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# A voting model of privatization\*

Rim Lahmandi-Ayed<sup>†</sup>      Didier Laussel<sup>‡</sup>

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## Abstract

We consider a general equilibrium model with vertical preferences, where workers and consumers are differentiated respectively by their sensitivity to effort and their intensity of preference for quality. We consider a public monopoly, i.e. which is owned equally by all individuals. The question is under which conditions the firm will be privatized and at which rate/price. The decisions are taken through majority vote in a plurality system. When the firm is controlled by the State, the price is determined through a vote among all the population. Otherwise, the price is the one which maximizes the profit. We prove that, under some conditions on the dispersion of consumers relative to workers, privatization may emerge as a possible choice of the majority, even if no hypothesis is made on the efficiency of a private management relative to a public one.

**Keywords:** democracy, general equilibrium, privatization, vertical preferences, majority vote, public monopoly.

**JEL codes:** D4, D5, L2.

## 1 Introduction

The main object of the present paper is to study the relation between democracy and privatization, i.e. whether majority voting may or not lead to the privatization of a public firm.

A great wave of privatization began in Great Britain and the USA under the Thatcher and Reagan administrations<sup>1</sup>, lasting until the recent years. Mass privatizations affected many British firms in the eighties and nineties (British

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<sup>1</sup>A few privatizations had occurred earlier: privatization of common lands in Britain, known as the enclosure (1760-1820), in the thirties, privatization by the Nazi government of many state-owned enterprises in several sectors (Railways, mining, steel, ship-lines, banking, local public utilities, shipyard, see Bel, 2010), privatizations by democratic governments in the fifties and the sixties such as of Volkswagen in 1961 by the West german government.

Rail, British Aerospace, Rover Group, British Telecom, Sealink ferries, British Petroleum, Rolls-Royce, British Steel Corporation, British Gas among others). Many others took place in Latin America at the same period (water management, transportation, telecommunication enterprises being sold off to the private sector) as part of liberal economic policies. In France, privatizations began under the Chirac government (1986) and continued over all the following administrations until now, affecting a great number of important enterprises (Saint-Gobain, Paribas, Société Générale, Havas, Renault, Total to name only a few). In Russia, Eastern and Central Europe, the transition from socialist to market economies was also accompanied in the nineties by massive privatizations of State-Owned Enterprises.

A greater efficiency of private firms was generally invoked to motivate privatization but efficiency may be intended in different ways, as noticed by Willner (2001) “Attitudes to ownership are however often based on [...] confusion about the meaning of efficiency.” (page 723).

A first, unambiguous, meaning is cost-efficiency. Because the managers of public firms are less easily monitored than the managers of private firms, they are supposed to have less incentives to exert efforts to reduce costs. For instance Laffont and Tirole (1991) write that “the cost of public ownership is a sub-optimal investment by the firm’s managers in those assets that can be redeployed to serve social goals pursued by the public” (page 84).

But there is also a second, more disputable, sense of the word, namely State-Owned Enterprises are often considered inefficient because they pursue other objectives than profit-maximization, leading to levels of output and/or employment which are considered as too high compared to their profit-maximizing levels.<sup>2</sup> The problem with this use of the word is that profit-maximizing output and employment levels are not necessarily welfare-maximizing. In particular, regarding SOEs which are generally large firms having substantial market power, profit-maximization is well-known to generally lead to too small output and employment levels. For some services such as water provision or health, profit maximization does not generally coincide with welfare maximization. Regarding the management of SOEs and their possible privatization, labor and/or access to the output are fundamental issues and may legitimately be considered to make the “right” decision. Room has also to be made for democracy and the opinion of majority. The existing theoretical literature makes room for labor but is scarce on output. The access to such outputs may nonetheless be fundamental, even vital in some cases, for instance water, electricity, transportation... A study of the World Bank (Jones et al., 2008) assessed the impact of privatization in 4 sub-saharian countries on workers as well as on consumers and concluded that, if privatization is done properly “it can lead to substantial welfare gains that are reasonably and equitably distributed across stakeholders -consumers, workers, governments, and owners or operators.” However, taking as an example water provision, according to Lobina et al. (2014), “Cities, regions and countries

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<sup>2</sup>This is in line with the findings of Duanmu and Pittman (2019) suggesting that “SOEs may have ‘multitask’ responsibilities in terms of protecting employment as well as achieving efficiency.”

worldwide are increasingly choosing to close the book on water privatization and to “remunicipalize” services by taking back public control over water” because of the failures of the private sector in terms of output provision in quantity and quality. “Where near-universal access to water has been achieved, it has virtually always been through a public commitment” (The Guardian <sup>3</sup>).

These considerations may explain why, even when decided by elected governments, privatizations are often unpopular. In France for instance, according to a YouGov poll<sup>4</sup>, 75% of respondents were ready to sign a petition to stop the privatization of ADP, the Paris airports society, which was decided by the Macron administration. This fear from privatization brings us back to the debate between the nineteenth century economists on the effect of democracy, in particular of universal suffrage, on property rights. The fear of some economists then was that the universal extension of voting would lead to “expropriation of capital” for redistribution purposes (what may take place nowadays in the form of nationalization).<sup>5</sup> Even Ricardo who supported the expansion of suffrage in fact did not support “extending the elective franchise [ ] universally to all the people, but to that part of them which cannot be supposed to have any interest in overturning the rights of property.”<sup>6</sup> In the same spirit, Hayek (1982) proposed to limit drastically the powers of future political majorities in order notably to avoid any infringement of property rights.

Though privatizations are not decided by majority voting, they are easier to implement and more likely to take place if they have a majority support. In the present paper, assuming for heuristic purposes no superior cost efficiency of private relative to public management and considering explicitly employment and output provision, we determine under which circumstances a majority support is more likely to occur for the privatization of a public firm and under which ones it is likely to be opposed by a majority of voters.

To this end, we adapt the small general equilibrium model developed in Kahloul et al. (2017) with an initially public monopoly and an exogenous quality of the product, supposing that the firm’s output price is determined through majority voting whenever the State retains a majority of shares and is fixed so as to maximize profits when the private investors take over the control<sup>7</sup>. Any sale of shares of the firm is supposed to take place at a price which reflects the post-privatization firm’s value<sup>8</sup> and may lead either to a situation when the firm is privately managed (“radical” privatization) or to a situation where the

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<sup>3</sup>Water privatisation: a worldwide failure? <https://www.theguardian.com/global-development/2015/jan/30/water-privatisation-worldwide-failure-lagos-world-bank#top>

<sup>4</sup>[https://www.huffingtonpost.fr/entry/referendum-sur-adp-75-des-francais-pret-a-signer-une-petition-pour-le-reclamer-sondage-exclusif\\_fr\\_5d1b5d02e4b07f6ca5841d1d](https://www.huffingtonpost.fr/entry/referendum-sur-adp-75-des-francais-pret-a-signer-une-petition-pour-le-reclamer-sondage-exclusif_fr_5d1b5d02e4b07f6ca5841d1d)

<sup>5</sup>Concerning this debate, see for instance Robbins (1978), in particular pp. 201.

<sup>6</sup>in his letter to Trower, 20th December 1818 (Ricardo, Sraffa’s ed. 2005).

<sup>7</sup>Privatization is broadly intended here as the transfer of the ownership of a fraction of the capital of a public sector enterprise to the private sector. We shall speak more specifically of private take-overs following which the State retains only a minority of the shares as “radical privatizations”.

<sup>8</sup>In the case when the firm would make losses after being privatized, we shall consider alternatively that either the State is able to commit to cover the losses of private shareholders or that no (partial) privatization leading to losses is feasible.

enterprise remains publicly managed. As already noticed, though it is often argued that private firms are more cost efficient because of greater incentives of private owners than politicians to keep costs low by more effectively monitoring and/or motivating appointed managers (see for instance Bishop, Kay and Mayer (1994)), we suppose here for heuristic reasons that there is no intrinsic superiority or inferiority of private management over public management: the only difference between the two is that a public firm may choose a price which differs from the profit-maximizing one. The economy encompasses a single firm producing a vertically differentiated product using labour as the unique input and a population of workers/consumers/shareholders characterized by two parameters: preference for product quality and sensitivity to effort. Each individual decides whether to work or not and decides whether to purchase one unit of the product or not, in order to maximize his/her utility given his/her income. When the price is set at some given level, the salary adjusts in order to equalize demand and supply on the labor market.

We study a three-stage game. In the first step, the individuals vote in order to decide the fraction of the shares to be sold to private investors (thus whether to privatize the firm and at which rate).<sup>9</sup> In the second step, the price per share is determined through a vote. Finally, in the third step, the output price is selected.<sup>10</sup> If the private investors hold a majority of shares, the equilibrium price is the profit-maximizing one. When the enterprise is (or remains) controlled by the State, the price is the one which is preferred by a majority of voters. In this latter case, we show that this is the profit-maximizing price iff the public share in the capital of the firm is large enough and/or working in the firm and consuming its output is not very attractive for most agents; but it is otherwise a low price (involving financial losses which have to be covered by the State budget) which is intended to please the, then more important, part of the population which consumes the good and is employed by the enterprise.

Our main results are as follows. When the maximum disutility of working in the firm is greater than the maximum utility of consuming its output, implying that initially (i.e. when all the shares belong to the State) a (at least relative) majority of voters neither consume the good nor work in the public firm, any ownership structure such that the firm maximizes its profit is an equilibrium structure. That of course includes the cases where the State retains less than one half of the shares (private control) but also the cases where it retains a fraction of the shares important enough so that the profit-maximizing output price is preferred by a majority of voters. An immediate consequence of this first result is that any cost-efficiency advantage, even infinitesimal, of private over public control is then enough to entail a majority support for a radical privatization. When, on the contrary, the maximum disutility of working in the firm is smaller than the maximum utility of consuming its output, implying that there is initially an at least relative majority of the population who consumes the good and works in the firm, an absolute majority of voters oppose any sale of

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<sup>9</sup>We do not consider the distribution of shares to voters (for such an analysis see Biais and Perotti (2002)).

<sup>10</sup>The equilibrium wage rate being a function of the price, as noticed above.

shares to private investors. Moreover they strictly prefer that the firm remains totally public to a radical privatization, so that a possible small cost-efficiency superiority of private over public management would not be enough to reverse the voting result. So our intuitive conclusion is that the privatization of a public firm is likely to be opposed by a majority of voters when they have a direct stake in it as consumers and workers.

Though the main part of the paper is about investigating the conditions under which there may or not exist a majority support for the privatization of a SOE, we also analyze here the welfare effects of privatization. Given the quasi-linear utility functions in our model, welfare is expressed in units of numeraire and is the sum of the utilities of the voters.<sup>11</sup> The conclusions of this analysis are surprisingly similar to the results obtained about majority voting, hinging as well on the comparison between the maximum utility of consuming one unit of the firm's output and the maximum disutility of producing it. When the latter is greater than the former, any ownership structure such that the firm maximizes its profit, including the initial one (all shares belonging to the State), gives the same welfare level, which is greater than an ownership structure leading to a smaller price. When the former is greater than the latter, any privatization reduces welfare.

It is also interesting to understand the factors which are driving the results. Basically the agents have preferences over the ownership structure, namely over the fraction of shares which is held by the State, only insofar as it determines the level of the output price, which, in turn, determines the levels of wage and profits. For instance, the agents who have no stake in the firm as consumers or workers or both want it to maximize profits and do not care whether this is as a public or a private entity. On the contrary, the agents who have a stake in the firm favor an output price lower than the profit-maximizing one, which occurs necessarily at the expense of the agents who have no interest in the enterprise. When considering the desirability of privatization, one has to consider whether initially the public firm's output price is the profit-maximizing price or a smaller one and to what output price the privatization would lead. In the first case, i.e. when the maximum disutility of working in the firm is greater than the maximum utility of consuming its output, an absolute majority of the population always prefers an ownership structure leading to profit-maximizing price to a structure which would imply the choice of a smaller price. In the second case, i.e. when the maximum disutility of working in the firm is smaller than the maximum utility of consuming its output, an absolute majority refuses any sale of shares, preferring an output price below the profit-maximizing one.

**The literature review** There is an abundant literature on privatization. Two main streams exist (Cavaliere, 2006). The first one applies the principal-agent theory to the question of privatization. Comparing State Owned Enterprises (SOE) and regulated private firms, Sappington and Stiglitz (1987) shed

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<sup>11</sup>This definition is rigorous iff there may exist non-distortionary transfers allowing "winners" to possibly compensate "losers" of a contemplated privatization.

light on the role of information asymmetry, risky production and the attitude toward risk of private producers. Shapiro and Willig (1990) assumed that a public framer has to make a choice between operating production through a SOE by delegating administrative power to a “malevolent agent” who pursues a private agenda, or with a regulated private firm facing an asymmetry of information on production. Another group of articles assume benevolent governments. Laffont and Tirole (1991) extended their previous model with incomplete information to compare public and private firms in the framework of incomplete contracts. Schmidt (1996), concerned with the soft budget constraint, compared SOEs with managers weakly motivated by reducing costs and regulated private firms with asymmetry of information.

The second stream of literature deals with privatization from a political economy point of view. Vickers and Yarrow (1988) pointed out the importance of labour in the decision of privatization which will avoid excess employment. Politicians may nevertheless refrain from privatization even if it is Pareto efficient because it will not be supported politically. The strong resistance of workers who will lose their employment will not be outweighed by the support of winners because privatization benefits are widespread. Labor is also at the core of the analysis of Shleifer and Vishny (1994) and Boycko, Shleifer and Vishny (1996). Because politicians will try to maintain excess employment even in privatized firms, privatization will not necessarily lead to reduce labor costs. Robinson and Torvik (2005) examined the soft budget constraint from another perspective. Politicians want “bad projects” to be supported by voters who will benefit from the redistribution of subsequent resources. Bortolotti and Pinotti (2003) compared the motivation for privatization of “majoritarian” political systems and “consensual-corporatist” democracies and conclude that the former are more likely to privatize. Biais and Perotti (2002) showed the difference in motivation for privatization by right wing and left wing politicians. Right wing politicians are motivated by future support from the constituency of shareholders of newly privatized firms, and left wing by redistribution of revenues accruing from privatization. Borner (2004) distinguished between privatization and restructuring of SOEs on the one hand, and between different types of governments, welfare or voter oriented, on the other hand. While a welfare maximizer will trade-off efficiency gains following privatization with the choice of the socially optimal employment level, a voter-oriented government will weigh the possibility to transfer to citizens the revenues accruing from privatization against the possibility to increase employment level in SOEs, with the objective of being re-elected.

The present paper belongs to the second stream, as we a voting model of privatization with no asymmetry of information. The employment level is central in our analysis, but also the level of consumption of the good produced by the monopoly. Willner (2001) is probably the paper which is the closest to ours. He also considered the output and employment levels. In his paper, the public firm’s output level is indeed fixed through a Nash bargaining process in which output and profits have complementary weights. This situation is then compared to a private Cournot oligopoly characterized by a given value of the

Herfindahl index. Welfare is greater in the public monopoly than under privatization if the output weight in the bargaining process and/or the Herfindahl index are important. The main differences with the present paper is that (i) we use a general equilibrium model and (ii) we consider that the public firm’s price decisions as well as the privatization one are taken through majority voting.

Finally, loosely related to the present paper, there is an abundant literature on privatization in a mixed oligopoly framework, when competition exists initially between public and private firms, generally in partial equilibrium settings (to name only a few, Lin et al., 2018; Capuano & De Foe, 2010; Matsumura, 1998; Matsumura & Okumura, 2013).

The paper is organized as follows. Section 2 describes the model. Section 3 provides the outcome of a democratic choice in terms of privatization. Section 4 analyzes the welfare effects of privatization. Section 5 concludes. All proofs are given in Appendix.

## 2 The model

We build on the basic model introduced by Kahloul et al. (2017) and used by Lahmandi-Ayed and Laussel (2018).

The economy encompasses 3 goods: labour as the unique input, a numeraire and an indivisible “differentiated” good as the unique output (“differentiated” in the sense that it may possibly be of different qualities perceived differently by consumers).

There is a population of workers/consumers/shareholders. Each individual is endowed with an indivisible unit of labour and a given quantity  $e$  of a (composite) numeraire good. Denote by  $\lambda$  the share of the individual in the firm. Each agent  $(\alpha, \theta)$  is doubly characterized by:

- a “working parameter”  $\alpha \in [0, \bar{\alpha}]$  which captures the worker’s sensitivity to effort,
- a “consumption parameter”  $\theta \in [0, \bar{\theta}]$  which measures the intensity of the consumer’s preference for the product’s quality.

Individuals are uniformly distributed over  $[0, \bar{\alpha}] \times [0, \bar{\theta}]$  with a density normalized to 1. Each worker/consumer  $(\alpha, \theta)$  chooses sequentially:

- first, whether to remain idle ( $\bar{W}$ ) or to work ( $W$ ) in the differentiated sector,
- then to compose his/her consumption bundle, in particular whether to consume ( $C$ ) or not ( $\bar{C}$ ) one unit of the differentiated product.

One firm produces the differentiated product. One unit of the differentiated good requires one unit of labour.

Individuals derive their utility from the consumption of these two goods as follows:



$$V(x, t) = \theta qx + t,$$

where

- $x$  is the consumption of the differentiated product of exogenous quality  $q$ ,
- $t$  is the consumption of the numeraire good,
- the consumption bundle  $(x, t)$  must belong to the consumption set  $\{0, 1\} \times \mathbb{R}$ .

If the individual chooses to work, he/she receives a salary  $\omega$  and must incur a training cost (or disutility)<sup>12</sup>  $\alpha q$ . If he/she chooses not to work, he/she receives no salary (and does not have to be trained), his/her revenue being limited to the initial endowment in the numeraire and to his/her share in the firm's profit.

Denote by  $\nu$  the share of the "State" in the firm. This means that every individual owns the share  $\lambda = \frac{\nu}{\alpha\theta}$ .

Initially the firm is entirely public, i.e. it is fully owned by the State.

The share to be privatized (thus the share  $\nu$  to be kept), the price per share and the price of the output, are determined as follows:

1. In a first stage, the fraction  $\nu$  of the firm which is to remain public is determined through majority voting.
2. Given the value of  $\nu$  selected in the first stage, the price per share  $z$  is determined through a vote. We impose  $z \geq 0$ .
3. Given the choice of  $\nu$  and  $z$ , two cases are distinguished:
  - If  $\nu \geq 1/2$ , the firm is controlled by the State. Then the output price is determined through majority voting.
  - If  $\nu < 1/2$ , the firm is controlled by a private owner. We then suppose that the firm chooses the price so as to maximize its profit<sup>13</sup>.

Under some conditions, when controlled by the State, the firm may incur losses at equilibrium. The firm is thus unsalable unless the State commits to cover the losses. Covering the losses of a private owner is not always acceptable and politically feasible. We shall however derive our results under the two possible assumptions.

- $\mathcal{H}_1$ : The State cannot cover the possible losses a private shareholder may incur.

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<sup>12</sup>This cost may also be interpreted as an opportunity cost, the agents having heterogenous productivities outside the differentiated sector, i.e. in producing the numeraire.

<sup>13</sup>This assumption may seem arbitrary. But any other assumption will be more complicated and nonetheless equally arbitrary.

- $\mathcal{H}_2$ : The State commits to cover the possible losses a private shareholder may incur.<sup>14</sup>

Helpfully, we denote by  $\gamma = \frac{\bar{\alpha}}{\bar{\theta}}$  (as in Lahmandi-Ayed and Laussel, 2018). This is the ratio between the maximum disutility of manufacturing one unit of output and the maximum utility of consuming it. It is going to play a crucial role in the results.

### 3 Main results

The game is solved by backward induction. We first determine the equilibrium output price of stage 3, then the price per share of the firm, and finally the share of the firm to be kept by the State.

#### 3.1 The output price

When  $\nu < 1/2$ , the firm is controlled by a private owner. The chosen price is  $p(\bar{C}, \bar{W})$  given by Equation 4 which maximizes the firm's profit.

When  $\nu \geq 1/2$ , the firm is controlled by the State and the price of the output is determined through a vote by all individuals. In this case, we have only to apply the results obtained by Lahmandi-Ayed and Laussel (2018) replacing  $\mu$  by  $\frac{1}{\nu}$ .

The prices candidate to be chosen are given by:

$$p(C, W) = \frac{q\bar{\theta}(\nu(2\bar{\alpha} + \bar{\theta}) - (\bar{\alpha} + \bar{\theta}))}{2\nu(\bar{\alpha} + \bar{\theta})}, \quad (1)$$

$$p(C, \bar{W}) = \frac{q\bar{\theta}(\bar{\theta}(\nu - 1) + 2\nu\bar{\alpha})}{2\nu(\bar{\alpha} + \bar{\theta})}, \quad (2)$$

$$p(\bar{C}, W) = \frac{q\bar{\theta}(\nu(\bar{\theta} + 2\bar{\alpha}) - \bar{\alpha})}{2\nu(\bar{\theta} + \bar{\alpha})}, \quad (3)$$

$$p(\bar{C}, \bar{W}) = \frac{q\bar{\theta}(\bar{\theta} + 2\bar{\alpha})}{2(\bar{\theta} + \bar{\alpha})}; \quad (4)$$

We rewrite Proposition 1 of Lahmandi-Ayed and Laussel (2018) replacing  $\mu$  by  $\frac{1}{\nu}$ . Then we obtain our first result. Let

$$R(\nu) = r\left(\frac{1}{\nu}\right) = \frac{6\nu^2 + \nu + \sqrt{128\nu^4 + 4\nu^2 + 12\nu + 1}}{8\nu(2\nu - 1)}, \quad (5)$$

$$S(\nu) = s\left(\frac{1}{\nu}\right) = \frac{1}{2\nu - 1}, \quad (6)$$

$$T(\nu) = t\left(\frac{1}{\nu}\right) = \frac{3(2\nu + 1) - 16\nu^2 + \sqrt{128\nu^4 + 36\nu^2 - 12\nu - 7}}{4(4\nu^2 - 1)}. \quad (7)$$

<sup>14</sup>In this case it sells shares at a zero price but, given the commitment, this is equivalent to a negative price

Note that all the functions defined above are decreasing, as depicted in Fig 1, and helpfully that  $S(\nu = 1) = 1$ .

**Proposition 1** *When  $\nu < 1/2$ ,  $p(\bar{C}, \bar{W})$  is chosen.*

*For  $\nu \geq 1/2$ , the winning price in a plurality vote is either  $p(\bar{C}, \bar{W})$  or  $p(C, W)$ . More precisely, we have the following.*

1. *The profit-maximizing firm's price  $p(\bar{C}, \bar{W})$  is the ideal price of an absolute majority if and only if*

$$\gamma \geq R(\nu). \quad (8)$$

*In this case,  $p(\bar{C}, \bar{W})$  is the Condorcet winner.*

2. *The profit-maximizing firm's price  $p(\bar{C}, \bar{W})$  is the ideal price of only a relative majority if and only if*

$$S(\nu) \leq \gamma < R(\nu). \quad (9)$$

3. *Price  $p(C, W)$  is preferred by only a relative majority if and only if*

$$T(\nu) < \gamma \leq S(\nu). \quad (10)$$

4. *Price  $p(C, W)$  is preferred by an absolute majority if and only if*

$$\gamma \leq T(\nu). \quad (11)$$

*In this case,  $p(C, W)$  is the Condorcet winner.*

In Lahmandi-Ayed and Laussel (2018), to establish Proposition 1, we proceeded as follows. Depending on the output price and  $\theta$ , an individual chooses to consume or not the differentiated good. This determines the demand of the output as function of the output price, and at the same time the demand of labour. Depending on the wage and  $\alpha$ , an individual chooses to work or not, which determines the labour supply as function of the wage. Balancing offer and demand on the labour market allows to express the wage as function of the output price, so that:

$$\omega = \bar{\alpha}(q - \frac{p}{\theta}). \quad (12)$$

The firm's profit is then the following function of its price<sup>15</sup>:

$$\pi = \bar{\alpha}[p(1 + \frac{\bar{\alpha}}{\theta}) - \bar{\alpha}q][\bar{\theta} - \frac{p}{q}]. \quad (13)$$

As  $p$  varies, the individual goes through different regimes in terms of consumption and working, resulting in different expressions of the corresponding

<sup>15</sup>For details on the derivation of the results see Kahloul et al. (2017).

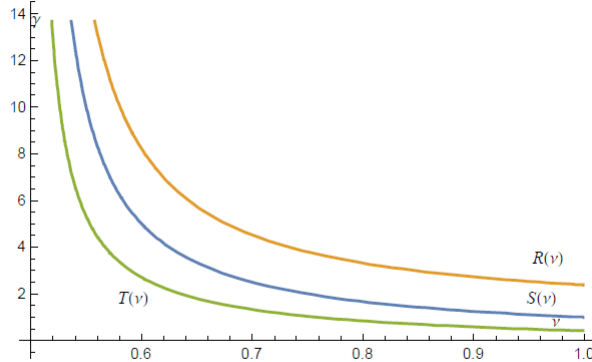


Figure 1: Absolute/Relative majority vote for the output price in the  $(\nu, \gamma)$ -space

indirect utility. We then identify the prices which are candidates to be local maxima of these utilities, determine the voters for each candidate price. Finally, we count the number of voters for each candidate price, thus obtaining the winner in a plurality system. Further details may be found in Kahloul et al. (2017) and Lahmandi-Ayed and Laussel (2018).

### 3.2 The price per share

The price per share  $z$  of the firm depends on what is expected to occur at stage 3, i.e. in terms of the output price, thus the firm's profit, which in turn depend on  $\gamma$  and  $\nu$  as stated in Proposition 1. It may also depend on whether  $\mathcal{H}_1$  or  $\mathcal{H}_2$  is supposed, as the anticipated firm's profit may be negative. The formal result is provided in Proposition 2.

**Proposition 2 (The price per share)** *At equilibrium of step 2, the price per share of the firm,  $z$ , depends on  $\nu$  and  $\gamma$  as follows:*

1. If  $\nu < 1/2$ , then  $z = \frac{q\bar{\theta}^2}{4(\bar{\alpha}+\bar{\theta})}$  and the output price is given by  $p(\bar{C}, \bar{W})$ .
2. If  $\nu \geq 1/2$ , two cases are distinguished.
  - If  $\gamma \geq S(\nu)$ , then  $z = \frac{q\bar{\theta}^2}{4(\bar{\alpha}+\bar{\theta})}$  and the output price is given by  $p(\bar{C}, \bar{W})$ .
  - If  $\gamma < S(\nu)$ , then, under  $\mathcal{H}_1$ , the firm is unsalable and  $z$  is not defined. Under  $\mathcal{H}_2$ ,  $z = 0$ .

When the anticipated firm's profit is negative, either the firm is unsalable (under  $\mathcal{H}_1$ ) or the price per share is null (under  $\mathcal{H}_2$ ). In the latter case, the private purchaser is completely indifferent between buying or not. If we suppose that the State gives  $\epsilon$  to urge the purchaser to accept, the transaction will occur, whatever low  $\epsilon$  is. The above results are thus obtained when  $\epsilon$  converges to zero.

### 3.3 The share to be kept by the State

We are going to establish the results under each possible assumption concerning the possibility for the State to cover the losses a private stakeholder may incur after buying her shares. It turns out that the results are independent of this assumption. But the proofs are not the same under  $\mathcal{H}_1$  or  $\mathcal{H}_2$ . In other words, we obtain the same result under each one of the two possible hypotheses but for different reasons.

**Proposition 3** *Whether under  $\mathcal{H}_1$  or under  $\mathcal{H}_2$ , at equilibrium,  $\nu$  and the output price depend on  $\gamma$  as follows:*

- *When  $\gamma > 1$ , there exists a unique  $\tilde{\nu} = \frac{\gamma+1}{2\gamma} \in ]1/2, 1]$  satisfying  $\gamma = S(\tilde{\nu})$ . At equilibrium, a continuum of values of  $\nu$  may emerge. That is:  $\nu \in [0, 1/2[ \cup ]\tilde{\nu}, 1]$ ; and the output price is in all cases  $p(\overline{C}, \overline{W})$ .*
- *When  $\gamma \leq 1$ ,  $\nu = 1$  and the output price is  $p(C, W)$ .*

In the first case (high  $\gamma$ ), the status quo and a privatization giving control to a private owner are equivalent and may both emerge at equilibrium. Thus a privatization may emerge at equilibrium even when we do not suppose any superior efficiency of a private management relative to a public one. However, if we introduce a difference in efficiency between the two types of management, whatever small this difference is, the indeterminacy will disappear and the privatization will be the only outcome at equilibrium.

Although the result obtained is the same, independently of whether the State covers or not the losses a private purchaser may make after transaction, the proof depends on this hypothesis. The proof thus encompasses two parts, supposing first  $\mathcal{H}_1$ , then  $\mathcal{H}_2$ . Under each one of the two hypotheses, the analysis depends of the output price chosen at the last step, which according to Proposition 1, depends on  $\gamma$  and  $\nu$ . More precisely, it depends on whether  $\gamma$  is greater or smaller than  $S(\nu)$ . Since  $S$  is decreasing with 1 as the minimal value, we have to compare  $\gamma$  to 1.

When  $\gamma < 1$ , when the State controls the firm ( $\nu > 1/2$ ), the price  $p(C, W)$  is chosen at the last stage of the game, yielding a negative profit. Under  $\mathcal{H}_1$ , the firm is unsalable and only  $\nu = 1$  may emerge at equilibrium among the values of  $\nu$  giving control to the State. Therefore  $\nu = 1$  has to be compared to the values  $\nu < 1/2$  transferring the control of the firm to private owners and yielding the price  $p(\overline{C}, \overline{W})$  at the last stage of the game. Under  $\mathcal{H}_2$ , the State covers the losses, which means that each individual has to bear his/her share of the losses.

When  $\gamma \geq 1$ ,  $\gamma$  may be greater or smaller than  $S(\nu)$ , depending on whether  $\nu$  is greater or smaller than  $\tilde{\nu}$ . We have to study the indirect utilities for  $\nu \in [0, 1/2[\cup[\tilde{\nu}, 1]$ , yielding  $p(\bar{C}, \bar{W})$  at the last stage and a positive profit; and for  $\nu \in [1/2, \tilde{\nu}]$  yielding  $p(C, W)$  and a negative price, in which case we have to distinguish  $\mathcal{H}_1$  and  $\mathcal{H}_2$  as the indirect utilities are different under each of these assumptions. The local maxima in each case are compared to determine the majority voting outcome in terms of  $\nu$ .

## 4 Privatization and Social Welfare

It is interesting to know whether or not the majority choice coincides with the welfare enhancing one. By welfare we intend the sum of the individuals' utilities measured in units of numeraire, as is usual in models with semi-linear utility functions.<sup>16</sup> Notice that this definition supposes the possibility of non-distortionary transfers between “gainers” and “losers” of any contemplated change. In their absence, the procedure of “*treating a dollar as a dollar, no matter to whom it accrues*” has been criticized (see, e.g., Hendren, 2014; Saez and Stantcheva, 2013).

Consider the initial situation when  $\nu = 1$ . It is convenient to express the social welfare as a function of the output price  $p$ . Notice that, from the labor market equilibrium condition, the equilibrium wage

$$w = \gamma(q\bar{\theta} - p).$$

The firm's profit as a function of the output price is accordingly equal to

$$\pi = \bar{\alpha}(p - w)(\bar{\theta} - \frac{p}{q}) = \bar{\alpha}(p(1 + \gamma) - \gamma q\bar{\theta})(\bar{\theta} - \frac{p}{q}).$$

The social welfare is simply the aggregate utility from consuming the good,

$$\bar{\alpha} \int_{\frac{p}{q}}^{\bar{\theta}} \theta q d\theta,$$

minus the aggregate disutility from working

$$\bar{\theta} \int_0^{\frac{w}{q}} \alpha q d\alpha.$$

Subtracting and substituting  $\omega$  for its value, we obtain the social welfare as a function of the output price as follows:

$$W_0 = \frac{1}{2q} [\bar{\theta} (q\bar{\theta} - p_0) \gamma (p_0 + q\bar{\theta} + p_0\gamma - q\bar{\theta}\gamma)],$$

<sup>16</sup>We deal here with a general equilibrium model so that profits are distributed and appear in the individuals' utilities each in proportion of his/her share. When private investors buy shares we proved that they do that at a price which leaves them with no net profit.

where  $p_0$  stands for the initial output price.

Notice that  $W_0$  is a concave function of  $p_0$  which takes its maximum at  $p^{opt} = q\bar{\theta} \frac{\gamma}{1+\gamma}$ . It is interesting, though not surprising, to remark that  $p(C, W) < p^{opt} < p(\bar{C}, \bar{W})$ .

Let us now consider the social welfare after a privatization leading to some share of the State  $\nu < 1$  and to some new output price  $p_1$ . We are going to prove that the welfare keeps the same expression as a function of the output price. Indeed the social welfare following the privatization equals the aggregate utility from consuming the good minus the aggregate disutility from working *plus the money from the sale of a fraction  $1-\nu$  of the shares to private investors*, (namely a fraction  $1-\nu$  of the post-privatization profits) *minus the same term as it corresponds to the part of the profits which will not be distributed as dividends after transaction*. As for the private investors, whether they belong to the considered population or not, amounts to the same, as the money they spend to acquire the share  $(1-\nu)$  of the firm corresponds exactly to what they earn as dividends after transaction.

$$W_1 = \frac{1}{2q} [\bar{\theta} (q\bar{\theta} - p_1) \gamma (p_1 + q\bar{\theta} + p_1\gamma - q\bar{\theta}\gamma)].$$

**Proposition 4** *Suppose that  $\nu$  is chosen so as to maximize the total welfare and that the other decisions are made as in the original model.*

- *When  $\gamma < 1$ , the unique share that maximizes the welfare is  $\nu^* = 1$ .*
- *When  $\gamma \geq 1$ , all the values of  $\nu^* \in [0, 1/2 \cup [\tilde{\nu}, 1]]$  are equivalent and maximize the total welfare.*

The optimal choice of  $\nu$  in terms of global welfare coincides with the outcome of majority voting. In other words, the centralized and decentralized choices amount to the same in this setting.

For low values of  $\gamma$  ( $\gamma < 1$ ), privatization is desirable neither from a majority of voters' point of view nor in terms of global welfare. In both cases, the firm remains entirely under the control of the State, the output price is below its profit-maximizing level and the equilibrium profit is negative. For high values of  $\gamma$  ( $\gamma \geq 1$ ), the welfare-maximizing choice, as well as the outcome of majority voting, involves profit maximization, what may follow either from a limited sale of shares to private stockholders, the State retaining a large enough majority of shares, or from a radical privatization.

To obtain this result, we compare two types of privatization: privatization such that the firm remains under the control of the state (with  $p(C, W)$  as the output price) and radical privatization transferring the control to the private investors (with  $p(\bar{C}, \bar{W})$ ). The analysis is not the same depending on whether the State can or cannot commit to cover or not the private investors' losses after privatization. Supposing  $\mathcal{H}_1$  excludes the values of  $\nu < 1$  yielding a negative profit. The difference between high and low values of  $\gamma$  is that when  $\gamma < 1$ , for

all  $\nu \in [1/2, 1]$ , the output price is always given by  $p(C, W)$  yielding a negative profit, whereas when  $\gamma \geq 1$ , the output price and the resulting profit depend on the position of  $\nu$  relative to  $\tilde{\nu}$ .

## 5 Conclusion

In this paper we have tried to determine whether there will be a majority support for the privatization of a State monopoly and, incidentally, whether it can increase social welfare. Supposing that there is no superior nor inferior cost-efficiency of private management, we have focused on the output and employment effects of privatization. Under public control the output price and hence the output and employment levels are assumed to be the ones which please a majority of voters. The equilibrium price turns out to be either the profit-maximizing one, which is greater than the welfare-maximizing one, or the price which maximizes the utility of the voters who work in the firm and consume its product, a price smaller than the welfare-maximizing one. Under private control, the profit-maximizing price always follows. In this framework, the only possible relevant effect of privatization is to allow a switch from one price to the other (and from one wage to the other). What we found is that privatization is neither strictly better than the status quo for a majority of voters nor strictly welfare increasing. However when the maximum disutility of working in the firm is greater than the maximum utility of consuming its output, so that a minority of voters have a direct stake in the firm as consumers/workers, an infinitesimal cost-efficiency superiority of private over public control is enough to entail a majority support for a radical privatization which, at the same time, is welfare enhancing. In the reverse case, the only equilibrium has the State retaining all shares of the firm, what is also the best outcome from the social welfare point of view, a result which only a substantial cost-efficiency advantage of private control is likely to reverse.

The results on the welfare analysis have been obtained under the hypothesis that there is no shadow price on the government's deficit, which is questionable, in particular in the developing world. A natural extension would be to relax this assumption by considering a positive shadow price.<sup>17</sup>

All the analysis has been conducted assuming that, under private control, the previously State Owned Enterprise is not regulated.<sup>18</sup> An obvious extension would be to analyze what would be the equilibrium regulatory policy for the private firm decided through majority voting and then to see whether, in these circumstances, a privatization inducing a switch from a SOE to a private firm regulated in this way would find a majority support.

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<sup>17</sup>Since this model is a general equilibrium one, it would be natural to endogenize the shadow price of public funds. This would require to identify a specific (distortionary) tax system.

<sup>18</sup>On the contrary, Laffont and Tirole (1991) compare a public enterprise and a private regulated firm.



## References

- [1] Bel, G. (2010). Against the mainstream: Nazi privatization in 1930s Germany 1. *The Economic History Review*, 63(1), 34-55.
- [2] Biais, B., Perotti, E. C. (2002), Machiavellian privatization, *American Economic Review*, 92, pp. 240-58.
- [3] Bishop, M., Kay, J., & Mayer, C. (1994). *Privatization and economic performance*. Oxford University Press.
- [4] Borner, K. (2004). The Political Economy of Privatization: Why Do Governments Want Reforms?. *Nota di Lavoro 106.2004*, Fondazione Eni Enrico Mattei.
- [5] Bortolotti, B., Pinotti, P. (2003). The Political Economy of Privatization. *Nota di Lavoro 45.2003*, Fondazione Eni Enrico Mattei.
- [6] Boycko, M., Shleifer, A., Vishny, R. W. (1996). A Theory of Privatization. *The Economic Journal*, 106, pp. 309-19.
- [7] Capuano, C., & De Feo, G. (2010). Privatization in oligopoly: the impact of the shadow cost of public funds. *Rivista Italiana Degli Economisti*, 15(2), 175-208.
- [8] Cavaliere, A. (2006). Privatization and Efficiency: from Principals and Agents to Political Economy. *Nota di Lavoro, No. 99.2006*, Fondazione Eni Enrico Mattei (FEEM), Milano.
- [9] Duanmu, J-L. & R. Pittman (2019). The response of State-Owned Enterprises to import competition : evidence from chinese manufacturing firms. *Annals of Public and Cooperative Economics* 90(4), 577-613.
- [10] Hayek, F. A. (1982). *Law, legislation and liberty, volume 3: The political order of a free people*. London: Routledge.
- [11] Hendren, (2014). The inequality deflator: interpersonal comparisons without a social welfare function. NBER working paper No. 20351.
- [12] Jones , L., Y. Jammal, N. Gokgur, & S. Kikeri (2008). Doing privatization right (English). Public policy for the private sector. Note no. 319. Washington, DC: World Bank. <http://documents.worldbank.org/curated/en/695081468004817496/Doing-privatization-right>
- [13] Kahloul, A., Lahmandi-Ayed, R., Lasram, H., & Laussel D. (2017). Democracy and Competition: Vertical Differentiation and Labor in a General Equilibrium Model. *Journal of Public Economic Theory*, 19(4), 860-874.
- [14] Laffont J-J. and J. Tirole (1991). Privatization and Incentives. *Journal of Law, Economics and Organization*, vol. 7, 84-105.

- [15] Lahmandi-Ayed, R. et D. Laussel (2018). When Do Imperfectly Competitive Firms Maximize Profits? The Lessons from a Simple General Equilibrium Model with Shareholders' Voting. *Journal of Mathematical Economics*, 78, 6-12.
- [16] Lin, M. H. and T. Matsumura (2018). Optimal privatization and uniform subsidy policies: a note. *Journal of Public Economic Theory*, 20: 416–423.
- [17] Lobina, E., S. Kishimoto & O. Petitjean (2014). Here to stay: Water Remunicipalisation as a global trend. Public Services International Research Unit (PSIRU), Transnational Institute (TNI) and Multinational Observatory.
- [18] Matsumura, T. (1998). Partial privatization in mixed duopoly. *Journal of Public Economics*, 70(3), 473–483.
- [19] Matsumura, T., & Okumura, Y. (2013). Privatization neutrality theorem revisited. *Economics Letters*, 118(2), 324-326.
- [20] Ricardo, D. (2005) *The Works and Correspondence of David Ricardo*, ed. Piero Sraffa with the Collaboration of M.H. Dobb (Indianapolis: Liberty Fund, 2005). Vol. 7 Letters 1816-1818. 01/05/2020. [https://oll.libertyfund.org/titles/208#Ricardo\\_0687-07\\_921](https://oll.libertyfund.org/titles/208#Ricardo_0687-07_921)
- [21] Robbins, L. (1978). *The theory of Economic Policy*, in english classical political economy. second edition. MacMillan Press LTD.
- [22] Robinson, J. A., Torvik, R. (2005). *A Political Economy Theory of the Soft Budget Constraint*. Harvard Research Paper.
- [23] Saez and Stantcheva, (2013). Generalized social marginal welfare weights for optimal tax theory. NBER working paper No. 18835.
- [24] Sappington, D. E. M., Stiglitz, J. E. (1987). Privatization, Information and Incentives. *Journal of Policy Analysis and Management*, vol. 6, n. 4, pp. 567-82.
- [25] Schmidt, K. M. (1996). The Costs and Benefits of Privatization- An Incomplete-Contracts Approach. *Journal of Law, Economics and Organization*, 12, pp. 1-24.
- [26] Shapiro, C., Willig, R. D. (1990). Economic Rationales for the Scope of Privatization, in Suleiman, E. N., Waterbury, J. (eds), *The Political Economy of Private Sector Reform and Privatization*, Boulder: Westview Press.
- [27] Shleifer, A., Vishny, R. W. (1994), Politicians and Firms, *Quarterly Journal of Economics*, 109, pp. 995-1025.
- [28] Vickers, J., Yarrow, G. (1988). *Privatization. An Economic Analysis*, Cambridge, Massachusetts: MIT Press.
- [29] Willner, J (2001). Ownership, efficiency, and political interference. *European Journal of Political Economy*, Vol. 17, 723–748.

## Appendix

### Proof of Proposition 1

In the present paper,  $\nu$  plays the role of  $\frac{1}{\mu}$  in Lahmandi-Ayed and Laus-  
sel (2018). We then just replace in Proposition 1,  $\mu$  by  $\frac{1}{\nu}$ , to have the price  
candidates, Figure 1 and the outcomes of votes.

**Proof of Proposition 2** Suppose  $\nu < 1/2$ . Then the firm will be under  
the control of a private owner who will choose price  $p(\bar{C}, \bar{W})$ , yielding the firm's  
profit

$$\bar{\pi} = \frac{q\bar{\alpha}\bar{\theta}^3}{4(\bar{\alpha} + \bar{\theta})},$$

which is positive.

Whatever the type of the individual, its indirect utility to be considered at  
this step is the sum of four terms:

1. a consumption term,
2. a working term,
3. a term corresponding to the share in the firm's profit,
4. a term corresponding to the share of the individual in the revenues from  
selling a part of the firm, equal to  $z(1 - \nu)$ , which is obviously increasing  
with  $z$ , for all  $\nu < 1$ .

The three first terms do not depend on the price per share  $z$ . The indirect  
utility is thus obviously increasing with  $z$ , for all  $\nu < 1$ . Therefore, all individuals  
prefer the highest value possible of  $z$ .

A potential private owner will not accept to buy a part of the firm if the price  
per share exceeds the dividend per share. Hence the maximal value possible of  
 $z$  is simply given by  $\frac{\bar{\pi}}{\bar{\alpha}\bar{\theta}}$ .

Suppose now that  $\nu \geq 1/2$ . We have to distinguish two cases.

- If  $\gamma \geq S(\nu)$ , it is price  $p(\bar{C}, \bar{W})$  which is voted by a majority, yielding the  
same profit as the case  $\nu < 1/2$ . We continue the reasoning similarly.
- If  $\gamma < S(\nu)$ , it is price  $p(C, W)$  which is voted, yielding the profit

$$\pi = \frac{q\bar{\alpha}\bar{\theta}(\nu^2\bar{\theta}^2 - (\bar{\alpha} + \bar{\theta})^2)}{4\nu^2(\bar{\alpha} + \bar{\theta})}, \quad (14)$$

which is negative for all  $\nu$ . This means that the firm is unsaleable unless  
the State commits to cover the losses after transaction. Therefore,

- under  $\mathcal{H}_1$ , there is no transaction, thus the price per share is not  
defined.

– under  $\mathcal{H}_2$ , the price per share is null.

### Proof of Proposition 3

#### Part 1: Under $\mathcal{H}_1$

Suppose first that  $\gamma > 1 = S(1)$ .

Given the decreasingness of  $S$ , when  $\nu < \bar{\nu}$ ,  $\gamma < S(\nu)$ , the firm's output is  $p(C, W)$ , which yields a negative profit. The firm is thus unsaleable under  $\mathcal{H}_1$ . Thus such a value of  $\nu$  cannot emerge at equilibrium. At equilibrium, we have necessarily  $\nu \geq \nu_0$  (thus  $\gamma > S(\nu)$ ) or  $\nu < 1/2$ , and an indifference of all the individuals between all these values of  $\nu$ .

Suppose now that  $\gamma \leq 1 = S(1)$ , since  $S$  is a decreasing function with  $\nu$ , then  $\gamma < S(\nu)$  for all  $\nu \in [1/2, 1]$ . The output price chosen by the majority is thus  $p(C, W)$  and the firm is unsaleable. Under  $\mathcal{H}_1$ , among the segment  $[1/2, 1]$ , only  $\nu = 1$  is eligible at equilibrium.

For  $\nu < 1/2$ , the private owner has the control and the output price is  $p(\bar{C}, \bar{W})$ .

At price  $p(\bar{C}, \bar{W})$ , the indirect utilities of the individuals when we integrate the revenues of the firm's sale, are given by:

$$U(C, W)(p(\bar{C}, \bar{W})) = (\theta - \alpha)q - \frac{q\bar{\theta}(2\bar{\alpha} + \bar{\theta})}{4(\bar{\alpha} + \bar{\theta})}, \quad (15)$$

$$U(\bar{C}, W)(p(\bar{C}, \bar{W})) = -\alpha q + \frac{q\bar{\theta}(2\bar{\alpha} + \bar{\theta})}{4(\bar{\alpha} + \bar{\theta})}, \quad (16)$$

$$U(C, \bar{W})(p(\bar{C}, \bar{W})) = \theta q - \frac{q\bar{\theta}(2\bar{\alpha} + \bar{\theta})}{4(\bar{\alpha} + \bar{\theta})}, \quad (17)$$

$$U(\bar{C}, \bar{W})(p(\bar{C}, \bar{W})) = \frac{q\bar{\theta}^2}{4(\bar{\alpha} + \bar{\theta})}. \quad (18)$$

The used notation is self-explaining. For instance,  $U(C, W)(p(\bar{C}, \bar{W}))$  is the utility at price  $p(\bar{C}, \bar{W})$  of an individual choosing to consume the differentiated good and to work. All these utilities do not depend on  $\nu$ . All the values  $\nu < 1/2$  are eligible to be chosen.

At price  $p(C, W)$  for  $\nu \geq 1/2$ , the utilities are given by:

$$U_{\mathcal{H}_1}(C, W)(p(C, W)) = (\theta - \alpha)q + \frac{q(\bar{\alpha} + (1 - \nu)\bar{\theta})^2}{4\nu(\bar{\alpha} + \bar{\theta})} \quad (19)$$

$$U_{\mathcal{H}_1}(C, \bar{W})(p(C, W)) = \theta q - \frac{q\bar{\theta}(\nu(2\bar{\alpha} + \bar{\theta}) - (\bar{\theta} + \bar{\alpha}))}{2\nu(\bar{\alpha} + \bar{\theta})} + \frac{q((\nu - 1)\bar{\theta} - \bar{\alpha})(\bar{\theta}(\nu + 1) + \bar{\alpha})}{4\nu(\bar{\alpha} + \bar{\theta})} \quad (20)$$

$$U_{\mathcal{H}_1}(\bar{C}, W)(p(C, W)) = -\alpha q + \frac{q\bar{\alpha}(\bar{\alpha} + (\nu + 1)\bar{\theta})}{2\nu(\bar{\alpha} + \bar{\theta})} + \frac{q((\nu - 1)\bar{\theta} - \bar{\alpha})(\bar{\theta}(\nu + 1) + \bar{\alpha})}{4\nu(\bar{\alpha} + \bar{\theta})} \quad (21)$$

$$U_{\mathcal{H}_1}(\bar{C}, \bar{W})(p(C, W)) = \frac{q((\nu - 1)\bar{\theta} - \bar{\alpha})(\bar{\theta}(\nu + 1) + \bar{\alpha})}{4\nu(\bar{\alpha} + \bar{\theta})} \quad (22)$$

Therefore, we have to compare the number of individuals choosing  $\nu = 1$  and those choosing some  $\nu < 1/2$ . This comparison stems from the comparison for each individual between the two options. To do so, we have first to determine the decisions of each individual in terms of consumption and work at each option, i.e. for each price. The determination of the consumption and work decisions at each price ( $p(C, W)$  or  $p(\bar{C}, \bar{W})$ ) amounts to position the considered price relative to  $\theta q$  and  $\bar{\theta}q(1 - \frac{\alpha}{\bar{\alpha}})$ . Doing this the consumption and work decisions of individuals are depicted in Figures 2 and 3, denoting by:

$$\hat{\alpha} = \frac{\bar{\alpha}\bar{\theta}}{2(\bar{\theta} + \bar{\alpha})}, \quad (23)$$

$$\hat{\theta} = \frac{\bar{\theta}(\bar{\theta} + 2\bar{\alpha})}{2(\bar{\theta} + \bar{\alpha})}, \quad (24)$$

$$\tilde{\alpha}(\nu) = \frac{\bar{\alpha}(\bar{\theta}(\nu + 1) + \bar{\alpha})}{2\nu(\bar{\theta} + \bar{\alpha})}, \quad (25)$$

$$\tilde{\theta}(\nu) = \frac{\bar{\theta}(\bar{\theta}(\nu - 1) + (2\nu - 1)\bar{\alpha})}{2\nu(\bar{\theta} + \bar{\alpha})}. \quad (26)$$

As the comparison must be done between  $\nu = 1$  and  $\nu < 1/2$ , Figures 2 and 3 have to be superimposed making  $\nu = 1$  in Figure 3. Denoting by  $\tilde{\alpha} = \tilde{\alpha}(\nu = 1)$  and  $\tilde{\theta} = \tilde{\theta}(\nu = 1)$  and noting that  $\hat{\alpha} < \tilde{\alpha} < \bar{\alpha}$  and  $\tilde{\theta} < \hat{\theta} < \bar{\theta}$ , Figure 4 shows the 9 zones of interest for the comparison of the utilities.

We then compare on each zone the utility of the individual at  $p(\bar{C}, \bar{W})$  and at  $p(C, W)$  making  $\nu = 1$ . Simple calculations lead to Figure 5, where

$$\theta_1 = \frac{2\bar{\alpha}\bar{\theta} + \bar{\theta}^2 - \bar{\alpha}^2}{4(\bar{\alpha} + \bar{\theta})} \quad (27)$$

$$\theta_2 = \frac{4\bar{\alpha}\bar{\theta} + \bar{\theta}^2 + \bar{\alpha}^2}{4(\bar{\alpha} + \bar{\theta})} \quad (28)$$

noting that for  $\gamma < 1$ , we have  $\tilde{\theta} < \theta_1 < \theta_2 < \hat{\theta}$ .

Now we calculate the area corresponding to the choice  $\nu = 1$  and the one corresponding to the choice  $\nu < 1/2$  and compare them. The comparison shows that for  $\gamma < 1$ ,  $\nu = 1$  always wins a majority.

## Part 2: under $\mathcal{H}_2$ .

If the output price  $p(C, W)$  wins, we just have to add to the expressions of the utilities given in Equations from 19 to 22, the share of each individual in the

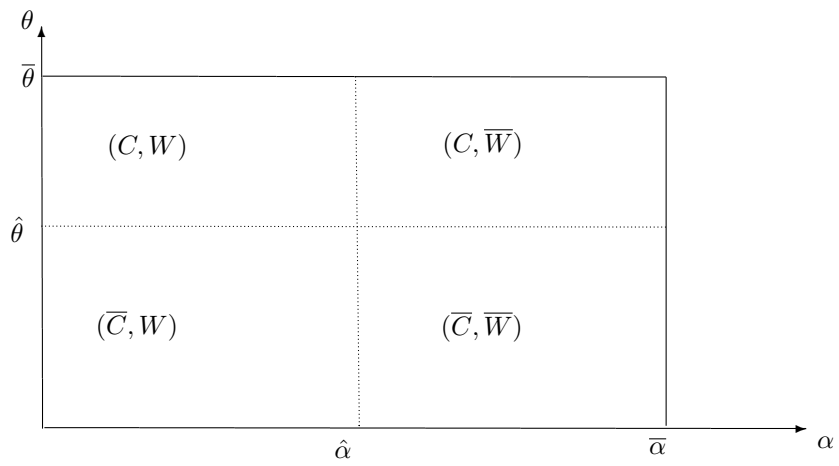


Figure 2: The choice of each individual in terms of consumption and work at  $p(\bar{C}, \bar{W})$ .

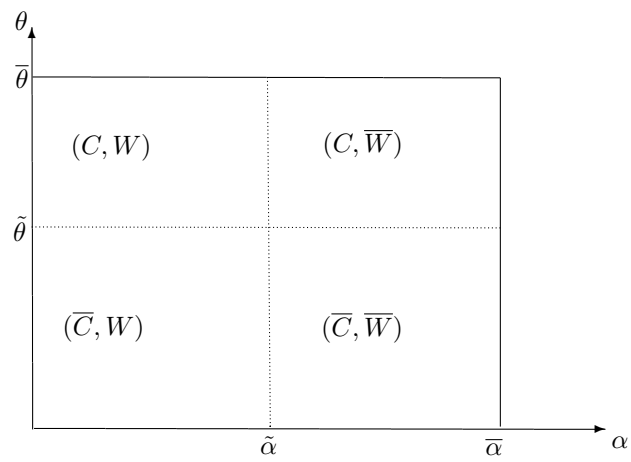


Figure 3: The choice of each individual in terms of consumption and work at  $p(C, W)$ .

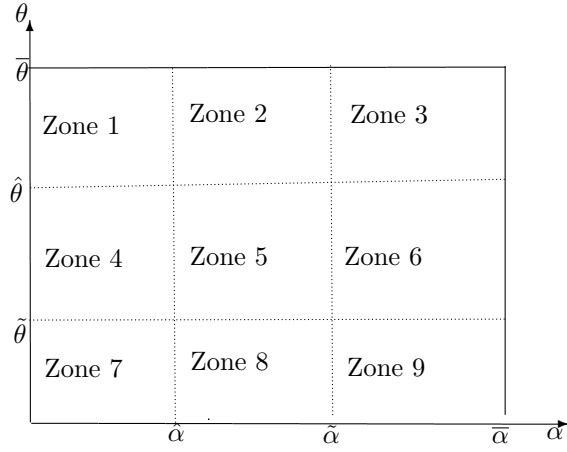


Figure 4:

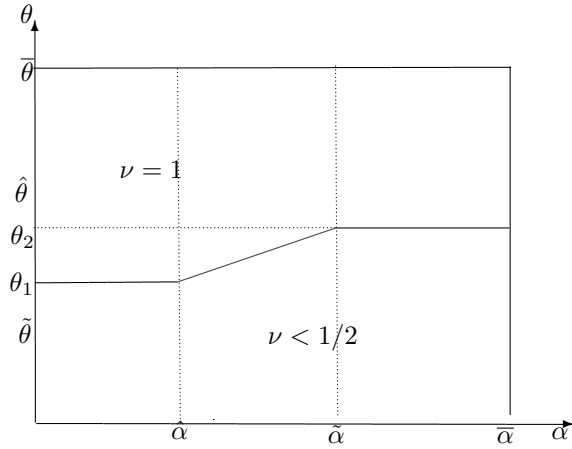


Figure 5: Votes in terms of  $\nu$  for  $\gamma \leq 1$  under  $\mathcal{H}_1$ .

losses, equal to  $(1 - \nu) \frac{\pi}{\alpha \bar{\theta}}$ . Recall that the expression of  $\pi$  is given by Equation 14.

We prove easily that all the obtained utilities  $U_{\mathcal{H}_2} = U_{\mathcal{H}_1} + (1 - \nu) \frac{\pi}{\alpha \bar{\theta}}$  at  $p(C, W)$  are increasing in  $\nu$ .

For  $\gamma \leq 1$ , for all  $\nu \geq 1/2$ ,  $p(C, W)$  is chosen by a majority. Since  $U_{\mathcal{H}_2}$  is always increasing in  $\nu$ , among  $\nu \geq 1/2$ , only  $\nu = 1$  is eligible. We have thus to compare the options  $\nu = 1$  using  $U_{\mathcal{H}_2}$  and the option  $\nu < 1/2$ . But at  $\nu = 1$ ,  $U_{\mathcal{H}_1}$  and  $U_{\mathcal{H}_2}$  are equal. The comparison has already been made under  $\mathcal{H}_1$  and we concluded that  $\nu = 1$  wins a majority.

For  $\gamma > 1$ , there is a change for the output price at  $\tilde{\nu}$  satisfying  $\gamma = S(\tilde{\nu})$ , thus given by:

$$\tilde{\nu} = \frac{\gamma + 1}{2\gamma}.$$

For  $1/2 \leq \nu \leq \tilde{\nu}$ , the output price  $p(C, W)$  is chosen; and for  $\nu > \tilde{\nu}$ , the output price  $p(\bar{C}, \bar{W})$  is chosen. For  $\nu < 1/2$ , the output price  $p(\bar{C}, \bar{W})$  is also chosen and the situation amounts for everybody to  $\nu > \tilde{\nu}$ . Given the increasingness of the utilities  $U_{\mathcal{H}_2}$ , the comparison to be made relevantly is between  $\nu = \tilde{\nu}$  and  $\nu = 1$  (equivalent to any other  $\nu > \tilde{\nu}$  or  $\nu < 1/2$  but requiring no transaction). As done previously, we have to superimpose Figures 3 and 2, but this time making  $\nu = \tilde{\nu}$  in Figure 3, and obtain the equivalent of Figure 4, to determine the choice of each individual in terms of consumption and work. We have

$$\tilde{\alpha}(\tilde{\nu}) = \frac{\bar{\theta}\gamma(1 + 2\gamma)}{2(\gamma + 1)},$$

and

$$\tilde{\theta} = \frac{\bar{\theta}}{2(\gamma + 1)}.$$

As for the utilities, we have to determine  $U_{\mathcal{H}_2}(p(C, W))$  at  $\nu = \tilde{\nu}$ . They are given by the following.

$$U_{\mathcal{H}_2}(C, W)(p(C, W)) = (\theta - \alpha)q + \frac{\bar{\theta}q\gamma(2\gamma + 1)}{2(\gamma + 1)} - \frac{q\bar{\theta}}{2(\gamma + 1)} + \frac{q\bar{\theta}(1 - 4\gamma^2)}{4(\gamma + 1)}, \quad (29)$$

$$U_{\mathcal{H}_2}(C, \bar{W})(p(C, W)) = \theta q - \frac{q\bar{\theta}}{2(\gamma + 1)} + \frac{q\bar{\theta}(1 - 4\gamma^2)}{4(\gamma + 1)}, \quad (30)$$

$$U_{\mathcal{H}_2}(\bar{C}, W)(p(C, W)) = -\alpha q + \frac{\bar{\theta}q\gamma(2\gamma + 1)}{2(\gamma + 1)} + \frac{q\bar{\theta}(1 - 4\gamma^2)}{4(\gamma + 1)}, \quad (31)$$

$$U_{\mathcal{H}_2}(\bar{C}, \bar{W})(p(C, W)) = \frac{q\bar{\theta}(1 - 4\gamma^2)}{4(\gamma + 1)}. \quad (32)$$

The comparison on each zone among the 9 zones, between the utilities leads to Figure 6.

The area corresponding to  $\nu = 1$  is equal to  $\frac{\gamma^2 \bar{\theta}^2}{\gamma + 1}$ , while the area corresponding to  $\nu = \tilde{\nu}$  is equal to  $\frac{\bar{\theta}^2 \gamma (4\gamma + 3)}{4(\gamma + 1)^2}$ . The comparison between the two expressions



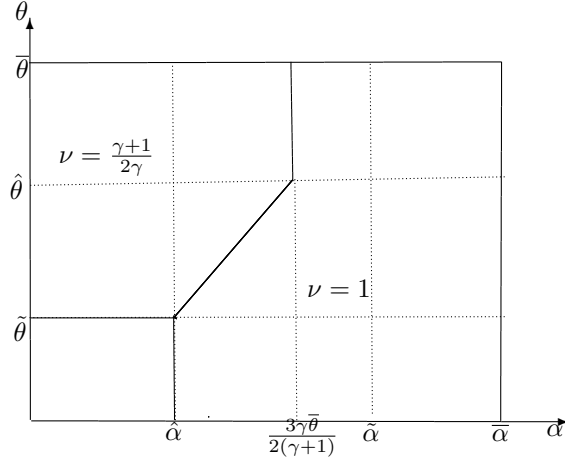


Figure 6: Votes in terms of  $\nu$  for  $\gamma > 1$  under  $\mathcal{H}_2$ .

shows that the area corresponding to  $\nu = 1$  is larger than the second one for all  $\gamma > 1$ .

#### Proof of Proposition 4

**First case**  $\gamma < 1$ . We have  $p_0 = p(C, W)$  at  $\nu = 1$ . Two types of privatization have to be considered.

The first one is such that the State retains the control of the firm, thus  $\nu \in [1/2, 1]$ , which implies price  $p(C, W)$  at step 3 and a negative profit. Under  $\mathcal{H}_1$ , the firm is unsalable and the only eligible  $\nu \in [1/2, 1]$  is  $\nu = 1$ .

Under  $\mathcal{H}_2$ , one has simply to replace  $p_1$  by  $p(C, W)$  (with an arbitrary  $\nu \in [1/2, 1]$ .) We obtain

$$W_1(\nu) = \frac{q\bar{\theta}^3(3\nu - 1 - \gamma)(1 + \gamma + \nu)}{8\nu^2(1 + \gamma)}. \quad (33)$$

This is an increasing function of<sup>19</sup>  $\nu$ , which takes its maximum value at  $\nu = 1$ . Any partial privatization where the State retains the control of the firm decreases social welfare as defined here.

To conclude partially, whether under  $\mathcal{H}_1$  or  $\mathcal{H}_2$ ,  $\nu = 1$  is the local maximum or the only eligible value among  $\nu \in [1/2, 1]$ .

The second type of privatization is a radical one transferring the control of the firm to private investors who maximize profits. Therefore we have to compare the welfare at  $\nu = 1$  with the value of welfare for  $\nu < 1/2$ .

Evaluating  $W_0$  at  $p_0 = p(C, W)$  and  $\nu = 1$ , we obtain

<sup>19</sup>Indeed the derivative of this function w.r.t.  $\nu$  is of the same sign as  $\gamma + 1 - \nu$ , which is positive as  $\nu \leq 1$ .

$$\frac{q\bar{\theta}^3(4-\gamma^2)}{8(1+\gamma)}. \quad (34)$$

A radical privatization would lead to  $W_1$  evaluated at  $p_1 = p(\bar{C}, \bar{W})$ , i.e;

$$\frac{3q\bar{\theta}^3\gamma}{8(1+\gamma)}. \quad (35)$$

Subtracting (34) from (35), one obtains

$$\frac{3q\bar{\theta}^3\gamma}{8(1+\gamma)} - \frac{q\bar{\theta}^3(4-\gamma^2)}{8(1+\gamma)} = q\bar{\theta}^3 \frac{(\gamma-1)(\gamma+4)}{8(\gamma+1)}$$

so that we can conclude that a radical privatization would reduce social welfare.

**Second case**  $\gamma > 1$ . In the initial situation, the firm maximizes profits and  $p_0 = p(\bar{C}, \bar{W})$ . Accordingly  $W_0$  equals (35). Any radical privatization or a partial privatization toward some  $\nu > \frac{1+\gamma}{2\gamma}$  obviously yields  $W_1 = W_0$  since the firm remains a profit-maximizer. Thus the situation would be equivalent in terms of welfare.

Let us now consider a partial privatization toward  $\nu \in \left[1/2, \frac{1+\gamma}{2\gamma}\right]$ .

Under  $\mathcal{H}_1$  this privatisation is not possible as the firm's profit is negative making the firm unalable. Thus the possible values of  $\nu$  are only the ones belonging to  $\in [0, 1/2 \cup [\tilde{\nu}, 1]$  and the proof ends here.

Under  $\mathcal{H}_2$ , a partial privatization toward  $\nu \in \left[1/2, \frac{1+\gamma}{2\gamma}\right]$  leads to a social welfare given by (33) which has been shown to be increasing in  $\nu$ . So we have to compare  $W_0$  given by (35) with the  $W_1$  given by (33) for  $\nu = \frac{1+\gamma}{2\gamma}$ . The latter is:

$$W_1\left(\frac{1+\gamma}{2\gamma}\right) = q\bar{\theta}^3 \frac{(3-2\gamma)(2\gamma+1)}{8(\gamma+1)}.$$

The difference

$$W_1 - W_0 = q\bar{\theta}^3 \frac{(3+\gamma-4\gamma^2)}{8(1+\gamma)} = q\bar{\theta}^3 \frac{(-4\gamma-3)(\gamma-1)}{8(\gamma+1)} < 0.$$

Thus such a partial privatization will decrease the total welfare.