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# A Novel Method for Accelerating the Analysis of Nonlinear Behaviour of Power Grids using Normal Form Technique

Nnaemeka Sunday UGWUANYI<sup>1</sup>, Xavier KESTELYN<sup>1</sup>, Olivier THOMAS<sup>2</sup> and Bogdan MARINESCU<sup>3</sup>

**Abstract**—Today’s power systems are strongly nonlinear in essence and are becoming more complex with the large penetration of power-electronics interfaced generators. Conventional Linear Modal Analysis does not adequately study such a system with complex nonlinear behavior. Inclusion of higher-order terms in small-signal (modal) analysis, associated with the Normal Form theory proposes a nonlinear modal analysis as an efficient way to improve the analysis. However, heavy computations involved make Normal Form method tedious, and unamenable to large power system application. In this paper, we present a less rigorous and speedy approach for obtaining the required nonlinear coefficients of the nonlinear equations modelling of a power system, without actually going through all the usual high order differentiation involved in Taylor’s expansion. The proposed method is demonstrated on the conventional IEEE 9-bus system and 68-bus New England/New York system.

**Index Terms**—Nonlinear coefficients, nonlinear modal analysis, Normal Form, quadratic and cubic coefficients, Hessian matrices, small signal analysis.

## I. INTRODUCTION

Power system is composed of several generators working in parallel to supply common load. It is nonlinear in essence and usually modelled by a set of nonlinear differential algebraic equations. An inherent problem associated with interconnected power systems is the presence of oscillations that could have dangerous effects on the system. The multiplication of distributed generation units usually composed of renewable-energy-based generators coupled with the increase of energy exchanges through long distance lead to highly stressed power systems and consequent poor damping and inter-area oscillations [1], [2]. As a result of the nonlinearity increase in the system, the conventional Linear Modal Analysis can be deceptive since its validity becomes very small [3].

The addition of higher-order terms in small signal (modal) analysis, has led to Nonlinear Modal Analysis tools which proved to be more efficient than their linear counterpart. The two currently used techniques for nonlinear modal analysis are method of Normal Form and Modal Series method [4]. There seems to be argument in literature regarding which method is better [1], [5], [6] but one limitation common to these methods is the heavy computation required. Due to the computational burden, most works reported are based on

inclusion of second order term of Taylor’s series expansion. Though, few works have been recently reported on inclusion of third order terms [6]–[10]. One of the difficulties in computation with these nonlinear analysis tools is the need to compute numerous coefficients even with system of small size. These coefficients increase geometrically with increase in the model. In order to reduce the computational burden, it was suggested in [11], [12] that accurate determination of the interacting modes by higher order spectra (HOS) and prony analyses will significantly reduce the computational burden associated with the method of Normal Form since several of the computations can be restricted to the interacting modes. This suggestion calls for a convenient method for selectively computing these coefficients. Normal Form computations are generally uninteresting irrespective of the size because many results of the computation are not very useful in the end. In the study of modal interaction for instance, even though all these coefficients are computed, only few of them reveal the interacting modes at the end. In order to get these coefficients, higher order differentiation of the system equations is performed either by using symbolic tools or manually defining the derivatives for the Hessian matrices in advance. Symbolic computations are generally slow. The challenge of defining the Hessian derivatives in advance is that when a model is modified the whole process is overhauled to account for the new model. Moreover, modal interaction depends on the operating condition making it difficult to pre-define the derivatives for the needed coefficients since the interaction can change with the change in operating condition. The authors in [3], [13] acknowledged the challenges in getting these coefficients.

This paper proposes an easy way to compute the nonlinear coefficients of a power system nonlinear model in modal coordinate without all the needed Taylor’s series expansions and associated Hessian matrices. The rest of the paper is organized as follows. Section II is a review of the Normal Form technique. Section III details the proposed method used in this paper to compute the needed coefficients. The proposed method is demonstrated on the IEEE 9-bus and IEEE 68-bus systems in section IV. Finally, a conclusion section shows the interesting perspectives proposed by the method.

## II. REVIEW OF NORMAL FORM TECHNIQUE

The Normal Form technique transforms a set of nonlinear differential equation up to a desirable order into a set of linear differential equations and then handle the system as linear provided certain conditions are met. This is achieved

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by sequential nonlinear coordinate transformations which simplify the equations. The system is then said to be in their simple form (Normal Form) [14]. As in Linear Modal Analysis, a nonlinear function is Taylor-expanded around a stable operating point (SEP); however the expansion is performed beyond first-order. Consider a power system modelled as

$$\mathbf{M}\dot{\mathbf{X}} + \mathbf{f}_s(\mathbf{X}) = \mathbf{P}_m \quad (1)$$

Where  $\mathbf{M}$ ,  $\mathbf{X}$  and  $\mathbf{P}_m$  are inertia constant, state vector and mechanical power respectively. Though,  $\mathbf{M}$  and  $\mathbf{P}_m$  do not apply to all the equations if the system is in first order differential equations, the idea is to put element of  $\mathbf{M} = 1$  and element of  $\mathbf{P}_m = 0$  where they do not apply. This representation is chosen to facilitate the understanding of proposed method. (1) is the same as

$$\dot{\mathbf{X}} + \mathbf{f}(\mathbf{X}) = \mathbf{P}_m/\mathbf{M}. \quad (2)$$

where  $\mathbf{f}(\mathbf{X}) = \mathbf{f}_s(\mathbf{X})/\mathbf{M}$ .

The system in third order Taylor expansion can be written in Einstein notation as

$$\dot{x}_i + A_{ij}x_j + H2_{ijk}x_jx_k + H3_{ijkl}x_jx_kx_l + H.O.T_i = \Delta P_{m_i}/M_i. \quad (3)$$

where  $A_{ij} = \frac{\partial f_i}{\partial X_j}|_{\mathbf{x}=\mathbf{x}_0}$ ,  $H2_{ijk} = \frac{1}{2} \frac{\partial^2 f_i}{\partial X_j \partial X_k}|_{\mathbf{x}=\mathbf{x}_0}$ ,

$H3_{ijkl} = \frac{1}{6} \frac{\partial^3 f_i}{\partial X_j \partial X_k \partial X_l}|_{\mathbf{x}=\mathbf{x}_0}$  ( $i, j, k = 1, 2, \dots, N$ ).

$\mathbf{A}$ ,  $\mathbf{H2}$ , and  $\mathbf{H3}$  are respectively the Jacobian of size  $(N \times N)$ , second order Hessian of size  $(N \times N \times N)$  and third order Hessian of size  $(N \times N \times N \times N)$  corresponding to first, second and third order terms evaluated at the initial operating point vector  $\mathbf{X}_0$ . The small case letter  $\mathbf{x}$  is a vector of the perturbed state variable ( $\Delta\mathbf{X}$ ), while  $\mathbf{X}$  is the original unperturbed state variable both of length  $N$ . H.O.T is the higher order terms.

Neglecting the higher order terms and performing a linear transformation  $\mathbf{x} = \mathbf{U}\mathbf{y}$  in (3) and pre-multiplying the resulting equation by the left vector, the  $j^{th}$  equation of (3) yields

$$\dot{y}_j + \lambda_j y_j + \sum_{k=1}^N \sum_{l=1}^N C_{kl}^j y_k y_l + \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N D_{pqr}^j y_p y_q y_r = 0. \quad (4)$$

Where  $C_{kl}^j = \frac{1}{2} \sum_{i=1}^N v_{ji}^T [\mathbf{U}^T \mathbf{H}_2^i \mathbf{U}]$ ,

$D_{pqr}^j = \frac{1}{6} \sum_{i=1}^N v_{ji} \sum_{k=1}^N \sum_{l=1}^N \sum_{m=1}^N H_{3pqr}^i u_k^p u_l^q u_m^r$ .

$\lambda$ ,  $\mathbf{U}$  and  $\mathbf{V}$  are respectively eigenvalue, right and left eigenvector of  $\mathbf{A}$ .  $v_{ji}$  is  $j^{th}$  element of  $\mathbf{V}$  and  $u_k^p$  is the  $k^{th}$  row and  $p^{th}$  column of  $\mathbf{U}$  and so on.  $C_{kl}^j$  and  $D_{pqr}^j$  are the required second and third order nonlinear coefficients in modal coordinate. In modal analysis, the mechanical power is assumed to be constant for small disturbance and therefore, the right hand side of (3) is usually assumed zero.

From (4), it is observed that the linear part is decoupled and simplified but the second and third order terms are yet coupled. Also the system is now in a new coordinate (modal coordinate). Normal Form theory requires simplification of the system by introducing nonlinear transformation given as

$$\mathbf{y} = \mathbf{z} + \mathbf{h2}(\mathbf{z}) + \mathbf{h3}(\mathbf{z}). \quad (5)$$

Where

$$y_j = z_j + \sum_{k=1}^N \sum_{l=1}^N h2_{kl}^j z_k z_l + \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N h3_{pqr}^j z_p z_q z_r. \quad (6)$$

The vector  $\mathbf{z}$  is the state vector in the new coordinate,  $\mathbf{h2}$  and  $\mathbf{h3}$  are respectively complex valued quadratic and cubic polynomials in  $\mathbf{z}$ . If  $h2_{kl}^j$  and  $h3_{pqr}^j$  coefficients are determined and certain conditions apply, the second and third order terms can be annihilated. Then the system is simplified to a linear problem in another coordinate (Normal Form coordinate) as in (7).

$$\dot{z}_j = \lambda_j z_j \quad (7)$$

The quadratic and cubic Normal Form coefficients of (6) are given by [14]

$$h2_{kl}^j = \frac{C_{kl}^j}{\lambda_k + \lambda_l - \lambda_j}, h3_{pqr}^j = \frac{D_{pqr}^j}{\lambda_p + \lambda_q + \lambda_r - \lambda_j}. \quad (8)$$

Where  $h2_{kl}^j$  and  $h3_{pqr}^j$  are the quadratic and cubic Normal Form coefficients of (3) in Normal Form coordinates.

From (8) it can be seen that if the denominators are very close to zero, the values of the coefficients will be high. Therefore, large values of these Normal Form coefficients indicate potential strong interactions among the modes that form the denominator. This information is very important in achieving a proper control design and siting of power system stabilizers PSS [11]. The system can be reversed to its original coordinate by an inverse Normal Form transformation.

The whole work in applying Normal Form transformation to study power system revolves around finding the so called Normal Form coefficients  $h2_{kl}^j$  and  $h3_{pqr}^j$ . However, manipulations of (3) to get coefficients in (4) are very cumbersome to follow due to multivariate higher order differentiation involved. For every model, one has to either work out the expressions for all the needed derivatives in advance or resort to multivariate numerical differentiation where possible. The former is indeed awful and the latter is based on careful selection of step size which offers approximate result with increased error as the order of differentiation increases.

For linear mode base of size  $N$  (i.e  $N$  differential equations), the number of coefficients is given by

$$N_c = \frac{N^4}{6} + \frac{5N^2}{6} + N^3. \quad (9)$$

The above equation clearly shows that the computational burden will increase in the power of four.

### III. THE PROPOSED METHOD

In this paper we propose a method that utilizes only the information from the state matrix to obtain simultaneously the coefficients of the higher order terms of (3). The idea was introduced in [15] in the of study geometric nonlinearities of second order mechanical systems. To the author's best knowledge, this is the first time the method is being formulated in order to study power system in first order. In the following paragraphs, step by step derivation of the proposed method is presented.

*Step 1 - Run power flow and linear analysis:* The equations for the nonlinear model are written, power flow performed and linear analysis done to obtain the state matrix ( $\mathbf{A}$ ), the right eigenvector ( $\mathbf{U}$ ) and the left eigenvector ( $\mathbf{V}$ ).

*Step 2 - Change variables and transform to modal coordinate:* The nonlinear equation is written as a combination of linear and nonlinear part and then transformed to Jordan form by linear transformation. Notice that (3) is made up of linear, quadratic and cubic part. Therefore, it could be written as comprising linear and nonlinear parts in (10).

$$\dot{\mathbf{x}} + \mathbf{A}\mathbf{x} + \beta(\mathbf{x}) = \mathbf{P} \quad (10)$$

Where  $\mathbf{P} = \Delta\mathbf{P}_m/\mathbf{M}$  and  $\beta(\mathbf{x})$  is a collection of all the nonlinear parts. Applying linear transformation to (10) and pre-multiplying by left vector yields

$$\dot{\mathbf{y}} + \mathbf{V}\mathbf{A}\mathbf{U}\mathbf{y} + \mathbf{f}_{NL}(y_1, y_2, \dots, y_N) = \mathbf{V}\mathbf{P}. \quad (11)$$

where  $\mathbf{f}_{NL} = \mathbf{V}\beta$ .

*Step 3 - Prescribe some arbitrary modal deviations:* From (4) we know the expression for  $\mathbf{f}_{NL}$  in the Taylor series for  $r^{th}$  equation to be

$$f_{NL}^r(y_1, y_2, \dots, y_N) = \sum_{j=1}^N \sum_{k=1}^N C_{jk}^r y_j y_k + \sum_{j=1}^N \sum_{k=1}^N \sum_{l=1}^N D_{jkl}^r y_j y_k y_l. \quad (12)$$

Where  $r = (1, 2, \dots, N)$ .

But avoiding the Hessian evaluations, we can find  $f_{NL}^r$  by prescribing small deviation in modal space to the original nonlinear and linear functions under static considerations (i.e all derivative = 0) and estimate the forcing term  $\mathbf{P}$ . The assumption made for this method is that the mechanical input power is no longer constant. But this assumption is only to compute the coefficients. That is to say the system is disturbed by varying the state variables arbitrarily and the forcing vector that would have caused such variation is estimated. For instance, using (2) let us prescribe a very small deviation  $\mathbf{x}$ ; then the right hand side can be written as

$$\mathbf{P}_T = \mathbf{f}(\mathbf{X}_0 + \mathbf{x}) \approx \mathbf{f}(\mathbf{X}_0) + \mathbf{A}\mathbf{x} + \beta(\mathbf{x}) = \mathbf{P}_0 + \mathbf{P}_L + \mathbf{P}_{NL}. \quad (13)$$

$\mathbf{P}_T$  is the total forcing vector estimated which can cause such deviation ( $\mathbf{X}_0 + \mathbf{x}$ ),  $\mathbf{P}_0$  is the original value at equilibrium,  $\mathbf{P}_L$  is linear contribution of the forcing vector while  $\mathbf{P}_{NL}$  is the nonlinear contribution. Notice that this is a restorative process since the system of (2) should have been driven by its right hand side ( $\mathbf{P}_T = \mathbf{P}_m/\mathbf{M} + \Delta\mathbf{P}_m/\mathbf{M}$ ).

To ensure that the deviation is in the modal space, the value of this deviation is assigned using the linear eigenvector. We choose arbitrary small amplitude of deviation say  $\alpha$  and multiply with right eigenvector since any multiple of eigenvector is still an eigenvector. Hence  $\mathbf{x} = \mathbf{U}\alpha$ ; where  $\alpha$  is the amplitude of the deviation (*note that  $\alpha$  means arbitrary value of  $y$* ).

*Step 4 - Evaluate the nonlinear contribution:* From (13) we can write  $\mathbf{P}_{NL} = \mathbf{P}_T - \mathbf{P}_L - \mathbf{P}_0$ . (14)

Where  $\mathbf{P}_L = \mathbf{A}\mathbf{x}$ ,  $\mathbf{P}_0 = \mathbf{f}(\mathbf{X}_0)$ .

*Step 5 - Formulate linear equations:* Passing  $\mathbf{x} = \mathbf{U}_i\alpha_i$  to linear and nonlinear static models will have effect only on the node  $ii$  since only mode  $i$  will be excited,  $\mathbf{x} = \mathbf{U}_i\alpha_i + \mathbf{U}_j\alpha_j$  will have effect on nodes  $ii$ ,  $jj$  and  $ij$  since both mode  $i$  and  $j$  will be excited. By prescribing series of  $\mathbf{x}$  each time getting the linear and nonlinear static contributions, a set of linear equations is formulated and solved simultaneously to get the coefficients.

For example, if  $C_{11}^r$  and  $D_{111}^r$  ( $r = 1, 2, N$ ) are needed, we can prescribe  $\mathbf{x} = +\mathbf{U}_1\alpha_1$  and  $\mathbf{x} = -\mathbf{U}_1\alpha_1$ . The essence of the negative part is to create two equations in order to solve simultaneously. By prescribing as above and solving for linear and nonlinear static solutions,  $\mathbf{P}_{NL}$  is obtained from (14). Then from (4) and (11) we can write set of linear equations as

$$\begin{cases} \mathbf{V}_r\mathbf{P}_{NL}^+ = \mathbf{V}_r\beta(+\mathbf{U}_1\alpha_1) = C_{11}^r\alpha_1\alpha_1 + D_{111}^r\alpha_1\alpha_1\alpha_1 \\ \mathbf{V}_r\mathbf{P}_{NL}^- = \mathbf{V}_r\beta(-\mathbf{U}_1\alpha_1) = C_{11}^r\alpha_1\alpha_1 - D_{111}^r\alpha_1\alpha_1\alpha_1 \end{cases}. \quad (15)$$

Or generally as  $\mathbf{A}_c\mathbf{X}_c = \mathbf{B}_c$ .

Where

$$\mathbf{A}_c = \begin{bmatrix} \alpha_1^2 & \alpha_1^3 \\ \alpha_1^2 & -\alpha_1^3 \end{bmatrix}, \mathbf{X}_c = \begin{bmatrix} C_{11}^r \\ D_{111}^r \end{bmatrix}, \mathbf{B}_c = \begin{bmatrix} \mathbf{V}_r\mathbf{P}_{NL}^+ \\ \mathbf{V}_r\mathbf{P}_{NL}^- \end{bmatrix}.$$

In this way all  $C_{11}$  &  $D_{111}$  coefficients are computed. The approach is same for all  $C_{22}$  and  $D_{222}$  only that  $\mathbf{x}$  becomes  $\mathbf{x} = \mathbf{U}_2\alpha_2$ . All  $C_{ii}$  and  $D_{iii}$  can be calculated this way. For coupled terms such as  $C_{ij}^r, D_{ijj}^r$  &  $D_{jji}^r$ ,  $\mathbf{x}$  may be

$$\mathbf{x} = \begin{cases} \pm\mathbf{U}_i\alpha_i \\ \pm\mathbf{U}_j\alpha_j \\ \mathbf{U}_i\alpha_i + \mathbf{U}_j\alpha_j \\ \mathbf{U}_i\alpha_i - \mathbf{U}_j\alpha_j \\ -\mathbf{U}_i\alpha_i + \mathbf{U}_j\alpha_j \end{cases}.$$

Following the same approach, we get similar expression  $\mathbf{A}_c\mathbf{X}_c = \mathbf{B}_c$ , with

$$\mathbf{A}_c = \begin{bmatrix} \alpha_i^2 & \alpha_i^3 & 0 & 0 & 0 & 0 & 0 \\ \alpha_i^2 & -\alpha_i^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_j^2 & \alpha_j^3 & 0 & 0 & 0 \\ 0 & 0 & \alpha_j^2 & -\alpha_j^3 & 0 & 0 & 0 \\ \alpha_i^2 & \alpha_i^3 & \alpha_j^2 & \alpha_j^3 & \alpha_i\alpha_j & \alpha_i^2\alpha_j & \alpha_i\alpha_j^2 \\ \alpha_i^2 & \alpha_i^3 & \alpha_j^2 & -\alpha_j^3 & -\alpha_i\alpha_j & -\alpha_i^2\alpha_j & \alpha_i\alpha_j^2 \\ \alpha_i^2 & -\alpha_i^3 & \alpha_j^2 & \alpha_j^3 & -\alpha_i\alpha_j & \alpha_i^2\alpha_j & -\alpha_i\alpha_j^2 \end{bmatrix}$$

$$\mathbf{X}_c = \begin{bmatrix} C_{ij}^r \\ D_{jii}^r \\ C_{jj}^r \\ D_{jjj}^r \\ C_{ij}^r \\ D_{ijj}^r \\ D_{jji}^r \end{bmatrix}, \mathbf{B}_c = \begin{bmatrix} \mathbf{V}_r\mathbf{P}_{NLi}^+ \\ \mathbf{V}_r\mathbf{P}_{NLi}^- \\ \mathbf{V}_r\mathbf{P}_{NLj}^+ \\ \mathbf{V}_r\mathbf{P}_{NLj}^- \\ \mathbf{V}_r\mathbf{P}_{NLij}^{++} \\ \mathbf{V}_r\mathbf{P}_{NLij}^{+-} \\ \mathbf{V}_r\mathbf{P}_{NLij}^{-+} \\ \mathbf{V}_r\mathbf{P}_{NLij}^{--} \end{bmatrix}.$$

For coupled terms such as  $D_{ijk}^r, k \neq i \neq j$ ,  $\mathbf{x}$  can be prescribed as:  $\mathbf{x} = \mathbf{U}_i\alpha_i + \mathbf{U}_j\alpha_j + \mathbf{U}_k\alpha_k$  and then the size of  $\mathbf{A}_c$  increases accordingly.

With the nonlinear coefficients determined, (8) is then used to determine the Normal Form coefficients  $h_{kl}^j$  and  $h_{pqr}^j$ . With the Normal Form coefficients and Normal Form initial conditions known, the indices defined in [11] can reveal

many information about the system behaviour under stress.

#### IV. APPLICATION TO IEEE TEST SYSTEMS

The proposed method is demonstrated on a 3-machine 9-bus power system [16] in Fig. 1 and 16-machine 68-bus power system [17] in Fig. 2. The system of Fig.1

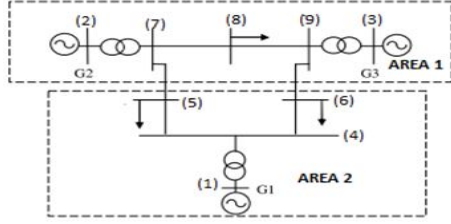


Fig. 1. 3-Machine 9-Bus System

is represented in two-axis model with 4 state variables per machine 16-19.

$$\dot{\delta}_i = \omega_s(\omega_i - 1) \quad (16)$$

$$T'_{d0}\dot{E}'_{qi} = -E'_{qi} - (x_{di} - x'_{di})I_{di} + E_{fdi} \quad (17)$$

$$T'_{q0}\dot{E}'_{di} = -E'_{di} + (x_{qi} - x'_{qi})I_{qi} \quad (18)$$

$$M_i\dot{\omega}_i = P_{mi} - (E'_{qi}I_{qi} + E'_{di}I_{di}) - D_i(\omega_i - 1) \quad (19)$$

Where  $\omega_s$  is the rated speed of the synchronous generator,  $\omega$  is the rotor speed deviation in p.u, D is damping coefficient, M is the inertial constant,  $\delta$  is the rotor angle.

If one of the generators is used as a reference, the number of differential equations will be 11. From equation 9, it implies we expect at least 3,872 coefficients. In this paper we show randomly some of the coefficients in order to compare with the usual direct Hessian approach.

The quadratic and cubic coefficients of  $r^{th}$  equation of system 16-19 using the proposed method are presented in Table II and Table IV. The results from the conventional direct Hessian approach are also presented (Table I and Table III) for comparison. As could be seen from the tables, the two results highly agree.

TABLE I  
QUADRATIC COEFF. HESSIAN METHOD

$r^{th}$	$C_{11}$	$C_{23}$	$C_{45}$
1	0.3249 + 0.2378i	0.5124 + 0.3663i	0.6631 - 3.0947i
4	-0.1323 - 0.0191i	0.0641 + 0.1335i	0.3234 - 0.8295i
7	-0.1193 + 0.0151i	0.0264 - 0.0059i	-0.0597 + 0.0051i
8	0.1065 + 0.0439i	-0.0052 - 0.0020i	-0.1101 - 0.0258i
11	0.0140 + 0.0099i	-0.0469 + 0.0075i	-0.2191 - 0.0653i

TABLE II  
QUADRATIC COEFF. PROPOSED METHOD

$r^{th}$	$C_{11}$	$C_{23}$	$C_{45}$
1	0.3249 + 0.2376i	0.5124 + 0.3661i	0.6631 - 3.0948i
4	-0.1323 - 0.0189i	0.0641 + 0.1337i	0.3234 - 0.8293i
7	-0.1191 - 0.0151i	0.0264 - 0.0059i	-0.0597 + 0.0051i
8	0.1065 + 0.0439i	-0.0051 - 0.0020i	-0.1100 - 0.0258i
11	0.1088 + 0.0439i	-0.0468 + 0.0075i	-0.2191 - 0.0653i

The maximum error taking into account all the coefficients were computed as  $Error_2^{Max} = |C_{kl}^{j_{Hessian}} - C_{kl}^{j_{Proposed}}|$  and  $Error_3^{Max} = |D_{3^{pqr}}^{j_{Hessian}} - D_{3^{pqr}}^{j_{Proposed}}|$  for quadratic and

TABLE III  
CUBIC COEFF. HESSIAN METHOD

$r^{th}$	$D_{222}$	$D_{4,8,8}$	$D_{9,10,11}$
1	-0.3191 - 0.3763i	-0.0015 - 0.0188i	-0.2008 - 1.0589i
4	0.0034 + 0.0091i	0.0077 - 0.0214i	0.0210 - 0.2150i
7	-0.0046 - 0.0104i	0.0104 + 0.0015i	0.0633 + 0.0000i
8	-0.0162 - 0.0334i	0.0189 + 0.0004i	0.1378 + 0.0000i
11	-0.0022 - 0.0067i	0.0013 - 0.0000i	0.0071 + 0.0000i

TABLE IV  
CUBIC COEFF. PROPOSED METHOD

$r^{th}$	$D_{222}$	$D_{4,8,8}$	$D_{9,10,11}$
1	-0.3191 - 0.3763i	-0.0015 - 0.0188i	-0.2010 - 1.0584i
4	0.0034 + 0.0091i	0.0077 - 0.0213i	0.0209 - 0.2150i
7	-0.0046 - 0.0104i	0.0104 + 0.0015i	0.0632 + 0.0000i
8	-0.0162 - 0.0334i	0.0189 + 0.0004i	0.1378 + 0.0000i
11	-0.0022 - 0.0067i	0.0013 - 0.0000i	0.0071 + 0.0000i

cubic coefficients respectively. These errors were found to be 0.024% and 1.8% for quadratic and cubic coefficients respectively.

The time cost was investigated on an Inter CoreTM i7-3520M 2.9GHz desktop computer, where the nonlinear coefficients computation using the Symbolic Math Toolbox in MATLAB takes about 40.7 seconds while the proposed method takes about 10.5seconds. No time optimization was considered in coding the algorithm. All coefficients were selectively computed each time as if there had not been any previous computations. Hence, several computations were repeated. Much time will be saved if it is programmed to use the previously computed coefficients in the linear equations. This can be improved upon in future work.

As earlier stated, the proposed method was also applied to 16-machine 68-bus New England/New York power system shown in Fig.2. Classical representation of generator was used and one generator used as a reference, making the total number of differential equations 31. It follows from equation (9) that there are 184,512 different coefficients to be computed. The direct Hessian approach was performed with the help of Symbolic MATLAB Toolbox in the same computer as before and the total time for all the coefficients was 21,924.095 seconds ( $\approx$  6hrs). This same coefficients were all computed with proposed method in about 3540 seconds ( $\approx$  1hr) with same computer.

As shown in Table V and Table VI, the results are highly

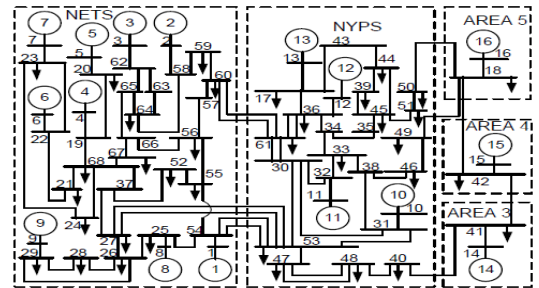


Fig. 2. 16-Machine 68-Bus System

TABLE V  
QUADRATIC & CUBIC COEFF.HESSIAN METHOD

$r^{th}$	$C_{14,14}$	$C_{1,20}$	$D_{2,10,16}$	$D_{30,30,30}$
1	-0.0260i	-0.0154i	0.0073i	0.0655i
5	-0.0085i	0.0006i	0.0013i	-0.0015i
7	-0.0060i	0.0025i	0.0004i	0.0007i
20	-0.0024i	-0.0520i	0.0015i	-0.0019i

TABLE VI  
QUADRATIC & CUBIC COEFF.PROPOSED METHOD

$r^{th}$	$C_{14,14}$	$C_{1,20}$	$D_{2,10,16}$	$D_{30,30,30}$
1	-0.0261i	-0.0153i	0.0075i	0.0655i
5	-0.0090i	0.0006i	0.0014i	-0.0016i
7	-0.0061i	0.0025i	0.0004i	0.0007i
20	-0.0025i	-0.0510i	0.0016i	-0.0020i

in agreement. We have stated that amplitude of the modal deviation is chosen arbitrarily, however, it should neither be too small nor too big. Very small value of  $\alpha$  does not trigger the nonlinearity very well; hence, the system is more or less linear. On the other hand too large value of  $\alpha$  leads to higher nonlinearity, hence the domain of validity of 3rd order approximation is exceeded. A value in the range of  $0.001 \leq \alpha < 0.9$  gives good result.

Fig. 3 shows the evolution of some coefficients with variation of  $\alpha$ . As pointed out earlier, the coefficients are

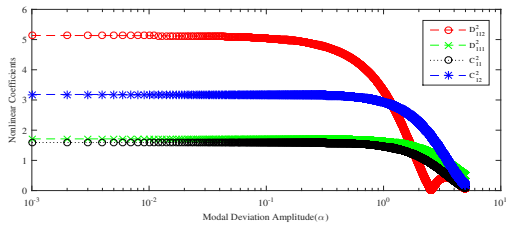


Fig. 3. Sensitivity of Modal deviation amplitude ( $\alpha$ )

consistent for  $\alpha$  within certain range as seen in Fig. 3. As  $\alpha$  becomes large, the nonlinearity increases beyond the validity of third order approximation. This is evident from Fig.3 as the curves deviate from the actual results.

The benefits of the proposed method are highlighted as follows: (1) Both 2<sup>nd</sup> and 3<sup>rd</sup> order nonlinear coefficients are computed simultaneously. (2) With good knowledge of the coefficients of interest, they can be selectively computed rapidly. (3) It requires only the state matrix used for the linear analysis. No further differentiation needed. (4) It is easy to implement and can easily adapt to model variations. (5) It maybe convenient to integrate in a commercial software.

## V. CONCLUSION

In this paper a method for selectively determining the nonlinear coefficients of power system represented with third order Taylor expansion has been proposed. The method is easy requiring only the state matrix of the system and direct substitution of arbitrary values in the static equations of the system. The proposed method has been demonstrated on conventional IEEE 9-bus and IEEE 68-bus systems but can

be extended to other types of grid. The method is convenient and fast with results in agreement with the conventional direct Hessian approach.

Since many commercial time-domain simulation software can give the state matrix and the static computations correspond to system's steady state after the small modal deviation introduced, we will investigate in future work the possibility of obtaining the static computation directly from commercial time domain simulation software. In future work we will also investigate a criterion for determining a priori the most relevant coefficients and then use the proposed method to compute just these coefficients in order to apply Normal Form to some selected modes in a large system.

## REFERENCES

- [1] H. Modir Shanechi, N. Pariz, and E. Vaahedi, "General Nonlinear Modal Representation of Large Scale Power Systems," *IEEE TRANSACTIONS ON POWER SYSTEMS*, vol. 18, no. 3, 2003.
- [2] S. You, G. Kou, Y. Liu, X. Zhang, Y. Cui, M. Till, W. Yao, and Y. Liu, "Impact of High PV Penetration on the Inter-Area Oscillations in the U.S. Eastern Interconnection," *IEEE Access*, vol. 5, pp. 4361–4369, 2017.
- [3] T. Tian, X. Kestelyn, O. Thomas, H. Amano, and A. R. Messina, "An Accurate Third-Order Normal Form Approximation for Power System Nonlinear Analysis," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 2128–2139, 2018.
- [4] Q. Huang, Z. Wang, and C. Zhang, "Evaluation of the effect of modal interaction higher than 2nd order in small-signal analysis," *2009 IEEE Power and Energy Society General Meeting, PES '09*, no. 2, pp. 1–6, 2009.
- [5] F. X. Wu, H. Wu, Z. X. Han, and D. Q. Gan, "Validation of power system non-linear modal analysis methods," *Electric Power Systems Research*, vol. 77, no. 10, pp. 1418–1424, 2007.
- [6] Q. Huang, Z. Wang, and C. Zhang, "Evaluation of the effect of modal interaction higher than 2nd order in small-signal analysis," *2009 IEEE Power and Energy Society General Meeting, PES '09*, no. 2, pp. 1–6, 2009.
- [7] I. Martínez, A. R. Messina, and E. Barocio, "Perturbation analysis of power systems: Effects of second- and third-order nonlinear terms on system dynamic behavior," *Electric Power Systems Research*, vol. 71, no. 2, pp. 159–167, 2004.
- [8] I. C. Martínez, A. R. Messina, and E. Barocio, "Higher-order normal form analysis of stressed power systems: A fundamental study," *Electric Power Components and Systems*, vol. 32, no. 12, pp. 1301–1317, 2004.
- [9] H. Amano, T. Kumano, and T. Inoue, "Nonlinear stability indexes of power swing oscillation using normal form analysis," *IEEE Transactions on Power Systems*, vol. 21, no. 2, pp. 825–834, 2006.
- [10] H. Amano and T. Inoue, "A New PSS Parameter Design Using Nonlinear Stability Analysis," *2007 IEEE Power Engineering Society General Meeting*, pp. 1–8, 2007.
- [11] J. J. Sanchez-Gasca, V. Vittal, M. J. Gibbard, A. R. Messina, D. J. Vowles, S. Liu, and U. D. Annakkage, "Inclusion of Higher Order Terms for Small-Signal (Modal) Analysis: Committee Report/Task Force on Assessing the Need to Include Higher Order Terms for Small-Signal (Modal) Analysis," *IEEE TRANSACTIONS ON POWER SYSTEMS*, vol. 20, no. 4, 2005.
- [12] A. R. Messina and V. Vittal, "Assessment of nonlinear interaction between nonlinearly coupled modes using higher order spectra," *IEEE Transactions on Power Systems*, vol. 20, no. 1, pp. 375–383, 2005.
- [13] B. I. N. Wang and K. A. I. Sun, "Nonlinear Modal Decoupling of Multi-Oscillator Systems With Applications to Power Systems," *IEEE Access*, vol. 6, pp. 9201–9217, 2018.
- [14] A. H. Nayfeh, *The Method of Normal Forms*. Weinheim: WILEY-VCH Verlag GmbH&Co. KGaA, second ed., 2011.
- [15] A. A. Muravyov and S. A. Rizzi, "Determination of nonlinear stiffness with application to random vibration of geometrically nonlinear structures," *Computers & Structures*, vol. 81, pp. 1513–1523, 7 2003.
- [16] P. W. Sauer and M. A. Pai, *Power system dynamics and stability*. New Jersey: Prentice Hall, 1998.
- [17] "http://www.sel.eesc.usp.br/ieee/NETS-NYPS 68 Bus System."