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J.F. Charpentier, Yvan Lefèvre, Hubert Piquet. An original and natural method of coupling electromagnetic field equations with circuit equations put in a state form.. IEEE Transactions on Magnetics, 1998, 34 (5), pp.2489-2492. 10.1109/20.717573 . hal-02514387

**HAL Id: hal-02514387**

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### To cite this version:

Charpentier, Jean-Frédéric and Lefèvre, Yvan and Piquet, Hubert An original and natural method of coupling electromagnetic field equations with circuit equations put in a state form. (1998) IEEE Transactions on Magnetics, 34 (5). 2489-2492. ISSN 0018-9464

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# An Original and Natural Method of Coupling Electromagnetic Field Equations with Circuit Equations Put in a State Form

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**Abstract**— In this paper a general method of coupling static converters with electromagnetic devices is presented. This method is based on the simultaneous solution of the converter equations and the field equations in the electromagnetic devices. The equations are established in an automatic way: the circuit equations are put in a state form and the field equations are discretized using a 2D finite element method. An original way of linking the two systems of equations is proposed.

**Index terms**— Finite element method, eddy currents, static converters, power electronics, coupled problems.

## I. INTRODUCTION

As the switching frequency of the power electronics circuits increases, parasitic phenomena due to eddy currents in the magnetic devices like transformers or coils have to be taken into account. In this paper a general method based on the coupling of the electromagnetic field equations and the circuit equations is presented. A 2D finite element method is used to discretize the field equations in the electromagnetic device. Formulations accounting for the eddy currents in different types of conductor are considered. The circuit equations are set up in a state form. An original way to link the two systems into a global one is presented. To illustrate the global capabilities of the obtained software, results of the simulations of different associations of static converters and magnetic devices are shown.

## II. ELECTROMAGNETIC FIELD EQUATIONS

The magnetic field equations in the electromagnetic devices are determined from the geometry of the structure and the properties of the materials. The final differential system of equations in matrix form is mounted by a 2D nodal finite element method. We obtain the following system :

$$[M]\{A\} + [N]\frac{d}{dt}\{A\} - [P]\{I_w\} - \{S_{ma}\} = \{0\} \quad (1)$$

$$\{U_w\} - [Q]\frac{d}{dt}\{A\} = \{0\} \quad (2)$$

Where  $\{A\}$  is the magnetic vector potential at the nodes of the mesh,  $\{I_w\}$  is the vector of the currents in the windings.  $\{U_w\}$  is the vector of the voltage drops on windings.

Matrices  $[M]$ ,  $[N]$ ,  $[P]$  and  $[Q]$  are assembled by the nodal 2D finite element method.  $\{S_{ma}\}$  is a vector due to permanent magnets [1]. This is a general form of the discretized magnetic field equations which takes into account all kinds of windings: with stranded or solid conductors [2], [1]. These equations can be written in different ways depending on how the magnetic structure is supplied. It can be supplied by currents or voltages. To put in evidence the kind of supply, the main equations, which contain the main unknowns and the real sources of the whole system, must be distinguished from the secondary or output equations which allow to calculate the other variables as a function of the real sources and the main unknowns.

### A. Supplying magnetic devices with currents

If the magnetic device is supplied by currents, the sources of the magnetic field are the permanent magnets and the currents in the windings. In this case, the unknowns of the magnetic equations are the magnetic vector potential on nodes. The voltage drop on each winding can be calculated from these unknowns by means of equations (2) which then constitute the output equations. It is then convenient to write the main equations as follows:

$$[M]\{A\} + [N]\frac{d}{dt}\{A\} = [P]\{I_w\} + \{S_{ma}\} \quad (3)$$

and the output equations in this manner:

$$\{U_w\} = [Q]\frac{d}{dt}\{A\} \quad (4)$$

This form of the equations puts in evidence the source in the righthand side of the main equations and the output variables in the lefthand side of the secondary equations.

### B. Supplying magnetic devices with voltages

In this case, the sources of the whole system are the permanent magnets and the voltage drops on the windings. The unknowns are the vector potential on nodes and the currents in the windings. The final system is composed of the two equations (1) and (2). It is convenient to write them in the following way:

$$[M]\{A\} + [N]\frac{d}{dt}\{A\} - [P]\{I_w\} = \{S_{ma}\} \quad (5)$$

$$[Q]\frac{d}{dt}\{A\} = \{U_w\} \quad (6)$$

Equations (5) and (6) are the main equations and they put in evidence the sources of the whole system in the righthand side.

Remarks. These two manners of writing the discretised magnetic field equations show that supplying a magnetic device by currents is more natural than supplying it with voltages. In the case of supplying by currents, the sources of the magnetic fields, which are the currents in the windings and the permanent magnets, are homogenous because they are added to the right hand side of the main equations (3). In the case of supplying by voltages they are not homogenous because they don't interact in the same equations. They are acting in two different equations (5) and (6). In fact, it is already a case of coupling magnetic field equations with a very simple electric circuit as shown in the next paragraph.

### C. Coupling the field equations with simple circuit equations

In practise, the voltages are applied on the windings by means of thin wires whose impedances are not null. Each of these wires can be taken into account by means of a resistor and an inductor in series with each winding. These electric components associated with the supply device and the magnetic device constitute a simple electric circuit. The equations are in this case:

$$[M]\{A\} + [K] \frac{d}{dt}\{A\} - [P]\{I_w\} = \{S_{ma}\} \quad (7)$$

$$[R_s]\{I_w\} + [L_s] \frac{d}{dt}\{I_w\} + [Q] \frac{d}{dt}\{A\} = \{U\} \quad (8)$$

$[R_s]$  and  $[L_s]$  are diagonal matrices that represent the resistors and inductors of the connecting wires. In the preceding paragraph, the obtained system, in the case of a supply by means of voltages, is a particular case of this simple system where the voltages are applied directly on the windings:

$$\{U_w\} = \{U\} \quad (9)$$

### III. STATE EQUATIONS OF AN ELECTRIC CIRCUIT

From the description of the topology of the circuit, Welsh's algorithm allows to determine a normal tree [3], [4], [5], [1]: the circuit is subdivided into branches and links. The branches are in priority voltage sources, capacitors and resistors. The Kirchhoff's laws give the following equations:

$$V_l = -[B_1]V_b \quad I_b = [B_1]^* I_l \quad (10)$$

Where  $V_l$  and  $I_l$  are the voltages and currents of links,  $V_b$  and  $I_b$  are the voltages and currents of branches. The voltage-current relations of the passive components are :

$$U = RI \quad U = LdI/dt \quad I = CdV/dt \quad (11)$$

With the relations (10) and (11), the circuit equations can be put in a state form:

$$\frac{d}{dt}\{X\} = [A]\{X\} + [B]\{E\} + [D] \frac{d}{dt}\{E\} \quad (12)$$

$\{X\}$  is the vector of state variables of the circuit which are the voltage drops on branch capacitors and the currents in inductors in links.  $\{E\}$  is the vector of voltage and current sources of the circuit. Any vector  $\{Y\}$  of variables of the electric circuit can be expressed easily by means of the state variables and the vector of the sources.

$$\{Y\} = [F]\{X\} + [G]\{E\} + [H] \frac{d}{dt}\{E\} \quad (13)$$

### IV. COUPLING THE FIELD AND CIRCUIT EQUATIONS

When the circuit equations are expressed in state form, there are two ways to account for the windings of the magnetic devices on the circuit level. They can be considered as current or voltage sources.

#### A. Windings considered as current sources

If the windings are considered as current sources on the circuit level, the currents  $\{I_w\}$  in these windings are part of the vector of the sources in the state equations of the circuit. Separating the real sources from the current sources representing the windings, the circuit equations can be written in the following way:

$$\begin{aligned} \frac{d}{dt}\{X\} = [A]\{X\} + [B_e^i]\{E\} + [B_b^i]\{I_w\} \\ + [D_e^i] \frac{d}{dt}\{E\} + [D_b^i] \frac{d}{dt}\{I_w\} \end{aligned} \quad (14)$$

The matrices  $[B]$  and  $[D]$  are separated into two blocks, marked 'e' for the one concerned by the real sources of the circuit and marked 'b' for the one concerned by the current sources representing the windings. The voltage drops on the windings are part of the output vector:

$$\begin{aligned} \{U\} = [F^i]\{X\} + [G_e^i]\{E\} + [G_b^i]\{I_w\} + \\ [H_e^i] \frac{d}{dt}\{E\} + [H_b^i] \frac{d}{dt}\{I_w\} \end{aligned} \quad (15)$$

On the other hand, the sources of the magnetic field equations are in part the voltage drops on the windings and the currents in them are part of the unknowns. This corresponds to the case of supplying the magnetic device by voltages and the magnetic field equations are written in the form of equations (7) and (8) [5].

After the elimination of the voltage drops  $\{U\}$  on the windings from equations (8) and (15), we obtain the system of differential equations:

$$[M]\{A\} + [K] \frac{d}{dt}\{A\} - [P]\{I_w\} = \{S_{ma}\} \quad (16)$$

$$\begin{aligned} [Q_b] \frac{d}{dt}\{A\} + ([R_s] - [G_b^i])\{I_w\} + ([L_s] - [H_b^i]) \frac{d}{dt}\{I_w\} \\ - [F^i]\{X\} = [G_e^i]\{E\} + [H_e^i] \frac{d}{dt}\{E\} \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{d}{dt}\{X\} - [A]\{X\} - [B_b^i]\{I_w\} - [D_b^i] \frac{d}{dt}\{I_w\} \\ = [B_e^i]\{E\} + [D_e^i] \frac{d}{dt}\{E\} \end{aligned} \quad (18)$$

The right hand side of the system is composed of the permanent magnets, the vector of the real, current and voltage, sources of the circuit and its time derivative. The unknowns are the potential vector on nodes, the currents in the windings of the magnetic device and the state variables of the circuit. The voltage drops on the windings are calculated by the output equations of the circuit (15) or the second equations of the magnetic field (8).

Reference [5] shows that with some hypotheses this coupled system can be simplified by eliminating, in the equations (18), the time derivative of the currents in the windings. We can see that between the magnetic field equations (16) and the electric circuit equations (18) there is a third equation which can be called coupling equations (17). Reference [6] shows that this equation cannot be eliminated if the above hypotheses hold.

#### B. Windings considered as voltage sources

In this case, the voltage drops on the windings are part of the sources of the circuit. The circuit equations can then be written as follows:

$$\begin{aligned} \frac{d}{dt}\{X\} = [A]\{X\} + [B_e^u]\{E\} + [B_v^u]\{U_w\} \\ + [D_e^u]\frac{d}{dt}\{E\} + [D_v^u]\frac{d}{dt}\{U_w\} \end{aligned} \quad (19)$$

The matrices  $[B]$  and  $[D]$  are splitted into two blocs, marked 'e' when they are related with the real sources of the circuit, marked 'v' when they are related with the voltage sources representing the windings. The currents in the windings are part of the output vector:

$$\begin{aligned} \{I_w\} = [F^u]\{X\} + [G_e^u]\{E\} + [G_v^u]\{U_w\} + \\ [H_e^u]\frac{d}{dt}\{E\} + [H_w^u]\frac{d}{dt}\{U_w\} \end{aligned} \quad (20)$$

On the other hand, the currents in the windings are part of the sources of magnetic field and the voltage drops on windings among the output. This situation corresponds to the case of supplying the magnetic device with currents and the magnetic field is governed by the equations (3):

$$[M]\{A\} + [K]\frac{d}{dt}\{A\} = [P]\{I_w\} + \{S_{ma}\} \quad (21)$$

To this equation, the output equations (4) must be added:

$$\{U_w\} = [Q]\frac{d}{dt}\{A\} \quad (22)$$

After the elimination of  $\{I_w\}$  and of the output circuit equations (20), we obtain the coupled equations:

$$[M]\{A\} + [K]\frac{d}{dt}\{A\} - [P][F^u]\{X\} - [P][G_v^u]\{U_w\} -$$

$$\begin{aligned} [P][H_v^u]\frac{d}{dt}\{U_w\} = \\ \{S_{ma}\} + [P][G_e^u]\{E\} + [P][H_e^u]\frac{d}{dt}\{E\} \end{aligned} \quad (23)$$

$$\{U_w\} = [Q]\frac{d}{dt}\{A\} \quad (24)$$

$$\begin{aligned} \frac{d}{dt}\{X\} - [A]\{X\} - [B_v^u]\{U_w\} - [D_v^u]\frac{d}{dt}\{U_w\} \\ = [B_e^u]\{E\} + [D_e^u]\frac{d}{dt}\{E\} \end{aligned} \quad (25)$$

Reference [1] shows that the time derivative of  $\{U_w\}$  can be eliminated in equations (23) and (25) if the resistor and the inductor in series with the windings are taken into account at the circuit level. We remark that between the magnetic field equation (23) and the electric equation (25) there is a third equation (24) which is the output equations of the magnetic field. This equation and the term  $\{U_w\}$  can be eliminated together. These eliminations are possible because the coupling in this case is perfect: one of the sources of the magnetic field, the currents in the windings, are part of the circuit output equations and the voltage drop on winding, which are a part of the source of the circuit, are the output of the magnetic field [6].

A global and unique system of differential equations can be obtained [1]. The source of the whole system are the real sources of the circuit and the permanent magnets in the magnetic device. The unknowns are the vector potential on nodes and the state variables of the circuit. The currents and the voltage drops on the windings are calculated by means of the two output equations (20) and (24).

The switches are modelled by binary value resistors. The evolution of the sequences of the converter is managed by a colored Petri net representation of the switches. The places in the Petri net characterise the state of the switches and the validation of the transitions of the net are function of the switching conditions [3]. This method allows to simulate the association between the circuit and the electromagnetic device without knowing 'a priori' the operating sequences of the converter.

#### V. SIMULATION RESULTS

We present here the results of simulations of a series resonant inverter obtained by the presented method. The inverter is shown on figure 1.

The value of the components of the converter are :  $C = 0.150\mu F$ ,  $E = 10V$  and  $R = 15\Omega$ . The switching frequency of the converter is  $20kHz$ . Figure 2 shows the current in the coil ( $I_{coil}$ ) and the voltage across the capacitor ( $V_{capa}$ ) in two different cases: in the first case the conductivity of the core is high and in the second case this conductivity is null.

At the initial time all the variables are equal to zero. We can see the influence of the value of the conductivity

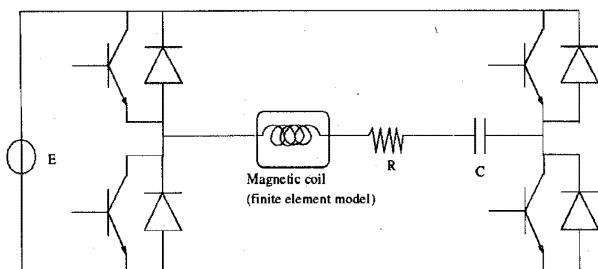


Fig. 1. Series resonant inverter with magnetic coil

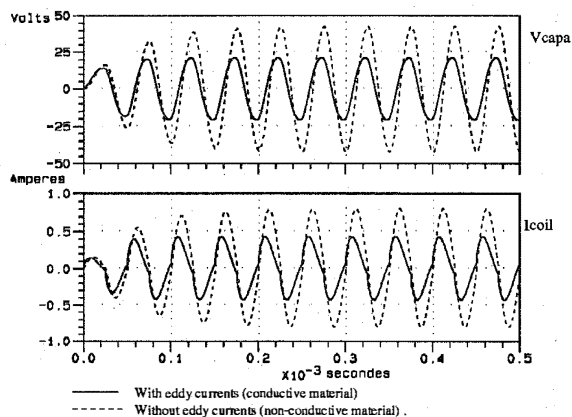


Fig. 2. Effect of the conductivity of the core: Dashed line: the conductivity is null, Solid line: conductivity is high

of the core on the waveforms of the current in the coil and of the voltage drop on the capacitor. We have another simulation. In this simulation the core of the inductance has been removed and the stranded conductors winding is replaced by a winding made of 1 mm square circular solid conductors. This windings has 20 coils of 21 mm radius. The other conditions remaining the same, we obtain the results shown in figure (3). We can see the effect of the induced currents in the conductors of the windings.

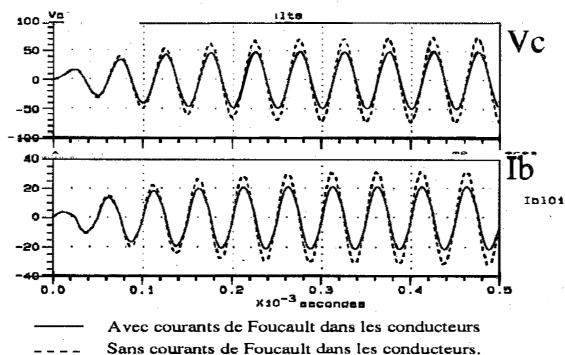


Fig. 3. Effect of the conductivity of the winding solid conductors: Dashed line: stranded conductors, Solid line: solid conductors

## VI. CONCLUSION

In this paper a general method to simulate the association of converters with magnetic devices is presented. This method is based on the simultaneous solution of the field equations in the electromagnetic devices and the circuit equations of static converters. An original and natural way of coupling the two sets of equations is proposed. This solution allows to take into account the eddy currents in different kind of conductors as the solid conductors in windings. An example of application is presented. It deals with a series resonant inverter associated with different magnetic coils. The quality of the simulation results illustrates the capabilities of the proposed modelling and offers very interesting perspectives in the analysis of the association of converters with electromagnetic devices.

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