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► To cite this version:

Jean Auriol, Ulf Jakob F Aarsnes, Roman Shor. Self-Tuning Torsional Drilling Model for Real-Time Applications. ACC 2020 - American Control Conference, Jul 2020, Denver, United States. 10.23919/acc45564.2020.9147242 . hal-02524244

HAL Id: hal-02524244

<https://hal.science/hal-02524244>

Submitted on 30 Mar 2020

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Self-Tuning Torsional Drilling Model for Real-Time Applications*

Jean Auriol¹, Ulf Jakob F. Aarsnes² and Roman Shor¹

Abstract—A self tuning model construct is presented which includes a field validated torsional drillstring model and a bit-rock interaction law. The drillstring model includes side-forces from borehole contact, where the kinematic and static friction coefficients are tuned when the drillstring begins rotating but prior to the bit contacting the bottom. Subsequently, when tagging bottom and drilling ahead, the estimation of side-forces is suspended and changes in measured torque is used to update the parameters in the bit-rock interaction model. This approach allows to isolate the effects of side forces and bit torque, respectively, and consequently tune both these elements in torsional drilling model for real-time applications.

I. INTRODUCTION

In deep drilling applications, communication between subsurface tools and surface equipment typically has low bandwidth and high latency, so there exists a need for real-time, high fidelity estimation of distributed drillstring state based on surface measurements. With such estimation, the efficiency of directional drilling operations would increase, wellbore quality would improve and overall cost of well construction would be reduced [1]. In a previous work [2] we presented a torsional model of the drill string, with side forces, that gave a good compromise between model fidelity and complexity for certain applications [3], [4], [5]. However, this model had one significant limitation, which was the lack of model for torque on bit. The present paper is an extensions of this work to address this limitation by adding such a model and proposing an approach for tuning the magnitude of both the side forces and the torque on bit.

The goal is to create a simple, or minimal, real-time model by ignoring all but the dominating effects of interest, which in this case are the torsional dynamics of the drill string. As such, again we do not include the axial dynamics of the drill string, see e.g. [6], [7] for a distributed coupled axial-torsional model. Further improvements on models with side forces have been presented by [8]. Other effects that can be relevant to include, but at the cost of increased complexity, are discussed in [9]. For more perspective on drilling models we refer to [10].

*This work was in part supported by the Marie Skłodowska-Curie (MSCA) - Individual Fellowships through the Robust Estimation and Control of Infinite Dimensional Systems (RECIDS) Project. Additional support came from the University of Calgary's Canada First Research Excellence Fund Program, the Global Research Initiative in Sustainable Low Carbon Unconventional Resources and a National Science and Engineering Research Council (NSERC) Collaborative Research and Development (CRD) Grant with Precision Drilling.

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Our goal of a simple model amenable for self-tuning requires us to modify the torque on bit model. This is done by assuming a steady axial velocity of the bit. For a drillstring in constant rotation, this may be a valid first order approximation since axial stick-slip is reduced or eliminated due to the fact that there is no portion of the drillstring in zero axial velocity and thus no switching between static and kinematic friction. Given this assumption, we rigorously derive a relation for torque on bit from the classical model of [11], [12]. The derivation and evaluation of this simplified torque on bit model is one of the key contributions of the paper. Previous torque on bit models from the literature have either been based on the Stribeck effect, which is incorrect as the cutting action has no inherent velocity weakening effect [13], [14], or have been completely synthetic and not related to physical properties. Our proposed model finds the velocity weakening effect from a the experimentally validated principles of cutting and can be computed from the physical properties of the model.

The paper first recaps the distributed torsional model with side forces in Section II. Then, we propose our approach to a torque on bit model without axial dynamics. In Section III, the observer is presented for the bit-rock interaction law. Finally, in Section IV, we show simulation results with the full model and illustrate the estimation of the bit-rock interaction parameters.

II. MODEL

In this section we recap the main points of the torsional simulation model given in full in [2], and then extend the model to also account for torque at the bit from the cutting action. The model is relatively simple to facilitate its use regarding control and estimation applications. The main assumptions we use are the following:

- The torsional motion of the drill string is the dominating dynamic behaviour.
- Uniform axial motion. No distributed axial dynamics.
- The transition from static to dynamic Coulomb friction is modelled as a jump, i.e., the Stribeck curve is assumed negligible.
- The effects of along-string cuttings distribution on the friction is assumed to be homogeneous.
- The effect of the pressure differential, inside and outside the drill string, on the bending moment is not represented and is assumed to be negligible.

A. Torsional dynamics of the drill string

The torsional motion of the drill string is assumed to be the dominating dynamic behavior. We represent the torsional dy-

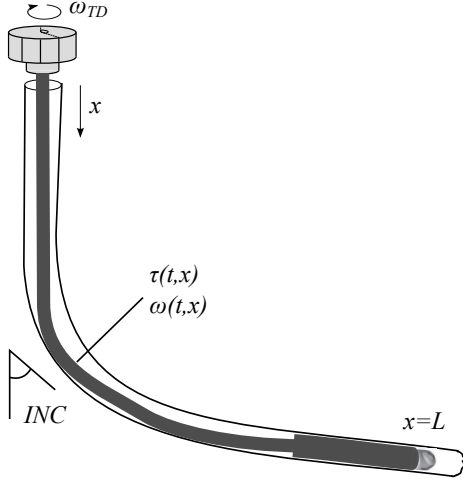


Fig. 1: Schematic indicating the distributed drill string of length L lying in deviate borehole.

namics with a distributed wave model where discontinuities in impedance can be included to model different sections of the drill string, such as a pipe and a collar section. This kind of representation is popular in the literature [6], [7], however we here only use the torsional dynamics. The parts of the model presented before is only covered cursory here, but we refer to [2] for the full model derivation.

We denote the angular velocity and torque as $\omega(t,x)$, $\tau(t,x)$, respectively, with $(t,x) \in [0,\infty) \times [0,L]$ (L being the length of the drill string). A schematic representation of the drill string is given in Fig. 1. We have

$$\frac{\partial \tau(t,x)}{\partial t} + JG \frac{\partial \omega(t,x)}{\partial x} = 0 \quad (1)$$

$$J\rho \frac{\partial \omega(t,x)}{\partial t} + \frac{\partial \tau(t,x)}{\partial x} = S(\omega,x), \quad (2)$$

where we model the source term S as

$$S(\omega,x) = -k_t \rho J \omega(t,x) - \mathcal{F}(\omega,x), \quad (3)$$

where the damping constant k_t represent the viscous shear stresses and where $\mathcal{F}(\omega)$ is a differential inclusion that represent the Coulomb friction between the drill string and the borehole, also known as the side force.

The side force is implemented using the following inclusion

$$\begin{cases} \mathcal{F}(\omega,x) = r_o(x) \mu_k F_N(x), & \omega > \omega_c, \\ \mathcal{F}(\omega,x) \in \pm r_o(x) \mu_s F_N(x), & |\omega| < \omega_c, \\ \mathcal{F}(\omega,x) = -r_o(x) \mu_k F_N(x), & \omega < -\omega_c, \end{cases} \quad (4)$$

where ω_c is the threshold on the angular velocity where the Coulomb friction transits from static to dynamic, $r_o(x)$ is the outer drill string radius, and μ_s, μ_k are the static and kinetic friction coefficients. The function $\mathcal{F}(\omega) \in \pm r_o(x) \mu_s F_N(x)$ denotes the inclusion where

$$\begin{aligned} \mathcal{F}(\omega,x) &= -\frac{\partial \tau(t,x)}{\partial x} - k_t \rho J \omega(t,x) \\ &\in [-r_o(x) \mu_s F_N(x), r_o(x) \mu_s F_N(x)], \end{aligned} \quad (5)$$

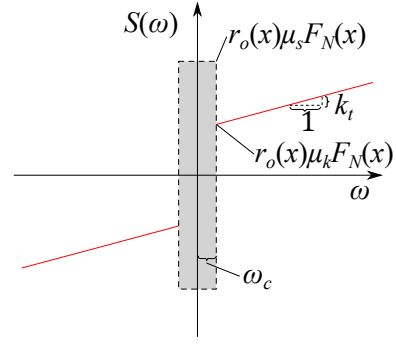


Fig. 2: Schematic illustrating the four parameters determining the friction: the coulomb friction parameters ω_c , F_c , F_d and the viscous friction coefficient k_t . The shaded region represents the angular velocities for which a constant value of static torque is assumed and the red curve indicates the dynamic torque as a function of angular velocity.

and take the boundary values $\pm \mu_s F_N(x)$ if this relation does not hold. The shape of the friction source term is illustrated in Fig. 2. Using the torque model of [15] it is possible to derive the normal force profile $F_N(x)$. Assuming a planar well and torsional rotation of the drill string the normal force in terms of the tension profile σ_e writes:

$$\sigma_e(x) = \int_L^x W_b \cos \theta(\xi) d\xi \quad (6)$$

where $W_b(x) = gA(x)(\rho - \rho_{mud})$ is the buoyed weight per meter. The normal force profile, F_N , is obtained as

$$F_N(x) = \left(\sigma_e(x) \frac{\partial \theta}{\partial x} + W_b \sin(\theta) \right). \quad (7)$$

B. Top-drive boundary condition

The top drive at the topside boundary is actuated by a motor torque, τ_m , that is controlled using a PI control law [16] to a desired velocity set-point ω_{SP} :

$$e = \omega_{SP} - \omega_{TD} \quad (8)$$

$$I_e = \int_0^t e(\xi) d\xi \quad (9)$$

$$\tau_m = k_p e + k_i I_e, \quad (10)$$

where k_p is a proportional gain and k_i an integral gain. We denote J_{TD} the topdrive inertia. We have the following equation

$$\frac{\partial \omega_{TD}}{\partial t} = \frac{1}{J_{TD}} (\tau_m - \tau_0). \quad (11)$$

Finally, the angular velocity at the top of the drill string verifies $\omega_0 = \omega_{TD}$.

C. Torsional bit-rock interaction (BRI) law

We now consider the downhole boundary condition. More precisely, we derive the torsional bit-rock interaction (BRI) law which is a consequence of the axial motion of the drill-string while drilling. The model we propose in this paper is

a simplified version of the classical bit-rock interaction law described in [12] as we assume that the Rate Of Penetration (ROP) is constant. This approximation allows us to represent the torque on bit without the complexity of including a full axial model of the drill string. To ease the notations, the torque on bit will be denoted $\tau_b = \tau(t, L)$ and the bit RPM will be denoted $\omega_b = \omega(t, L)$. Moreover, we denote $v_b(t)$ the axial motion of the drill-bit (known as the **Rate of Penetration**, ROP) and $f_b(t)$ the force exerted at the bit (known as the **Weight on Bit**, WOB). In this work, we assume uniform axial motion such that the ROP is equal to block-velocity at the surface. In reality, a one to two second transfer delay, due to the speed of sound, exists between the surface and the bit, but this is constant for a specified drilling depth. Inspired by [12], we can now give the boundary condition satisfied by τ_b . The torque on bit τ_b can be decomposed as

$$\tau_b(t) = \tau_c(t) + \tau_f(t), \quad (12)$$

where τ_c is the component of torque associated with the cutting process, and τ_f is the component for the frictional process. Regarding the cutting action, we have

$$\tau_c(t) = \frac{1}{2} a^2 \varepsilon N d(t), \quad (13)$$

where N is the number of cutter blades, a is the bit radius and ε the rock intrinsic specific energy. The function $d(t)$ is known as the *depth of cut*. It is defined by

$$d(t) = \frac{2\pi v_b}{N \omega_b(t)}. \quad (14)$$

Using (13), we obtain

$$\tau_c(t) = \frac{\pi v_b}{\omega_b(t)} a^2 \varepsilon. \quad (15)$$

Regarding the frictional component of the torque τ_f , we can write

$$\tau_f(t) = \tau_f^0 g(\omega_b), \quad (16)$$

where the non-linear function g (colloquially referred to as the $g(\cdot)$ non-linearity) enforces torsional stick if the torque applied to the bit by the drill-string cannot overcome the torque induced by the bit-rock interaction. If it is higher than that torque-on-bit, the bit starts to slip torsionally. Following Filippov's solution concept, and as described in [17] it can be represented by the convex set-valued map:

$$g(\omega_b) = \frac{1 - \text{Sign}(\omega_b)}{2} = \begin{cases} 0, & \omega_b > 0, \\ [0, 1], & \omega_b = 0, \\ 1, & \omega_b < 0, \end{cases} \quad (17)$$

where $\text{Sign}(\cdot)$ is the set-valued sign function. Due to potential stick-slip oscillations, the bit RPM can potentially be equal to zero. To avoid any problem, we will consider that the expression (15) holds only if $|\omega_b| > \omega_c$, where ω_c is a threshold on the angular velocity. If this is not satisfied we will set

$$\tau_c(t) = \pm \frac{\pi v_b}{\omega_c} a^2 \varepsilon \quad (18)$$

All in all, we get the following boundary condition for the torque on bit

$$\tau_b(t) = \begin{cases} a_2 g(\omega_b(t)) + a_1 \frac{v_b}{\omega_b(t)}, & \text{if } |\omega_b| > \omega_c, \\ a_2 g(\omega_b(t)) \pm a_1 \frac{v_b}{\omega_c}, & \text{if } |\omega_b| < \omega_c, \end{cases} \quad (19)$$

where a_1 and a_2 are real positive coefficients that depend on the different physical parameters.

D. Derivation of Riemann invariants

On each section of the drill string, we define the Riemann invariants as

$$\alpha_i = \omega_i + \frac{(c_t)_i}{J_i G_i} \tau, \quad \beta_i = \omega_i - \frac{(c_t)_i}{J_i G_i} \tau, \quad (20)$$

where $(c_t)_i = \sqrt{\frac{\rho_i}{J_i}}$ is the velocity of the torsional wave and where the index $i = c$ if we consider the collar section and $i = p$ if we consider the pipe section. These new states satisfy the diagonalized PDE [18] system

$$\frac{\partial \alpha_i}{\partial t} + (c_t)_i \frac{\partial \alpha_i}{\partial x} = -(k_t(\alpha_i + \beta_i) + \frac{1}{J_i \rho_i} \mathcal{F}) \quad (21)$$

$$\frac{\partial \beta_i}{\partial t} - (c_t)_i \frac{\partial \alpha_i}{\partial x} = -(k_t(\alpha_i + \beta_i) + \frac{1}{J_i \rho_i} \mathcal{F}). \quad (22)$$

with the boundary conditions

$$\frac{\partial \omega_0}{\partial t} = \frac{1}{I_{TD}} \left(\tau_m + \frac{GJ}{c_t} (\beta_p(t, 0) - \omega_0(t)) \right). \quad (23)$$

$$\beta_c(t, L) = \alpha_c(t, L) - \frac{2c_t}{J_c G_c} \quad (24)$$

$$\beta_p(t, L_p) = \frac{1}{1 + \bar{Z}} (\alpha_p(t, L_p)(1 - \bar{Z}) + 2\bar{Z}\beta_c(t, L_p)) \quad (25)$$

$$\alpha_c(t, L_p) = \frac{1}{1 + \bar{Z}} (2\alpha_p(t, L_p) - (1 - \bar{Z})\beta_c(t, L_p)), \quad (26)$$

where the two last conditions are due to the discontinuity in the drill string impedance, between the pipe and collar sections, where we have denoted the relative magnitude of the impedance as

$$\bar{Z} = \left[\frac{c_t}{JG} \right]^{\text{collar}} \left/ \left[\frac{c_t}{JG} \right]^{\text{pipe}} \right. \quad (27)$$

III. ESTIMATION

A. Observer design

In this section we design an observer that combines measurements from physical sensors with the proposed model of the system dynamics. This observer relies on the measured output of the system that corresponds to the top-drive angular velocity ω_0 . It should provide reliable estimates of the states (torque and RPM), of the friction coefficients related to the side forces (μ_{sta} and μ_{kin}) and an estimate of the BRI law parameters (a_1, a_2). The observer we design in this paper is an extension of the one that was presented in [19], which was shown to obtain good estimates of the torque and RPM states and of the side forces friction parameters when the bit is off bottom.

Let us denote with the $\hat{\cdot}$ supercript the estimated states and $e = \hat{\omega}_0 - \omega_0$ the measured estimation error of the top-drive angular velocity. The observer equations given in [19] in terms of Riemann invariants read as follows

$$\dot{\hat{\omega}}_0 = a_0 \left(\hat{\beta}_p(t, 0) - \hat{\omega}_0 \right) + \frac{1}{I_{TD}} \tau_m - p_0 e, \quad (28)$$

$$\frac{\partial \hat{\alpha}_i}{\partial t}(t, x) + c_i \frac{\partial \hat{\alpha}_i}{\partial x}(t, x) = \hat{\mathcal{J}}_i(t, x) - p_\alpha^i(x) e, \quad (29)$$

$$\frac{\partial \hat{\beta}_i}{\partial t}(t, x) - c_i \frac{\partial \hat{\beta}_i}{\partial x}(t, x) = \hat{\mathcal{J}}_i(t, x) - p_\beta^i(x) e, \quad (30)$$

the source term in each section being computed from the estimated states and friction factor ($\mathcal{F}(t, x)$ being defined in (5))

$$\hat{\mathcal{J}}_i(t, x) = k_i (\hat{\alpha}_i(t, x) + \hat{\beta}_i(t, x)) + \frac{1}{J_i \rho} \mathcal{F}(t, x), \quad (31)$$

Finally, the boundary conditions at the top and bottom, and between the drill string sections, are

$$\hat{\alpha}_p(t, 0) = 2\hat{\omega}_0(t) - \hat{\beta}_p(t, 0) - P_0 e, \quad (32)$$

$$\hat{\beta}_p(t, L_p) = \frac{\hat{\alpha}_p(t, L_p)(1 - \bar{Z}) + 2\bar{Z}\hat{\beta}_c(t, L_p)}{1 + \bar{Z}} - P_1 e, \quad (33)$$

$$\hat{\alpha}_c(t, L_p) = \frac{2\bar{Z}\hat{\alpha}_p(t, L_p) - \hat{\beta}_c(t, L_p)(1 - \bar{Z})}{1 + \bar{Z}}, \quad (34)$$

$$\hat{\beta}_c(t, L) = \hat{\alpha}_c(t, L), \quad (35)$$

and the estimates of the friction factor is updated according to

$$\dot{\hat{\mu}}_s(t) = \begin{cases} -l_s e, & |\hat{\omega}_{L_c}| \leq \omega_c, \\ 0, & |\hat{\omega}_{L_c}| > \omega_c, \end{cases} \quad (36)$$

$$\dot{\hat{\mu}}_k(t) = \begin{cases} 0, & |\hat{\omega}_{L_c}| \leq \omega_c, \\ l_k e, & |\hat{\omega}_{L_c}| > \omega_c, \end{cases} \quad (37)$$

Finally, the following saturation is used to improve robustness of the method: $\hat{\mu}_s = \max(\hat{\mu}_s, \hat{\mu}_k)$. The different observer gains $p_\alpha^i, p_\beta^i, p_0, p_1, P_0, P_1, l_s, l_k$ are given in [19]. As explained above, this observer provides a good estimation of the states and of the side forces friction parameters in the situation of an off-bottom bit. To take the effect of the BRI into account when we are drilling; equation (35) is changed to

$$\hat{\beta}_c(t, L) = \hat{\alpha}_c(t, L) + \hat{\tau}_b(t), \quad (38)$$

where

$$\hat{\tau}_b(t) = \begin{cases} \hat{a}_2 g(\omega_b(t)) + \hat{a}_1 \frac{v_b}{\hat{\omega}_b(t)}, & \text{if } |\hat{\omega}_b| > \omega_c, \\ \hat{a}_2 g(\omega_b(t)) \pm \hat{a}_1 \frac{v_b}{\omega_c}, & \text{if } |\hat{\omega}_b| < \omega_c, \end{cases} \quad (39)$$

where the estimates of the BRI estimation parameters are updated according to

$$\dot{\hat{a}}_1(t) = \begin{cases} 0, & |\hat{\omega}_{L_c}| > \omega_c, \text{ or } v_b = 0, \\ -l_{a_1} e, & |\hat{\omega}_{L_c}| < \omega_c, \text{ and } v_b \neq 0, \end{cases} \quad (40)$$

$$\dot{\hat{a}}_2(t) = \begin{cases} 0, & v_b = 0, \\ -l_{a_2} e, & v_b \neq 0, \end{cases} \quad (41)$$

with l_{a_1} and l_{a_2} being two tunable gains.

Remark 1: We have implicitly assumed in (39) that the variable v_b is known. Sometimes, measurements of v_b are available, however, if not the top-drive velocity can be used in stead when drilling ahead. Hence, we assume an uniform axial motion of the drill string.

B. Overview of the estimation strategy

In [19] it has been shown how the observer (28)-(41) can estimate the friction coefficients of the side forces with the bit off-bottom. However, when the bit goes on bottom the total torque acting on the drill string equals the sum of the side forces and the torque from the cutting action of the bit. The friction coefficients of the side forces and the coefficients in the BRI law have a similar effects on the topside measured output of the drilling system. Hence, the BRI law parameters and the static and kinematic friction coefficients may not be distinguishable by our observer. Consequently, we propose an approach wherein these two sets of parameters are estimated separately.

Consider the following strategy:

- 1) After a connection, rotation is restarted with the bit off-bottom. Any discrepancy between the model and the measured data is used to estimate the side force friction factors μ_{kin} and μ_{sta} using the observer (28)-(37).
- 2) When the axial motion of the block is initiated, we stop the estimation of the side force friction factors.
- 3) Finally, when the bit tags the bottom of the well and when the quasi-steady drilling is resumed, we can start estimating the BRI parameters.

This strategy will be applied on simulated data in the next section.

IV. NUMERICAL RESULTS

The observer we have presented in the previous section may be used to provide online estimation for the BRI law parameters a_1 and a_2 but can also be used to estimate BHA rotation and torque. More precisely, the contact between the bit and the rock implies a change of period and amplitude for the RPM and the torque that can be captured by our observer. Estimating BHA angular orientation is of particular usefulness for directional drilling scenarios and feedforward stick-slip mitigation systems. Our observer is tested against the simulation model given in [6] using the wellbore survey given in Figure 4. The side forces static friction term is chosen to be equal to 0.42, while the kinematic friction term is equal to 0.29. We set a reference for the top-drive RPM of 65. As described in Figure 3, the scenario is the following: we rotate with the bit off-bottom for $t \in [0, 30]$. Then, at $t = 30$, ROP slowly increases before reaching its final value. The BRI law parameters are chosen as $a_1 = 100$ and $a_2 = 600$. As the purpose of this paper is to estimate the coefficients a_1 and a_2 , we will assume that the side-forces kinematic and static friction are known a priori and will not be updated. As explained above, they can be estimated when, after a connection, rotation is restarted off-bottom (see [19])

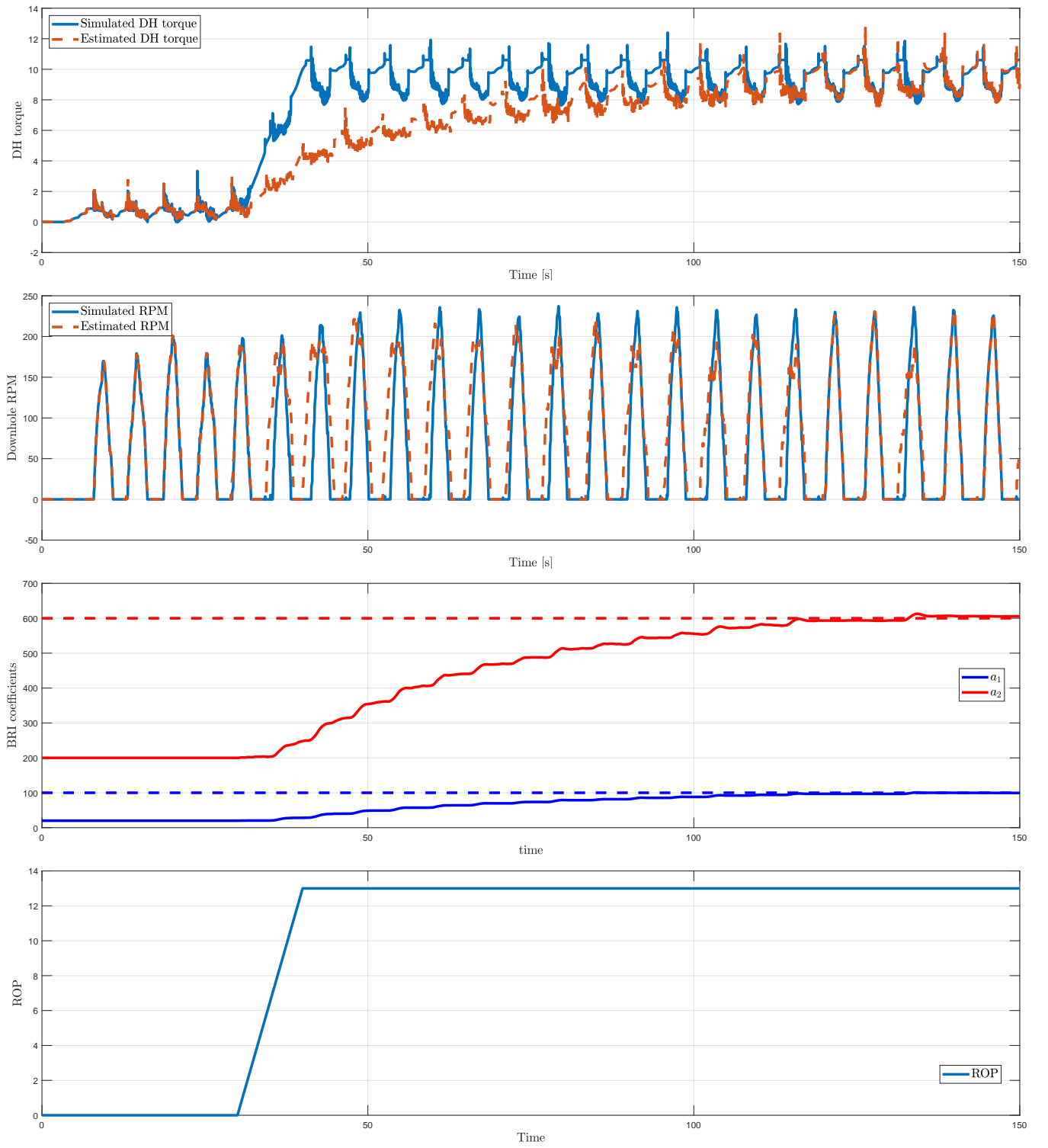


Fig. 3: Simulated and Estimated BHA torque, $\tau(t, L)$ (top), BHA RPM, ω_b (second), estimation of the BRI coefficients a_1 and a_2 (third) and rate of penetration (bottom) for the case of known side force friction coefficients.

for details). We have pictured in Fig 3, the estimation given by the observer of the BHA torque and RPM as well as the estimation of the friction parameters (as well as the ROP profile). One can notice the previously mentioned change of amplitude and period after $t > 30$. As the side-forces friction

parameters are perfectly known, our estimations match perfectly before the initiation of the axial motion. Then, after some time, the estimated states converge towards the real ones. Finally, our observer provides a reliable estimation of the BRI law coefficients.

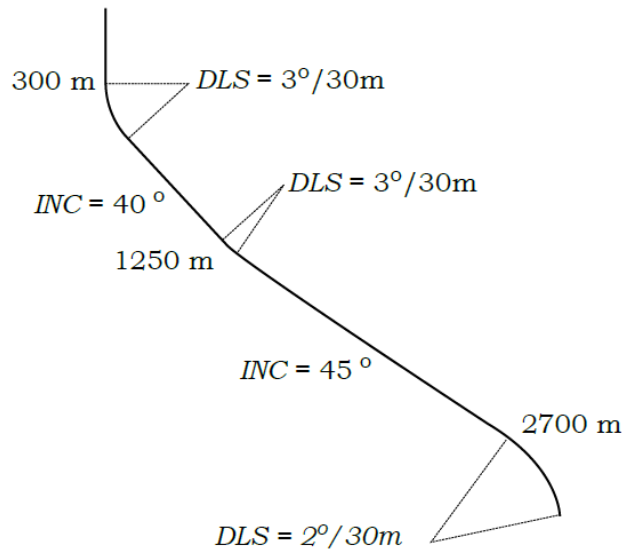


Fig. 4: Wellbore survey of the well. The length of the drill-string is 1750m.

V. CONCLUSION

Real-time estimation of bit-rock interaction parameters is a well known challenge in the drilling industry and, if understood, presents the opportunity to improve drilling operations through feedforward or model predictive control and to improve the knowledge of formation properties. We present an extension to a field validated torsional drillstring model with distributed friction which includes a bit-rock interaction law. We have presented method to estimate bit-rock interaction parameters, first through the derivation of an observer and secondly as a procedure to differentiate between drillstring effects and bit-rock interaction effects. The formulation is tested on simulated data modelling a field scenario, where it is assumed that friction coefficients along the drillstring are known, and convergence to modelled bit-rock interaction coefficients is shown. The computational efficiency and simplicity of this approach presents an appealing candidate for an online, real-time formation sensing system for field applications.

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