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# State Dynamic Estimation of a Grid Connected Generator

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**Abstract**—The work presented in this paper deals with observability analysis of a synchronous machine connected to an infinite bus. The dynamical behaviour of the latter is represented by a one axis fifth order model including regulators with unknown reference signals. Different possible outputs scenarios are checked to achieve observability conditions. Then, an observer based on Extended Kalman Filter (EKF) algorithm is proposed by considering the reference signals as a part of its state. The observer performance is tested with and without regulators saturations.

## I. INTRODUCTION

The determination of a physical system state from measurement is crucial to ensure its monitoring and proper functioning. The state reconstruction problem for grid elements based on the available measurements is referred to as *observer synthesis problem*, and its feasibility as *observability analysis*. In the power system industry, the dynamic security analysis is needed. For this, all the states of the power system have to be estimated, in particular the variables of the generators and their regulators. The time updating should be much faster for this task than for the static *state estimator* (for this notion see [5], [6]).

Our aim in this paper is to set up an algorithm which considers a complete model representing the overall grid behaviour. The regulators of the generators are considered along with their nonlinearities due to saturation of actuating variables. Moreover, the reference signals of regulators are supposed unknowns.

As a first step, in this paper, we address the observability analysis of a nonlinear Ordinary Differential Equations (ODE) model of power systems using different measurement sets. This give us answers on which model outputs selection we get a best observability and about observers feasibility. In a second step, we propose an alternative algorithm based on the EKF proposed in [1]. Instead of approaches that uses canonical form transformations such as *high gain observers* (see [9]). In general, these last approaches are based on diffeomorphism inversion that is difficult to calculate numerically in the case of large systems.

In simulation, we apply our algorithm for dynamical state estimation of a Synchronous Machine connected to an Infinite Bus (SMIB) as well as the estimation of the state of its voltage and frequency local regulators. Furthermore, we consider the saturation phenomenon of internal regulators' variables, which

has not been considered previously due to the complexity of the analysis.

The reminder of the paper is organized as follows: In section II, we give the model details of SMIB. Next, a problem statement is presented in section III. The fourth section addresses the observability analysis and observer synthesis followed by simulation validation. Finally, a general conclusion is given in the last section.

## II. MODEL OF THE SYNCHRONOUS MACHINE CONNECTED TO AN INFINITE BUS

The SMIB system is described in Fig. 1. Here, we consider the dynamical model with three state variables (see, [2]):

*differential equations:*

$$\dot{e}'_q = -\frac{1}{T'_{d0}} (e'_q - (x_d - x'_d)i_d + E_{fd}) \quad (1)$$

$$\dot{\omega} = \frac{1}{2H\omega} (P_m - i_q (e'_q + i_d(x_q - x'_d))) \quad (2)$$

$$\dot{\delta} = \omega - \omega_s \quad (3)$$

*algebraic equations:*

$$i_q = -\frac{(X + x'_d)V_s \sin(\delta) - (R + r)(V_s \cos(\delta) - e'_q)}{(R + r)^2 + (X + x'_d)(X + x_q)} \quad (4)$$

$$i_d = \frac{X + x_q}{X + r} i_q - \frac{1}{R + r} V_s \sin(\delta) \quad (5)$$

*voltage and frequency regulators equations:*

$$\begin{aligned} \dot{E}_{fd} &= -\frac{E_{fd}}{T_a} \\ &+ \frac{K_a}{T_a} (V_{ref} - \sqrt{(x_q i_q - r i_d)^2 + (e'_q - r i_q - x'_d i_d)^2}) \end{aligned} \quad (6)$$

$$\dot{P}_m = \frac{1}{T_g} (-P_m + P_{ref} + K_g(\omega_{ref} - \omega)) \quad (7)$$

Equations (1)-(7) belong to the class of systems described by Differential Algebraic Equations (DAE):

$$\begin{aligned} \dot{x} &= f(x, z, u) \\ 0 &= g(x, z) \\ y &= h(x, z), \end{aligned} \quad (8)$$

where

- The intermediate vector state  $z$  represents currents ( $i_d, i_q$ ) in the  $qd0$  base (see [2] for this notation).

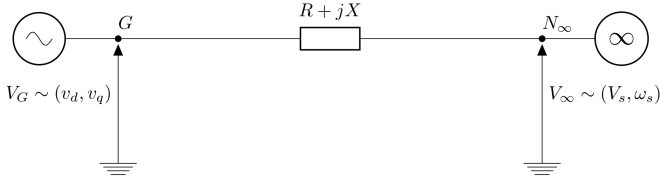


Fig. 1: Application example: synchronous machine (G) connected to an infinite bus ( $N_\infty$ ).

- The state  $x$  is a vector of five elements representing respectively the generating e.m.f. ( $e'_q$ ), the generator angular speed  $\omega$ , the rotation angle  $\delta$ , the rotor voltage  $E_{fd}$  and the mechanical power  $P_m$  supplied to the generator.
- The system's inputs  $u$  are :  $V_{ref}$  and  $P_{ref}$ .
- The other variables are constant parameters which are given in Tab. I

Parameter	Value	Parameter	Value	Parameter	Value
$T'_{d0}$	9.67	$x_d$	2.38	$x'_d$	0.336
$x_q$	1.21	$H$	3	$r$	0.002
$\omega_{ref}, \omega_s, V_s$	1	$R, T_a$	0.01	$X$	1.185
$T_g$	0.4	$K_g$	0.5	$K_a$	70

TABLE I: Parameters values of model (1)-(7) in per-unit (pu).

**Remark 1:** In this example, intermediate signals  $z$  ( $i_d, i_q$ ) can be written in terms of the state  $x$  ( $e'_q, \omega, \delta, E_{fd}$  et  $P_m$ ). Thus, the model is transformed into an ODE system which belong to the following class:

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= h(x) \end{aligned} \quad (9)$$

**Remark 2:** For simplification reasons, from now on, the new simplified ODE system is given in the following form:

$$\begin{aligned} \dot{e}'_q &= f_1(e'_q, \delta, E_{fd}) \\ \dot{\omega} &= f_2(e'_q, \omega, \delta, P_m) \\ \dot{\delta} &= f_3(\omega) \\ \dot{E}_{fd} &= f_4(e'_q, \delta, E_{fd}, V_{ref}) \\ \dot{P}_m &= f_5(\omega, P_m, P_{ref}) \end{aligned} \quad (10)$$

### III. PROBLEM STATEMENT

In this paper our aim is to estimate asymptotically the state  $x = (e'_q, \omega, \delta, E_{fd}, P_m)$  of system (10) using a *state observer* approach with a priori knowledge of measurements  $y$  assuming that the inputs ( $P_{ref}, V_{ref}$ ) are unknowns and that the regulators dynamics can be saturated. More precisely, we have:

$$\dot{E}_{fd} = \begin{cases} 0 & \text{if } (E_{fd} \geq E_{max} \text{ and } f_4(*) > 0) \\ & \text{or } (E_{fd} \leq E_{min} \text{ and } f_4(*) < 0) \\ f_4(*) & \text{otherwise} \end{cases} \quad (11)$$

$$\dot{P}_m = \begin{cases} 0 & \text{if } (P_m \geq P_{max} \text{ and } f_5(*) > 0) \\ & \text{or } (P_m \leq P_{min} \text{ and } f_5(*) < 0) \\ f_5(*) & \text{otherwise} \end{cases} \quad (12)$$

For measurements variables  $y$ , the following scenarios are considered:

A - Measurement of terminal voltage,

$$V_G = \sqrt{(x_q i_q - r i_d)^2 + (e q' - r i_q - x'_d i_d)^2}.$$

B - Measurement of the active power,

$$P_a = i_q (e q' - r i_q - x'_d i_d) + i_d (x_q i_q - r i_d).$$

C - Measurement of the reactive power,

$$Q = i_d (e q' - r i_q - x'_d i_d) - i_q (x_q i_q - r i_d).$$

D - Measurement of generator angular speed  $\omega$ .

E - A + B.

F - A + B + C.

G - A + B + C + D.

H - Measurement of the full state  $e'_q, \omega, \delta, E_{fd}$  and  $P_m$ .

Mathematically, we are searching for a mapping  $\mathcal{F} : \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^+ \rightarrow \mathbb{R}^n$  such that the following auxiliary system:

$$\dot{\hat{x}}(t) = \mathcal{F}(\hat{x}(t), y(t), t), \quad (13)$$

satisfies:

- $\hat{x}(0) = x(0) \Rightarrow \hat{x}(t) = x(t), \forall t \geq 0;$
- $\|\hat{x}(t) - x(t)\| \rightarrow 0$  when  $t \rightarrow \infty$ .

The synthesis of state observers for systems (9) with unknown inputs is a complex problem. However, since the inputs  $V_{ref}$  and  $P_{ref}$  are considered as a reference signals respectively of the voltage and frequency regulators, they usually have piecewise constant values. In this case, in order to simplify the study, we assume them as a constant unknown parameters and we add them to the state vector. The resulting augmented (autonomous) system is given by equations (14).

$$\Sigma \begin{cases} \dot{e}'_q = f_1(e'_q, \delta, E_{fd}) \\ \dot{\omega} = f_2(e'_q, \omega, \delta, P_m) \\ \dot{\delta} = f_3(\omega) \\ \dot{E}_{fd} = \begin{cases} 0 & \text{if } (E_{fd} \geq E_{max} \text{ and } f_4(*) > 0) \\ & \text{or } (E_{fd} \leq E_{min} \text{ and } f_4(*) < 0) \\ f_4(*) & \text{otherwise} \end{cases} \\ \dot{P}_m = \begin{cases} 0 & \text{if } (P_m \geq P_{max} \text{ and } f_5(*) > 0) \\ & \text{or } (P_m \leq P_{min} \text{ and } f_5(*) < 0) \\ f_5(*) & \text{otherwise} \end{cases} \\ \dot{V}_{ref} = 0 \\ \dot{P}_{ref} = 0 \end{cases} \quad (14)$$

Once the observer's synthesis is done for system (14), its robustness will be checked in the presence of variations in  $V_{ref}$  and  $P_{ref}$  signals.

The necessary conditions for the existence of state observers is referred to as *observability*. In the case of linear time invariant systems, the observability is equivalent to satisfying Kalman criterion [3]. In the nonlinear case, the most important property in practice is the *rank condition* which is an extension of Kalman criterion. The rank condition is equivalent to the

notion of locally weak observability which is a geometric property (see [1] for more details).

**Proposition 1:** *we consider the following particular case of autonomous systems :*

$$\begin{aligned}\dot{x} &= f(x) \\ y &= h(x).\end{aligned}\quad (15)$$

Then system (15) is locally weakly observable if the Jacobian  $J_x(\Phi(x))$  of  $\Phi(x)$  defined by:

$$\Phi(x) = \begin{bmatrix} h(x) \\ L_f h(x) \\ \vdots \\ L_f^{n-1} h(x) \end{bmatrix}, \quad (16)$$

is a full rank column matrix along the trajectories  $x(t)$  solutions of (15), where  $L_f h(x)$  is the Lie derivative of  $h(x)$  and  $L_f^i h(x)$  is the Lie derivative of  $L_f^{i-1} h(x)$  in the direction of vector field  $f(x)$ .

#### IV. OBSERVER SYNTHESIS

##### A. Observability test

To check if the Jacobian matrix of  $\Phi(x)$  is of full rank column, one can calculate its Smallest Singular Value (SSV). Since the singular values of a matrix with elements in  $\mathbb{R}$  are positive then we say that a dynamical system (15) is locally weakly observable if the SSV is non-zero for all values of  $x \in X$ , where  $X$  in our test, is the set of values taken on the solution trajectories of equations (1)-(7) at given times  $t$ .

In order to take into account the saturations effect of voltage and frequency regulators as well as the case of the unknown input signals ( $V_{ref}$ ,  $P_{ref}$ ), two different tests are presented:

**Test 1 (Model with unknown  $V_{ref}$ ,  $P_{ref}$  without saturations of  $E_{fd}$ ,  $P_m$ ):** In this test, the expression of  $\Phi(x)$  is obtained using the model given by (14) but without saturations of  $E_{fd}$  and  $P_m$ .

**Inputs signals:** we recall that signals ( $V_{ref}$ ,  $P_{ref}$ ) are assumed as a part of state and must be constants. We choose them such that  $E_{fd}$  and  $P_m$  remain out of saturations. The values  $V_{ref} = 0.98$  pu and  $P_{ref} = 0.56$  pu satisfy this condition.

The simulation results presented in Tab. II show that for the measurement scenario A, observability is the weakest compared to scenarios B, C and D. However, for scenarios E, F, G and H, observability is even better which is consistent because we get more informations when we have more measures.

**Test 2 (Model with unknown  $V_{ref}$ ,  $P_{ref}$  with saturation of  $E_{fd}$ ,  $P_m$ ):** In this case, equations (14) are replaced by the following equations:

$$\begin{aligned}\dot{e}'_q &= f_1(e'_q, \delta, E_{fd}) \\ \dot{\omega} &= f_2(e'_q, \omega, \delta, P_m) \\ \dot{\delta} &= f_3(\omega) \\ \dot{E}_{fd} &= 0 \\ \dot{P}_m &= 0 \\ \dot{V}_{ref} &= 0 \\ \dot{P}_{ref} &= 0\end{aligned}\quad (17)$$

This change generates a change in the expression of  $\Phi(x)$  and obviously a change in the observability. Indeed, based on the fundamental geometric notion of *indistinguishability*, we can prove analytically (without going through the numerical calculation of  $J(\Phi(x))$ ) that system (17) is a non observable one.

**Definition 1 (indistinguishability [1]):** *Let the following nonlinear autonomous system :*

$$\begin{aligned}\dot{x} &= f(x), \\ y &= h(x)\end{aligned}\quad (18)$$

Then, a pair  $(x_0, x'_0) \in \mathbb{R}^n \times \mathbb{R}^n$  is indistinguishable if

$$\forall t \geq 0, h(\chi(t, x_0)) = h(\chi(t, x'_0)),$$

where  $\chi$  is the solution of system.

Now, if we choose for the model (17) two different initial values

$$x_0 = (e'_{q_0}, \omega_0, \delta_0, E_{fd_0}, P_{m_0}, P_{ref_0}, V_{ref_0})$$

and

$$x'_0 = (e'_{q_0}, \omega_0, \delta_0, E_{fd_0}, P_{m_0}, P_{ref_0}^*, V_{ref_0}^*)$$

with

$$P_{ref_0} \neq P_{ref_0}^* \quad \text{et} \quad V_{ref_0} \neq V_{ref_0}^*.$$

Note that in all the measurement scenarios cited above, the output  $y$  does not depend explicitly on  $V_{ref}$  and  $P_{ref}$ . In other words:

$$y(t) = h(e'_q(t), \omega(t), \delta(t), E_{fd}(t), P_m(t))$$

Therefore, it results:

$$y(x_0, t) = y(x'_0, t).$$

Since the dynamics of  $V_{ref}$  and  $P_{ref}$  are decoupled from the rest of equations (14), we conclude that the system (17) is indistinguishable. We can even conclude that the system is not locally weakly observable because for all  $x_0 \in \mathbb{R}^n$ , there is always a neighbourhood  $V$  and a point  $x'_0 \in V$  such that  $(x_0, x'_0)$  are indistinguishable (see definition in [1]). The question arises intuitively, is it possible to construct an asymptotic observer (as defined above) for estimating the state  $x = (e'_q, \omega, \delta, E_{fd}, P_m)$  even if the observability condition of the augmented system (17) is not verified. In other words, are the unobservable states only  $V_{ref}$  and  $P_{ref}$ ? To answer this question, we apply the observability test on the non-augmented system (with  $V_{ref}$  and  $P_{ref}$  in saturation):

$$\begin{aligned}\dot{e}'_q &= f_1(e'_q, \delta, E_{fd}) \\ \dot{\omega} &= f_2(e'_q, \omega, \delta, P_m) \\ \dot{\delta} &= f_3(\omega) \\ \dot{E}_{fd} &= 0 \\ \dot{P}_m &= 0\end{aligned}\quad (19)$$

By choosing  $V_{ref} = 1.5$  pu et  $P_{ref} = 0.84$  pu, the state of system (19) go into saturation and remains there during the simulation time. Now, if we use the same measurement

SSV	$t = 10$	$t = 20$	$t = 30$	$t = 40$	$t = 50$	$t = 60$	$t = 70$	$t = 80$	$t = 90$	$t = 100$
A	$3.2 * 10^{-5}$	$1.8 * 10^{-5}$	$2.5 * 10^{-6}$	$9.5 * 10^{-6}$	$5.1 * 10^{-7}$	$2.3 * 10^{-6}$	$9.5 * 10^{-8}$	$3.0 * 10^{-5}$	$9.8 * 10^{-6}$	$3.6 * 10^{-7}$
B	$4.0 * 10^{-4}$	$7.1 * 10^{-4}$	$1.9 * 10^{-4}$	$3.2 * 10^{-4}$	$1.7 * 10^{-4}$	$8.4 * 10^{-5}$	$5.9 * 10^{-5}$	$1.1 * 10^{-4}$	$5.0 * 10^{-5}$	$7.0 * 10^{-6}$
C	$6.4 * 10^{-4}$	$8.4 * 10^{-4}$	$5.9 * 10^{-5}$	$2.8 * 10^{-4}$	$1.5 * 10^{-4}$	$4.7 * 10^{-5}$	$5.0 * 10^{-5}$	$1.3 * 10^{-4}$	$5.6 * 10^{-5}$	$5.1 * 10^{-6}$
D	$4.0 * 10^{-5}$	$3.6 * 10^{-5}$	$4.2 * 10^{-5}$	$1.5 * 10^{-5}$	$1.2 * 10^{-5}$	$6.9 * 10^{-7}$	$2.9 * 10^{-6}$	$2.5 * 10^{-6}$	$2.3 * 10^{-6}$	$8.2 * 10^{-7}$
E	0.06	0.05	0.06	0.05	0.06	0.05	0.06	0.06	0.05	0.06
F	0.07	0.08	0.08	0.07	0.08	0.07	0.08	0.07	0.08	0.08
G	0.15	0.14	0.14	0.15	0.14	0.14	0.14	0.14	0.14	0.14
H	1	1	1	1	1	1	1	1	1	1

TABLE II: Test 1: SSV of Jacobian matrix  $J(\Phi(x))$  for values of  $x$  taken on trajectories of the dynamic model solution at time instants  $t$  in sec.

scenarios as above, we obtain the results presented in Fig. 2-3. Then, the same remarks as in the case of test 1 are observed. We can conclude then, that only  $V_{ref}$  and  $P_{ref}$  are the non observable variables.

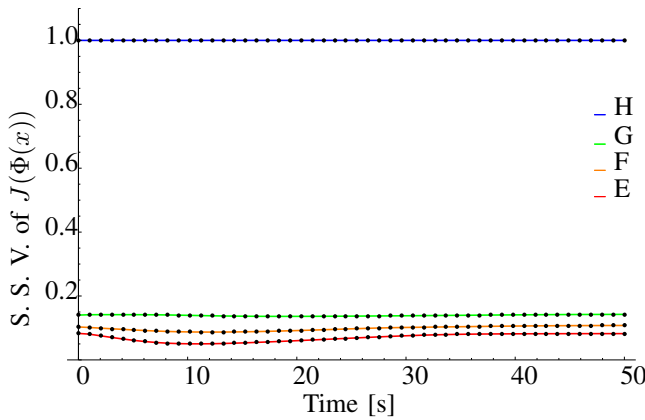


Fig. 2: Test 2 – SSV values of Jacobian matrix  $J(\Phi(x))$  for scenarios: H (blue line), G (green line), F (orange line) and E (red line).

### B. Extended Kalman Filter (EKF) for autonomous nonlinear Systems

The Kalman Filter (KF) is a dynamic gain observer which is basically designed for the state estimation of Linear Time Variant (LTV) systems (see [1]):

$$\dot{x} = A(t)x + B(t)u \quad (20)$$

$$y = C(t)x + D(t)u. \quad (21)$$

The concept of *uniform observability* is the necessary condition for asymptotic convergence of KF which is proved in many works (see [3], [4]). However, for nonlinear systems (14), the non-stationary gain of KF is adapted by replacing matrices  $A(t)$  and  $C(t)$  by respectively the instantaneous values of Jacobian of  $f(x)$  and  $h(x)$ , then the algorithm is called EKF. However, one of EKF drawbacks is its convergence guarantee which is in general proved only locally (see [7], [8]). On the other hand, the advantage of EKF is its simple structure and ease of implementation compared to the other approaches that are applied on nonlinear systems with canonical forms such as high gain observers. But the most

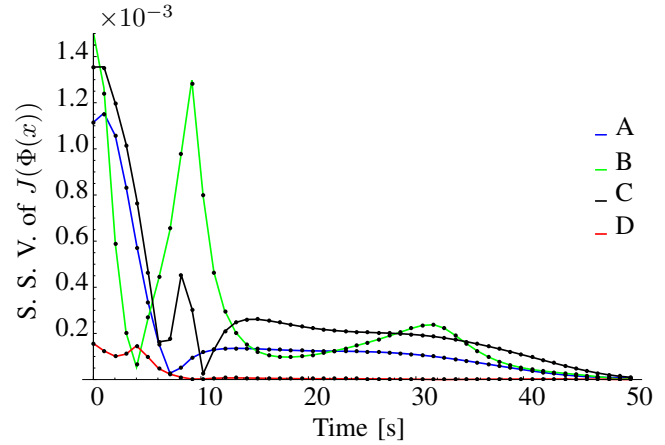


Fig. 3: Test 2 – SSV values of Jacobian matrix  $J(\Phi(x))$  for scenarios: A (blue line), B (green line), C (black line) and D (red line).

important advantage that prompted us to choose the FKE is its ability to fit measurement noise making it the most used algorithm in the industry and real applications.

1) *Algorithm*: The observer proposed for state estimation of system  $\Sigma$  (see equations (14)) is given by system  $\Sigma_o$  defined as following:

$$\Sigma_o \left\{ \begin{array}{l} \dot{\hat{e}}'_q = f_1(\hat{e}'_q, \hat{\delta}, \hat{E}_{fd}) + k_1(t)(y - \hat{y}) \\ \dot{\hat{\omega}} = f_2(\hat{e}'_q, \hat{\omega}, \hat{\delta}, \hat{P}_m) + k_2(t)(y - \hat{y}) \\ \dot{\hat{\delta}} = f_3(\hat{\omega}) + k_3(t)(y - \hat{y}) \\ \dot{\hat{E}}_{fd} = \begin{cases} k_4(t)(y - \hat{y}) & \text{if } cond_1 \\ f_4(\hat{*}) + k_4(t)(y - \hat{y}) & \text{otherwise} \end{cases} \\ \dot{\hat{P}}_m = \begin{cases} k_5(t)(y - \hat{y}) & \text{if } cond_2 \\ f_5(\hat{*}) + k_5(t)(y - \hat{y}) & \text{otherwise} \end{cases} \\ \dot{\hat{V}}_{ref} = k_6(t)(y - \hat{y}) \\ \dot{\hat{P}}_{ref} = k_7(t)(y - \hat{y}) \end{array} \right. \quad (22)$$

with  
 $cond_1 = (\hat{E}_{fd} \geq E_{max} \& f_4(\hat{*}) > 0) \vee (\hat{E}_{fd} \leq E_{min} \& f_4(\hat{*}) < 0)$ ,  
 $cond_2 = (\hat{P}_m \geq P_{max} \& f_5(\hat{*}) > 0) \vee (\hat{P}_m \leq P_{min} \& f_5(\hat{*}) < 0)$ .

and when

$$\hat{P}_m(t) = P_{max} \ \& \ \hat{P}_m(t_+) - P_{max} < -\epsilon_1, \quad 0 < \epsilon_1 \lll 1$$

$$\hat{E}_{fd}(t) = E_{max} \ \& \ \hat{E}_{fd}(t_+) - E_{max} < -\epsilon_2, \quad 0 < \epsilon_2 \lll 1$$

we have respectively

$$\hat{P}_{ref}(t_+) = \hat{P}_m - K_g(\omega_{ref} - \hat{\omega}) \quad (23)$$

$$\hat{V}_{ref}(t_+) = \hat{E}_{fd}/K_a + \sqrt{(x_q \hat{i}_q - r \hat{i}_d)^2 + (\hat{e}q' - r \hat{i}_q - x_d' \hat{i}_d)^2} \quad (24)$$

$k_i(t)$  for  $i = 1, \dots, 7$  are the rows of observer matrix gain  $K(t)$  which is given by the following auxiliary system:

$$\dot{S} = -A^\top(t)S - SA(t) + C^\top(t)W^{-1}C(t) - \theta S - SVS \quad (25)$$

$$K(t) = S^{-1}(t)C^\top(t)W^{-1}, \quad (26)$$

where  $W$ ,  $\theta$  and  $V$  are robustness and speed of convergence setting parameters. Matrices  $A(t)$  and  $C(t)$  are given for all  $t > 0$  as follows:

$$A(t) = \left. \frac{\partial f(x)}{\partial x} \right|_{x=\hat{x}}, \quad C(t) = \left. \frac{\partial h(x)}{\partial x} \right|_{x=\hat{x}}, \quad (27)$$

with

$$f(x) = \begin{bmatrix} f_1(e'_q, \delta, E_{fd}) \\ f_2(e'_q, \omega, \delta, P_m) \\ f_3(\omega) \\ 0 \\ f_4(e'_q, \delta, E_{fd}, V_{ref}) \\ 0 \\ f_5(\omega, P_m, P_{ref}) \\ 0 \\ 0 \end{bmatrix}, \quad h(x) = \begin{bmatrix} V_t \\ P_a \\ Q \end{bmatrix}$$

if  $cond_1$  otherwise

if  $cond_2$  otherwise

**Assumption 1:** Since the convergence of the EKF is guaranteed only locally, we assume that the simulations start from an equilibrium point, which makes it possible to calculate the observer initial values very close to the real system initial values. Thus, we solve the following system of equations:

$$\begin{aligned} 0 &= f(\hat{x}_0) \\ y_0 &= h(\hat{x}_0) \end{aligned} \quad (28)$$

where  $y_0 \in \mathbb{R}^p$  are the measurements initial values at time  $t_0$  and  $\hat{x}_0$  are the initial values of the EKF state. From a practical point of view, the hypothesis that the real system starts from an equilibrium point is feasible since the measurements are available over a very large window of time. On the other hand, it is difficult to ensure that the initial point  $\hat{x}_0$  will be identical to the real value  $x_0$  of the internal state due to the model uncertainties.

**Remark 3:** Since  $\hat{x}_0 \in \mathbb{R}^{n+2}$  and  $f : \mathbb{R}^{n+2} \rightarrow \mathbb{R}^n$ , the outputs dimension  $p$  must be  $p \geq 2$  to obtain a unique solution of equations (28). We assumed measuring three variables:  $(V_t, P_a, Q)$  to satisfy both the latter condition and the practical constraints (availability and reliability of the measures).

**Remark 4:** From the observability study we know that when the regulators dynamic reach saturation levels, the reference signals  $P_{ref}$  and  $V_{ref}$  will no longer be observable and their values will not be properly estimated during the saturation

time. Nevertheless, when  $\hat{P}_m$  and/or  $\hat{E}_{fd}$  leaves saturation, at this time precisely, we can quickly update the values of  $P_{ref}$  and  $V_{ref}$  using respectively (23) and (24) instead of waiting for the observer correction which is slower. Of course, this technique is only effective when the estimates  $\hat{P}_m$  and  $\hat{E}_{fd}$  are good.

2) *Simulation:* To be more realistic, the simulations were made considering the effect of an additive white noise (of magnitude =  $2.10^{-2}$  p.u.) to the output signals. Starting from initial values of Tab. III and using input signals of Tab. IV, we get simulation results given in Fig. 4–6. Signals  $V_{ref}$ ,  $P_{ref}$  change so that the dynamics of the regulators go into saturation at the instants  $t = 100$  and  $t = 200$ . When  $V_{ref}$ ,  $P_{ref}$  change, we can see that the observer states  $\hat{E}_{fd}$  and  $\hat{P}_m$  does not exactly follow the true system state. It makes sense because we assumed in the observer model that the reference signals are constants and when they change to another constant values it takes some time to compensate the model errors and/or moving away from a non observability region (like, for example, when  $E_{fd}$  and/or  $P_m$  are saturated). However, in general, the accuracy of results is acceptable.

State	$e'_q$	$\omega$	$\delta$	$E_{fd}$	$P_m$	$P_{ref}$	$V_{ref}$
System initial values	1.3	1.0	1.6	2.5	0.7	0.7	1.0
Observer initial values	0.5	0.5	1.0	1.6	0.3	0.1	0.7

TABLE III: Simulation – system and observer initial values

Inputs (p.u.)	$t : 0 \rightarrow 100$	$t : 100 \rightarrow 200$	$t : 200 \rightarrow 500$
$V_{ref}$	1.0	$1 + 5\%$	$1 + 5\%$
$P_{ref}$	0.7	0.7	$0.7(1 + 2.75\%)$

TABLE IV: Simulation – reference signals (Inputs) evolution.

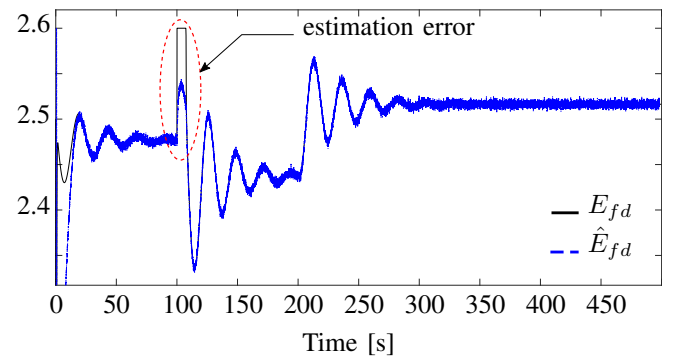


Fig. 4: Simulation results – comparison between  $E_{fd}$  (black line) and estimated  $\hat{E}_{fd}$  (blue dashed line).

In order to show the effectiveness of the method presented in Remark 4 we have carried out a simulation scenario so that one of the regulators dynamics ( $P_m$  in this case) will be saturated and during this time,  $P_{ref}$  changes. In the ideal case (without noise effect) shown in Fig. 7, we note that the estimate  $\hat{P}_{ref}$  at the end of saturation joins instantaneously the true value of  $P_{ref}$ . The results obtained in the presence of noise are given in Fig. 8.

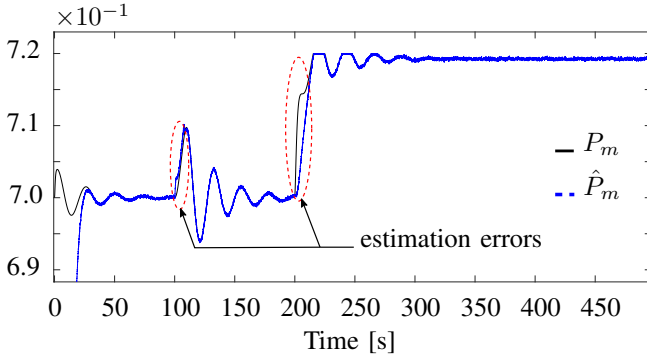


Fig. 5: Simulation results – comparison between  $P_m$  (black line) and estimated  $\hat{P}_m$  (blue dashed line).

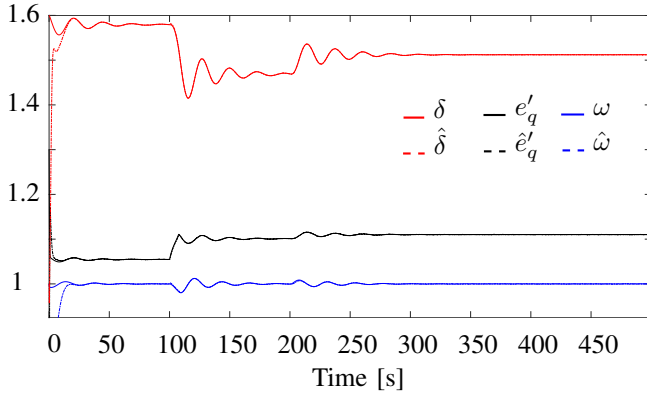


Fig. 6: Simulation results – comparison between  $(e'_q, \omega, \delta)$  and estimations  $(\hat{e}'_q, \hat{\omega}, \hat{\delta})$ .

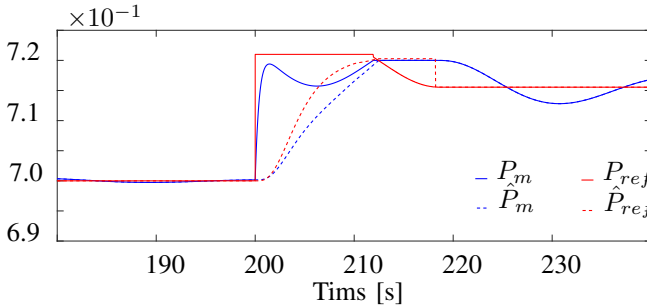


Fig. 7: Without noise – (red) detection of  $P_{ref}$  value when  $P_m$  leaves saturation/ (blue) estimation precision of  $P_m$ .

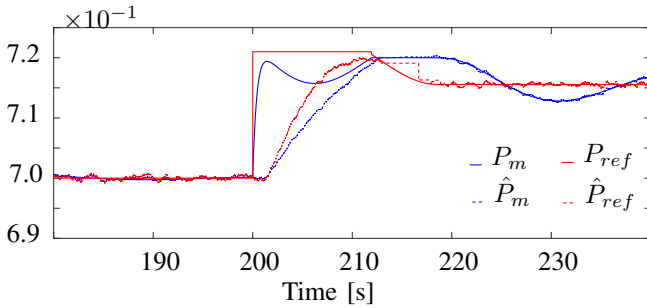


Fig. 8: With noise – (red) detection of  $P_{ref}$  value when  $\hat{P}_m$  leaves saturation/ (blue) estimation precision of  $P_m$ .

## V. CONCLUSION

In this paper, based on the notion of rank condition, the observability of a dynamical model (14) for a synchronous machine connected to an infinite bus is studied. We concluded that during the saturated modes of regulators, the reference signals are no longer observable. However, an observer is synthesized using the EKF to estimate asymptotically the state of the non-augmented model (1)-(7). The results obtained show that the estimates provided by the observer converge and remain centered around the model values in the presence of measurement noises. However, some estimation errors occur when the reference signals change to another constant values, but the values of the EKF dynamical gain remain bounded. The most critical situation is when this happens during a saturation mode. In this case, the lack of observation can be overcome by instantaneously and algebraic computation of the values of the reference signals when the dynamics of the regulators come out from saturation modes, which greatly improves the observer's precision. Future work will focus on extending the methodology to the case of non constant references and implementation on large-scale grid.

## REFERENCES

- [1] G. Besançon, *Nonlinear observers and applications*, Vol. 363. pp. 1-33, Berlin: Springer, 2007.
- [2] P. Kundur, *Power system stability and control*, Vol. 7. 1176 pages, New York: McGraw-hill, 1994.
- [3] R. E. Kalman, *A new approach to linear filtering and prediction problems*, 82(1) . pp. 35-45, Journal of basic Engineering, 1960.
- [4] R. E. Kalman and R. S. Bucy, *New Results in Linear Filtering and Prediction Theory*, 83(1). pp. 95-108, Journal of basic Engineering, Mar 01, 1961.
- [5] A. Abur and A. G. Exposito, *Power system state estimation: theory and implementation*, CRC press, 2004.
- [6] F. C. Schweppe and R. D. Masiello, *A tracking static state estimator*, pp. 3(1025-1033), In IEEE Transactions on Power Apparatus and Systems, 1971.
- [7] M. Boutayeb, H. Rafaralhy and M. Derouach, *Convergence analysis of the extended Kalman filter used as an observer for nonlinear deterministic discrete-time systems*, 42(4). pp. 581-586, IEEE transactions on automatic control, 1997.
- [8] Y. Song and J.W. Grizzle, *The extended Kalman filter as a local asymptotic observer for nonlinear discrete-time systems*, pp. (3365-3369), In IEEE American Control Conference, 1992.
- [9] G. Bornard and H. Hammouri, *A high gain observer for a class of uniformly observable systems*, pp. 1494-1496, In 30th IEEE Conference on Decision and Control, 1991.