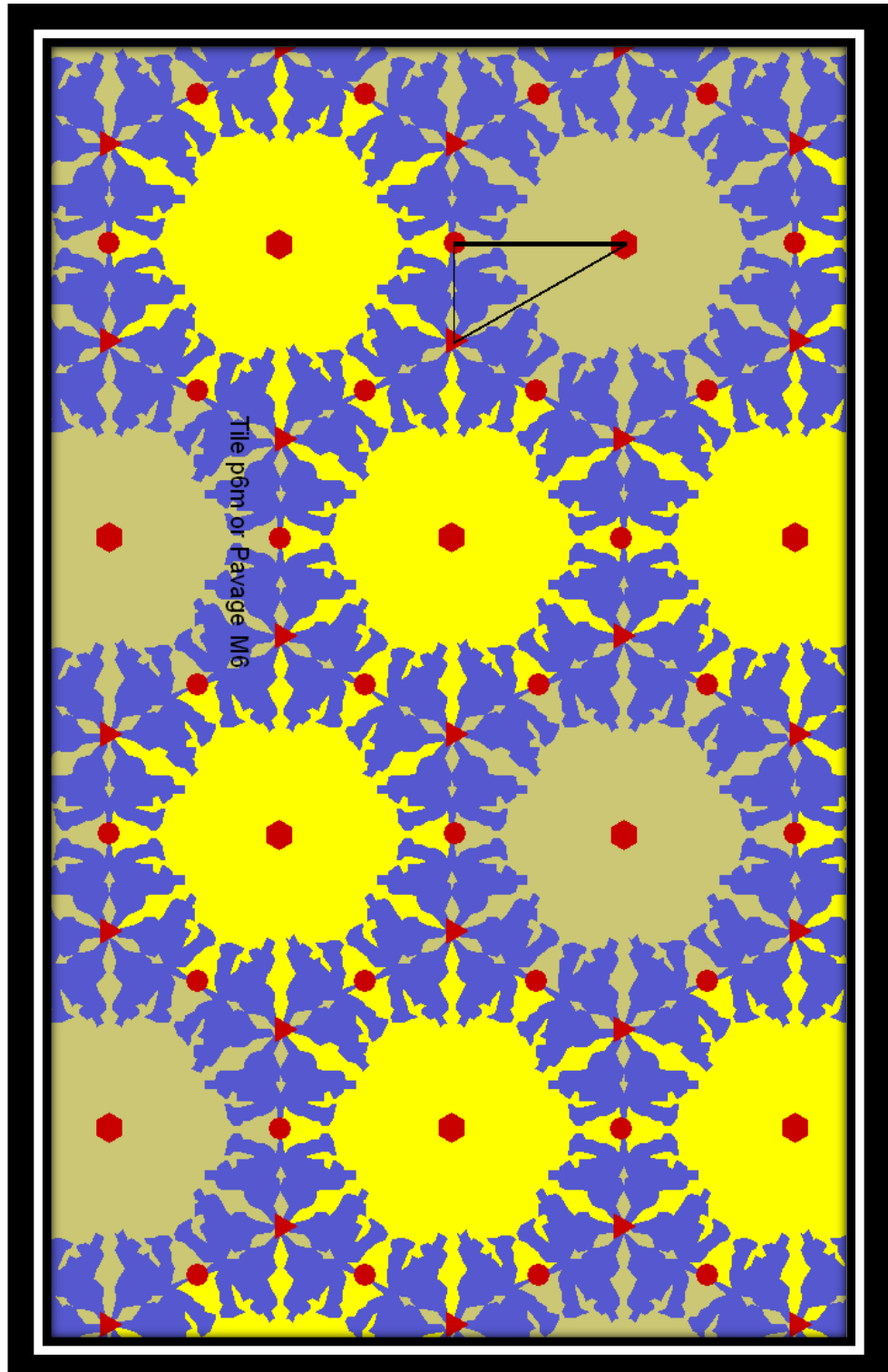
We fulfill the plane by using symmetries with axis the sides of the rectangle triangle:



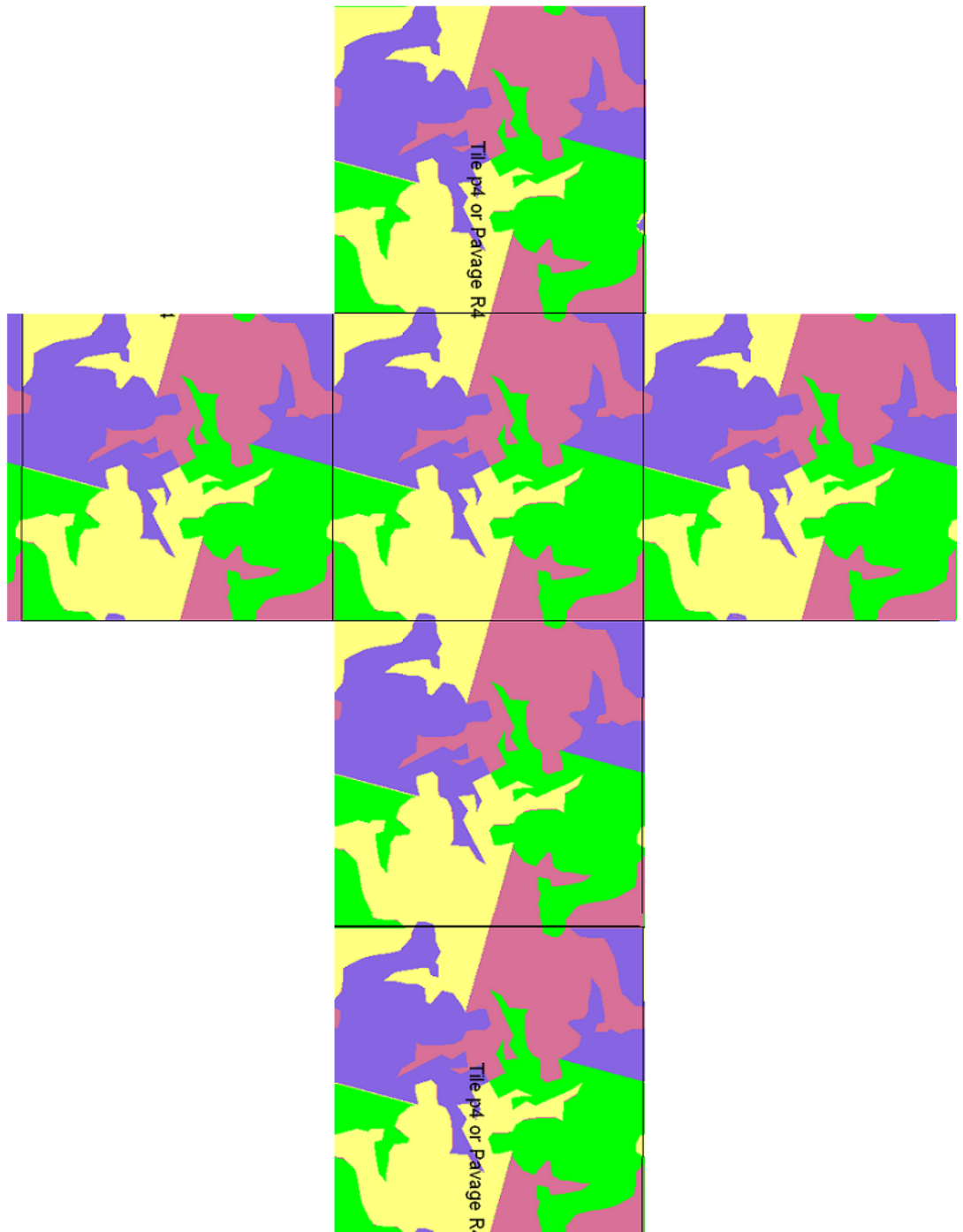


Lattice and fundamental region for p6m (M6)

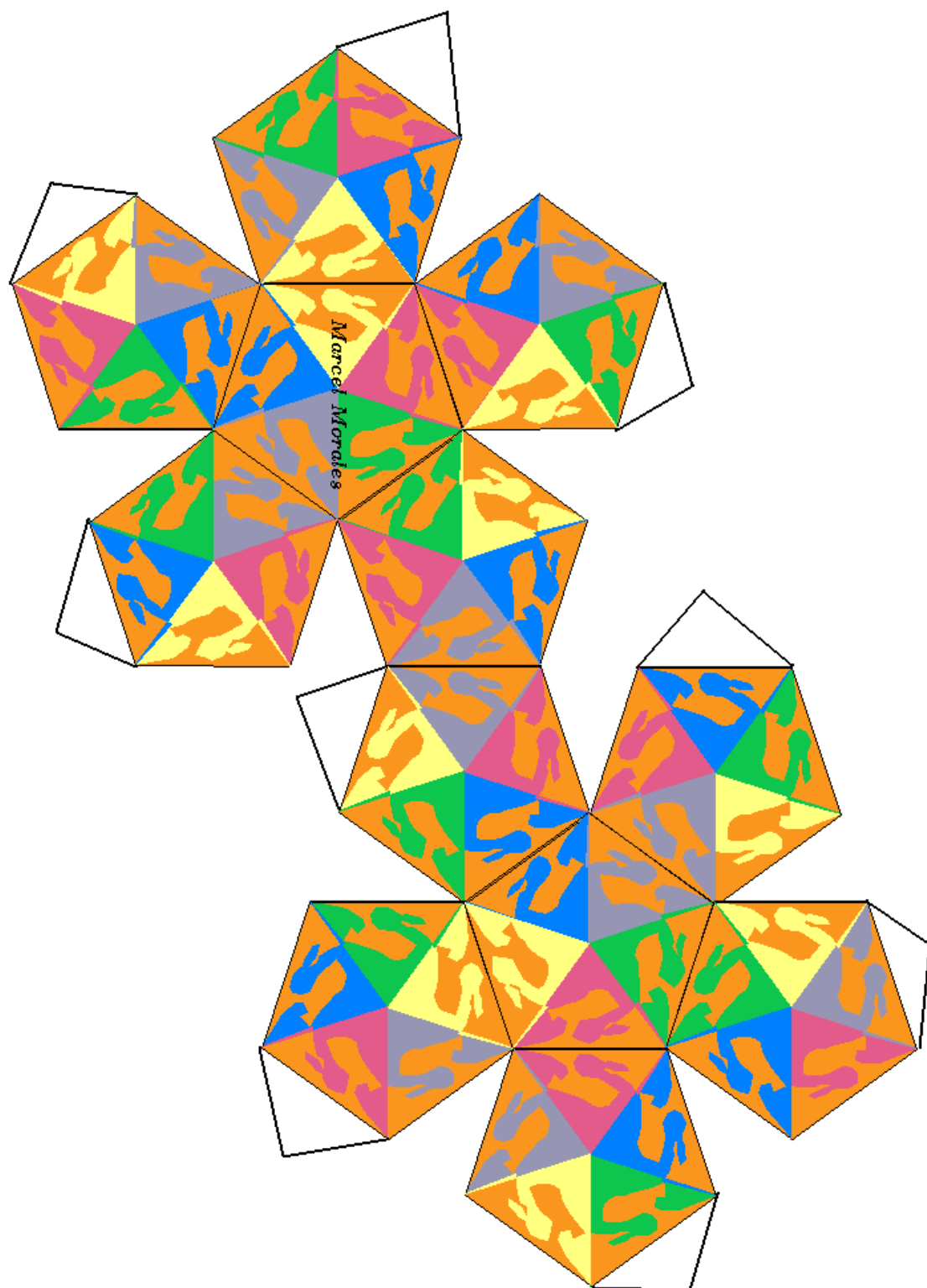
The following picture is the lattice associated to the Tiling p31m (M3R3), the fundamental region is equilateral triangle, and as you can check there are translations following the sides of the triangles. The lattice is the same as the tiling p3 (R3), and we must add the mirror lines, that is the sides of the triangles.

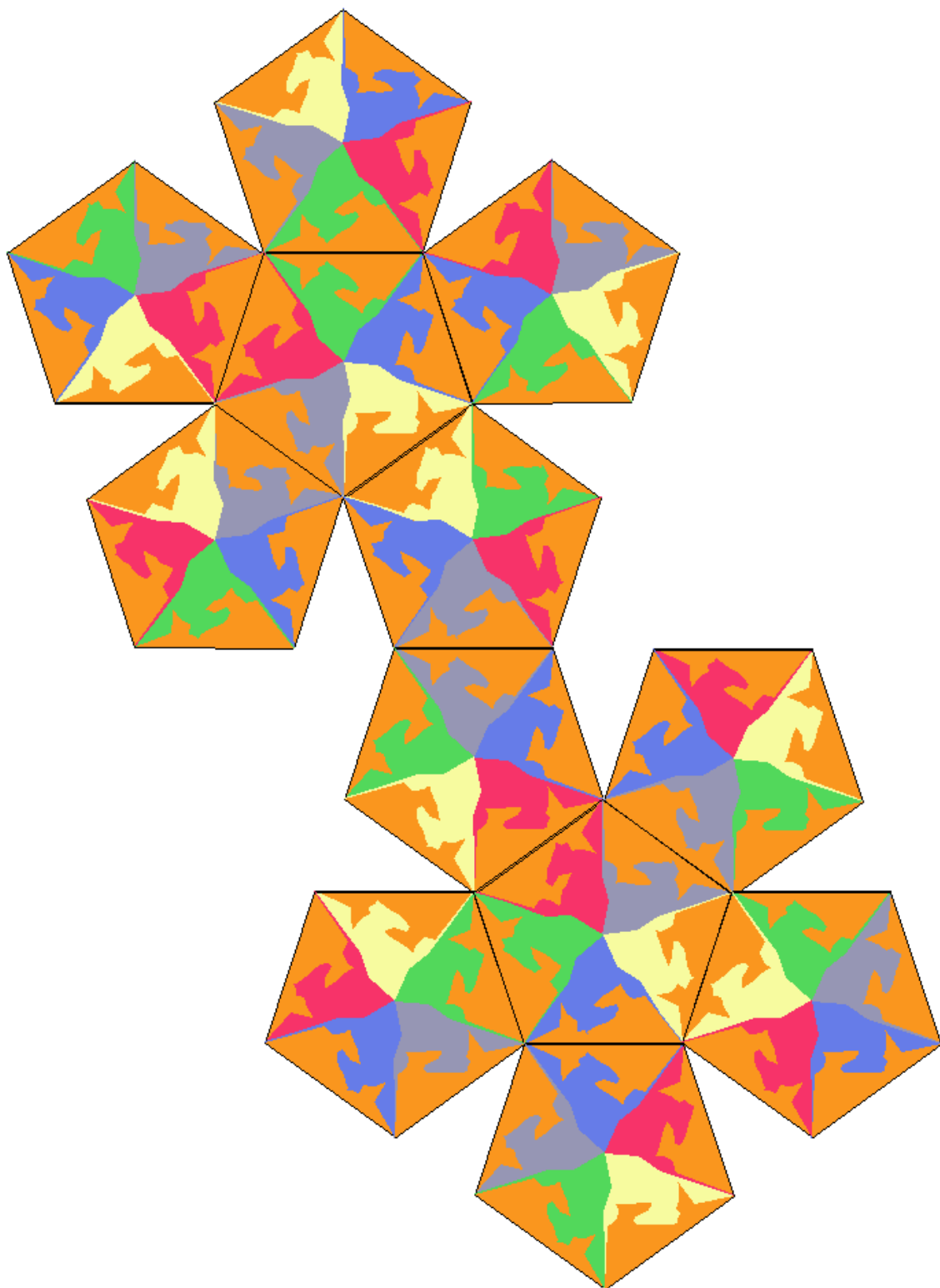


Pattern of a tiling of a cube

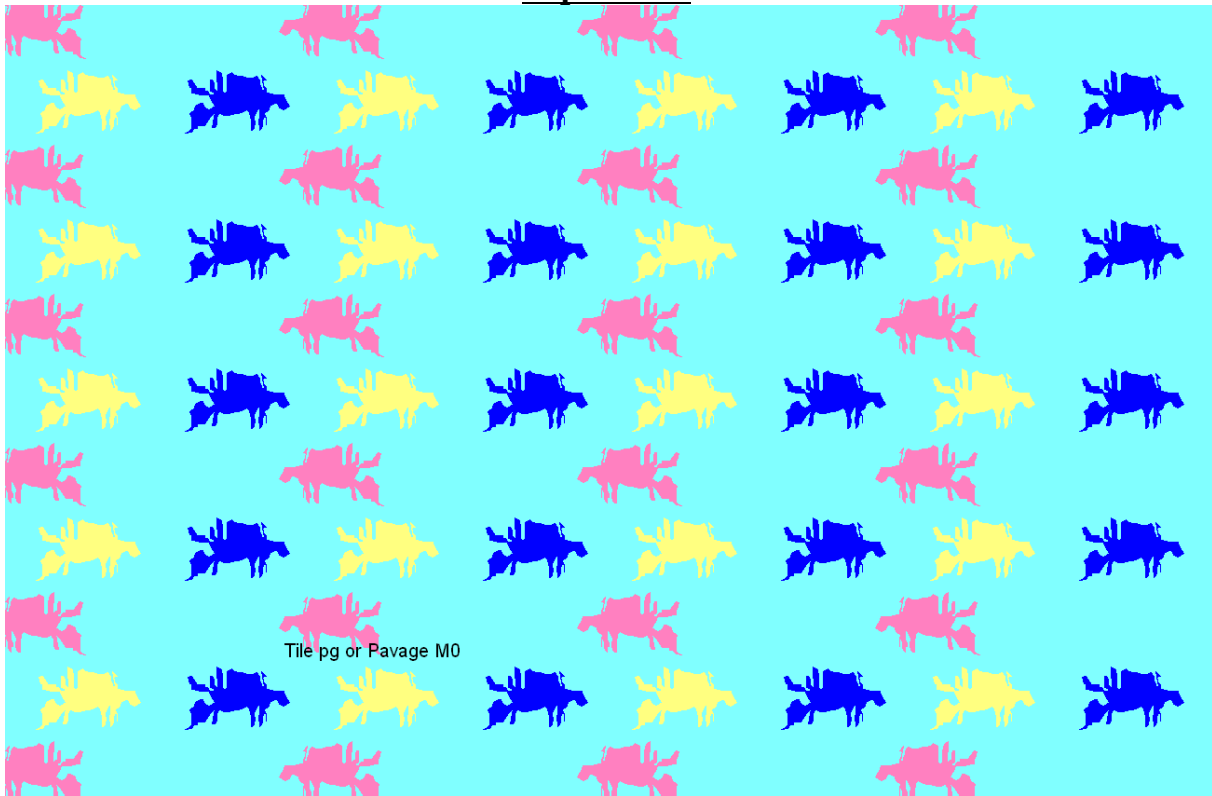


Pattern of a tiling of a dodecahedron





Paper wall



The hyperbolic Tilings

We understand by hyperbolic geometry, the drawn of pictures and transformations by using the hyperbolic distance. The Hyperbolic plane is represented by a disk without the border, the border of the disk is the infinite; figures in the hyperbolic plane seem smaller when we approach to infinite. We give some examples of hyperbolic Tiling of the hyperbolic plane, drew by my software. My software is the first in the world to draw automatically hyperbolic tiling and allows anybody to realize its own hyperbolic tiling. By contrast with Euclidian geometry there are infinitely many hyperbolic tiling groups.



