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# Theoretical study of a high-permittivity dielectric ring resonator for Magnetic Resonance Microscopy applications

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**Abstract** – A novel type of probe for Magnetic Resonance Microscopy, based on high-permittivity and low-loss ceramic ring resonators, is investigated from a theoretical point of view. An analytical study of the first Transverse Electric eigenmode of such resonator is provided to confirm the relevance of the corresponding magnetic field distribution in terms of signal-to-noise ratio.

## I. INTRODUCTION

The goal of Magnetic Resonance Microscopy (MRM) is to image tiny biological samples by analyzing the NMR signal of hydrogen atoms [1]. The sample is placed in a static magnetic field  $B_0$  to initiate spins precession around the  $B_0$  axis at the Larmor frequency, resulting in a net magnetization. A radiofrequency excitation field  $B_1$  is then applied within the sample volume to flip the magnetization in the transverse plane. The excitation field must be orthogonal to the static magnetic field at the Larmor frequency  $f_L$ , which is proportional to  $B_0$ . After spins relaxation, the decay of the transverse magnetization is processed to evaluate the local protons environment. The highest resolution attainable is limited, in practice, by the Signal-to-Noise Ratio (SNR) defined as the ratio of the magnetic field amplitude in the sample over the noise induced by the sample and the probe. The solenoid coil is the reference volumetric detector for MRM. The work of Minard and Wind [2, 3] provides detailed guidelines for solenoid microcoil design. The main contributions to the SNR limitation are metal losses induced by the winding and dielectric losses due to interactions of the conservative electric field with the sample [4]. An alternative design of microscopy probe was proposed to overcome these limitations. This design is based on the resonance of dielectric materials [5], which are in practice ferroelectric ceramics with a high relative permittivity and low dielectric losses [6]. Such a probe was more recently designed, built and experimentally tested in [7] for MR microscopy at 17.2 T ( $f_L \approx 730$  MHz) and it consists of two coupled high-permittivity ring dielectric resonators (DRs). The first coupled mode is adjusted to  $f_L$  by changing the distance between the resonators. In this paper, the first transverse electric mode of one DR is studied from an analytical point of view in order to pave the way to further dielectric losses estimation and then SNR comparison with the reference probe. Each DR has an outer radius  $a = 9$  mm, an inner radius  $b = 2.8$  mm, a height  $L = 10$  mm, and as electromagnetic properties, a relative permittivity  $\epsilon_r$  equal to 530 and loss tangent  $\tan \delta = 8.10^{-4}$ .

## II. $TE_{01\delta}$ MODE OF A HIGH-PERMITTIVITY RING DIELECTRIC RESONATOR

The first transverse electric eigenmode of a high permittivity disk resonator, represented on Fig. 1 is the so-called  $TE_{01\delta}$  mode [8, 9]. In the center of the disk, the magnetic field is parallel to the disk axis and its magnitude reaches its maximum. This orientation along one direction enables the excitation field  $B_1$  to be orthogonal to the static field  $B_0$ . On the other hand the centre of the disk resonator in its first transverse electric mode is associated with low electric field values. As a result, dielectric losses in this region are limited. The corresponding ring resonator (disk resonator with a hole in the centre) also has a resonant mode classified as  $TE_{01\delta}$ [8] which field distribution is

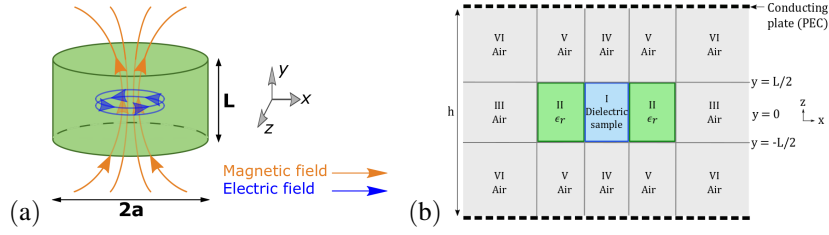


Fig. 1:  $TE_{01\delta}$  mode: (a) schematic of the field lines in the disk resonator; (b) space subdivision for field distribution computation in the ring resonator.

very similar to that of the disk. A semi-analytical method, involving to numerically solve transcendental equations accounting for boundary conditions, is proposed to compute the field distribution of the ring resonator.

The  $TE_{01\delta}$  eigenmode has a cylindrical symmetry and is represented by three nonzero field components  $H_y$ ,  $H_\rho$  and  $E_\theta$ . Due to the high permittivity of the considered dielectric materials, the electromagnetic field is mainly confined within it. However, it does not vanish on the boundaries and the field leakages should be considered. These leakages are included in the method used to evaluate the field distribution, based on the work of [11] and illustrated on Fig. 2(a). This configuration involves Perfect Electric Conductor boundary conditions, the so-called conducting plates on Fig. 2, above and under the resonator. The distance  $h$  between those plates is taken equal to the MR device bore diameter. Region entitled II is the dielectric material of real relative permittivity  $\epsilon_r$ , while region I represents the sample of real relative permittivity 50. In the field distribution estimation, losses due to the imaginary part of the complex permittivity are not considered. However the materials conductivity is involved in the dielectric losses computation (not presented in this paper). Eqs. 1 provide analytical expressions for the axial component of the H-field [8]. Other nonzero field components are deduced from Eq. 1 through Helmholtz equation [10].  $J_0$  and  $Y_0$  are the Bessel functions of the first and second kinds, and  $K_0$  the modified Bessel function of the second kind, of zero order.  $a_{I \rightarrow IX}$  represent relative field amplitudes in regions  $I \rightarrow IX$ ,  $\alpha_{I \rightarrow II}$  the propagative radial wavenumbers,  $\nu_{III}$  the evanescent radial wavenumber, and  $\beta$  and  $\gamma_{1,2}$  the propagative and evanescent axial wavenumbers, respectively.  $y_{1,2}$  and  $\xi$  are constants depending on the distance between the resonators and the conducting plates. These variables are related to each others through the dispersion equations and the boundary conditions. More specifically, the wavenumbers are computed by numerically solving equations of tangential field continuity at the boundaries II-V, I-II and II-III.

$$\begin{aligned}
 H_y^I &= a_I J_0(\alpha_I \rho) \cos(\beta y) \\
 H_y^{II} &= a_{II} [J_0(\alpha_{II} \rho) + \xi Y_0(\alpha_{II} \rho)] \cos(\beta y) \\
 H_y^{III} &= a_{III} K_0(\nu_{III} \rho) \cos(\beta y) \\
 H_y^{IV} &= a_{IV} J_0(\alpha_{IV} \rho) \sinh[\gamma_1(y - y_1)] \\
 H_y^V &= a_V [J_0(\alpha_{II} \rho) + \xi Y_0(\alpha_{II} \rho)] \sinh[\gamma_1(y - y_1)] \\
 H_y^{VI} &= a_{VI} K_0(\nu_{III} \rho) \sinh[\gamma_1(y - y_1)] \\
 H_y^{VII} &= a_{VII} J_0(\alpha_I \rho) \sinh[-\gamma_2(y - y_2)] \\
 H_y^{VIII} &= a_{VIII} [J_0(\alpha_{II} \rho) + \xi Y_0(\alpha_{II} \rho)] \sinh[-\gamma_2(y - y_2)] \\
 H_y^{IX} &= a_{IX} K_0(\nu_{III} \rho) \sinh[-\gamma_2(y - y_2)]
 \end{aligned} \tag{1}$$

The analytically calculated electromagnetic field is represented in Fig. 2 (b, c) in the case of the studied probe with a sample of relative permittivity 50. The magnetic field decreases within the sample volume but remains strong compared to its distribution within the dielectric material. In the meantime the electric field has low values within the sample, which is of great interest in the framework of imaging as noise in the acquisition chain is, among others, caused by electric field - sample interactions. The obtained electromagnetic field maps were compared to numerical simulations performed with the eigenmode solver of CST Microwave Studio. As represented on Figs. 2(c,d), the analytical and computed field distributions demonstrated similar patterns especially within the sample, and also in the dielectric material despite underestimated leakages at the boundaries with the analytical method.

### III. CONCLUSION

The  $TE_{01\delta}$  mode of disk resonators has been extensively addressed in the literature, either through accurate numerical methods or approximated approaches. This abstract focuses on the study of ring resonators' first transverse

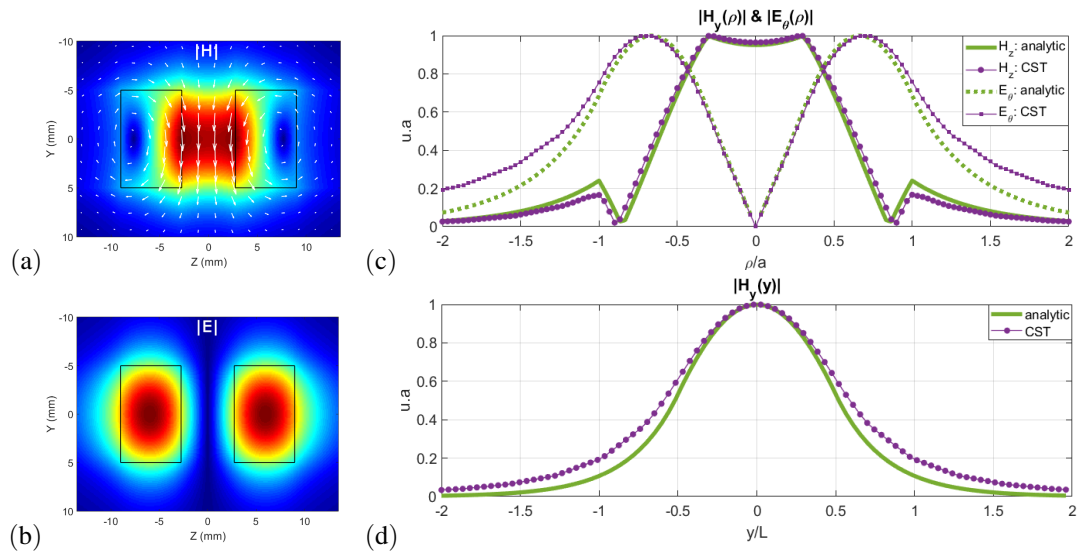


Fig. 2: Analytical field distribution: normalized (a) H-field and (b) E-field distributions (YZ plane) filled with a sample of relative permittivity 50 and diameter 5.6 mm. White arrows on the H-field map illustrate the magnetic field lines (the E-field is normal to the YZ plane). (c) and (d): comparison of analytical and numerical (CST Microwave Studio) field distributions along  $z = 0$  and  $\rho = 0$  lines respectively.

electric mode. A semi-analytical method is proposed to estimate the field distribution which is exploited as excitation source and reception probe in the context of MRM. This work enables further characterization of dielectric resonators used as microscopy probes through the possibility to estimate the induced dielectric power losses and then compare the SNR to the reference probe. This perspective is very promising as both the low-loss ceramics and the possibility of tuning with no electrical components ensure lower losses than in the case of the solenoid.

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