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► **To cite this version:**

Rihem Nasfi, Arsenia Chorti. Non-orthogonal multiple access networks under QoS delay constraints. ICUFN: 11th International Conference on Ubiquitous and Future Networks, Jul 2019, Zagreb, Croatia. pp.1-5. hal-02517474

**HAL Id: hal-02517474**

**<https://hal.science/hal-02517474v1>**

Submitted on 24 Mar 2020

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# Non-Orthogonal Multiple Access Networks Under QoS Delay Constraints

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**Abstract**—The multiple access technique allows users to share the allotted spectrum in the most effective manner. It can be classified in the orthogonal multiple access (OMA) techniques which is based on the time/frequency/code domain (TDMA, OFDMA, ...) or in the non-orthogonal multiple access (NOMA) techniques. Nowadays, with The exponential growth of mobile data traffic in 5G and IoT wireless networks which imposes requirements in terms of quality of service, low latency etc., the OMA technique may not meet this stringent emerging requirements. So, NOMA principle emerges as a solution to improve the spectral efficiency under delay quality of service constraints. Unlike previous work, in this paper, we study the performance of NOMA in the uplink transmission, in particular the performance in terms of the effective capacity of the channel (EC). We derive the closed-form expression of EC per user and prove the accuracy using Monte Carlo method.

**Index Terms**—NOMA, Quality-of-service, low latency, effective capacity, closed-form expressions, Monte Carlo.

## I. INTRODUCTION

The tremendous increase in the number of users has led to the emergence of the fifth generation in the radio networking (5G). This latter is based on maximizing the data rate of the channel and minimizing the latency between the network terminals. Multiple access techniques are used to solve the concerns of the 5G network, mainly classified in two approaches: the orthogonal multiple access (OMA) and the non-orthogonal multiple access (NOMA). An OMA scheme is based on allocating resources to the network users by either dividing the transmission in time slots and assign each slot to a user (time division multiple access TDMA), either by dividing the frequency band in slots and assign it to the different users (orthogonal frequency division multiple access OFDMA). On the other hand, NOMA schemes have proven better than the OMA in terms of low latency, massive connectivity and user fairness. While the OMA technique gives the user, with a good channel condition, the high priority to be served and so on. On the contrary, the NOMA technique serves all users simultaneously. Also, NOMA assigns the same frequency resource to multiple users at the same time which enhances the the spectral efficiency and the throughput, while a specific frequency resource in OMA is assigned to each user whatever the state of the channel. Theoretically, the boundary

of achievable rate pairs (two users) with NOMA is outside the OMA capacity region [1].

Besides, as many applications involves randomness in wireless channel conditions and user mobility, previous works showed that the power domain NOMA has a better efficiency in meeting the delay QoS requirement in terms of effective capacity (EC) of the channel. The effective capacity is a QoS-aware link-layer wireless channel model. In [2], the work was based on the NOMA downlink transmission and the analysis of the EC over many cases of the signal-noise-rate (SNR). However, our focus will be on studying the EC of a NOMA uplink transmission.

The rest of this paper is organized as follows. The related works about the NOMA technique, the effective capacity and the delay quality of service is given in Section II. Then, we start to analyze and investigate the NOMA uplink transmission under the delay QoS constraints in Section III, which includes the closed-form expressions for the link-layer rates in a two-user network. Simulation results are given in Section IV, followed by conclusions in Section VI.

## II. RELATED WORKS

### A. The Noma Scheme

The non orthogonal multiple access (NOMA) was proposed as a promising radio access scheme to enhance the spectral efficiency [3]. The Basic technique used in the power-domain NOMA is the superposing of the power of multiple users to transmit the signal on the same bandwidth (same time-frequency). The receiver then tries to extract the desired signal by using different algorithms, known as the successive interference cancellation (SIC) [3].

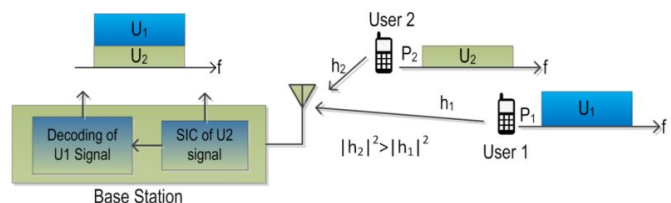


Fig. 1. Noma With SIC technique [1].

The SIC receiver decodes multi-user signals according to descending signal power and subtracts the decoded signal from the received multi-user signal, so as to improve the signal-to-interference ratio. The process is repeated until the signal of interest is decoded. The performance of SIC is related to the ordering of the received signal power which depends on the spatial distribution of the active transmitters and on the condition of the channels. In order the SIC works in the good way, the users should be sorted according to their channel gain. The SIC decodes successively the code of the weakest, subtracts it and then decodes its desired code. Thus, the weakest can recover its code first without the needing to parse the rest.

### B. The Effective Capacity

At the level of the transmitter in a wireless network, we investigate the queue overflow probability, i.e., the probability of the steady-state queue length process  $Q(t)$  at  $t$  exceeding a certain threshold  $x$  [2]:

$$\lim_{x \rightarrow \infty} \frac{\ln(\Pr[Q(x) > x])}{x} = \theta \quad (1)$$

given that  $\theta$  specifies the asymptotic exponential decay-rate of the overflow probability. So when the queue-overflow threshold  $x$  is large, the queue-overflow probability can be approximated to  $\Pr[Q(x) > x] \approx e^{-\theta x}$ . Furthermore, as mentioned in [10], we can approximate the delay violation probability by:

$$\Pr_{delay}^{out} = \Pr[D(t) > d] \approx \Pr[Q(t) > 0]e^{\theta \zeta d} \quad (2)$$

where  $\Pr[Q(t) > 0]$  denotes the probability of a non-empty buffer at time  $t$  [9],  $D(t)$  is the delay experienced by a source packet arriving at fading-block  $t$  and  $d$  is the maximum delay bound [8].  $\zeta$  equals to the EC satisfying the statistical delay QoS metrics [2]. EC model was designed in the link layer to find the maximum constant arrival rate that a service process can handle in order to guarantee a statistical QoS requirement specified by the QoS exponent  $\theta$ . EC is the combination of two functions namely, i.e., QoS exponent and probability of non-empty buffer. [5] formulated the effective capacity as:

$$C(\theta) = \lim_{t \rightarrow \infty} \frac{1}{\theta t} \ln E[e^{-\theta S[t]}](bits/s) \quad (3)$$

where  $S[t] = \sum_{i=1}^t s[i]$  is the time-accumulated service process, and  $s[i], i = 1, 2, \dots$  denotes the discrete-time stationary and ergodic stochastic service process.

The notion of the delay exponent  $\theta$  establishes the relationship between the maximum queue length and the buffer overflow probability, it expresses how strict the QoS constraints are [3]. A slower decay rate can be represented by a smaller  $\theta$ , which indicates that the system is tolerant towards delay-latency, while a larger  $\theta$  means that it has more stringent QoS requirement

In order to simplify the analysis, the wireless channel follows the Rayleigh Fading distributions, so, during the frame duration  $T$ , we have  $s[i] = TR[i]$ , with  $R[i]$  is the

instantaneous service rate in the  $i^{th}$  frame duration [6]. For the rest of the paper, the effective capacity used is normalized by bandwidth  $B$ :

$$E_c = \frac{C(\theta)}{B} = \lim_{t \rightarrow \infty} \frac{1}{\theta t} \ln E[e^{-\theta S[t]}](bits/s) \quad (4)$$

### III. UPLINK TRANSMISSION NOMA UNDER DELAY QoS CONSTRAINTS

The up-link transmission, in the digital communication, is the link from one user equipment (U) or more to the base station (BS). Considering that we have two users that will send a message to BS. The power of the transmitted signal given by the two users  $U_1$  and  $U_2$  are respectively:  $E[|S_1|^2] = P_1$  and  $E[|S_2|^2] = P_2$ . At the Base station, the signal received has the form of:

$$Z = \sqrt{P_1}h_1S_1 + \sqrt{P_2}h_2S_2 + w \quad (5)$$

given that  $w$  is the Gaussian white noise. So when both users sends their signals, supposedly very synchronized and each signal has the same bandwidth  $B$  as the other. The BS will first decode the stronger channel which is of the  $U_2$  and think of the part that came from the other user  $U_1$  as interference. Once done, it will decode the signal of the  $U_1$ . This process results that the uplink achievable rates, in b/s/Hz, for both users  $U_1$  and  $U_2$  have respectively the form [4]:

$$R_1 = \log_2[1 + \rho P_1 |h_1|^2] \quad (6)$$

$$R_2 = \log_2\left[1 + \frac{\rho P_2 |h_2|^2}{1 + \rho P_1 |h_1|^2}\right] \quad (7)$$

where  $P_i$  is a power allocated per-user and  $\rho = \frac{1}{\sigma_w^2}$  is the SNR.

Because of many factors such as the multipath fading, the noise disturbance or the channel interference, the wireless channels have a low-reliability and time-varying capacities which may cause a real violations of the delay QoS. In order, to analyze this issue, the effective capacity (EC) of the  $m^{th}$  channel is defined as a metric of delay QoS [6]:

$$E_c^m = -\frac{1}{\theta_m T_f B} \ln E[e^{-\theta_m T_f B R_m}](b/s/Hz) \quad (8)$$

by assuming that the Gartner-Ellis limit exists [11] and  $E[.]$  is the expected value of the probability density function (PDF) of the allocated channel,  $\theta_m$  is the exponential decay rate of the delay QoS violation probability with ( $\theta_m > 0$ ), . Replacing the  $R_m$  of both users by its formula, we get two expressions of the EC:

$$E_c^1 = -\frac{1}{\theta_1 T_f B} \ln E[(1 + \rho P_1 |h_1|^2)^{\beta_1}]$$

$$E_c^2 = -\frac{1}{\theta_2 T_f B} \ln E\left[\left(1 + \frac{\rho P_2 |h_2|^2}{1 + \rho P_1 |h_1|^2}\right)^{\beta_2}\right]$$

given that  $\beta_i = -\frac{\theta_i T_f B}{\ln 2}$  is the normalized QoS exponent and  $i=1,2$ .

### A. The Closed-Form Expressions Of EC

The channel in NOMA network are ordered according to the strength of their channel's gain. To do so, we use the order statistics to sort the channels [5]. So, supposedly a network of two users' channels having respectively  $|h_1|^2 < |h_2|^2$ , and each  $\gamma_i = \rho|h_i|^2$  ( $i = 1, 2$ ) are i.i.d random variables with a PDF each  $f(\gamma_i)$ . Ranking these random variables using the order statistics, result the PDFs of  $U_1$  and  $U_2$  :

$$f_{\gamma_{1:2}}(\gamma_1) = \psi_1 f(\gamma_1) F(\gamma_1) \quad (9)$$

$$f_{\gamma_{2:2}}(\gamma_2) = \psi_2 f(\gamma_2) (1 - F(\gamma_2)) \quad (10)$$

where  $\psi_i = \frac{1}{B(i, M-i+1)}$ , here we have  $M = 2$  and  $B(a, b)$  is the beta function  $B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ . The  $F(\gamma_i)$  functions are the CDF of the the random variables  $\gamma_i$ .

Given that the gains of the channel in NOMA are modeled as Rayleigh Fading distribution which makes the random variables  $\gamma_i$  exponentially distributed having a PDF and CDF respectively:  $f(\gamma) = \frac{1}{\rho} e^{-\frac{\gamma}{\rho}}$  and  $F(\gamma) = 1 - e^{-\frac{\gamma}{\rho}}$  [7]. This yields to:

$$f_{\gamma_{1:2}}(\gamma_1) = \frac{1}{B(1, 2)} f(\gamma_1) (1 - F(\gamma_1)) = \frac{2}{\rho} e^{-\frac{2\gamma_1}{\rho}} \quad (11)$$

$$f_{\gamma_{2:2}}(\gamma_2) = \frac{1}{B(2, 1)} f(\gamma_2) F(\gamma_2) = \frac{2}{\rho} e^{-\frac{\gamma_2}{\rho}} (1 - e^{-\frac{\gamma_2}{\rho}}) \quad (12)$$

In this context, the EC will be:

$$E_c^1 = \frac{1}{\beta_1} \log_2 \left( \int_0^\infty (1 + P_1 \gamma_1)^{\beta_1} f_{\gamma_{1:2}}(\gamma_1) d\gamma_1 \right) \quad (13)$$

$$E_c^2 = \frac{1}{\beta_2} \log_2 \left( \int_0^\infty \int_0^\infty \left(1 + \frac{P_2 \gamma_2}{1 + P_1 \gamma_1}\right)^{\beta_2} f_{\gamma_{2:2}}(\gamma_2) f_{\gamma_{1:2}}(\gamma_1) d\gamma_2 d\gamma_1 \right) \quad (14)$$

So, by replacing the two PDFs (11) and (12) into (13) and (14) with considering that  $\gamma_1$  and  $\gamma_2$  are two independent random variables, results the following effective capacities form of both users:

$$E_c^1 = \frac{1}{\beta_1} \log_2 \left( \frac{2}{\rho} \int_0^\infty (1 + P_1 \gamma_1)^{\beta_1} e^{-\frac{2\gamma_1}{\rho}} d\gamma_1 \right) \quad (15)$$

$$E_c^2 = \frac{1}{\beta_2} \log_2 \left( \frac{4}{\rho^2} \int_0^\infty \int_0^\infty \left(1 + \frac{P_2 \gamma_2}{1 + P_1 \gamma_1}\right)^{\beta_2} e^{-\frac{\gamma_2}{\rho}} (1 - e^{-\frac{\gamma_2}{\rho}}) e^{-\frac{2\gamma_1}{\rho}} d\gamma_2 d\gamma_1 \right) \quad (16)$$

By calculating (15), we get the mathematical expression:

$$E_c^1 = \frac{1}{\beta_1} \log_2 \left( \left(\frac{P_1 \rho}{2}\right)^{\beta_1} e^{\frac{2}{P_1 \rho}} \Gamma(1 + \beta_1, \frac{2}{P_1 \rho}) \right) \quad (17)$$

given that  $\rho > 0$ ;

For the second channel, due to the complex form of  $E_c^2$ , we study only particular values of the normalized QoS exponent

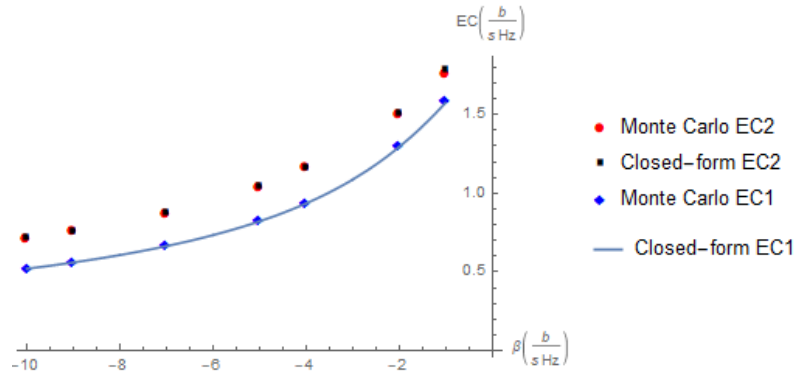


Fig. 2. The Effective Capacity EC Versus  $\beta$

$\beta_2$  by assuming that is non-positive integer and we use the Binomial expansion:

$$\left(1 + \frac{P_2 \gamma_2}{1 + P_1 \gamma_1}\right)^{\beta_2} = \sum_{k=0}^{-\beta_2} \binom{-\beta_2}{k} \left(\frac{-P_2 \gamma_2}{1 + P_1 \gamma_1 + P_2 \gamma_2}\right)^k \quad (18)$$

Inserting (18) into (16), we get:

$$E_c^2 = \frac{1}{\beta_2} \log_2 \left( \sum_{k=0}^{-\beta_2} \binom{-\beta_2}{k} (-1)^k \Gamma(k+1) \left( \int_0^\infty \frac{4}{\rho} e^{-\frac{2\gamma_1}{\rho}} U(k, 0, \frac{1 + P_1 \gamma_1}{P_2 \rho}) d\gamma_1 \right) + \int_0^\infty \frac{2}{\rho} e^{-\frac{2\gamma_1}{\rho}} U(k, 0, \frac{2(1 + P_1 \gamma_1)}{P_2 \rho}) d\gamma_1 \right) \quad (19)$$

where  $U(a, b, z)$  is the confluent hypergeometric function of the second kind and equal to:

$$U(a, b, z) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt, \quad (20)$$

for  $\text{Re } a, \text{Re } z > 0$ .

## IV. NUMERICAL RESULTS

In the following, we evaluate method, the accuracy of the given mathematical expressions of the effective capacity of both channel ( $E_c^1$ ) and ( $E_c^2$ ) by using Monte Carlo simulations. We plot the Effective capacity over three parameters ( $\beta, \rho, P$ ):

### A. The Effective Capacity Versus The Normalized QoS Exponent Coefficient $\beta$

Fig.2 shows the curves of the different EC in a scenario where the power of the user with stronger channel condition  $U_1$  is  $P_2 = 1\text{W}$  and with the weaker one  $U_2$  is  $P_1 = 0.8\text{W}$ . The plot of the closed form of both EC is versus discrete values of  $\beta$  due to the assumption of being non-positive integer. Further, more the transmission is delay-constrained ( $\theta$  becomes more stringent  $\Rightarrow \beta$  decreases), more the value of both EC decreases which means that the link-layer rates decreases.

The table I represents the values of the EC using Monte Carlo simulation

$\beta(\text{b/s/Hz})$	-10	-7	-5	-2	-1
$E_c^1$	0.522	0.665	0.822	1.302	1.584
$E_c^2$	0.702	0.867	1.029	1.494	1.750

TABLE I  
MONTE CARLO SIMULATION OF THE EC VERSUS  $\beta$

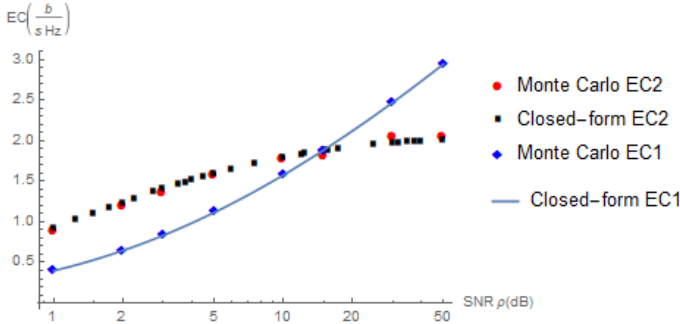


Fig. 3. The Effective Capacity  $E_c^1$  an  $E_c^1$  in NOMA versus the transmit SNR  $\rho$ .

### B. The Effective Capacity Versus SNR

Supposing the scenario where the QoS exponent under the delay constraint is the same for both users ( $\beta_1 = \beta_2 = -1$ ). Besides, the two users  $U_1$  and  $U_2$  transmit their signals with powers respectively  $P_1 = 0.8\text{W}$  and  $P_2 = 1\text{W}$ . Also, to simplify the work, we plotted the  $E_c^2$  overs discrete values of the SNR  $\rho$ .

SNR $\rho(\text{dB})$	1	5	10	30	50
$E_c^1$	0.883	1.570	1.772	2.03328	2.048
$E_c^2$	0.405	1.130	1.593	2.478	2.945

TABLE II  
MONTE CARLO SIMULATION OF THE EC VERSUS  $\rho$

As shown in Fig.3, the Monte Carlo numerical results agree with the evaluation of  $E_c^1$  and  $E_c^2$  obtained from the analysis presented in III.A. It is also clear at some value of the SNR ( $\rho \approx 30\text{dB}$ ),  $E_c^1$  becomes better than  $E_c^2$ .

### C. The Effective Capacity Versus Powers

We simulated various values of  $P_1$  between  $[10^{-2}\text{W}, 1\text{W}]$  and fixed the power  $P_2 = 2\text{W}$ ; The plot in Fig.4 confirms the accuracy between the Monte Carlo simulation of  $E_c^1$  and  $E_c^2$  (results in the Table III) and the closed-form expression in (17) and (19). Besides, the graph show that when the power of  $U_1$  and of  $U_2$  are equal ( $P_1 \approx P_1 \approx 1\text{W}$ ), then  $E_c^1$  and  $E_c^2$  get the same value.

$P_1(\text{W})$	0.001	0.01	0.1	0.8	1
$E_c^1$	0.003	0.062	0.47	1.581	1.741
$E_c^2$	3.261	3.191	2.810	1.772	1.63

TABLE III  
MONTE CARLO SIMULATION OF THE EC VERSUS  $P_1$

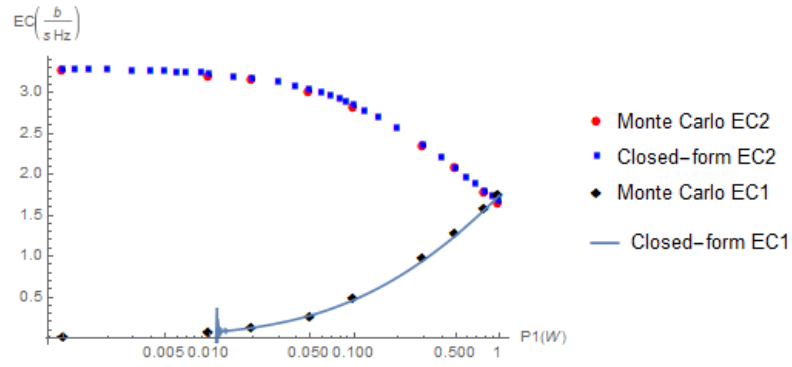


Fig. 4. Effective Capacity  $E_c^1(P_1)$  and  $E_c^2(P_1)$ .

## V. CONCLUSION

The effective capacity enables us to study the performance levels between the two extreme cases of delay limited capacity. In this paper, we have investigated the effective capacity in a uplink wireless transmission using the NOMA technique and have derived novel closed-form expressions for the effective capacity of the uplink of a two-user NOMA network using the order statistics tool. Due to the complex form of the user with stronger channel gain, we had to use some simplifications and get approximate results. Still, the comparison between the NOMA and the OMA uplink transmission, as well as the extension to an M-user NOMA network are left as future work.

## REFERENCES

- [1] O. A. Dobre S.M. R. Islam, N. Avazov and K. Kwak., "Power-domain non-orthogonal multiple access(noma) in 5g systems: Potentials and challenges", *IEEE Communications Surveys and Tutorials*, vol. 19, no. 2, pp. 721–739, 2017.
- [2] W. Yu, L. Musavian, Q. Ni, "Link-Layer Capacity of NOMA Under Statistical Delay QoS Guarantees", *IEEE transactions on wireless communications*, vol. 66, no. 10, pp. 4907–4922, October 2018.
- [3] Y. Saito, Y. Kishiyama, A. Benjebbour, T. Nakamura, A. Li, K. Higuchi, "Non-Orthogonal Multiple Access (NOMA) for Cellular Future Radio Access" in Proc. *IEEE Vehicular Technology Conference (VTC Spring)*, Dresden, Germany, pp.1-5, Jun. 2013.
- [4] L. Fan, S. Jin, C.K. Wen, H Zhang, "Uplink Achievable Rate for Massive MIMO Systems with Low-Resolution ADC", *IEEE Communications Letters*, vol.19, issue.12, pp.2186 - 2189, 2015.
- [5] Y.C. Yang, M.S. Alouini, *Order Statistics in wireless Communications, Diversity, Adaptation and Scheduling MIMO and OFDM Systems* Cambridge University Press, pp. 0-71, 2011.
- [6] Deli Qiao, *Wireless Communications under QoS Constraints: Energy Efficiency, Power and Rate Control, and Throughput* Spring, 5-2012.
- [7] Athanasios Papoulis, *Probability, Random Variables and Stochastic Processes* McGraw-Hill Inc., USA, 1991.
- [8] W. Yu, L. Musavian and Qiang Ni, "Tradeoff Analysis and Joint Optimization of Link-Layer Energy Efficiency and Effective Capacity Toward Green Communications", *IEEE transactions on wireless communications*, vol.15, no.5, may 2016.
- [9] D. Wu, R. Neg, "Effective Capacity: A Wireless Link Model for Support of Quality of Service", *IEEE transactions on wireless communications*, vol.2, no.4, july 2003.
- [10] J. Tang, Xi Zhang, "Cross-Layer Modeling for Quality of Service Guarantees over Wireless Links", *IEEE transactions on wireless communications*, vol.6, no.12, december 2007
- [11] J. A. Bucklew, *Introduction to Rare Event Simulation*. Berlin, Germany: Springer-Verlag, 2004.