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# Ingredientes para elaborar um resumo expandido

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10 de março de 2020

Divulgar a pesquisa é parte crucial de qualquer bom trabalho científico, sendo uma tarefa bem mais complexa que apenas escrever sobre resultados e metodologia. É essencial que a escrita seja bem elaborada, com coerência gramatical, clareza, concisão e objetividade, pois é através desta que o trabalho será defendido e registrado. Elaborar bem o texto é um dos ingredientes para ter seu trabalho aceito num bom veículo de divulgação. Portanto, dedicar-se à redação é extremamente importante para o sucesso de um trabalho acadêmico. Nesse sentido, o presente texto apresenta, de maneira simplista, algumas dicas de como escrever um resumo expandido, usualmente exigido em eventos científicos (congressos, conferências, simpósios e etc.) quando da submissão de um trabalho de pesquisa para apresentação.

## 1 Passos preliminares

É preciso ter em mente que um resumo expandido é mais extenso que o resumo simples e sua principal diferença está na estrutura, na disposição das informações e a permissão de comparar o trabalho com outros da literatura. O resumo expandido deve apresentar, de forma concisa, a ideia geral do trabalho, dando ao autor mais liberdade para desenvolver uma ideia do um resumo simples, pois permite a inclusão de tabelas e gráficos que facilitem a compreensão do texto. Dessa forma, é preciso estar ciente das regras do evento para estruturar o trabalho (popularmente conhecido como *template* do evento), verificando as exigências, como número máximo de palavras ou páginas, estilo e tamanho da fonte, formatação do texto e das referências, entre outros. Lembrando que cada evento possui suas particularidades, que devem ser seguidas fielmente. Lendo essas instruções o autor terá uma noção de como deverá organizar suas ideias para expor no texto atendendo as exigências. Esse é o primeiro dever de casa!

## 2 Organização de ideias

Essa etapa é basicamente escrever um esboço de tudo que será citado no trabalho. Existem algumas ferramentas como diagramas e mapas mentais que colaboram na organização das ideias. Deve-se inserir neste esboço a ideia central para cada parte do resumo, desde a introdução até a conclusão. Em suma, o autor deve ter ciência do irá escrever em todo o texto. Nesta etapa é importante o autor indicar a cronologia que irá discorrer no texto, lembrando que uma organização linear das ideias facilitará o entendimento do leitor.

## 3 Estrutura da redação

O resumo expandido inicia-se por uma breve introdução ao problema de interesse. Uma boa prática é iniciar contextualizando o tema a ser discutido, visando proporcionar ao leitor um cenário histórico, atual ou de tendências futuras. Essa contextualização facilitará ao autor introduzir motivação e justificativa(s). Essa parte tem que ser muito bem elaborada, pois é aqui que você irá atrair a atenção do leitor, e esse vai decidir se irá continuar a leitura. Em seguida, define-se o objetivo do trabalho, que é um ponto de importância central, pois está intimamente conectado à

contribuição que o trabalho traz ao estado da arte. Esse deve ser direto e escrito numa linguagem simples, para que o leitor tenha melhor entendimento. No desenvolvimento do trabalho apresenta-se, de forma breve, a metodologia utilizada, procurando referenciar trabalhos que detalham a explicação. Após toda a definição da metodologia, apresenta-se os resultados obtidos, sendo de grande valia sua organização na forma de figuras, tabelas ou gráficos, pois isso facilita a visualização e compreensão por parte do leitor. Entretanto, deve-se mencionar no texto o que essas figuras, tabelas e gráficos apresentam. Além disso, deve-se apresentar a interpretação dos resultados e enfatizar seus impactos, sendo enfático e buscando deixar evidente ao leitor os resultados obtidos. Por fim conclua o trabalho ressaltando a ideia principal e se a tese apresentada foi defendida.

## 4 O desafio do limite de palavras

A grande dificuldade para elaborar um resumo expandido é, sem sombra de dúvidas, a limitação no número de palavras e páginas (geralmente entre uma e duas). Dessa forma, o autor deve ser objetivo em suas frases, aproveitando apenas as ideias essenciais do trabalho. São consideradas boas práticas de redação: (i) evitar repetição de palavras; (ii) usar voz ativa (sujeito antes do verbo); (iii) correlacionar ideias em uma única frase; (iv) generalizar conceitos e utilizar conjunções adequadas. Uma técnica importante é usufruir ao máximo do espaço disponível, não permitindo que a última linha de um parágrafo esteja praticamente vazia. Utilizar figuras, tabelas e equações com alta resolução e excelente padrão tipográfico (sempre que for permitido, escrever o texto em  $\text{\LaTeX}$ ). Abusar da possibilidade de inserir informações gráficas numa mesma figura. Além disso, inserir legendas ricas em detalhes, descrevendo as informações disponíveis na figura, colaborando dessa forma para passar ao leitor informações que não cabem no texto.

## 5 Usando referências

Num resumo expandido, assim como em qualquer manuscrito acadêmico, referências bibliográficas são obrigatórias e servem para dar solidez ao trabalho, oferecendo ao leitor informações sobre o que já foi feito na área de interesse e o que se encontra disponível na literatura. Ressalva-se que, preferencialmente, as referências devem ser atuais (últimos 5 anos), apresentando ao leitor um contexto atualizado. Em se tratando de um resumo expandido, devido ao espaço limitado, as referências bibliográficas devem ser selecionadas com maior atenção, citando apenas aquelas de maior relevância para fundamentar o trabalho. Tenha atenção à formatação das referências segundo as regras de submissão do evento, que pode limitar até a quantidade de caracteres da mesma.

## Considerações finais

Essas são boas práticas, que podem auxiliar bastante na elaboração de um resumo expandido, porém, deve-se salientar que não é um procedimento rígido, cada trabalho tem particularidades que podem demandar considerações adicionais. A título de exemplo, em anexo a esse documento, o leitor encontra alguns resumos elaborados segundo as orientações acima [1, 2, 3, 4].

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# ANEXO

# AN APPLICATION OF THE GLOBAL SENSITIVITY ANALYSIS ON A BISTABLE ENERGY HARVESTER

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## ABSTRACT

Piezoelectric energy harvesting technologies are being explored by many researchers, due to their high potential to provide clean energy for low consumption devices. It consists of an electromechanical system capable of converting mechanical energy, dissipated in the environment, into electrical energy. Currently, one of the main challenges is to optimize the power harvesting over a wide frequency range and, according to [1], an alternative is the inclusion of non-linearities. Nonlinear systems require rigorous studies and analysis, as they can present chaotic behavior. This work presents an application of a global sensitivity analysis based on the Sobol indices, defined by [2], which consists of a decomposition of the total variation of the model's output, quantifying the individual and joint effect of each model's input variability. The piezo-magneto-elastic energy harvester (PMEH) system studied consists of a ferromagnetic cantilever beam attached in vertical to a rigid base excited by a harmonic force, a pair of magnets is placed at the rigid base in lower part, and two piezoelectric laminae are placed on beam's highest part, being also connected to an electrical circuit, as illustrated in Figure 1a. According to [3], the equation of motion is given by

$$\ddot{x} + 2\xi \dot{x} - 0.5x(1 - x^2) - \chi v = f \cos(\Omega t) \quad \text{and} \quad \dot{v} + \lambda v + \kappa \dot{x} = 0, \quad (1)$$

and the averaged power harvested, that is the quantity of interest, is given by

$$P = \frac{1}{T} \int_{t_i}^{t_f} \lambda v(t)^2 dt. \quad (2)$$

The global sensitivity analysis based on Sobol' indices employs an orthogonal decomposition in terms of conditional variances, making it possible to measure the (individual and/or joint) effects of the model parameters variability on the average power (quantity of interest) [2]. In Figure 1b, it is possible to verify that the electric piezoelectric coupling ( $\kappa$ ) and the frequency of the external force ( $\Omega$ ) have a higher degree of influence on the system's response, respectively. And in Figure 1c has the sensitivity of the joint effect that has less influence than first-order. Thus, this analysis becomes essential, since non-linear systems can have high sensitivity.

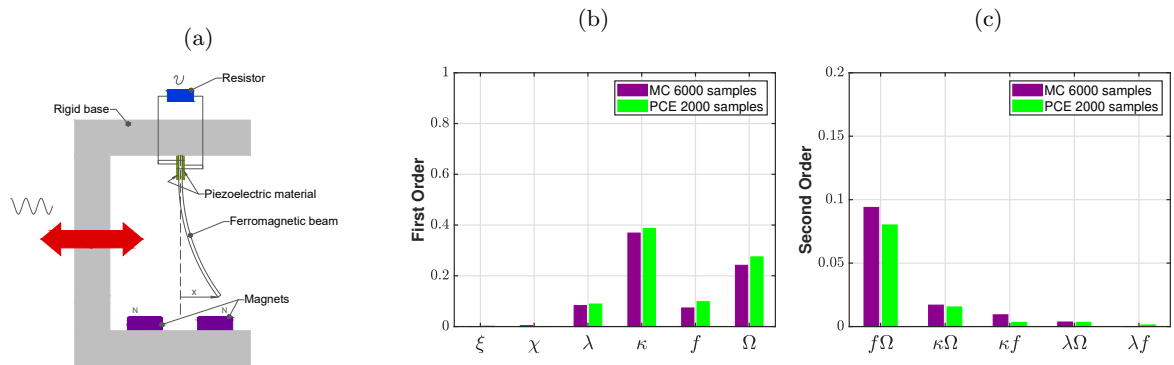


Figure 1: (a) PMEH schematic illustration (b) First-Order Sobol' Indices (c) Second-Order Sobol' Indices. System parameters:  $\xi = 0.01$ ,  $\chi = 0.05$ ,  $\lambda = 0.05$ ,  $\kappa = 0.5$ ,  $\Omega = 0.8$ ,  $f = 0.147$ ,  $(x_0, \dot{x}_0, v_0) = (1, 0, 0)$ , and  $\sigma = 20\%$ .

**Keywords:** energy harvesting, bistable system, global sensitivity analysis, Sobol' indices

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# Stochastic Dynamics of Inclined Risers Induced by Pulsating Internal Fluid Flow

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**Summary.** In this work, we numerically explore the stochastic dynamics of inclined marine risers subjected to pulsating internal fluid flow. The presence of geometric nonlinearities with static deflection makes the response of the inclined riser different from conventional top tension risers when subjected to pulsating flows. At first, the riser model is solved via Galerkin method and validated using perturbation approaches. Then, we study the propagation of uncertainties i.e. amplitude and frequency of pulsations in the stochastic model revealing rich and complex dynamics features.

## Introduction

Pulsating flow is a phenomenon that affects the oil and gas industries. It occurs due to abrupt perturbations and fluctuations in the internal fluid flow of the riser pipe which in return can affect and influence the vibrational motion of the structure. It occurs due to several reasons such as the nature of the multi-phase flow and sudden geometric changes [1]. Because the value of the excitation amplitude and frequency of fluctuation of the flow are uncertain i.e. stochastic, the influence of the flow can be sever especially if the frequency of these flows are near structural resonances of the riser making them prone to failure by fatigue.

## Problem Formulation

The inclined riser to be analyzed in this work is under mid-plane stretching and subjected to static deflection [2] and pulsating internal fluid flow. Then, the equation that describes the motion of the riser in dimensionless form can be written as

$$\begin{aligned} & \frac{\partial^2 y_d}{\partial t^2} + \frac{\partial^4 y_d}{\partial x^4} + 2\sqrt{\beta}v \frac{\partial^2 y_d}{\partial x \partial t} + \sqrt{\beta} \frac{\partial v}{\partial t} (1-x) \frac{\partial^2 y_d}{\partial x^2} + c \frac{\partial y_d}{\partial t} + c_d \frac{\partial y_d}{\partial t} \left| \frac{\partial y_d}{\partial t} \right| + (v^2 - T + \sigma(1-x)) \left( \frac{\partial^2 y_d}{\partial x^2} \right) - \sigma \left( \frac{\partial y_d}{\partial x} \right) \\ & - \left( \eta \left( \int_0^1 \left( \frac{dy_s}{dx} \right)^2 dx + \int_0^1 \left( \frac{\partial y_d}{\partial x} \right)^2 dx + \int_0^1 2 \frac{dy_s}{dx} \frac{\partial y_d}{\partial x} dx \right) \left( \frac{\partial^2 y_d}{\partial x^2} \right) - \eta \left( \int_0^1 2 \frac{dy_s}{dx} \frac{\partial y_d}{\partial x} dx + \int_0^1 \left( \frac{\partial y_d}{\partial x} \right)^2 dx \right) \left( \frac{d^2 y_s}{dx^2} \right) \right) = 0 \end{aligned} \quad (1)$$

where  $y_d$  is the deflection of the riser along position  $x$  and time  $t$ ,  $\beta$  is fluid mass parameter,  $\sigma$  is self-weight parameter,  $v$  is internal fluid flow velocity,  $\eta$  is nonlinear geometric parameter,  $c$  is structural damping,  $c_d$  is external fluid damping and  $T$  is the applied top tension. Due to flow fluctuations, the internal velocity is assumed to have the form  $v = V(1 + \gamma \cos(\Omega t))$  where  $V$  is the magnitude of the internal velocity,  $\gamma$  is a detuning parameter between 0 and 1 and  $\Omega$  is the excitation frequency. Equation (1) is solved via Galerkin method utilizing the procedures prescribed in [3] and [4] and validated using perturbation method defined in [5-7]. Next, we consider a probabilistic frame of work in which the amplitude  $\gamma$  and the excitation frequency  $\Omega$  are random variables. Then, we study the influence of type of different probability distributions (PDF) on the dynamic response of the structure using Monte Carlo (MC) simulations with  $2^{13}$  samples.

## Numerical Results

At first, the deterministic model is analyzed. Due to the quadratic nature of the internal fluid flow, the excitation frequency is expected to occur at  $\Omega$  and  $2\Omega$  because of the nature of the parametric excitation. The dynamic response of the lowest three modes is depicted in Fig.1

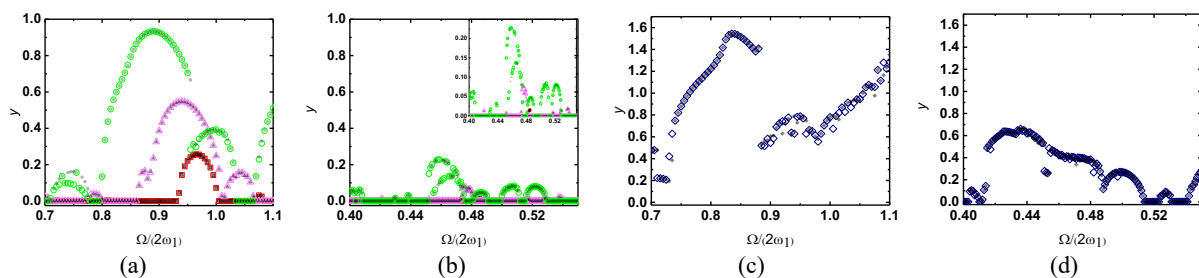


Figure 1: Multi-modal Frequency response curves around the first mode of vibrations for  $V = \sqrt{\sigma}$ ,  $c_d = 0.416$  at  $x=0.48$ . (a,b) Backward Sweep (  $\square$  )  $\gamma = 0.25$ , (  $\triangle$  )  $\gamma = 0.50$ , (  $\circ$  )  $\gamma = 0.75$ . The inset is magnified results for case (b). (c,d) (  $\diamond$  )  $\gamma = 1$ . Filled shapes denotes forward sweep.

We observe, in Fig. 1, the influence of the different components that exists in the system as a result of the interaction. The influence of the softening nonlinearity is less apparent due to the competing effects between the first mode and contributions from other modes that exist in the response. This is observed very well in Fig. 1b in comparison to other cases. In addition, the interaction of other resonances with the response of the solution near  $\Omega$  and  $2\Omega$  become more visible at higher fluctuating velocity as the solution demonstrate chaotic behavior.

Next, we consider the stochastic response of the riser. The results from the MC method are divided into ten categories considering the two excitation frequency ranges and the combination of the distributions for the amplitude utilizing Beta and uniform distributions where they are applicable. As an example, we demonstrate the response of the riser under Beta distribution for excitation at a frequency of  $\Omega/2\omega_1 = 0.464$  in Fig.2

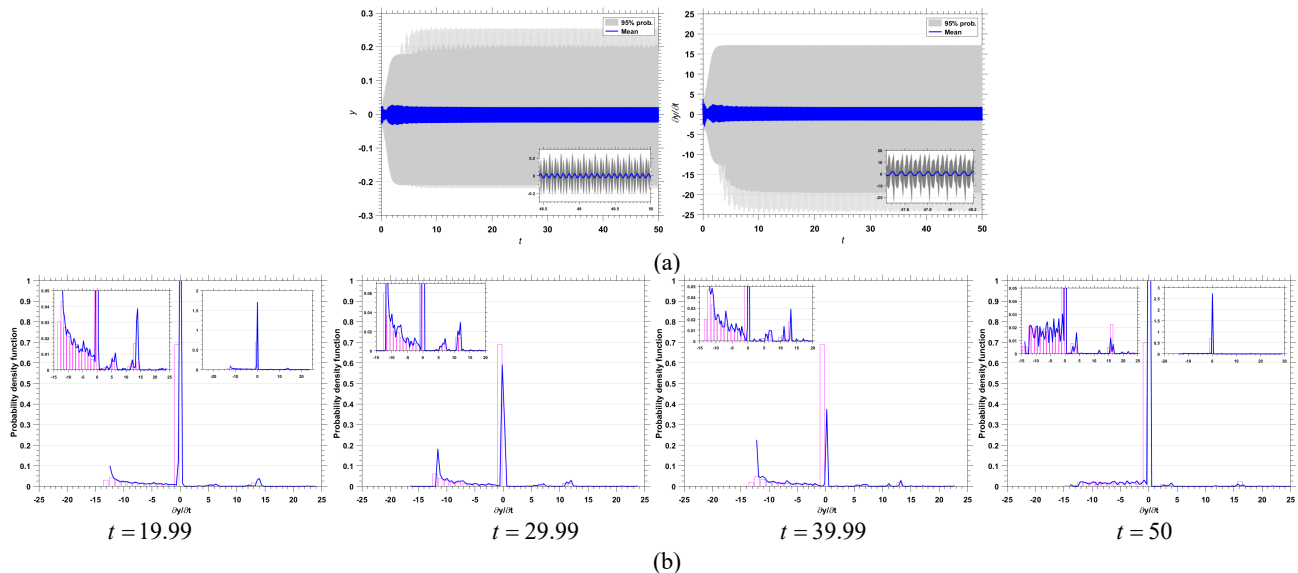


Figure 2:  $\Omega/2\omega_1 = 0.464$  (a) Illustration of the nominal value (Blue line) and the 95 % envelop (grey shadow) of the displacement (left) and the velocity (right) of the riser. The box in each figure is a magnified response of the riser. (b) Evolution of the normalized probability density function as a function of the rise velocity at different times. The box in each figure is a magnified distribution of the probability density function.

The response of the riser displacement is exhibited on the left Fig. 2a and we plot the velocity in order to have a better representation of the riser response. The distribution influences type of response because at this frequency the riser excite not only secondary terms due to quadratic nonlinearity but also primary due the squaring of terms. Due to this fact, the mean value is observed to be superseded by different excitation amplitudes constituting the envelope. The propagation of the probability density function at different time interval demonstrate that it is stationary. The main feature is attributed to the primary excitation of the riser structure.

## Conclusions

In this work, the stochastic dynamics of inclined risers is studied considering the influence of pulsating internal fluid flow. The presence of static deflection under the influence of geometric nonlinearity causes multiple resonances to exist. This influences the stochastic response of the riser examined under different distributions. As a result, the response of the riser in the Monte Carlo simulations revealed interesting complex and rich dynamic features

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# Application of a stochastic version of the restoring force surface method to identify a Duffing oscillator

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**Abstract.** This work deals with the identification of stochastic parameters in a nonlinear single degree-of-freedom system. For this purpose, a stochastic version of the restoring force surface method is proposed and used to identify the parameters of a clamped-free beam, with nonlinear effects induced by the presence of a magnet near to the free extremity. This system recalls a Duffing oscillator, which is used as mechanical-mathematical model. In order to validate the obtained stochastic model, experimental and theoretical responses are compared in time and frequency domains, taking into account a probabilistic band of confidence. The results show that obtained stochastic model adequately predict the beam's vibration responses, which ensure the robustness of identification of a stochastic model.

## Introduction

It is known that many engineering structures can present nonlinear behavior caused by geometric effects, severe operating conditions, materials with complex structure and others. So, to perform a reliable analysis of a structure, the nonlinear effects have to be taken into account [1]. Furthermore, most of the approaches used to describe the nonlinear phenomena are deterministic. Then, these models are not robust to variations in the system parameters, neither offer a confidence interval to the identified parameters. However, real systems are uncertain with respect to the project nominal values, due to material imperfections, boundary conditions, noise and others, becoming essential to consider the presence of uncertainties [2]. In this sense, the use of a stochastic version of the restoring force surface (RFS) method is proposed here, where the probability density functions (PDFs) of model parameters are identified, instead of the parameters deterministic values as obtained through the conventional RFS method [3]. In order to illustrate the results, a nonlinear system composed by a clamped-free beam is identified.

## Experimental Setup

The experimental setup is composed by a clamped-free beam ( $300 \times 18 \times 3$  [mm<sup>3</sup>]) with a steel mass glued in the free extremity, which is connected to cause a magnetic interaction between the beam and a magnet (fig. 1(a)). A shaker is used to excite the structure considering different levels of voltage amplitude. A vibrometer laser is also used to measure the beam free extremity velocity. The magnetic interaction of the system generates a nonlinear hardening shown in fig. 1(b). Additionally, the spectrogram of the system response can be seen in fig. 1(c) where is observed the presence of second and third order harmonics in the response.

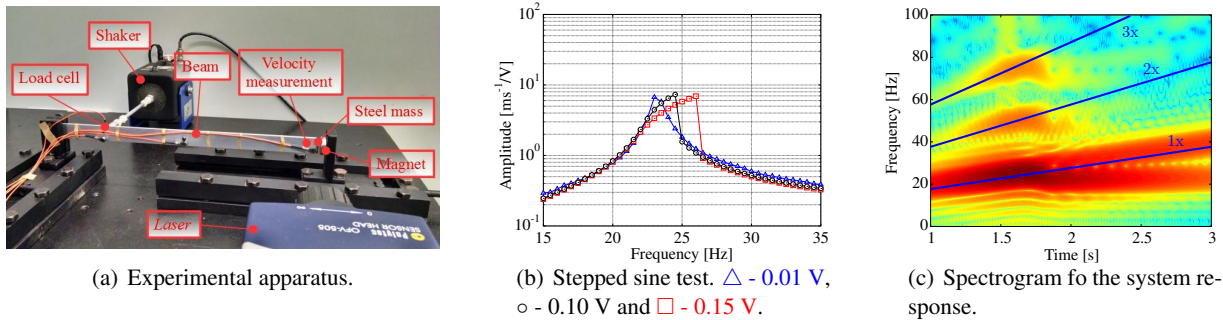


Figure 1: Illustration of the experimental apparatus and its nonlinear behavior.

## Stochastic model - A random Duffing oscillator

The model parameters subjected to uncertainties are described as random variables or random processes, defined on the probability space  $(\Theta, \mathcal{F}, \mathbb{P})$ , where  $\Theta$  is sample space,  $\mathcal{F}$  is a  $\sigma$ -algebra over  $\Theta$ , and  $\mathbb{P}$  is a probability measure. For the Duffing oscillator [4], the stochastic equivalent motion equation is given by

$$\ddot{\theta}(t) + \dot{\theta}(t) + \underbrace{1(\theta, t) + 2(\theta, t)^2 + 3(\theta, t)^3}_{\mathbb{F}(z(\theta, t), t) \rightarrow \text{random restoring force}} = U(t), \quad (1)$$



where the random processes  $(\theta, t)$ ,  $\dot{(\theta, t)}$ , and  $\ddot{(\theta, t)}$ , represent the displacement, velocity and acceleration, and  $(\theta)$  are the random mass and damping coefficient,  $\mathbb{F}((\theta, t), t)$  is the random process that represents the stochastic restoring force,  $k_1(\theta)$ ,  $k_2(\theta)$  and  $k_3(\theta)$  are the random linear, quadratic and cubic stiffness, and  $U(t)$  represents the input signal. The stochastic model of eq. (1) is used to describe the nonlinear random dynamics of the system. The PDFs of mass and damping are identified considering modal analysis. The linear and nonlinear stiffness are identified using the restoring force surface method and the polynomial regression based on the minimization of the squared error (least squares method), for each realization  $\theta$ .

### Parameters Estimation and Model Validation

Figure 2(a) shows the PDF of the quadratic stiffness. The mean value is equal to  $\mu_{k_2} = -30.867 \text{ N/m}^2$  and dispersion  $\delta_{k_2} = 2.72\%$ , with unimodal behavior. The PDF of the cubic stiffness (fig. 2(b)) has also unimodal distribution with mean value  $\mu_{k_3} = 39.859 \times 10^7 \text{ [N/m}^3\text{]}$  and dispersion  $\delta_{k_3} = 4.06\%$ . Figure 2(c) shows the experimental  $\mathbb{F}$  and the polynomial modeling identified with 99% of confidence bands. It is observed a satisfactory prediction of the restoring force. A validation test in the frequency domain was also performed, where the stochastic model is compared with new experimental data from the stepped sine test, considering two levels of input. Figure 3 shows that the stochastic model describes well the system behavior, both in linear as nonlinear regime of motion.

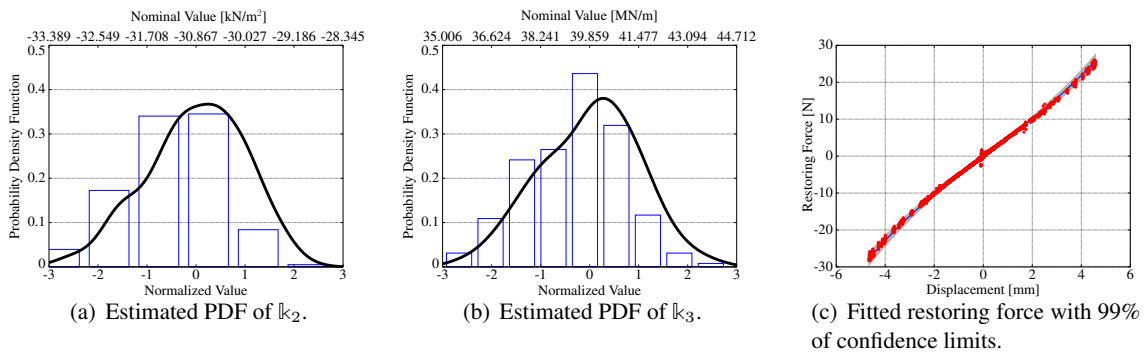


Figure 2: Estimated PDFs for mechanical system stiffnesses and the fitted restoring force.

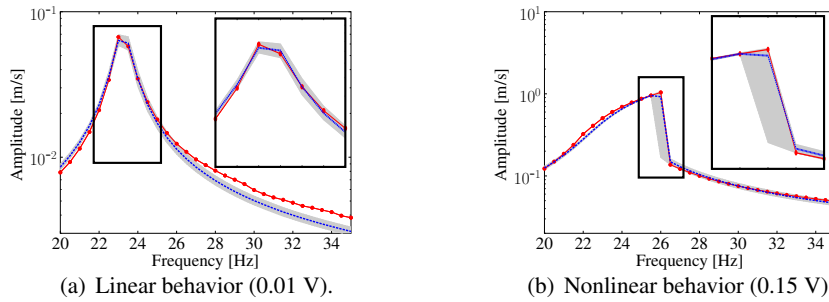


Figure 3: Comparison between experimental and theoretical beam velocity in frequency domain. The stochastic model mean value is presented as  $\text{---}$ , the 99% confidence band as grey shown, and the experimental realization as  $\text{---} \circ \text{---}$ .

### Final Remarks

The stochastic version of the Duffing oscillator identified based on a random restoring force surface method showed satisfactory performance to describe the nonlinear behavior of the system assuming uncertainties.

### Acknowledgements

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## A SEIR-SEI model calibration for the Zika virus epidemic in Brazil

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### EXTENDED ABSTRACT

#### Introduction

The Zika virus has been reported as autochthonously transmissible in over 30 countries and has reached more than 200,000 probable cases in Brazil. Thus, the use of mathematical models for epidemics become of great importance, since they are useful tools to predict the underlying outbreak numbers and allow one to test the effectiveness of different strategies used to combat the associated diseases. In previous works (Dantas, 2017; Tosin, 2017), the authors have identified reasonable parameters and initial conditions (IC) that fit a SEIR-SEI model to epidemic data of the Brazilian outbreak, by the deterministic solution of an inverse problem. This work uses Bayesian Inference to explore the uncertainties associated to the initial number of infectious humans, an IC that lacks reference data on the literature related to the epidemic, updating the associated stochastic model with experimental data provided by the Brazilian Ministry of Health. *Posterior* probability density functions are generated assuming uniform and gamma *prior* distributions over different conditions.

#### Mathematical Model

Figure 1 depicts schematically the evolution of human and mosquito (vector) populations through the compartmental model, with its dynamics governed by the following nonlinear system of ordinary differential equations

$$\begin{aligned} dS_h/dt &= -\beta_h S_h I_v, & dE_h/dt &= \beta_h S_h I_v - \alpha_h E_h, & dI_h/dt &= \alpha_h E_h - \gamma I_h, & dR_h/dt &= \gamma I_h, \\ dS_v/dt &= \delta - \beta_v S_v I_h/N - \delta S_v, & dE_v/dt &= \beta_v S_v I_h/N - (\alpha_v + \delta) E_v, & dI_v/dt &= \alpha_v E_v - \delta I_v, & dC/dt &= \alpha_h E_h, \end{aligned} \quad (1)$$

where the susceptible ( $S$ ), exposed ( $E$ ), infectious ( $I$ ) and recovered ( $R$ ) groups compartmentalize the populations at time  $t$  ( $h$  for human's groups, counted as individuals, and  $v$  for vector's, computed as a proportion of the total vector population);  $C(t)$  regards the cumulative number of infectious people;  $1/\alpha$  is the disease's incubation period;  $1/\gamma$ , the human infection period;  $1/\delta$ , the vector lifespan;  $\beta$ , the transmission rate; and  $N$  is the total human population.

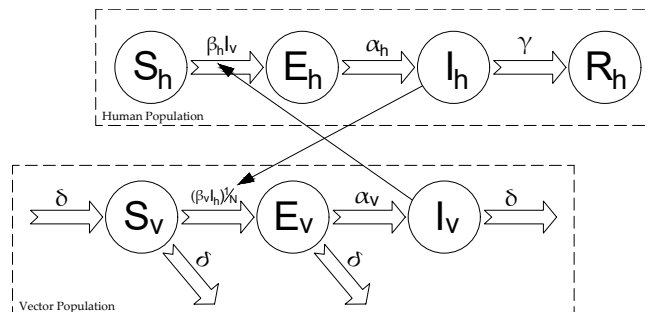


Figure 1: Schematic representation of the SEIR-SEI model for the Zika virus outbreak description.

Deterministic identification of the parameters and IC has been previously conducted (Dantas, 2017; Tosin, 2017), taking into account empirical data of the outbreak (Ministério da Saúde, 2017). Figure 2 displays one fitting of the model, achieved by solution of an inverse problem via a Trust-Region-Reflective method. The main issue of the deterministic fitting is the lack of reference data for some IC, in particular, the initial number of infectious humans ( $I_h^i$ ).

#### Results

Equation 2 summarizes the Bayesian formalism

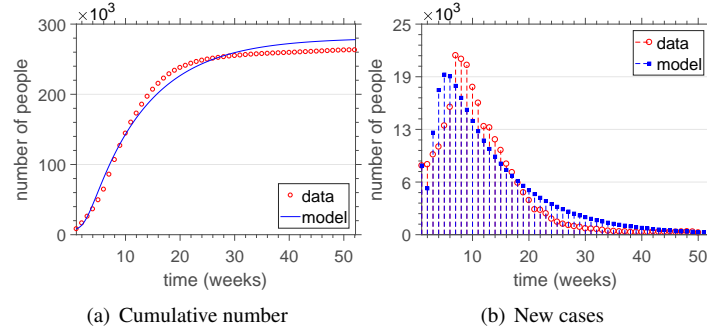


Figure 2: Outbreak data (red) and calibrated model response (blue). Cumulative number of infectious humans on the left, and the number of new infectious cases at each week on the right.

$$\pi(\text{model} \mid \text{data}) \propto \pi(\text{data} \mid \text{model}) \times \pi(\text{model}), \quad (2)$$

where, in this context,  $\pi(\text{model})$  is the *prior* distribution of  $I_h^i$ , analyzed as uniform and gamma distributions;  $\pi(\text{data} \mid \text{model})$  identifies the likelihood, modeled by assuming a Gaussian error on the system when comparing the empirical data with the model response of the fitting in Figure 2; and  $\pi(\text{model} \mid \text{data})$  is the *posterior* distribution, the probability density function of  $I_h^i$  updated by the empirical data. Figure 3 and 4 displays the analysis results for uniform and gamma *priors*, respectively. Different supports of the uniform *prior* were used ( $[0, 25,000]$ ,  $[0, 50,000]$ ,  $[0, 75,000]$ ,  $[0, 100,000]$ ), while on the gamma distributions 4 coefficient of variation were tried (20%, 40%, 60%, 80%) with the same mean of  $\mu = 10^3$ . The standard deviation of the likelihood was kept the same on all cases ( $\sigma = 10^4$ ).

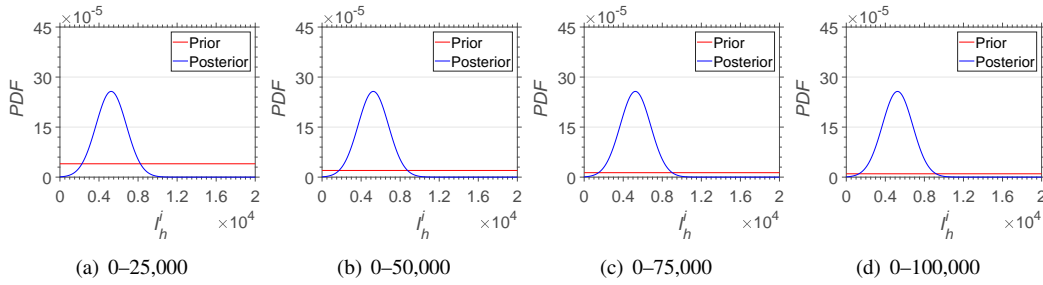


Figure 3: Uniform *priors* and *posteriors* for  $I_h^i$ .

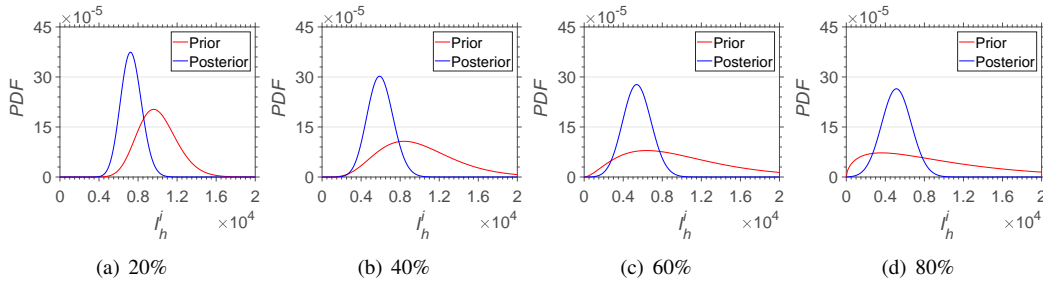


Figure 4: Gamma *priors* and *posteriors* for  $I_h^i$ .

## Conclusion

Multiple updated distributions were generated for  $I_h^i$ , allowing considerations over the effect of the *priors*. In future works, the authors intend to conduct the same procedure to other IC, and implement Markov chain Monte Carlo methods to better analyze the stochastic model adjacent to the SEIR-SEI.

## ACKNOWLEDGEMENTS

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