



**HAL**  
open science

# A systemic viewpoint on the approximation of a power transmission line model

Mohamed Belhocine, Bogdan Marinescu

► **To cite this version:**

Mohamed Belhocine, Bogdan Marinescu. A systemic viewpoint on the approximation of a power transmission line model. IFAC world, Jul 2017, Toulouse, France. 10.1016/j.ifacol.2017.08.2578 . hal-02512783

**HAL Id: hal-02512783**

**<https://hal.science/hal-02512783v1>**

Submitted on 19 Mar 2020

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# A systemic viewpoint on the approximation of a power transmission line model

M. Belhocine, B. Marinescu

*Ecole Centrale Nantes-LS2N, 1 rue de la Noë, 44300, Nantes, France  
(e-mails: mohamed.belhocine@ec-nantes.fr,  
bogdan.marinescu@ec-nantes.fr).*

---

**Abstract:** This paper explains, from a systemic viewpoint, the importance to take into account the dynamic structure of the whole system in order to simplify the distributed parameters model of the transmission lines. Usually, the latter is approximated without considering its connection with the rest of the system, by comparing only its input-output behaviour with the one of the simplified model. Here, it is shown that this way to do can lead to biased results. More precisely, it is shown that the short-lines hypothesis leads to a reduction link with the  $\pi$ -model but does not indicate clearly which dynamics have to be kept in the simplified model as well as their number. This is illustrated by considering the voltage collapse phenomenon. From this analysis, a more systemic approximation way is proposed to reduce subsystems of a general complex system. In power systems field, all these investigations can help to improve the models used for simulation and control synthesis. Especially, to better connect the models to the specific phenomena which have to be reproduced in simulation.

*Keywords:* Transmission lines, distributed parameter systems, spectral analysis, electric power systems, model reduction.

---

## 1. INTRODUCTION

The voltage and the current of power electric lines are both sum of two waves which travel with same finite velocity and in opposite directions along the line. Their dynamic is modelled by a Distributed Parameters (DP) model which consists of two Partial Differential Equations (PDE) giving these quantities in function of time and space. It is thus an *infinite-dimension* system. When it is connected to the rest of the power system, it leads to a full description of all the physical phenomena resulting from the different interconnections. However, for realistic applications, such a full and detailed model of the lines is difficult to be used and need to be simplified. For instance, in the case of short and medium length lines (up to 250 Km), the propagation phenomena is generally not taken into account and a simplified model, called  $\pi$ -model, is used. It is thus described by ordinary differential equations giving the voltages and the currents only at the ends.

In power systems, the models above are used to simulate different phenomena. The  $\pi$ -model is used, e.g., in (Meyer and Stubbe, 1992) to perform load-flow computations, to analyse transient stability and to design voltage controllers. To capture the wave propagation, ElectroMagnetic Transients Program (EMTP) simulators, like (Dommel, 1969), use a delay model obtained from the trajectories of the DP one (when the frequency dependence of the parameters can be neglected). Both models are based on physical considerations. However, their adequacy to reproduce a specific phenomenon, with the overall power system, was always checked *a posteriori* by experimental tests. The main reason of this is that only the input-output behaviours of the full and the proposed simplified models

are compared in order to validate the simplification. All the interactions with the other components of the system are not considered.

In this context, our goal is to establish a *systemic* link between the phenomena to be reproduced in simulation and the appropriate model of the transmission line. This firstly motivated us to investigate the dynamic properties (especially the modes and the transfer function) of both  $\pi$  and DP models in the setting of systems theory. This provides more informations on the relationship between their dynamic structures and their behaviours than a simple comparison of their trajectories. Indeed, the  $\pi$ -model was a priori adopted and it was afterwards checked that, in some particular situations, it provides trajectories comparable with the ones of the DP model. It is explained here the connection between the dynamic structures of the two models. More precisely, the  $\pi$ -model is shown to be close to a first order modal truncation of the DP model. Also, it is explained that the systematic use of the  $\pi$ -model as an approximation of the DP one, can lead to less satisfactory results in some situations when the line is connected to the rest of the system. Finally, a more systemic and appropriate way is proposed to well approximate the DP model of the transmission lines. All these investigations can be extended to other components of a power system and give a basis to further study the adequacy of the models to each typical power system dynamics (like, e.g., voltage response inter-area oscillations, sub-synchronous resonance,...).

The paper is organised as follows: in Section 2, the mathematical background used in our developments is recalled. In Section 3, the dynamic structures of both DP and  $\pi$

models of the line are analytically developed and their poles and trajectories are compared. A reduction link between them is established in the case of open lines. In Section 4, is explained how the use of the  $\pi$ -model, based on the hypothesis of short-lines can lead to less satisfactory results. Section 5 presents the proposed technique to well approximate the DP model of the transmission lines. Conclusions and ways in which our results can be exploited to improve general models used for simulations, analysis and control of power systems are presented in Section 6.

## 2. MATHEMATICAL BACKGROUND

Let  $Z$  be a Hilbert space, and  $\mathcal{A} : D(\mathcal{A}) \subset Z \rightarrow Z$  a linear operator with the domain  $D(\mathcal{A})$ . If  $\mathcal{A}$  is compact, the set of complex values  $\lambda \in \mathbb{C}$  for which the operator  $(\mathcal{A} - \lambda I)$  is not invertible is said the *spectrum* of  $\mathcal{A}$  and it is denoted by  $\rho(\mathcal{A})$ . Then the set of eigenvalues of  $\mathcal{A}$  is a subset of  $\rho(\mathcal{A})$  for which the following equation is satisfied

$$\mathcal{A}\phi_n = \beta_n\phi_n, \quad (1)$$

where  $\{\beta_n, n \geq 1\}$  are the eigenvalues of  $\mathcal{A}$  and  $\{\phi_n, n \geq 1\}$  the corresponding eigenvectors (eigenfunctions). Notice that  $n$  is not necessarily finite. In the case where  $\mathcal{A}$  is a self-adjoint operator,  $\{\phi_n, n \geq 1\}$  forms an *orthonormal basis* and then each element  $z \in Z$  can be uniquely written as  $z = \sum_{n=0}^{\infty} \langle z, \phi_n \rangle_Z \phi_n$  where  $\langle \cdot, \cdot \rangle_Z$  is the inner product in  $Z$ . Otherwise, if  $\mathcal{A}$  is non-self-adjoint but a Riez spectral operator, then each  $z \in Z$  can be written in a unique way as  $z = \sum_{n=0}^{\infty} \langle z, \psi_n \rangle_Z \phi_n$  where  $\psi_n$  are the eigenvectors of the adjoint of  $\mathcal{A}$  noted  $\mathcal{A}^*$ .

Also, the following representation

$$\begin{cases} \frac{dz(t)}{dt} = \mathfrak{A}z(t), & z(t)|_{t=0} = z_0, \\ \mathfrak{B}z(t) = u(t). \end{cases} \quad (2)$$

is a general form of an *abstract boundary control problem* where  $\mathfrak{A} : D(\mathfrak{A}) \subset Z \rightarrow Z$  is an operator,  $u(t) \in U$  the input and  $\mathfrak{B} : D(\mathfrak{B}) \subset Z \rightarrow U$  is called *boundary operator* with  $D(\mathfrak{A}) \subset D(\mathfrak{B})$ . More explanations and details can be found, e.g., in Curtain and Zwart (1995) or Tucsnak and Weiss (2009).

## 3. ANALYTIC DEVELOPMENTS

To start, consider the distributed parameters model of the line. If the transverse conductance is neglected, it can be written as

$$\begin{cases} \frac{\partial v(x,t)}{\partial x} = -L \frac{\partial i(x,t)}{\partial t} - Ri(x,t), \\ \frac{\partial i(x,t)}{\partial x} = -C \frac{\partial v(x,t)}{\partial t}, \end{cases} \quad (3)$$

where  $R$ ,  $L$ , and  $C$  are positive constant parameters given per unit length (see, e.g., (Miano and Maffucci, 2001)). To connect (3) with the other components of the system, boundary conditions are needed. They define the inputs and the outputs of the line model and then describe the behaviour at the extremities of a line of length  $\ell$ . Here, the line is considered open at the extremity  $x = \ell$  and submitted to an ideal voltage source  $v_0(t)$  at extremity  $x = 0$ . Thus, when the equations of (3) are combined to

eliminate the current  $i(x,t)$  and the boundary conditions are added, one gets the system

$$\begin{cases} \frac{\partial^2 v(x,t)}{\partial x^2} - LC \frac{\partial^2 v(x,t)}{\partial t^2} - RC \frac{\partial v(x,t)}{\partial t} = 0, \\ v(x,t)|_{t=0} = 0, \quad \frac{\partial v(x,t)}{\partial t}|_{t=0} = 0, \\ v(x,t)|_{x=0} = v_e(t), \quad \frac{\partial v(x,t)}{\partial x}|_{x=\ell} = 0, \end{cases} \quad (4)$$

defined for  $x \in \Omega = [0 \ \ell]$  and  $t \in [0 \ +\infty)$ . Its first equation is called *damped wave equation* or *telegrapher's equation*. From a systemic viewpoint, (4) can be formulated as an abstract boundary control problem of form (2). Indeed,

by considering  $z_1(t) = v(x,t)$  and  $z_2(t) = \frac{\partial v(x,t)}{\partial t}$  one has

$$\begin{cases} \frac{d}{dt} \underbrace{\begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix}}_{z(t)} = \underbrace{\begin{pmatrix} 0 & 1 \\ \alpha \frac{\partial^2}{\partial x^2} & \beta \end{pmatrix}}_{\mathfrak{A}} \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix}, \\ \mathfrak{B} \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix} = v_e(t), \quad z(0) = 0, \end{cases} \quad (5)$$

where  $\alpha = \frac{1}{LC}$  and  $\beta = -\frac{R}{L}$ . In the sequel, the Laplacian operator  $\frac{\partial^2}{\partial x^2}$  will be noted  $A_0 = -\frac{d^2}{dx^2}$ . Now, to get the trajectories of (5), some definitions are needed. First, let us define  $Z = D\left(A_0^{\frac{1}{2}}\right) \oplus L^2(\Omega)$ , as the Hilbert space to which belong the solutions of (5). It is equipped with the following inner product

$$\left\langle \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}, \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} \right\rangle_Z = \left\langle \sqrt{\alpha} A_0^{\frac{1}{2}} f_1, \sqrt{\alpha} A_0^{\frac{1}{2}} g_1 \right\rangle_{L^2(\Omega)} + \left\langle f_2, g_2 \right\rangle_{L^2(\Omega)} \quad (6)$$

where  $L^2(\Omega)$  is the complex space of square-integrable functions and  $D\left(A_0^{\frac{1}{2}}\right)$  the definition domain of the operator  $A_0^{\frac{1}{2}}$ , i.e.,

$$D\left(A_0^{\frac{1}{2}}\right) = \left\{ f \in L^2(\Omega) \mid f \text{ absolutely continuous, } \frac{df}{dx} \in L^2(\Omega) \text{ and } f(0) = 0 \right\}.$$

Next, two conditions have to be satisfied. The first one, is that the operator  $\mathcal{A} = \mathfrak{A}z$  defined by  $\mathcal{A} = \mathfrak{A}|_{\text{Ker}(\mathfrak{B})}$  (i.e.,  $\mathfrak{A}$  restricted to the kernel of  $\mathfrak{B}$ ), for  $f \in D(\mathcal{A})$ , is the infinitesimal generator of a  $C_0$ -semigroup  $\mathcal{S}(t)$  on  $Z$ . The second one, is that there exist an operator  $\mathcal{B}$  in the set  $\mathcal{L}(U, Z)$  of linear applications from  $U$  to  $Z$ , so that for  $u \in U$  and  $\mathcal{B}u \in D(\mathfrak{A})$ , the operator  $\mathfrak{A}\mathcal{B}$  belong to  $\mathcal{L}(U, Z)$  and  $\mathfrak{A}\mathcal{B}u = u$ . For (5), both conditions are satisfied with  $U = \mathbb{C}$  and  $\mathcal{B} = (1 \ 0)^T$ . From this point, a change of variable  $h(t) = z(t) - \mathcal{B}u(t)$  leads (5) to the following *homogeneous* (i.e.,  $\mathfrak{B}h(t) = 0$ ) problem

$$\frac{dh(t)}{dt} = \mathcal{A}h(t) - \mathcal{B} \frac{du(t)}{dt} + \mathfrak{A}\mathcal{B}u(t). \quad (7)$$

Equation (7) is well posed in  $Z$  and has a unique *classical* solution (see Theorem 3.3.3 of (Curtain and Zwart,

1995)) if  $h(0) = (z(0) - \mathcal{B}u(0)) \in D(\mathcal{A})$  and  $u(t) \in C^2([0, \tau]; \mathbb{C})$  for every  $\tau > 0$ . Thus, the trajectories of (5) can be written in our case as

$$z(t) = \mathcal{S}(t)z_0 - \mathcal{A} \int_0^t \mathcal{S}(t-\tau) \mathcal{B}u(\tau) d\tau, \quad (8)$$

since  $\mathfrak{A}\mathcal{B} = 0$ . An explicit expression of (8) is given below.

### 3.1 Trajectories

The eigenvalues and the eigenvectors of  $\mathcal{A}$  are provided by the equation of form (1):

$$\underbrace{\begin{pmatrix} 0 & 1 \\ -\alpha A_0 & \beta \end{pmatrix}}_{\mathcal{A}} \underbrace{\begin{pmatrix} \phi_{1n}(x) \\ \phi_{2n}(x) \end{pmatrix}}_{\Phi_n} = \lambda_n \underbrace{\begin{pmatrix} \phi_{1n}(x) \\ \phi_{2n}(x) \end{pmatrix}}_{\Phi_n}.$$

This leads to

$$\phi_{2n}(x) = \lambda_n \phi_{1n}(x), \quad \frac{d^2 \phi_{1n}(x)}{dx^2} + \left( \frac{\beta \lambda_n - \lambda_n^2}{\alpha} \right) \phi_{1n}(x) = 0,$$

with  $\phi_{1n}(0) = \frac{d\phi_{1n}(x)}{dx}|_{x=\ell} = 0$ . The second equation is called *Sturm-Liouville* equation (see, e.g., (Myint-U and Debnath, 2007)). It leads to a pair  $(\lambda_{n_1}, \lambda_{n_2})$  of complex conjugate eigenvalues to which correspond two families  $\{\phi_{n_1}\}_{n \geq 1}, \{\phi_{n_2}\}_{n \geq 1}$ , i.e.,  $\Phi_n = \{\phi_{n_1}, \phi_{n_2}\}$ , of eigenvectors. They are given by

$$\phi_{n_1} = \begin{pmatrix} \phi_{1n_1} \\ \lambda_{n_1} \phi_{1n_1} \end{pmatrix} \text{ and } \phi_{n_2} = \begin{pmatrix} \phi_{1n_1} \\ \lambda_{n_2} \phi_{1n_1} \end{pmatrix},$$

with

$$\lambda_{n_{1,2}}(n) = \frac{\beta}{2} \pm \frac{j}{2} \sqrt{4\alpha \left( \frac{(2n-1)\pi}{2\ell} \right)^2 - \beta^2},$$

and

$$\phi_{1n_1} = \frac{\ell \sqrt{2\alpha}}{\alpha(2n-1)\pi} \sqrt{\frac{2}{\ell}} \sin\left(\frac{(2n-1)\pi}{2\ell} x\right).$$

In our case,  $\phi_{n_1}$  and  $\phi_{n_2}$  do not form an orthonormal basis since the operator  $\mathcal{A}$  is non-self-adjoint. However, they form a Riez basis  $\Psi_n = \{\psi_{n_1}, \psi_{n_2}\}$  and lead to

$$\mathcal{S}(t) \underbrace{\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}}_z = \sum_{n=1}^{\infty} (e^{\lambda_{n_1} t} \langle z, \phi_{n_1} \rangle_Z \psi_{n_2} + e^{\lambda_{n_2} t} \langle z, \phi_{n_2} \rangle_Z \psi_{n_1}).$$

In this way, the trajectories (8) of (5) become

$$z(t) = \mathcal{S}(t)z_0 + \sum_{n=1}^{\infty} \int_0^t e^{A_n(t-\tau)} B_n v_e(t) d\tau, \quad (9)$$

for which

$$A_n = \begin{pmatrix} 0 & 1 \\ -\left(\frac{(2n-1)\pi}{2\ell\sqrt{LC}}\right)^2 & -\frac{R}{L} \end{pmatrix},$$

and

$$B_n(x) = \begin{pmatrix} 0 \\ \frac{(2n-1)\pi}{2} \left(\frac{2}{\ell^2 LC}\right) \sin\left(\frac{(2n-1)\pi}{2\ell} x\right) \end{pmatrix}.$$

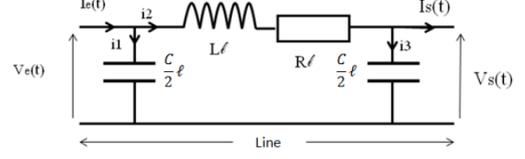


Fig. 1. Physical structure of the  $\pi$ -model

From (9), one can notice that the trajectories of (5) are a *superposition* of an infinite number of elementary trajectories. Each of them corresponds to a *second order* system described by the matrices  $A_n$  and  $B_n$ . Thus, it is then possible to write the transfer function between the input voltage  $v_0(t)$  and a chosen output for all  $x \in \Omega$ . In particular, when  $x = \ell$ , the transfer function  $G^\infty(s)$  between  $v_0(t)$  and the voltage  $v(\ell, t)$  is given below.

### 3.2 Transfer function

One way to deduce the transfer function is to use directly (9). As the later is an infinite sum of trajectories, the transfer function  $G^\infty(s)$  is the sum of the transfer functions corresponding to each elementary system. More precisely,

$$G^\infty(s) = \sum_{n=1}^{\infty} G_n(s),$$

where  $G_n(s)$  is the transfer function of the  $n^{\text{th}}$  second order system  $\Sigma_n(A_n, B_n(\ell), C_n)$  with  $C_n = (1 \ 0)$  (since the output is  $v(\ell, t)$ ). This gives,

$$G^\infty(s) = \frac{\hat{v}(\ell, s)}{\hat{v}_e(s)} = \sum_{n=1}^{\infty} \frac{\frac{(2n-1)\pi}{\ell^2 LC} (-1)^{n+1}}{(s - \lambda_{n_1})(s - \lambda_{n_2})}. \quad (10)$$

Moreover, (10) can be also written as (see (Belhocine and Marinescu, 2013) for details)

$$\frac{\hat{v}(\ell, s)}{\hat{v}_e(s)} = \frac{1}{\prod_{n=1}^{\infty} \left( \frac{4\ell^2 LC}{(2n-1)^2 \pi^2} s^2 + \left(\frac{R}{L}\right) \frac{4\ell^2 LC}{(2n-1)^2 \pi^2} s + 1 \right)}. \quad (11)$$

Both forms (10) and (11) are used in the next section to explain the connection with the  $\pi$ -model.

## 4. CONNECTION WITH THE $\pi$ -MODEL: A SYSTEMIC VIEWPOINT

The structure of the  $\pi$ -model is given in Fig. 1. Its behaviour is described by the following set of differential algebraic equations (DAE),

$$\begin{cases} \frac{C}{2} \ell \frac{dV_e(t)}{dt} = i_1(t), & \frac{C}{2} \ell \frac{dV_s(t)}{dt} = i_3(t) \\ V_e(t) = R\ell(I_e(t) - i_1(t)) + L\ell \frac{d}{dt}(I_e(t) - i_1(t)) \\ I_e(t) - i_1(t) = I_s(t) + i_3(t) = i_2(t) \end{cases} \quad (12)$$

where  $R, L$  and  $C$  are as in (3) and  $V_e(t), V_s(t)$  the voltages at both ends of the line of length  $\ell$ . Thus, in the case where the line is open at the extremity  $x = 0$ , one has  $I_s = 0$  and (12) leads to the following state space representation

$$\frac{d}{dt} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_x = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{2}{\ell^2 LC} & -\frac{R}{L} \end{bmatrix}}_A \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_x + \underbrace{\begin{bmatrix} 0 \\ \frac{2}{\ell^2 LC} \end{bmatrix}}_B V_e(t), \quad (13)$$

with  $x_1(t) = V_s(t)$  and  $x_2 = \frac{dx_1(t)}{dt}$ .

System (13) is linear time-invariant (LTI) of dimension 2. It has the poles

$$\lambda_{1,2} = -\frac{1}{2} \left( \frac{R}{L} \pm j \sqrt{8 \left( \frac{1}{\ell^2 LC} \right) - \left( \frac{R}{L} \right)^2} \right) \quad (14)$$

and the trajectories  $x(t) = e^{At}x_0 + \int_0^t e^{A(t-\tau)} B V_e(\tau) d\tau$ . (15)

From all these developments, the connection between (5) and the  $\pi$ -model (13) is investigated in two cases. In a more general case where the line is not necessarily of short-length and in the case of short and medium length lines.

#### 4.1 Case of general lines

When there is no particular condition on the length of the line, a way to explain the connection between (13) and (4) is the *modal* truncation (see, e.g., (Curtain and Morris, 2009)). Indeed, as (9) is a convergent sequence of trajectories related to the modes  $\{\lambda_{n1,2}, n \geq 1\}$ , for which  $\text{Real}(\lambda_{n1,2}) < 0$ , the truncation error to a finite order is bounded. For this, let us consider the first order (i.e.,  $n = 1$ ) truncation of (5), i.e., (9) with the first term only. This leads to a second order model  $\Sigma_1(A_1, B_1(\ell), C_1)$  for which

$$A_1 = \begin{pmatrix} 0 & 1 \\ -\frac{\pi^2}{8} \frac{2}{\ell^2 LC} & -\frac{R}{L} \end{pmatrix}, B_1(\ell) = \begin{pmatrix} 0 \\ \frac{\pi}{2} \left( \frac{2}{\ell^2 LC} \right) \end{pmatrix},$$

and  $C_1 = (1 \ 0)$ .

At this stage, it is important to notice that the poles of  $\Sigma_1$ , i.e., the eigenvalues of  $A_1$ , which are given by

$$\lambda_{n1,2}(1) = -\frac{1}{2} \left( \frac{R}{L} \pm j \sqrt{\frac{\pi^2}{8} \left( \frac{8}{\ell^2 LC} \right) - \left( \frac{R}{L} \right)^2} \right), \quad (16)$$

do not depend on the parameter  $x$ . Moreover, they are similar to  $\lambda_{1,2}$  of (14) and the only difference is the factor  $\frac{\pi^2}{8}$  between their imaginary parts which is thus not far from the unity.

Now, to compare the behaviours, i.e., the trajectories, one has also to compare  $B_1(\ell)$  with the matrix  $B$  of (13). Since  $B_1(\ell) = \frac{\pi}{2} B$ , one can then deduce that the difference with the trajectory (15) comes not only from the before mentioned factor  $\frac{\pi^2}{8}$  among the poles, but also from the factor  $\frac{\pi}{2}$  among the  $B$  and  $B_1(\ell)$  matrices. This shows that the  $\pi$ -model is not exactly the first order modal truncation of (10) but close to it, i.e. to  $\Sigma_1$ .

#### 4.2 Case of short-lines

The comparison done in (Kundur et al., 1994) has shown the similarity of the trajectories provided by (13) and (4) when the parameter  $\ell\sqrt{LC}$  is small enough (i.e., a short line). This can also be retrieved as a particular case of

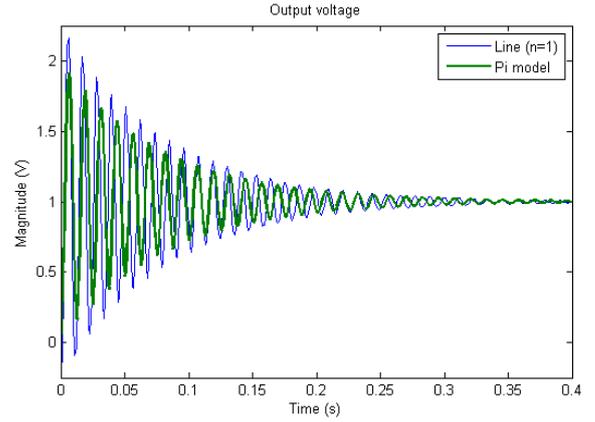


Fig. 2. Step response of the  $\pi$  and truncated models of the line

our results. More precisely, when  $\ell\sqrt{LC} \approx 0$ , the transfer function (11) of (5) can be approximated by that of the  $\pi$ -model (13). Indeed, the condition  $\ell\sqrt{LC}$  small enough in (11) leads to

$$\hat{v}_e(\ell, s) \approx \frac{1}{\frac{8}{\pi^2} \frac{\ell^2 LC}{2} s^2 + \frac{8}{\pi^2} \left( \frac{R}{L} \right) \frac{\ell^2 LC}{2} s + 1}, \quad (17)$$

since when  $\ell\sqrt{LC} \approx 0$ ,  $\left( \frac{\ell\sqrt{LC}}{2n-1} \right)^2 \approx 0$  for  $n > 1$ . Thus, as the transfer function of the  $\pi$ -model (13) is

$$\frac{\hat{V}_s(\ell, s)}{\hat{V}_e(s)} = \frac{1}{\frac{\ell^2 LC}{2} s^2 + \left( \frac{R}{L} \right) \frac{\ell^2 LC}{2} s + 1}, \quad (18)$$

the difference between (17) and (18) is the factor  $\frac{\pi^2}{8}$  among the denominators. As this factor is close to unity, the transfer (11) can thus be approximated by (18) when  $\ell\sqrt{LC}$  is small enough. In the time domain, this is shown in Fig. 2 for a transmission line of 200 km.

From a systemic viewpoint, this means that, in the case of short and medium length lines, only the first mode  $\lambda_{n1,2}(1)$  of (11) plays an important role in the input-output behaviour of the transmission line. In other words, the dynamic structure of (5) is, in this case, similar to its first order approximation, i.e., when  $n = 1$ . To compare this result with the one obtained in Section 4.1, let  $\bar{\Sigma}(\bar{A}, \bar{B}, \bar{C})$  with

$$A_1 = \begin{pmatrix} 0 & 1 \\ -\frac{\pi^2}{8} \frac{2}{\ell^2 LC} & -\frac{R}{L} \end{pmatrix}, B_1(\ell) = \begin{pmatrix} 0 \\ \frac{\pi}{2} \left( \frac{2}{\ell^2 LC} \right) \end{pmatrix},$$

and  $C_1 = (1 \ 0)$ .

be the state space model of the transfer function (17). It has thus the poles given by (16), and the difference with  $\lambda_{1,2}$  of (14) comes again from the factor  $\frac{\pi^2}{8}$  as in Section 4.1. Moreover, the difference between the matrices  $B$  and  $\bar{B}$  comes now from the factor  $\frac{\pi^2}{8}$ , i.e.,  $\bar{B} = \frac{\pi^2}{8} B$ . This similarity with the  $\pi$ -model is not fully preserved with the reduced model  $\Sigma_1$  of Section 4.1 where the factor among  $\bar{B}$  and  $B_1(\ell)$  is  $\frac{\pi}{4}$  (i.e.,  $\bar{B} = \frac{\pi}{4} B_1(\ell)$ ). Finally, as the differences between  $\Sigma_1$ ,  $\bar{\Sigma}$ , and (13) are not major, the trajectories provided by these models are close. This shows

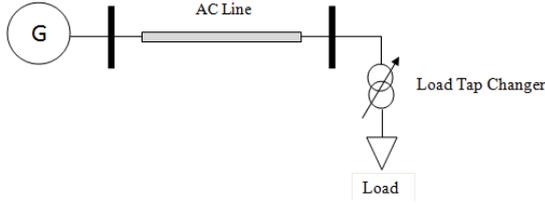


Fig. 3. Benchmark of voltage collapse

that the input-output behaviour of the full model (5) can be described by its first-order reduced model only in the case of short and medium length lines.

This result gives us a first view on the role of the systemic link to well understand the relationship between the dynamic structure of the system and its behaviour. Moreover, it leads us to another question which is the choice of the important dynamics and their number (i.e., the order of the reduced model). To see more clear, assume that  $\ell\sqrt{LC} \approx 0$  with a precision which vary between  $10^{-4}$  and  $10^{-3}$ . In this case, both precisions are not far from zero but the impact of this variation when the line is connected to the rest of the power system is not clear and well defined. More precisely, based only on the role of  $\ell\sqrt{LC}$  in (11), it is not easy to deduce *a priori* the number of modes which have to be kept in the reduced model of the transmission line in order to correctly reproduce the phenomena of the whole interconnected system. This is illustrated below by considering the voltage collapse phenomenon.

#### 4.3 Illustration example

The power system of Fig. 3 represents a structure mainly usually used to analyse the maximum amount of active power which can be transported through the AC line with stable dynamics. More specifically, when the load increases, the voltage diminishes slowly and the LTC (Load Tap Changer) plays the role of a voltage controller to fix it at a desired value. The power transfer from the generator to the load is limited (cannot increase) above a critical value called *power limit* in the sense that the operation of the system at a higher power transfer leads to a phenomenon called *voltage collapse*. It is characterised by a slow decrease of the load voltage from a desired value to zero in case of disturbance. This can lead, in a more general case (several generators, lines and loads interconnected), to a dynamic instability of the whole interconnected power system. Now, consider the transmission line embedded in the system of Fig. 3. It has a length of 300 km and  $\ell\sqrt{LC} = 0.0067$ . As the latter is small, its impact in (11) when  $n > 1$  is not very important (since  $(\ell\sqrt{LC})^2 \simeq 4,4 \cdot 10^{-5}$ ) and one can consider that only the first mode of (11) should be kept. However, when the  $\pi$ -model is used (i.e., the first mode based on the equivalence with (11) as mentioned above), Fig. 4 shows clearly that such a model cannot reproduce correctly the phenomenon of voltage collapse.

Indeed, one can see that at time 150 seconds, a step on the voltage reference value of the LTC, leads the load voltage obtained with both models ( $\pi$  and DP<sup>1</sup> model) to

<sup>1</sup> For numerical computations, the DP model given by (3) is approximated by a large-scale state space model of order 1000. Its inputs

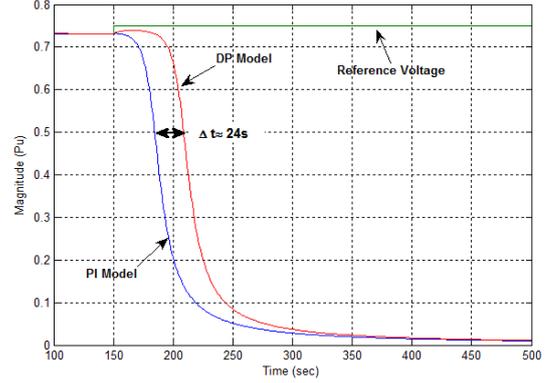


Fig. 4. Voltage collapse phenomenon

decrease slowly to zero. However, the important time shift (approx 24 seconds) between the two voltage trajectories, shows that the use of the  $\pi$ -model is not suited in this situation. In addition, it is not obvious, in this case, to improve this result by taking into account several modes by using, e.g., the model developed in Triezenberg (1979). More specifically, the number of modes to be retained to reproduce the phenomenon mentioned above is *a priori* unknown. It can be determined only *a posteriori* by trial and error tests.

## 5. TOWARDS A NEW APPROXIMATION TECHNIQUE OF A SUBSYSTEM OF A COMPLEX SYSTEM

The goal is to approximate only the model of a given subsystem (of a complex interconnected system) but in such way that, when the resulting simplified model is connected to the rest of the system, all phenomena of interest can be reproduced. For this, the structure of the overall system has to be taken into account. Indeed, the dynamics of the subsystem might not have the same contribution in the behaviour of the overall system. This is why, it is important to rate them first in order to keep only the most important ones in the simplified model of the subsystem. In this way, all the phenomena of interest can be reproduced.

To do this, we propose here a systemic approach based on the Hankel singular values (HSV) and the so-called *participation factors* (see, e.g., (Verghese et al., 1982)). More precisely, the HSV are, generally, helpful to rank the state variables following their impact on the input-output energy of the system. They are used in Moore (1981) to truncate the balanced system by keeping only its state variables corresponding to the largest values. Likewise, the participation factors were successfully used in power systems to make the link between the modes and the state variables related to the models of the electric components (see, e.g., (Verghese et al., 1982)). Thus, our strategy is to mix them in order to find the important modes of the subsystem. Such an idea was exploited successfully in (Belhocine and Marinescu, 2014) to approximate a complex model as a one input-output bloc. Here, the proposed procedure is as follows:

and outputs are, respectively, the currents and the voltages at the ends of the line.

- 1) Linearise the model of the overall system around an equilibrium point and choose an input-output transfer.
- 2) Compute the balanced realization as well as the HSV of the previous linearised model.
- 3) Find all the modes which have large participations (measured by the modulus of the participation factors) in the state variables (of the balanced realization) associated to the largest HSV and group them into a set  $\Lambda$ .
- 4) Among the modes of  $\Lambda$ , find all the ones of large participations in the state variables of the subsystem and group them into a set  $\Gamma$ .
- 5) From all the modes of the stand alone subsystem (i.e., disconnected from the overall system), find the ones which have close frequencies to each mode of  $\Gamma$ .
- 6) Construct the simplified model of the subsystem by keeping all the modes found in the previous step by using the modal truncation.

When the procedure above is applied to the power system of Fig. 3 (by choosing the reference voltage of the LTC as input and the load voltage as output), it follows that the largest HSV is related only to the LTC component. All the other values are very small compared to the largest one. In this situation, the set  $\Gamma$  is empty and there is no mode of the DP model which is involved in the voltage collapse. As a consequence, one can conclude that the appropriate simplified model for the line is, in this case, *algebraic*. More specifically, it corresponds to the static gain matrix  $G$  of the full DP model and not of its truncation at the first order. For the line embedded in the benchmark of Fig. 3, it is given by

$$G = \begin{pmatrix} 0, 2233 & 0, 1129 & -60, 8612 & 64, 5243 \\ 60, 8612 & -64, 5243 & 0, 2233 & 0, 1129 \\ -0, 1129 & -0, 2233 & -64, 5243 & 60, 8612 \\ 64, 5243 & -60, 8612 & -0, 1129 & -0, 2233 \end{pmatrix}.$$

The resulting simplified power system provides practically the same trajectories with the ones of the DP model shown in Fig. 4.

## 6. CONCLUDING REMARKS

In this paper, the dynamic structures of both  $\pi$  and distributed parameters (DP) models of a transmission lines are firstly analysed and compared in the setting of systems theory. It comes out from the comparisons that the  $\pi$ -model is not exactly a first order modal truncation of the DP model but close to it. More precisely, in the case of open lines, no much difference is found between the poles and the trajectories of the  $\pi$ -model and the ones corresponding to the modal truncation at the first order of the DP model. This fills the gap between the comparison of trajectories and the dynamic structure of models. Next, the validity condition of the  $\pi$ -model as an approximation of the DP one, in the case of short and medium length lines, is analysed. By considering the voltage collapse phenomenon, it is shown that when the line is connected to the rest of the system, the previous condition can leads to biased results. Indeed, it is not obvious to find an appropriate simplified model of the line without taking into account its interactions with the other components of the system. This is why, a new

approach to well approximate the model of a subsystem of a complex system is proposed. It is based on the Hankel singular values and the participation factors. Thus, it is more systemic and gives good results when it is applied to the case of voltage collapse. This method will be further developed for general systems. For power systems, all these results and investigations are a first step to improve the numerical simulations and the behaviour analysis. Indeed, we envisage to improve the actual simulators, used in power systems, by adjusting the simplified model of the transmissions lines as well as similar complex models (like the ones of magnetic circuits or power electronics) to each specific phenomenon which has to be reproduced in simulation. This will help the Transmission System Operators (TSO's) to adapt their simulation and analysis techniques to the modern power systems, i.e., with new components and new sources of energy.

## REFERENCES

- Belhocine, M. and Marinescu, B. (2013). On the connections between used models of power transmission lines. In *IEEE Power & Energy Society General Meeting*, 1–5.
- Belhocine, M. and Marinescu, B. (2014). A mix balanced-modal truncations for power systems model reduction. In *IEEE European Control Conference (ECC)*, 2721–2726.
- Curtain, R.F. and Morris, K. (2009). Transfer functions of distributed parameter systems: A tutorial. *Automatica*, 1101–1116.
- Curtain, R.F. and Zwart, H.J. (1995). *An Introduction to Infinite Dimensional Linear Systems Theory*. Springer-Verlag, New York, USA.
- Dommel, H.W. (1969). Digital computer solution of electromagnetic transients in single and multiphase networks. *IEEE Transactions on Power Apparatus and Systems*, PAS-88(4), 388–399.
- Kundur, P., Balu, N.J., and Lauby, M.G. (1994). *Power system stability and control*, volume 7. McGraw-hill New York.
- Meyer, B. and Stubbe, M. (1992). Eurostag, a single tool for power system simulation. *Transmission & Distribution International*, 47–52.
- Miano, G. and Maffucci, A. (2001). *Transmission Lines and Lumped Circuits*. Academic Press, USA.
- Moore, B.C. (1981). Principal component analysis in linear systems: Controllability, observability, and model reduction. *IEEE Transactions on Automatic Control*, 26(1), 17–32.
- Myint-U, T. and Debnath, L. (2007). *Linear Partial Differential Equations for Scientists and Engineers*. Birkhäuser, Boston, USA, fourth edition.
- Trietzenberg, D. (1979). An efficient state variable transmission line model. *Power Apparatus and Systems, IEEE Transactions on*, PAS-98(2), 484–492.
- Tucsnak, M. and Weiss, G. (2009). *Observation and Control for Operator Semigroups*. Birkhäuser Verlag AG, Basel, Switzerland.
- Vergheze, G.C., Perez-Arriaga, I.J., and Schweppe, F.C. (1982). Selective modal analysis with application to electric power systems, part i: Heuristic introduction, part ii: The dynamic stability problem. *IEEE Transactions*, PAS-101(9), 3117–3134.