

# No Better Time Than Now: Future Uncertainty and Private Investment Under Dictatorship

## Supporting Information

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# S1 Proof of Proposition 1

We first prove part (b) of Proposition 1.<sup>1</sup> Suppose  $\theta < \theta'$ , and denote the associated optimal strategies of those types by  $m = m^*(\theta)$  and  $m' = m^*(\theta')$ . Here, we show that  $m < m'$ . First, rewrite the potential autocrat's expected utility as:

$$\int \bar{p}a(m+u)g(u)du - C(m, \theta) = \bar{p}\pi(m) - C(m - \theta).$$

Let  $\theta < \theta'$ , and denote  $m = m(\theta)$  and  $m' = m(\theta')$ . By optimality,

$$\bar{p}\pi(m) - C(m - \theta) \geq \bar{p}\pi(m') - C(m' - \theta) \quad \text{and} \quad \bar{p}\pi(m') - C(m' - \theta') \geq \bar{p}\pi(m) - C(m - \theta').$$

Therefore, we can write  $C(m' - \theta) - C(m - \theta) \geq C(m' - \theta') - C(m - \theta')$ . By strict convexity of  $C(\cdot)$ ,  $m' > m$ , so the optimal messaging strategy  $m^*(\theta)$  is increasing in  $\theta$ .

Now we can show that the elite uses a threshold rule defined in part (a) of Proposition 1:

$$a^*(s) = \begin{cases} 1 & \text{if } s \geq k^* \\ 0 & \text{if } s < k^* \end{cases}$$

where the equilibrium threshold  $k^*$  is such that economic elites are indifferent between investing under the current autocrat and waiting for a potential autocrat turnover to invest whenever they receive a signal exactly at this threshold (i.e., when  $\mathbb{E}[\theta | s = k^*] = \bar{\theta}$ ).

The equilibrium message,  $m^*(\theta)$ , is increasing in the type  $\theta$ . Moreover, the distribution of the noise satisfies the monotone likelihood property. That is, for two messages  $m'$  and  $m''$ , if  $m' > m''$ , then  $g(s - m')/g(s - m'')$  increases in  $s$ . This implies that higher signals are “good news” (Milgrom 1981): higher signals imply a higher posterior distribution of types in the sense of first-order stochastic dominance, so that the expected message is strictly increasing in the signal. As a result, the expected type conditional on the signal is increasing in the

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<sup>1</sup>The formal setup in this paper draws from and adapts the framework in Cunningham & de Barreda (2015), which builds a noisy signaling model in which the sender's type is drawn from a continuous distribution and the receiver's action is binary. This model in turn builds on Caselli, Cunningham, Morelli & Barreda (2014). Some proofs and derivations therefore follow Cunningham & de Barreda (2015) and Caselli et al. (2014). The proof of Proposition 1 builds from the steps in Caselli et al. (2014, 413–414) and Cunningham & de Barreda (2015, 17–19).

signal: if  $s_1 < s_2$ , then  $\mathbb{E}[\theta|s_1] < \mathbb{E}[\theta|s_2]$ . Therefore, if  $\mathbb{E}[\theta|s = k^*] = \bar{\theta}$  has a solution, it is unique. Hence, economic elites follow a threshold rule by which  $a(s) = 1$  if and only if  $s \geq k^*$ .<sup>2</sup>

We can now prove part (c) of Proposition 1. From the strategy of economic elites,  $a(m + u) = 1 \iff s > k^* \iff u > k^* - m$ . Therefore, we can write the potential autocrat's expected utility as

$$\int_{k^*-m}^{\infty} \bar{p}g(u)du - C(m - \theta).$$

The first- and second-order conditions are:

$$\bar{p}g(k^* - m^*(\theta)) = C'(m^*(\theta) - \theta) \quad \text{and} \quad -\bar{p}g'(k^* - m^*(\theta)) - C''(m^*(\theta) - \theta) < 0.$$

**Assumption 1**  $\inf C'' > \sup \bar{p}g'$ .

By assumption 1, the second order condition is always satisfied as  $-\sup(-C'') = \inf C''$ , and  $g'$  is such that  $g(-x) = -g(x)$ . Moreover, assumption 1 ensures that the slope of the marginal cost  $C''$  is always larger than the slope of the marginal benefit  $\bar{p}g'$ . Intuitively, it implies that the cost function is sufficiently convex so that the marginal cost curve and marginal benefit curve cross only once, ensuring that the solution to the first-order condition is unique. This also implies that there is a unique  $k^*$  such that this equation is satisfied. In the case of the quadratic cost function  $C(x) = cx^2/2$ ,  $C''(x) = c$ . Moreover, in the case of a normally distributed noise,  $-g'(\cdot)$  attains a maximum at  $\sigma_u$ . To see why, note that  $g'(x) = -[x/\sigma_u^2]g(x)$ , and  $g''(x) = [(x^2 - \sigma_u^2)/\sigma_u^4]g(x)$ . So  $g'(\cdot)$  attains a minimum at  $x = \sigma_u$ . Thus, assumption 1 reduces to the following restriction on the cost function:  $c > \bar{p}e^{-1/2}/\sigma_u^2\sqrt{2\pi}$ .

## S2 Derivation of Equation 2

The probability of elite investment for a given threshold  $k$  is the probability that the signal economic elites receive is below the threshold for some realization of the noise:

$$\Pr(\text{elite investment}; k) = \Pr(m + u \leq k|k, u) = \Pr(m \leq k - u|k, u) = H(k - u, k|u),$$

<sup>2</sup>See Caselli et al. (2014, 414) for a similar argument.

where  $H$  is the cumulative distribution of messages.<sup>3</sup>

We first derive  $H$ . The potential autocrat's messaging strategy uniquely defines the optimal message,  $m(\theta, k)$ . However, it is defined implicitly. Nevertheless, we can invert this function to get an explicit expression for  $\theta(m, k)$ , as suggested by Cunningham & de Barreda (2015). This function is the type that would optimally send message  $m$  given a threshold  $k$ . Given the first-order condition  $\bar{p}g(k^* - m^*(\theta)) = C'(m^*(\theta) - \theta)$ , we can write

$$\theta(m, k) = m - (C')^{-1}(\bar{p}g(k - m)) = m - \frac{\bar{p}g(k - m)}{c},$$

as  $C(x) = cx^2/2$ , implying that  $(C')^{-1}(x) = x/c$ .

From there, we can compute the cumulative distribution of messages,  $H$ . For  $m \in [\mu - b, \mu + b]$ ,

$$\begin{aligned} H(m, k) &= \Pr(\tilde{m} \leq m) \\ &= \Pr(\tilde{\theta} \leq \theta(m)) \quad \text{as } m(\theta) \text{ is increasing in } \theta \\ &= \Pr(\tilde{\theta} \leq m - \bar{p}g(k - m)/c) \\ &= F(m - \bar{p}g(k - m)/c) \\ &= \frac{m - \bar{p}g(k - m)/c - \mu + b}{2b}. \end{aligned}$$

We can then derive the probability of elite investment for a given threshold as follows:

$$\begin{aligned} \Pr(\text{elite investment}; k) &= \Pr(m \leq k - u | k, u) \\ &= H(k - u, k | u) \\ &= \int H(k - u, k) g(u) du \\ &= \int \left[ \frac{k - u - \bar{p}g(u)/c - \mu + b}{2b} \right] g(u) du \\ &= \frac{1}{2} - \frac{\mu}{2b} + \frac{k}{2b} - \frac{\bar{p}}{2bc} \int [g(u)]^2 du \\ &= \frac{1}{2} - \frac{\nu}{2b} + \frac{k - \bar{\theta}}{2b} - \frac{\bar{p}}{2bc} \int [g(u)]^2 du, \end{aligned}$$

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<sup>3</sup>This derivation follows the steps in Cunningham & de Barreda (2015, 7–10).

where we use the facts that  $\int ug(u)du = 0$  as  $g(\cdot)$  is centered around zero, and  $\nu = \mu - \bar{\theta}$ .

### S3 Probability of Elite Investment and Noise Variance

We formally show that the probability of elite investment increases as the variance in the distribution of the noise increases. First, notice that:

$$\begin{aligned} \int [g(u)]^2 du &= \int \left[ \frac{1}{\sigma_u \sqrt{2\pi}} e^{-\frac{u^2}{2\sigma_u^2}} \right]^2 du \\ &= \frac{1}{2\sigma_u^2 \pi} \int e^{-\frac{u^2}{\sigma_u^2}} du \\ &= \frac{1}{2\sigma_u \pi} \int e^{-x^2} dx \\ &= \frac{1}{2\sqrt{\pi}\sigma_u^2}. \end{aligned}$$

Here, we use the fact that  $u$  is symmetric. Moreover, we operate a change in variables with  $x = u/\sigma_u$ , which implies that  $du = \sigma_u dx$ . Finally, we use the Gaussian integral defined by  $\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$ . Hence,

$$\Pr(\text{elite investment}; k) = \frac{1}{2} - \frac{\nu}{2b} + \frac{k - \bar{\theta}}{2b} - \frac{\bar{p}}{4bc\sqrt{\pi}\sigma_u^2}.$$

Therefore,

$$\frac{\partial \Pr(\text{elite investment}; k)}{\partial \sigma_u^2} = \frac{\bar{p}}{4bc\sqrt{\pi}(\sigma_u^2)^{1.5}} > 0.$$

### S4 Derivation of the Equilibrium Threshold

To derive the optimal threshold, we first show that the expected type conditional on a signal at the threshold can be expressed as:<sup>4</sup>

$$\mathbb{E}[\theta|s = k] = k + \frac{\bar{p}}{c} \int ug'(u)g(u)du,$$

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<sup>4</sup>The derivation of the expression for the equilibrium threshold follows the steps of the derivations in Cunningham & de Barreda (2015, 11–12).

under the assumption that the distribution of the noise has bounded support on  $[-D, D]$ , where  $D$  is defined as follows with  $k^0$  defined below:

**Assumption 2**  $D \leq \min \{1 - k^0, k^0\}$ .

Using Bayes' rule we can write:

$$\mathbb{E}[\theta | s = k] = \frac{\int \theta(m, k)g(k - m)h(m, k)dm}{\int g(k - m)h(m, k)dm},$$

where  $h$  is the density of messages obtained by differentiating  $H$ : for  $m \in [\mu - b, \mu + b]$ ,  $h(m, k) = [1 + \bar{p}g'(k - m)/c]/2b$ . Focus on the denominator first. Change the variable of integration  $m$  to  $u = k - m$ . Therefore,  $dm = -du$ . This gives the following expression for the denominator:

$$\int_0^1 \theta(m, k)g(k - m)h(m, k)dm = \frac{1}{2b} \int_k^{k-1} [k - u - \bar{p}g(u)/c] g(u) [1 + \bar{p}g'(u)/c] (-du).$$

Now, assumption 2 implies that for all  $u \in [-D, D]$ ,  $u \in [k - 1, k]$ . To see why, it is useful to compute the threshold used by economic elites when they perfectly observe the autocrat's type. It is the threshold  $k^0$  such that the probability of elite investment is equal to the probability of elite investment when there is no information asymmetry. This implies the following value for  $k^0$ :

$$k^0 = \bar{\theta} + \frac{\bar{p}}{c} \int [g(u)]^2 du.$$

This benchmark threshold is an upper bound for the equilibrium threshold. Intuitively, because the potential autocrat is able to send higher messages than his type, the elite will necessarily wait for an autocrat turnover more often under asymmetric information than when they can perfectly observe the autocrat's type. Hence, the optimal threshold  $k^*$  must be lower than  $k^0$ . Therefore, assumption 2 guarantees that for all  $u \in [-D, D]$ ,  $u \in [k - 1, k]$ . Then, inverting the bounds, we have:

$$\int \theta(m, k)g(k - m)h(m, k)dm = \frac{1}{2b} \int_{-D}^D [k - u - \bar{p}g(u)/c] g(u) [1 + \bar{p}g'(u)/c] du.$$

Moreover,  $g(\cdot)$  is symmetric around zero, so  $\int ug(u)du = 0$ . Also,  $g(\cdot)$  is an odd function, so its derivative is an even function. Thus,  $g(u)g'(u)$  is an odd function as well as  $[g(u)]^2 g'(u)$ .

Hence,  $\int g(u)g'(u)du = \int [g(u)]^2 g'(u)du = 0$ . Therefore,

$$\int \theta(m, k)g(k - m)h(m, k)dm = \frac{1}{2b} \left[ k - \frac{\bar{p}}{c} \int [g(u)]^2 du - \frac{\bar{p}}{c} \int ug'(u)g(u)du \right].$$

Using the same argument, we get that the denominator equals  $1/2b$ . Therefore,

$$\mathbb{E}[\theta|s = k] = k - \frac{\bar{p}}{c} \int [g(u)]^2 du - \frac{\bar{p}}{c} \int ug'(u)g(u)du.$$

$$\mathbb{E}[\theta|s = k] = k + \frac{\bar{p}}{c} \int ug'(u)g(u)du.$$

From there, we get the following expression for the equilibrium threshold:

$$k^* = \bar{\theta} - \frac{\bar{p}}{c} \int ug'(u)g(u)du.$$

## S5 Equilibrium Elite Investment and Elite Uncertainty

Using the expression of the probability of elite investment in equation 5, we can write the probability of elite investment as

$$\Pr(\text{elite investment}) = \frac{1}{2} - \frac{\nu}{2b} + \frac{\bar{p}}{2bc} \int ug'(u)g(u)du.$$

We start by proving the first comparative statics,  $\partial \Pr(\text{elite investment})/\partial \sigma_\theta^2 > 0$ :

$$\frac{\partial \Pr(\text{elite investment})}{\partial \sigma_\theta^2} = \frac{\partial \Pr(\text{elite investment})}{\partial b} \frac{\partial b}{\partial \sigma_\theta^2} = \left[ \frac{\nu}{2b^2} - \frac{\bar{p}}{2b^2c} \int ug'(u)g(u)du \right] \frac{\partial b}{\partial \sigma_\theta^2}.$$

$\partial b/\partial \sigma_\theta^2 = 3/2(3\sigma_\theta^2)^{1/2} > 0$  and  $\int ug'(u)g(u)du < 0$ , so  $\partial \Pr(\text{elite investment})/\partial \sigma_\theta^2 > 0$ .

We next prove the second comparative statics,  $\partial \Pr(\text{elite investment})/\partial \sigma_u^2 > 0$ . The distribution of the noise has bounded support on  $[-D, D]$ . Therefore, the distribution  $g(\cdot)$  can be expressed as:

$$g(u) = \frac{\frac{1}{\sigma_u} \phi\left(\frac{u}{\sigma_u}\right)}{\Phi\left(\frac{D}{\sigma_u}\right) - \Phi\left(-\frac{D}{\sigma_u}\right)} = \frac{e^{-\frac{u^2}{2\sigma_u^2}}}{\sigma_u \sqrt{2\pi} \operatorname{erf}\left(\frac{D}{\sqrt{2\sigma_u^2}}\right)},$$

where we use the error function defined by  $\text{erf}(x) = (2/\sqrt{\pi}) \int_0^x e^{-t^2} dt$ . Using a change of variable with  $x = u/\sigma_u$  and the fact that the error function is an odd function, we can compute the following:

$$\begin{aligned}
\int_{-D}^D u g'(u) g(u) du &= -\frac{1}{2\pi\sigma_u^2 \left[ \text{erf} \left( \frac{D}{\sqrt{2\sigma_u^2}} \right) \right]^2} \int_{-D}^D \frac{u^2}{\sigma_u^2} e^{-\frac{u^2}{\sigma_u^2}} du \\
&= -\frac{1}{2\pi\sigma_u^2 \left[ \text{erf} \left( \frac{D}{\sqrt{2\sigma_u^2}} \right) \right]^2} \int_{-\frac{D}{\sigma_u}}^{\frac{D}{\sigma_u}} \sigma_u x^2 e^{-x^2} dx \\
&= \frac{1}{4\pi\sigma_u \left[ \text{erf} \left( \frac{D}{\sqrt{2\sigma_u^2}} \right) \right]^2} \int_{-\frac{D}{\sigma_u}}^{\frac{D}{\sigma_u}} x \left( -2xe^{-x^2} \right) dx \\
&= \frac{1}{4\pi\sigma_u \left[ \text{erf} \left( \frac{D}{\sqrt{2\sigma_u^2}} \right) \right]^2} \left[ xe^{-x^2} \Big|_{-\frac{D}{\sigma_u}}^{\frac{D}{\sigma_u}} - \int_{-\frac{D}{\sigma_u}}^{\frac{D}{\sigma_u}} e^{-x^2} dx \right] \\
&= \frac{1}{4\pi\sigma_u \left[ \text{erf} \left( \frac{D}{\sqrt{2\sigma_u^2}} \right) \right]^2} \left[ \frac{2D}{\sigma_u} e^{-\frac{D^2}{\sigma_u^2}} - \sqrt{\pi} \text{erf} \left( \frac{D}{\sqrt{\sigma_u^2}} \right) \right] \\
&= \frac{1}{\left[ \text{erf} \left( \frac{D}{\sqrt{2\sigma_u^2}} \right) \right]^2} \left[ \frac{D}{2\pi\sigma_u^2} e^{-\frac{D^2}{\sigma_u^2}} - \frac{1}{4\sqrt{\pi\sigma_u^2}} \text{erf} \left( \frac{D}{\sqrt{\sigma_u^2}} \right) \right].
\end{aligned}$$

Therefore, we can write explicitly the probability of elite investment as

$$\text{Pr}(\text{elite investment}) = \frac{1}{2} - \frac{\nu}{2b} + \frac{\bar{p}}{2bc} \frac{\frac{D}{2\pi\sigma_u^2} e^{-\frac{D^2}{\sigma_u^2}} - \frac{1}{4\sqrt{\pi\sigma_u^2}} \text{erf} \left( \frac{D}{\sqrt{\sigma_u^2}} \right)}{\left[ \text{erf} \left( \frac{D}{\sqrt{2\sigma_u^2}} \right) \right]^2}.$$



Let  $A$  denote the last term of the above equation:

$$A = \frac{\frac{D}{2\pi\sigma_u^2} e^{-\frac{D^2}{\sigma_u^2}} - \frac{1}{4\sqrt{\pi\sigma_u^2}} \operatorname{erf}\left(\frac{D}{\sqrt{\sigma_u^2}}\right)}{\left[\operatorname{erf}\left(\frac{D}{\sqrt{2\sigma_u^2}}\right)\right]^2}. \quad (\text{S5.1})$$

We then have the following comparative statics:

$$\frac{\partial \Pr(\text{elite investment})}{\partial \sigma_u^2} = \frac{\bar{p}}{2bc} \frac{\partial A}{\partial \sigma_u^2}.$$

We can show numerically that under the restriction that  $D$  and  $\sigma_u^2$  are positive (see the Mathematica Notebook in Appendix I below) :

$$\arg \min_{\{D, \sigma_u^2\} \in \mathbb{R}_+^*} \frac{\partial A}{\partial \sigma_u^2} > 0. \quad (\text{S5.2})$$

More specifically, the numerical solution for the minimization problem is:

$$\arg \min_{\{D, \sigma_u^2\} \in \mathbb{R}_+^*} \frac{\partial A}{\partial \sigma_u^2} = 5.8 \times 10^{-58}, \text{ as } D \longrightarrow 1.4 \times 10^{23} \text{ and } \sigma_u^2 \longrightarrow 2.5 \times 10^{37}.$$

Therefore, for all values of the parameters,  $\partial \Pr(\text{elite investment})/\partial \sigma_u^2 > 0$ .

## S6 Elite Investment with Multiple Periods

Introducing an arbitrary number of periods does not change the fundamental implications of the model. There are  $T + 1$  periods from  $t = 0$  to  $t = T$ . Economic elites have a unit endowment, and they can choose in which period  $t$  to invest it ( $a = t$ ). In each period, a potential autocrat is drawn from the distribution  $F(\cdot)$ , and has a probability  $\bar{p}$  to replace the current autocrat. Draws are i.i.d. across periods.

Suppose first that the potential autocrat cannot signal his type to economic elites. Then, the likelihood of elite investment under the current autocrat—the probability that  $a = 0$ —is the probability that economic elites' expected payoff is higher than investing their unit endowment in any other period:

$$\Pr(\text{elite investment}) = \Pr(u_0(a = 0) \geq u_1(a = 1) \cap \dots \cap u_0(a = 0) \geq u_T(a = T))$$

$$= \Pr(u_0(a=0) \geq u_1(a=1)) \times \dots \times \Pr(u_0(a=0) \geq u_T(a=T)),$$

where  $u_t$  denotes economic elites' expected payoff in period  $t$ . We show in footnote 17 that

$$\begin{aligned} \Pr(u_0(a=0) \geq u_1(a=1)) &= \Pr((1+r)\bar{\theta} \geq (1-\bar{p})(1+r)\bar{\theta} + \bar{p}(1+r)\theta) \\ &= \Pr(\bar{\theta} \geq \theta). \end{aligned}$$

More generally,

$$\begin{aligned} \Pr(u_0(a=0) \geq u_t(a=t)) &= \Pr((1+r)\bar{\theta} \geq (1-\bar{p})^t(1+r)\bar{\theta} + [1-(1-\bar{p})^t](1+r)\theta) \\ &= \Pr(\bar{\theta} \geq \theta). \end{aligned}$$

Therefore,

$$\Pr(\text{elite investment}) = [\Pr(\theta \leq \bar{\theta})]^T = \left[ \frac{1}{2} - \frac{\nu}{2b} \right]^T \quad (\text{S6.1})$$

With a similar logic, and allowing for signaling, we have:

$$\Pr(\text{elite investment}) = \left\{ \frac{1}{2} - \frac{\nu}{2b} - \frac{\bar{p}}{2bc} \left[ \int [g(u)]^2 du + \int ug'(u)g(u)du \right] \right\}^T. \quad (\text{S6.2})$$

All the comparative statics in appendix S5 therefore still hold.

## S7 Elite Investment when Discounting the Future

Discounting the future only marginally changes economic elites' incentives to invest by making their outside option  $\bar{\theta}\delta$  rather than  $\bar{\theta}$ , with  $\delta = [1 - \beta(1 - \bar{p})] / \beta\bar{p}$ . Compared to the baseline model, this only changes the equilibrium threshold  $k^*$  to  $\bar{\theta}\delta - (\bar{p}/c) \int ug'(u)g(u)du$ .

As a result, the equilibrium probability of elite investment is monotonically decreasing in  $\beta$ :

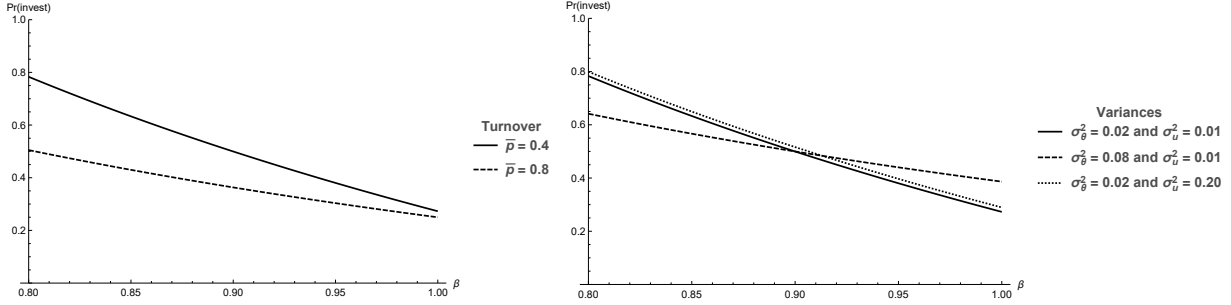
$$\Pr(\text{elite investment}) = \frac{1}{2} - \frac{\mu - \bar{\theta}\delta}{2b} - \frac{\bar{p}}{2bc} \left[ \int [g(u)]^2 du + \int ug'(u)g(u)du \right]. \quad (\text{S7.1})$$

We provide in Figure S1 simulations of the equilibrium elite investment for various sets of parameters as a function of the discount rate  $\beta$ .<sup>5</sup>

With these parameter restrictions along with the equilibrium probability of elite investment in equation S7.1, the comparative statics of equation 6 still hold—see appendix S5.

<sup>5</sup>As before, we consider cases in which elite investment is possible yet puzzling in the sense that the expected

Figure S1: Elite Investment and Discount Rate



Parameters:  $\mu = 0.5$ ,  $\bar{\theta} = 0.4$ ,  $c = 50$ ,  $p = 0.4$ ,  $\sigma_\theta^2 = 0.02$ , and  $\sigma_u^2 = 0.01$  otherwise noted. These parameters imply  $0.8 < \beta \leq 1$ .

## S8 Elite Investment with a Democratic Alternative

Having a potential democrat of type  $1 - \tau$  rather than a potential autocrat of type  $\theta$  as the alternative to the current autocrat does not change the logic of the model. It only marginally changes economic elites' incentives to invest by making their default option  $[(1 + r)\bar{\theta} - 1] / r$  instead of  $\bar{\theta}$ . Compared to the baseline model, this only changes the equilibrium threshold  $k^*$  to  $[(1 + r)\bar{\theta} - 1] / r - (\bar{p}/c) \int u g'(u) g(u) du$ . As a result, the equilibrium probability of elite investment under democratic alternative is:

$$\Pr(\text{elite investment}) = \frac{1}{2} - \frac{\mu - [(1 + r)\bar{\theta} - 1] / r}{2b} - \frac{\bar{p}}{2bc} \left[ \int [g(u)]^2 du + \int u g'(u) g(u) du \right].$$

The comparative statics described in equation 6 still hold—see appendix S5.<sup>6</sup>

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benefit to economic elites of investing under the potential autocrats relative to the current autocrat should be positive when the potential autocrat cannot signal his type, even after adjusting for the discount rate:  $\mu > \bar{\theta}\delta$ . This set of cases implies the following restriction on the discount rate:  $\beta > \bar{\theta} / [\mu\bar{p} + \bar{\theta}(1 - \bar{p})]$ .

Given that  $\bar{p} \in [0, 1]$ , this set of restrictions implies  $\bar{\theta}/\mu < \beta \leq 1$ .

<sup>6</sup>A sufficient condition is that  $1 > (1 + r)\bar{\theta} - r\mu$ .

## I Mathematica Code for the Proof of Equation E.2

$$\text{In[1]:= } A[d\_ , \sigma2\_ ] = \frac{\left( \frac{d \cdot \text{Exp}\left[-\frac{d^2}{\sigma2}\right]}{2 \cdot \pi \cdot \sigma2} - \frac{\text{Erf}\left[\frac{d}{\sqrt{\sigma2}}\right]}{4 \cdot \sqrt{\pi} \cdot \sqrt{\sigma2}} \right)}{\left( \text{Erf}\left[\frac{d}{\sqrt{2} \cdot \sqrt{\sigma2}}\right] \right)^2}$$

$$\text{Out[1]= } \frac{\frac{d e^{-\frac{d^2}{\sigma2}}}{2 \pi \sigma2} - \frac{\text{Erf}\left[\frac{d}{\sqrt{\sigma2}}\right]}{4 \sqrt{\pi} \sqrt{\sigma2}}}{\text{Erf}\left[\frac{d}{\sqrt{2} \sqrt{\sigma2}}\right]^2}$$

$$\text{In[2]:= } dA[d\_ , \sigma2\_ ] = D[A[d, \sigma2], \sigma2]$$

$$\text{Out[2]= } \frac{d e^{-\frac{d^2}{2 \sigma2}} \sqrt{\frac{2}{\pi}} \left( \frac{d e^{-\frac{d^2}{\sigma2}}}{2 \pi \sigma2} - \frac{\text{Erf}\left[\frac{d}{\sqrt{\sigma2}}\right]}{4 \sqrt{\pi} \sqrt{\sigma2}} \right)}{\sigma2^{3/2} \text{Erf}\left[\frac{d}{\sqrt{2} \sqrt{\sigma2}}\right]^3} + \frac{\frac{d^3 e^{-\frac{d^2}{\sigma2}}}{2 \pi \sigma2^3} - \frac{d e^{-\frac{d^2}{\sigma2}}}{4 \pi \sigma2^2} + \frac{\text{Erf}\left[\frac{d}{\sqrt{\sigma2}}\right]}{8 \sqrt{\pi} \sigma2^{3/2}}}{\text{Erf}\left[\frac{d}{\sqrt{2} \sqrt{\sigma2}}\right]^2}$$

$$\text{In[3]:= } \text{assumptions} = d > 0 \ \&\& \ \sigma2 > 0$$

$$\text{Out[3]= } d > 0 \ \&\& \ \sigma2 > 0$$

$$\text{In[4]:= } \text{NMinimize}[\{dA[d, \sigma2], \text{assumptions}\}, \{d, \sigma2\}]$$

$$\text{Out[4]= } \{5.76058 \times 10^{-58}, \{d \rightarrow 1.43497 \times 10^{23}, \sigma2 \rightarrow 2.46554 \times 10^{37}\}\}$$

## References

- Caselli, Francesco, Tom Cunningham, Massimo Morelli & Inés Moreno Barreda. 2014. “The Incumbency Effects of Signalling.” *Economica* 81(323):397–418.
- Cunningham, Tom & Inés Moreno de Barreda. 2015. “Equilibrium Persuasion.”.
- Milgrom, Paul R. 1981. “Good News and Bad News: Representation Theorems and Applications.” *The Bell Journal of Economics* 12(2):380–391.