



HAL
open science

Temperature Effects on Convection Speed and Steepened Waves of Temporally Developing Supersonic Jets

Pierre Pineau, Christophe Bogey

► **To cite this version:**

Pierre Pineau, Christophe Bogey. Temperature Effects on Convection Speed and Steepened Waves of Temporally Developing Supersonic Jets. *AIAA Journal*, 2020, 58 (3), pp.1227-1239. 10.2514/1.J058589 . hal-02504344

HAL Id: hal-02504344

<https://hal.science/hal-02504344>

Submitted on 24 Nov 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Temperature effects on convection speed and steepened waves of temporally-developing supersonic jets

Pierre Pineau^{*} and Christophe Bogey[†]

Univ Lyon, École Centrale de Lyon, INSA Lyon, Université Claude Bernard Lyon I, CNRS, Laboratoire de Mécanique des Fluides et d'Acoustique, UMR 5509, F-69134, Ecully, France

1 **Temperature effects on the convection speed of large-scale structures in supersonic temporally-**
2 **developing jets and on the steepened aspect of the acoustic waves generated by these structures**
3 **are investigated. For that, one isothermal jet at Mach 2 and four others at static temperatures**
4 **equal to 2 or 4 times that of the ambient medium are simulated at either the same exit velocity**
5 **or Mach number as the isothermal jet. For all temporally-developing jets, steepened acoustic**
6 **waves are emitted in the near field, leading to significant values of the pressure skewness and**
7 **kurtosis factors. Using conditional averages, their formation is directly linked to the supersonic**
8 **motion of large-scale structures at a convection speed whose ratio with the jet velocity is shown**
9 **to decrease with temperature. At a higher temperature, this leads to the generation of less**
10 **skewed acoustic waves for a constant jet velocity. For a constant Mach number, however, this**
11 **decrease is compensated by the rise of the jet speed, leading to steeper acoustic waves at a**
12 **higher temperature. Therefore, the modification of the steepened aspect of near-field acoustic**
13 **waves appears to be related to the change in the convection speed in both cases.**

Nomenclature

T	=	temperature
p	=	pressure
ρ	=	density
u	=	velocity
a	=	speed of sound
M	=	Mach number
F	=	Net thrust
\dot{m}	=	Mass flow rate
μ	=	dynamic viscosity
ν	=	kinematic viscosity

^{*}PhD, post-doctoral fellow at CNES, pierre.pineau@doctorant.ec-lyon.fr

[†]CNRS Research Scientist, AIAA senior member

r_s	=	specific ideal gas constant
γ	=	ratio of specific heats
r	=	radial coordinate
θ	=	azimuthal coordinate
z	=	axial coordinate
t	=	time
Δr	=	radial mesh spacing
$\Delta \theta$	=	azimuthal resolution
Δz	=	axial mesh spacing
D	=	jet diameter
r_0	=	jet radius
Re_D	=	diameter-based Reynolds number
δ_θ	=	shear-layer momentum thickness
$\dot{\delta}_\theta$	=	momentum thickness growth rate
$\dot{\delta}_{\theta,inc}$	=	momentum thickness growth rate for an incompressible mixing layer
δ_{05}	=	jet half-width
α	=	Mach wave radiation angle

Subscripts

∞	=	ambient quantity
j	=	initial value of the jet parameters
c	=	potential core closure
axis	=	centerline quantity
cond	=	conditionally averaged
trig	=	conditional averages trigger

I. Introduction

15

16 The effects of temperature on high-speed supersonic jet noise have been the topic of many investigations over the
 17 last decades [1–6]. In particular, the influence of this parameter on Mach wave radiation has been studied by Seiner at
 18 al. [3], Kearney-Fisher et al. [7], and Greska et al. [8], among others. Mach waves are generated when the convection
 19 speed u_c of coherent structures inside the jet is higher than the ambient sound speed a_∞ . In this case, straight, elongated

20 wavefronts are emitted from the jet and propagate downstream at the angle α given by

$$\cos \alpha = \frac{a_\infty}{u_c}. \quad (1)$$

21 Investigating the effects of temperature on Mach wave radiation is challenging given that the speed of sound a_j inside
22 the jet increases with the static temperature T_j as $a_j = \sqrt{\gamma r_s T_j}$, where γ is the ratio of specific heat and r_s is the specific
23 ideal gas constant. In the literature, two different approaches are usually adopted to study temperature effects. A first
24 possibility is to work at a fixed Mach number $M_j = u_j/a_j$, where u_j is the jet speed. This approach is natural from
25 an experimental perspective since it amounts to keeping the same nozzle pressure ratio while varying the stagnation
26 temperature. It has been followed in the studies of Seiner et al. [3], Krothapalli et al. [4], and Mora et al. [9], for instance.
27 In that case, the jet velocity increases with temperature, leading to a strengthening of the acoustic waves radiated by the
28 jet. In addition, the peak of sound emission is shifted toward higher angles relative to the jet direction due to the rise of
29 the convection speed. These effects are clearly visible in the results of Seiner et al. [3], who performed measurements in
30 the sound fields of Mach 2 jets. In this study, the acoustic Mach number of the jets, defined as $M_a = u_j/a_\infty$, however
31 varies from 1.3 to 3, which makes it difficult to isolate the effects of temperature given the strong dependence of jet
32 noise on the exhaust velocity. For that reason, it can be useful to investigate temperature effects for a fixed jet speed, as
33 in the work of Tanna et al. [1, 2] or of Panda [10] for jets at $M_a \simeq 1.5$. In that case, the pressure levels radiated at all
34 angles decrease with temperature, revealing a weakening of the sound sources.

35 The influence of temperature on the formation of steepened, nonlinear acoustic waves near the jets is currently
36 not well understood. When these waves propagate to the far field, they are expected to contribute to the formation
37 of crackle noise [11], which is an unpleasant perception effect. It is known that the pressure levels near highly
38 supersonic jets are sufficiently high that nonlinear propagation effects can occur, leading to a gradual steepening of the
39 wavefronts [12–14]. However, steepened waves have been reported in the near vicinity of the jet flow in experimental
40 measurements [9, 15, 16], as well as in numerical simulations [17–19], that is, too close to be only the result of nonlinear
41 propagation effects. This has led investigators [13, 20] to propose that these waves are produced, at least to a certain
42 extent, by high-intensity events occurring inside the turbulent flow. Unfortunately, there is currently no comprehensive
43 description of the mechanisms involved in their steepening at the source. Since the inclination angles of steepened
44 wavefronts are very close to that of Mach waves, it has been suggested that they constitute a particular, nonlinear case of
45 this phenomenon [18, 19, 21, 22]. Thus, given the influence of temperature on Mach wave radiation, it is reasonable to
46 expect that the mechanisms involved in wave steepening at the source are also affected by temperature.

47 Temperature effects on crackle noise have been studied by Mora et al. [9], who carried out measurements in the
48 near field of hot and cold supersonic jets at $M_j = 1.5$. For the hot jet, the skewness and kurtosis factors of the pressure
49 fluctuations and their time-derivative are higher than for the cold jet, indicating that the radiated acoustic waves are

50 more strongly nonlinear. These results are in agreement with those of Krothapalli et al. [4], who performed optical
51 visualizations of supersonic jets at $M_j = 2$ and at stagnation temperatures ranging from 580 K to 1250 K. At a higher
52 temperature, they observed that more and more steepened waves are present close to the jets. They suggested that
53 these shock-like waves are the consequence of micro explosions caused by the intrusion of cold lumps of ambient
54 fluid entrained in the hot jet core and convected at supersonic speeds. In the two aforementioned studies, the Mach
55 number of the jets is constant, so that the jet speed increases with the temperature. Therefore, it is difficult to conclude
56 whether these results constitute an actual effect of temperature, or if it merely results from the rise of exhaust speed
57 when the jet is heated. In their initial work on jet crackle, Ffowcs Williams et al. [11] reported no clear influence of jet
58 temperature on the formation of steepened acoustic waves for a constant velocity. They however recognized that the
59 range of temperature ratios considered is not sufficiently wide in order to make a definitive conclusion. Since then, very
60 few studies have focused on temperature effects on the formation of steepened waves for a constant jet speed. One
61 exception is the work of Nichols et al. [23], who performed simulations of two hot jets at an acoustic Mach number of
62 2.7 and at static temperature ratios T_j/T_∞ of 2.46 and 3.03. In these simulations, the skewness factor of the pressure
63 fluctuations obtained near the hottest jet is very close to that obtained for the other jet. However, given the proximity of
64 the two temperatures, further investigations are needed.

65 In the present study, the effects of temperature on the steepened waves formed near supersonic jets are therefore
66 investigated by considering the effects of a temperature rise at a constant Mach number, but also at a constant speed. In
67 particular, the importance of the convection speed, which has proven to be a key parameter for the study of crackle [18, 24]
68 is explored. Any parametric investigation of temperature effects in high-speed jets requires a meticulous control over the
69 flow conditions at the nozzle exit. Indeed, the shape of the velocity profile, the turbulence rates, as well as the state of the
70 nozzle boundary layer can affect noise generation [25–28]. In supersonic jets, an additional complexity comes from the
71 formation of shock cells in and downstream of the nozzle. These shock cells can alter the state of the nozzle boundary
72 layer, modify the location of the mixing layers, and cause variations of the convection speed. To prevent their formation,
73 one possibility is to consider jets exhausting at pressure matched conditions from nozzles that are designed using the
74 method of characteristics as in the study of Nonomura et al. [28] for jets at $M_j = 2$. This approach is however costly
75 when different Mach numbers are considered, since it requires the design and the modelling of one nozzle for each
76 Mach number. Furthermore, these nozzles will generate weak shocks since the method of characteristics does not take
77 into account the presence of boundary layers. Another possibility is to consider temporally-developing axisymmetric
78 mixing layers as simplified models for supersonic shock-free jets, as previously done by the authors to investigate sound
79 generation mechanisms [22, 29]. Indeed, these simulations allow us to vary the jet speed and temperature, hence Mach
80 number, by merely prescribing different initial conditions. This enables us to perform parametric studies at a reasonable
81 cost and with a better control over the initial conditions than in simulations of spatially-developing jets. Temporal
82 simulations are performed by imposing periodicity conditions at the axial boundaries of the computational domain, so

Table 1 Flow parameters

Jet	T_j/T_∞	M_j	M_a	M_c	Re_D	δ_θ/r_0	$u'_z(t=0)/u_j$	$t_f u_j/r_0$	n_{run}
jetT1Mj2	1	2	2	1	12,500	0.018	0.02	70	4
jetT2Mj2	2	2	2.8	1.17	12,500	0.018	0.02	100	3
jetT4Mj2	4	2	4	1.33	12,500	0.018	0.02	100	3
jetT2Ma2	2	1.4	2	0.82	12,500	0.018	0.02	60	3
jetT4Ma2	4	1	2	0.67	12,500	0.018	0.02	60	3

that the flow develops over time from a given initial state. Although these model flows are not jets, strictly speaking, as they do not exhaust from a nozzle nor spread in space, they will be designated as temporal jets in the following in order to facilitate the connection between the present results and those from the literature.

One isothermal and four hot temporally-developing jets are computed at a diameter based Reynolds number of 12,500 by large-eddy simulations. The isothermal jet is at $T_j = T_\infty$ and has a Mach number $M_j = M_a = 2$. The hot jets have static temperatures of $2T_\infty$ and $4T_\infty$, and either a Mach number $M_j = 2$ or an acoustic Mach number $M_a = 2$. This set of simulations allows us to investigate the effects of a temperature rise when the jet speed varies or remains constant. The temperature effects on the steepened aspect of the waves are explored by comparing the pressure levels, skewness and kurtosis factors in the near acoustic field. They are related to changes in the flow by computing, in particular, conditional averages of the generation process of steepened waves. These averages indeed allow us to highlight flow features which are directly related to noise generation, including the structures which produce the waves, as well as their convection speed [19].

The paper is organized as follows. First the different jet parameters are introduced and the numerical procedure used for their computations is described in section II. The simulations results are then presented in section III and include snapshots, statistical results such as the pressure spectra, skewness and kurtosis factors, as well as conditional averages. Finally, concluding remarks are given in section IV, and the conditional averaging procedure is applied to a simulation of a spatially-developing jet in appendix in order to verify that the flow structures in temporally-developing flow are similar to those obtained in spatially-developing jets.

II. Numerical methodology

A. Jet parameters

The five temporally-developing jets whose parameters are given in Table 1 are simulated. Their diameter-based Reynolds number $Re_D = u_j D/\nu_j$ is equal to 12,500, where $D = 2r_0$ is the initial jet diameter, and $\nu_j = \mu_j/\rho_j$ is the centerline kinematic viscosity, computed from the Sutherland law. Three of these jets have a Mach number M_j of 2. The static temperatures of these jets, which are labelled jetT1Mj2, jetT2Mj2 and jetT4Mj2, are equal to 1, 2, and 4 times the ambient temperature $T_\infty = 293$ K. The isothermal jet constitutes the reference case of this study, while jetT2Mj2 and

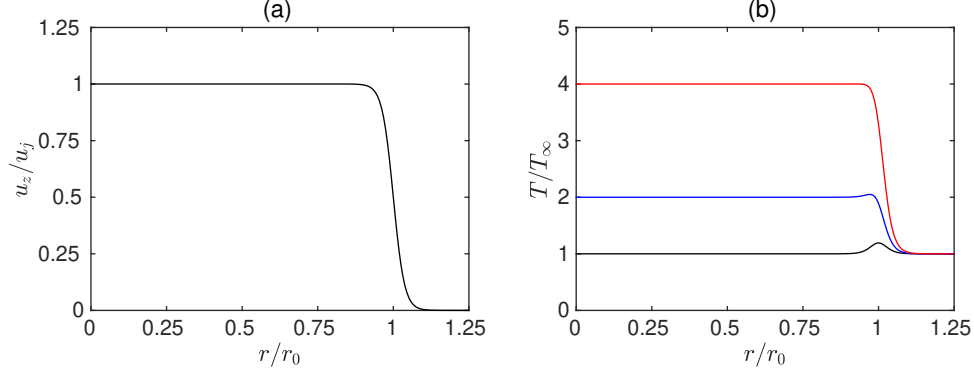


Fig. 1 Initial radial profiles of (a) axial velocity and (b) static temperature for $M_j = 2$ and ——— $T_j/T_\infty = 1$, ——— $T_j/T_\infty = 2$, and ——— $T_j/T_\infty = 4$.

jetT4Mj2 allow us to characterize the effects of a temperature rise at a constant Mach number M_j . In that case, the jet speed increases with the temperature, leading to acoustic Mach numbers M_a of 2.8 and 4 for jetT2Mj2 and jetT4Mj2. In order to isolate the effects of temperature, it is also useful to investigate the effects of a temperature rise at a constant jet speed. This is done here by simulating two hot jets at $T_j = 2T_\infty$ and $4T_\infty$ at the same speed as the isothermal jet, leading to $M_a = 2$ in both cases. Since the speed of sound increases with temperature, the Mach numbers of the jets, labelled jetT2Ma2 and jetT4Ma2, are equal to 1.4 and 1, respectively.

At initial time, the velocity field is initialized with the hyperbolic tangent profile represented in Fig. 1(a). The shear-layer momentum thickness δ_θ of this profile has been chosen such that $\delta_\theta/r_0 = 2/\sqrt{Re_D} = 0.018$, following an empirical relation obtained for initially-laminar subsonic jets [30]. The ambient pressure p_∞ is equal to 10^5 Pa, and the initial temperature profiles are determined from the Crocco-Busemann relation

$$\frac{T}{T_j} = \frac{T_\infty}{T_j} - \left(\frac{T_\infty}{T_j} - 1\right) \frac{u_z}{u_j} + \frac{\gamma - 1}{2} \left(\frac{u_j}{a_j}\right)^2 \frac{u_z}{u_j} \left(1 - \frac{u_z}{u_j}\right), \quad (2)$$

and are plotted in figure 1(b) for jetT1Mj2, jetT2Mj2 and jetT4Mj2. The initial temperature profiles for jetT2Ma2 and jetT4Ma2 are not provided, as they do not significantly differ from those for the jets at $M_j = 2$. For jetT2Mj2 and jetT4Mj2, the temperature is equal to T_j in the jet core and decreases down to T_∞ farther away from the flow. For the isothermal jet, a slight temperature excess is visible around $r = r_0$. It is caused by compressibility effects, as derived from relation (2). Finally, low-amplitude velocity perturbations are added at $t = 0$ inside the jet shear layers, in order to favor the flow transition from a laminar to a turbulent state. They are vortex rings of random azimuthal mode, phase and amplitude [31]. Their mean amplitude has been arbitrarily tuned so that the initial turbulence rate $u'_z(t = 0)/u_j$ is of 2% for each jets.

A summary of the different effects of increasing temperature on the flow properties is given in Fig. 2, where the initial conditions of the five simulated jets are represented in the velocity-temperature plane, along with isocontours of the Mach number M_j . This diagram highlights the difficulty to isolate any particular temperature effects. Notably,

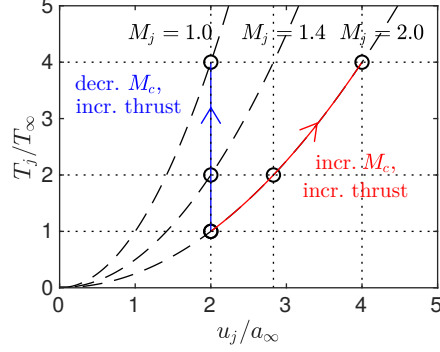


Fig. 2 Initial conditions for the jets at $M_a = 2$ and $M_j = 2$; $---$ isocontours of Mach number for $M_j = 1, 1.4$ and 2 .

129 the mass flow rate, defined as $\dot{m} = \pi D^2 \rho_j u_j / 4$, is not the same for the five jets, yielding different values of the net
 130 thrust $F = \dot{m} u_j$ which would be produced by spatially-developing jets at the same exhaust conditions. The latter can be
 131 expressed as

$$F = \frac{\pi}{4} Re_D^2 \frac{r_s}{p_\infty} T_j \mu_j^2(T_j), \quad (3)$$

132 and is only a function of the jet static temperature, since the five jets are at the same Reynolds number. Since the
 133 dynamic viscosity μ_j increases with the temperature, Eq. (3) implies that the net thrust is higher at a higher temperature.
 134 The temperatures of the jets also lead to different values for the speed of sound, since $a_j = \sqrt{\gamma r_s T_j}$. Therefore, the
 135 convective Mach number defined, following Papamoschou & Roshko [32], as

$$M_c = \frac{M_j}{1 + a_\infty / a_j}, \quad (4)$$

136 also varies. Its value, revealing the importance of compressibility on the development of the mixing layers, is equal
 137 to 1 for jetT1Mj2, 1.17 for jetT2Mj2, 1.33 for jetT4Mj2, 0.82 for jetT2Ma2 and 0.67 for jetT4Ma2. Therefore,
 138 compressibility effects are expected to strengthen with temperature for $M_j = 2$, but to weaken for $M_a = 2$.

139 The temperature in some of the present jets reach values higher than 1,000 K, which is beyond the temperature of
 140 600 K below which the ratio of specific heats γ can be considered as constant [33]. Despite this, for sake of simplicity,
 141 the gas in the jet core and the ambient medium is modelled as an ideal gas with $\gamma = 1.4$. This is supported by the
 142 observation in reference [34] that taking into account the variations with temperature of the specific heat ratio does not
 143 significantly affect the Mach wave radiation in supersonic jets, and by the finding of Buchta & Freund [35] that the
 144 pressure skewness near planar shear layer is only weakly affected by changes in the value of γ . Likewise, following
 145 Joseph et al. [36], it has been verified that the initial Mach numbers of the present jets differed from less than 3% from
 146 the ones computed assuming a temperature dependant specific heat ratio.

147 **B. Numerical methods**

148 The simulations are very well-resolved large-eddy simulations (LES) performed by solving the compressible
149 Navier-Stokes equations in cylindrical coordinates (r, θ, z) using high-order finite differences. In the computations, the
150 spatial derivatives are evaluated using a centered fourth-order eleven-point low-dispersion finite-difference scheme [37],
151 and time integration is carried out using a second-order, six-stage Runge-Kutta algorithm [37]. Near the jet axis, the
152 method proposed by Mohseni & Colonius [38] is applied to avoid singularity at $r = 0$. As a result, the first discretization
153 point is placed at a distance $r = \Delta r/2$ from the jet axis, where Δr is the radial mesh spacing close to the jet axis. The
154 azimuthal derivatives are also evaluated using fewer points than permitted by the grid, in order to reduce the constraint
155 on the time-step arising from the use of cylindrical coordinates near the centerline [39]. More precisely, the effective
156 azimuthal resolution is 16 for the row of points nearest to the jet axis. It progressively increases with the radial distance,
157 and reaches $n_\theta = 256$ for $r = 0.25r_0$. At the end of each time-step, a standard, twelfth-order selective filter is applied to
158 remove grid-to-grid oscillations. Another role of the filter is to relax the subgrid-scale energy near the grid cut-off
159 wavenumber. A shock-capturing scheme [40] based on the application of an adaptive filtering procedure is also applied
160 to damp Gibbs oscillations around shocks. Finally, the radiation conditions of Tam & Dong [41] are applied at the radial
161 boundary of the computational domain in order to allow the acoustic waves to leave the computational domain without
162 causing significant spurious reflections. At the axial inflow and outflow, periodicity conditions are imposed.

163 **C. Computational parameters**

164 The mesh used for the five simulations extends over a distance of $240r_0$ in the axial direction, and out to $13r_0$ in the
165 radial direction. It contains $n_r \times n_\theta \times n_z = 382 \times 256 \times 9600$ points, which yields a total number of 940 millions points.
166 The axial spacing Δz is constant and equal to $\Delta z = 0.025r_0$, yielding a maximum wavenumber $k_z r_0 \simeq 62.8$ for a wave
167 discretized using four points per wavelength. The radial spacing varies with the radial distance. It is such that $\Delta r = \Delta z$
168 on the jet axis, and reaches a minimum of $\Delta r = 0.0063r_0$ at $r = r_0$. It then progressively increases up to $r = 4r_0$, after
169 which it remains constant and equal to $0.05r_0$. Given their low Reynolds number, the computations can be considered as
170 very well resolved large-eddy simulations. Indeed, it has been verified that the sum of the different terms in the kinetic
171 energy budget is small. This high resolution is motivated by the need to accurately capture the steepened waves, but also
172 all the turbulence structures that may be involved in their generation process. The simulations are carried out until a
173 final time equal to $t_f = 70r_0/u_j$ for jetT1Mj2, to $60r_0/u_j$ for jetT2Ma2 and jetT4Ma2, and to $100r_0/u_j$ for jetT2Mj2
174 and jetT4Mj2. For all jets, approximately 10,000 time steps are necessary, which consumed 10,000 CPU hours.

175 **D. Space-averaged statistics**

176 Unlike for spatially-developing flows, which are statistically homogeneous in time but not in the axial direction,
177 the signals measured in and near the present temporally-developing jets are homogeneous in the z direction but are

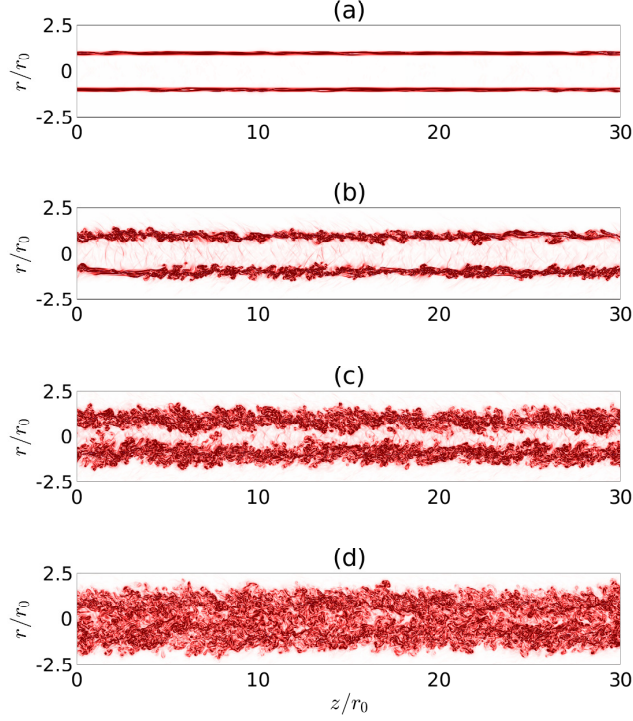


Fig. 3 Vorticity norm for jetT2Mj2 at (a) $t = 8r_0/u_j$, (b) $16r_0/u_j$, (c) $24r_0/u_j$, and (d) $32r_0/u_j$. The color scale ranges up to $4u_j/r_0$, from white to red.

178 not stationary. For this reason, as in other studies of temporally-developing flows [31, 42], all of the statistical results
 179 presented in section III, including the conditional averages in section III.D are computed, for a given time, over the
 180 homogeneous directions z and θ . In addition, the convergence of these space-averaged statistics is improved by averaging
 181 the results obtained from several runs of the same jet, which are performed by varying the random seed used in the
 182 excitation procedure. An illustration of the benefits of this method in the case of a temporal subsonic jet is provided in
 183 reference [31]. As reported in table 1, 4 runs are performed for jetT1Mj2 and 3 runs are performed for the other jets,
 184 yielding equivalent domain lengths of $4 \times 240r_0 = 960r_0$ and $3 \times 240r_0 = 720r_0$, respectively.

185 III. Results

186 A. Flow development

187 Snapshots of vorticity norm obtained for jetT2Mj2 are given in Fig. 3 at times $t = 8r_0/u_j$, $16r_0/u_j$, $24r_0/u_j$ and
 188 $32r_0/u_j$. At $t = 8r_0/u_j$, the jet core is surrounded by thin, laminar mixing layers in which slight perturbations are
 189 visible, indicating the development of instability waves. At $t = 16r_0/u_j$, the mixing layers are thicker than previously
 190 and contain more fine-scale turbulence, because of their transition from a laminar to a turbulent state. At subsequent
 191 times, the growth of the mixing layers continues until they interact with each other, which is visible at $t = 24r_0/u_j$
 192 where pockets of vortical flow coming from the shear layers intrude on the jet axis. Finally, at $t = 32r_0/u_j$, the mixing

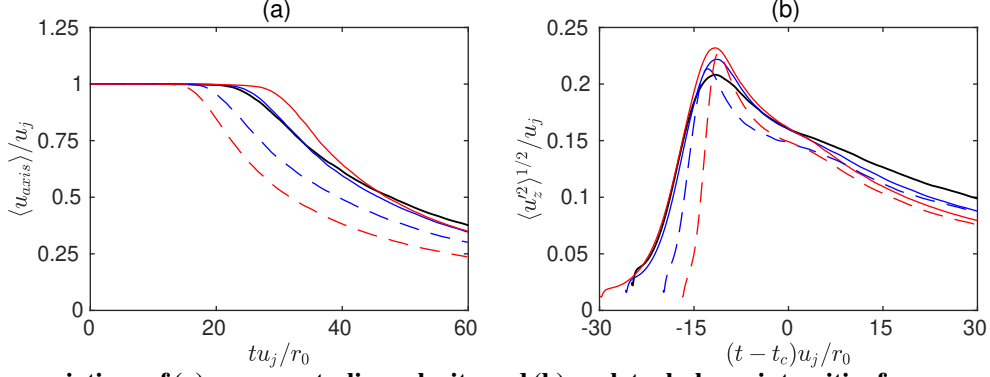


Fig. 4 Time variations of (a) mean centerline velocity and (b) peak turbulence intensities for — jetT1Mj2, — jetT2Mj2, — jetT4Mj2, - - - jetT2Ma2 and - - - jetT4Ma2.

193 layers join and merge and the inner, potential core of the jet disappears.

194 The time variations of the mean axial velocity are shown in Fig. 4(a). For all jets, the velocity is constant and equal
 195 to its initial value u_j at early times and as long as the centerline flow is irrotational. It rapidly decreases afterwards,
 196 as low-speed gas is entrained inside the jet core after the merging of the shear layers. The core-closing time t_c ,
 197 defined as the time when the centerline mean velocity $\langle u_{z,axis} \rangle$ is such that $\langle u_{z,axis} \rangle = 0.95u_j$, varies according to the
 198 initial conditions. It slightly increases with the temperature for $M_j = 2$, since $t_c = 25.3r_0/u_j$ for jetT1Mj2, $26.2r_0/u_j$
 199 for jetT2Mj2 and $29.8r_0/u_j$ for jetT4Mj2, whereas it diminishes at $M_a = 2$, as $t_c = 20.2r_0/u_j$ for jetT2T14, and
 200 $17.0r_0/u_j$ for jetT4Ma2. In order to explore the origin of these different trends, the time variations of the maximum
 201 root-mean-square value of axial velocity fluctuations are represented in Fig. 4(b) as a function of $(t - t_c)r_0/u_j$. In this
 202 figure, a peak of $0.2u_j$ is reached for all jets at $t \simeq t_c - 12r_0/u_j$. This peak is caused by pairings between vortices during
 203 the transition of the jet from a laminar to a turbulent state [43], which thus occurs in all cases approximately 12 time
 204 units before the closure of the potential core. The duration of the initial laminar phase before the pairings strongly
 205 depends on the jet considered, as it varies from $5.9r_0/u_j$ for jetT4Ma2 to $18.3r_0/u_j$ for jetT4Mj2. Consequently, the
 206 different core-closing times reported in Fig. 4(a) are mostly due to the different durations of this initial phase which can
 207 be explained by compressibility effects. Indeed, as reported in Table 1, the convective Mach number M_c of the jets
 208 ranges from 0.67 for jetT4Ma2 to 1.33 for jetT4Mj2. Since the mixing layers growth rate decreases with the convective
 209 Mach number [32], it is expected that the initial laminar phase of the jets would last longer for higher values of M_c ,
 210 which is observed in the present simulations.

211 The time variations of the momentum thickness δ_θ of the shear layers are provided in Fig. 5. Initially, for
 212 $t \leq t_c - 12r_0/u_j$, the growth is slow, as the shear layers are in a laminar state. It is faster for $t \geq t_c - 12r_0/u_j$, after their
 213 transition to a turbulent state. From that time, and until the closing of the potential core, the momentum thickness of the
 214 hot jets increases more rapidly than that of the isothermal one. Since the present jets have different Mach numbers,
 215 speeds and densities, the effects of temperature on the jet spreading are not straightforward to interpret. For that reason,

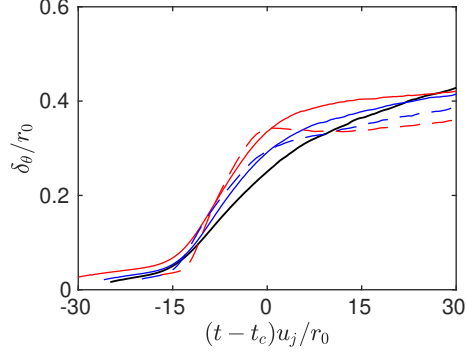


Fig. 5 Time variations of the shear-layer momentum thickness for — jetT1Mj2, — jetT2Mj2, — jetT4Mj2, - - - jetT2Ma2 and - - - jetT4Ma2.

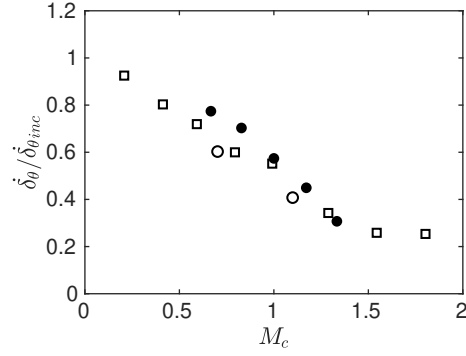


Fig. 6 Momentum-thickness growth rates obtained when $\delta_\theta = 0.08r_0$ normalized by their incompressible values; ● present results, □ Freund et al. [42] and ○ Pantano & Sarkar [45].

216 the momentum thickness growth rate $\dot{\delta}_\theta = (d\delta_\theta/dt)/u_j$ is normalized by that obtained for an incompressible mixing
 217 layer at the same density ratio, and plotted in Fig. 6 as a function of the convective Mach number M_c as in previous
 218 studies on compressible mixing layers at different density ratios [32, 44]. The growth rates for the incompressible cases
 219 have been determined by simulating three additional temporally-developing round jets at a convective Mach number
 220 of 0.15 and at static temperatures of T_∞ , $2T_\infty$ and $4T_\infty$. Since the shear-layer growth rates vary with time, they are
 221 displayed in Fig. 6 at a time where $\delta_\theta = 0.08r_0$ for all jets, following Freund et al. [42]. The normalized growth rate is
 222 shown to decrease with the convective Mach number, in agreement with experimental [32] and numerical [42, 45] data,
 223 which is a well-known effect of compressibility. The reported values are also in good agreement with those provided in
 224 references [42, 45], also plotted for comparison.

225 The effects of temperature on the mean velocity profiles are characterized by considering the jet half-width δ_{05} ,
 226 defined as the radial location where $\langle u_z \rangle = 0.5u_j$ in Fig. 7(a). For all jets, it is initially equal to $\delta_{05} = r_0$, as expected
 227 given the hyperbolic-tangent shape of the profiles at $t = 0$. At subsequent times, the half-width of the isothermal jet
 228 remains close to r_0 until $t = t_c$, whereas those of the hot jets decrease until they reach the values of $\delta_{05} = 0.80r_0$ for
 229 jetT2Ma2, $0.68r_0$ for jetT4Ma2, $0.84r_0$ for jetT2Mj2 and $0.74r_0$ for jetT4Mj2 at $t \simeq t_c$. Consequently, the half-velocity
 230 point is shifted toward the high-speed, low-density side of the mixing layers at a higher jet temperature. This effect is

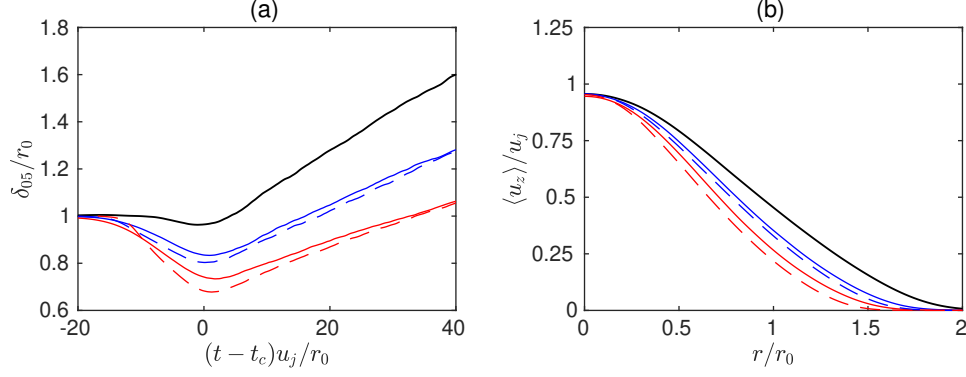


Fig. 7 Representations of (a) jet half-width and (b) mean velocity profile at core-closing time for ——— jetT1Mj2, ——— jetT2Mj2, ——— jetT4Mj2, - - - jetT2Ma2 and - - - jetT4Ma2.

231 illustrated in Fig. 7(b), where the mean velocity profiles obtained at the core-closing time are plotted. The profiles bend
 232 toward the jet axis when the temperature is raised, which has also been observed in studies of plane temporal mixing
 233 layers at different density ratios [45], as well as in experimental studies of spatially-developing hot jets at high Reynolds
 234 numbers [46].

235 B. Near pressure field

236 The time variations of the pressure levels computed at $r = 8r_0$ from the jet axis are plotted in figure 8(a) as a function
 237 of $(t - t_c)a_\infty/r_0$. For all jets, a peak is reached after the core closing time, at approximately $t = t_c + 7r_0/a_\infty$. The peak
 238 of pressure levels is of 1500 Pa for jetT1Mj2, 970 Pa for jetT2Ma2 and 650 Pa for jetT4Ma2. It thus decreases with
 239 temperature when the jet speed is constant, in agreement with the experimental results of Tanna et al. [2]. A different
 240 trend is however observed for a constant Mach number M_j . In that case, the peak level is of 1900 Pa for jetT2Mj2 and
 241 2100 Pa for jetT4Mj2, and is thus higher at a higher temperature, as observed by Seiner et al. [3] for jets at $M_j = 2$. This
 242 rise of the radiated pressure levels is caused, to a large extent, by the higher velocities of jetT2Mj2 and jetT4Mj2. In
 243 order to isolate the effects of temperature, the pressure levels are normalized by $M_a^{3/2}$ and plotted in Fig. 8(b) assuming,
 244 following previous investigators [8, 47], that the sound intensity radiated by supersonic jets increases with the third
 245 power of the jet speed. The pressure levels obtained for jets at the same static temperature are found to be very close to
 246 each others. In addition, the effect of temperature alone is to diminish the intensity of the acoustic waves radiated by the
 247 jets. For a constant Mach number M_j , this effect is however compensated by the rise of the jet speed, leading to higher
 248 pressure levels at a higher temperature.

249 Snapshots of pressure fluctuations and vorticity norm obtained for the five jets are provided in Fig. 9 at $t = t_c + 7r_0/a_\infty$,
 250 very close to the time of peak pressure levels at $r = 8r_0$. For all jets, straight, inclined wavefronts, whose associated
 251 wavelengths are of the order of the jet diameter are visible near the jets. This observation indicates that the jets radiate
 252 Mach waves during their development. For $M_j = 2$, in Fig. 9(a,b,d), the inclination angle of the acoustic waves

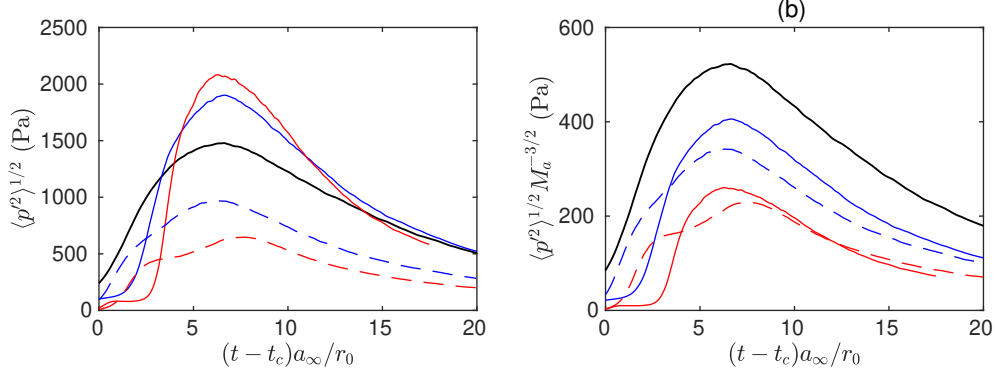


Fig. 8 Time variations of the rms value of pressure fluctuations at $r = 8r_0$ for — jetT1Mj2, — jetT2Mj2, — jetT4Mj2, - - - jetT2Ma2 and - - - jetT4Ma2.

253 significantly increases with temperature, as noticed in the numerical studies of Nonomura et al. [28] and Liu et al. [6],
 254 for instance. This increase is mainly explained by the higher velocities of jetT2Mj2 and jetT4Mj2 in comparison with
 255 jetT1Mj2. Indeed, no such effect is visible in the snapshots of Fig. 9(a,c,e), for a constant jet speed. In these figures, the
 256 inclination angle of the sound waves slightly diminishes with temperature, and the wavefronts radiated by jetT2Ma2 and
 257 jetT4Ma2 have a slightly curved aspect. This curved aspect is likely due to the faster decrease of the mean axial velocity,
 258 hence convection speed, in that case with respect to the jets at $M_j = 2$. Finally, it can be noted that steep pressure
 259 variations are present at the edges of the wavefronts radiated by the jets. These features are more clearly visible in the
 260 sound fields of jetT2Mj2 and jetT4Mj2 than in those of the jets at $M_a = 2$. This suggests that the hot jets at $M_j = 2$
 261 radiate more steepened waves than the ones at $M_a = 2$.

262 Steepened waves can also be observed in the axial variations of the pressure fluctuations at $r = 8r_0$ and $t = t_c + 7r_0/a_\infty$
 263 provided in Fig. 10 for the isothermal jet as well as for jetT4Mj2 and jetT4Ma2. In the three cases, the signals are
 264 normalized by their standard deviations in order to facilitate the comparison. For the isothermal jet, in Fig. 10(a), strong
 265 pressure peaks emerge as, for instance, at $z = 13r_0$ and $z = 19r_0$. These peaks consist of a fast compression phase,
 266 leading to a pressure peak whose value can exceed three times that of the standard deviation, followed by an expansion
 267 phase which is more gradual than the initial compression. The shapes of these shock-like structures are very similar to
 268 those depicted by Buchta & Freund [18] in their simulations of plane temporally-developing mixing layers. They also
 269 remind us of those observed in the near and far acoustic fields of spatially-developing supersonic jets [16, 23, 24]. In
 270 Fig. 10(b,c), those structures are also present in the sound field of the hot jets, for exemple at $z = 13r_0$ for jetT4Ma2,
 271 and at $z = 8r_0$ and $18r_0$ for jetT4Mj2. For the latter jet, the corresponding pressure peaks reach values as high as five
 272 standard deviations, which suggests that the steepened waves constitute stronger deviations from the mean pressure than
 273 those generated by the other, slower jets.

274 The shock-like structures in the pressure signals of Fig. 10 constitute strong deviations which cannot be easily
 275 characterized using basic statistics such as sound intensity and power spectrum densities. However, high-order statistics

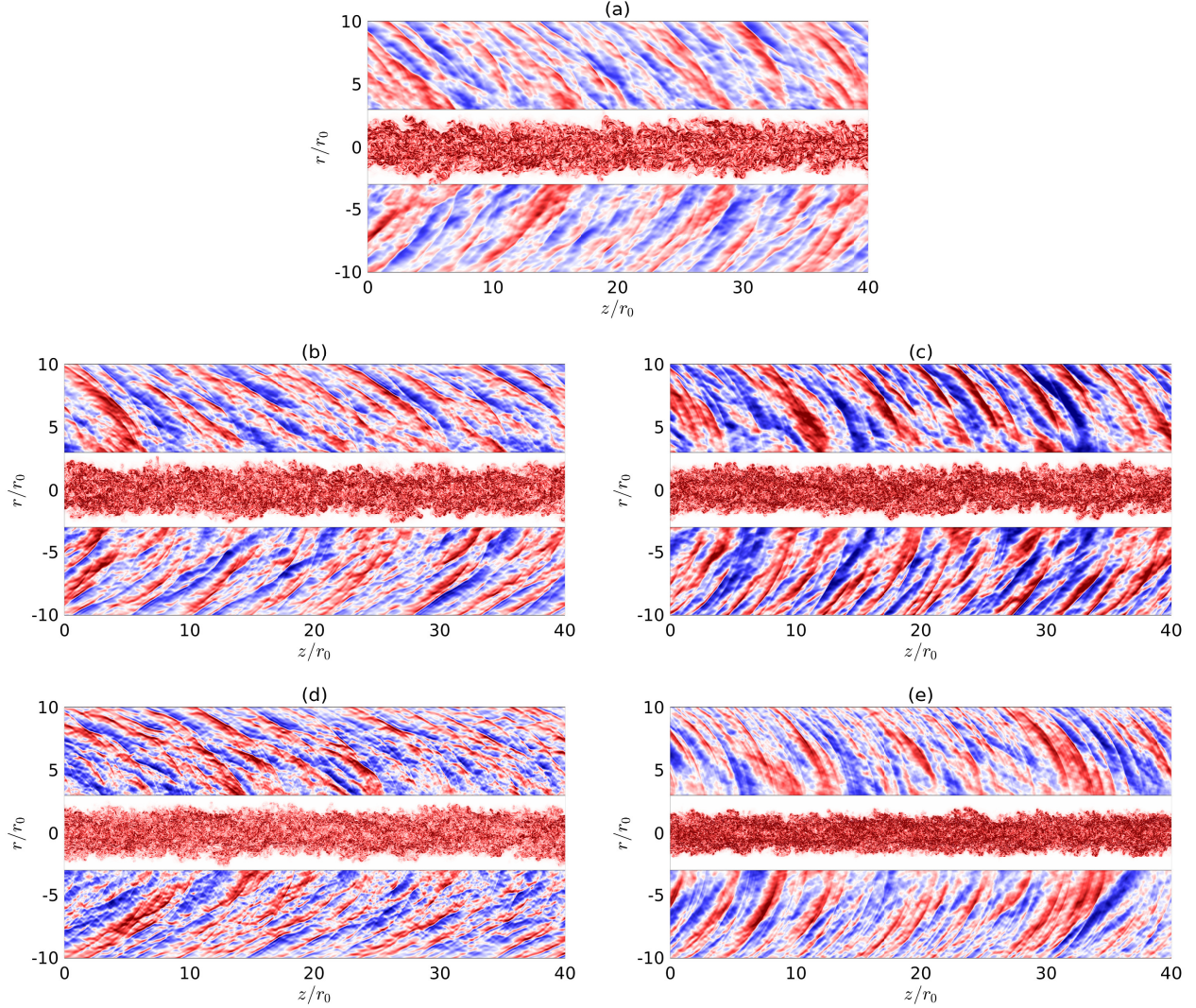


Fig. 9 Pressure fluctuations and vorticity norm at $t = t_c + 7r_0/a_\infty$ for (a) jetT1Mj2, (b) jetT2Mj2, (c) jetT2Ma2, (d) jetT4Mj2 and (e) jetT4Ma2. Color scales between (a,b,d) $\pm 6,000$ Pa and (c,e) $\pm 2,000$ Pa for the pressure and up to $3u_j/r_0$ for the vorticity.

276 such as the skewness and kurtosis factors provide useful insights on the properties of such events and have been used in
 277 the past to quantify the crackling behavior of high-speed jets [9, 11, 16]. The skewness factor of the pressure fluctuations,
 278 defined as

$$S(p) = \frac{\langle p'^3 \rangle}{\langle p'^2 \rangle^{3/2}} \quad (5)$$

279 is a measure of the asymmetry of the pressure fluctuations statistical distribution. Its values obtained at $r = 8r_0$ are
 280 plotted in Fig. 11 as a function of $(t - t_c)a_\infty/r_0$. For the jets at $M_j = 2$, in Fig. 11(a), the skewness factor is greater than
 281 zero for $t_c + 5r_0/a_\infty \leq t \leq t_c + 10r_0/a_\infty$, when the pressure levels are close to their maximum values. During that
 282 period, the skewness value is approximately constant and equal to 0.28 for jetT1Mj2, 0.37 for jetT2Mj2 and 0.41 for

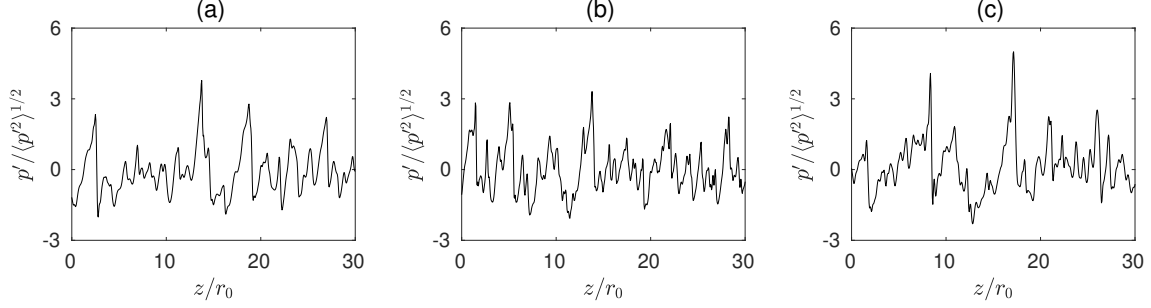


Fig. 10 Pressure fluctuations at $r = 8r_0$ and $t = t_c + 7r_0/a_\infty$ normalized by their standard deviation for (a) jetT1Mj2, (b) jetT4Ma2 and (c) jetT4Mj2.

jetT4Mj2. These positive values are caused by the pressure peak associated with the steepened shock-like structures and are higher at a higher temperature, indicating more prominent pressure peaks. No such increase is observed for $M_a = 2$ in Fig. 11(b). In this case, on the contrary, the skewness factor at time of peak levels, obtained by averaging between $t = t_c + 5r_0/a_\infty$ and $t_c + 10r_0/a_\infty$, is of 0.28 for jetT1Mj2, 0.26 for jetT2Ma2 and 0.18 for jetT4Ma2 and thus slightly decreases with the temperature. It can be noted that the variations of skewness in Fig. 11 occur over time scales of approximately $10r_0/a_\infty$ which are longer than the wave turnover time λ/a_∞ , where $\lambda \simeq 2r_0$ is a characteristic length of the pressure waves. Consequently, the skewness does not change significantly as a wave passes through the location of an observer. This allows us to compare the present values with the ones reported in the literature for spatially-developing jets. For instance, Petitjean et al. [24] measured skewness factors ranging from 0.1 to 0.5 in the far acoustic field of heat-simulated jets at acoustic Mach number between 1.2 and 2.5, close to the values reported above for the present temporally-developing jets.

The kurtosis factor of the pressure fluctuations is defined as

$$K(p) = \frac{\langle p'^4 \rangle}{\langle p'^2 \rangle^2} \quad (6)$$

and is a measure of the probability of occurrence of extreme pressure events, and hence of the signal intermittency. While $K(p) = 3$ is obtained for a signal with Gaussian statistics, values higher than 3 indicate that extreme deviations from the mean are more likely to occur. Previous experimental studies of supersonic jets have shown that the kurtosis factor can reach such values in the sound field of highly supersonic jets [9, 16] due to the presence of steepened waves. The time variations of the pressure kurtosis factors measured at $r = 8r_0$ are plotted in Fig. 12. For all jets, it is approximately constant between $t_c + 5r_0/a_\infty$ and $t_c + 12r_0/a_\infty$. For $M_j = 2$, in Fig. 12(a), the kurtosis factor is equal to 3.1 for jetT1Mj2, 3.4 for jetT2Mj2 and 3.5 for jetT4Mj2, and thus increases with temperature, in agreement with the measurements of Mora et al. [9]. However, it does not vary significantly at a constant jet speed, only varying between 3.1 and 3.2 for jetT1Mj2, jetT2Ma2 and jetT4Ma2 in Fig. 12(b).

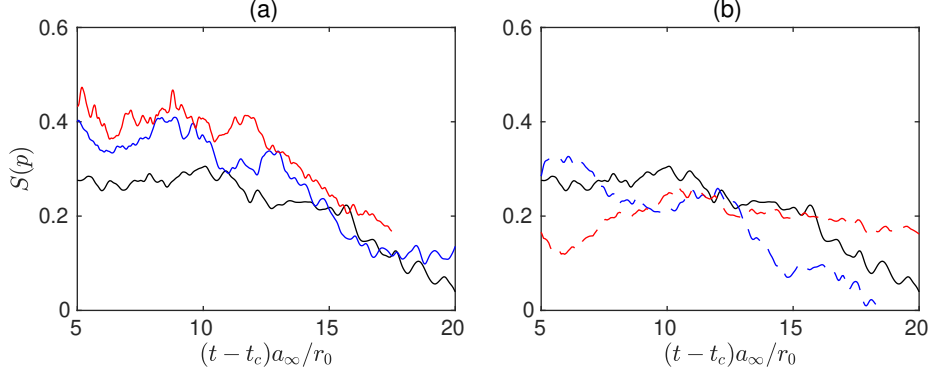


Fig. 11 Skewness factors of the pressure fluctuations at $r = 8r_0$ for (a) — jetT1Mj2, — jetT2Mj2 and — jetT4Mj2, and (b) — jetT1Mj2, - - - jetT2Ma2 and - - - jetT4Ma2.

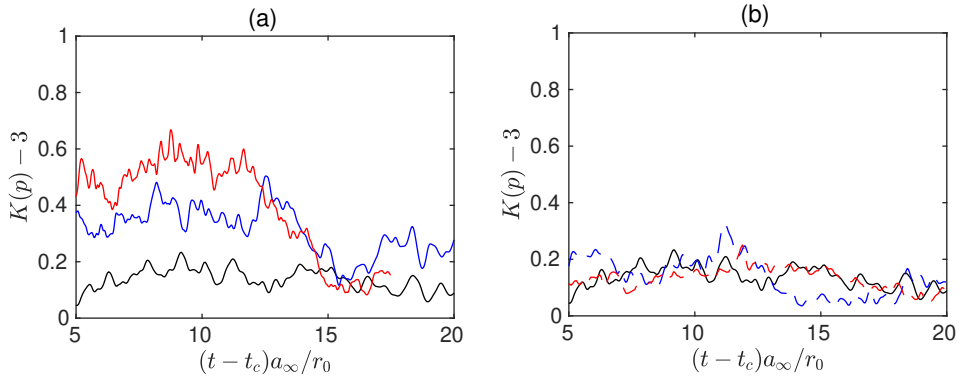


Fig. 12 Kurtosis factors of the pressure fluctuations at $r = 8r_0$ for (a) — jetT1Mj2, — jetT2Mj2 and — jetT4Mj2, and (b) — jetT1Mj2, - - - jetT2Ma2 and - - - jetT4Ma2.

304 Finally, in order to display the effects of the jet speed for a given temperature, the power spectrum densities of the
 305 pressure fluctuations at $r = 8r_0$ and $t = t_c + 7r_0/a_\infty$ are plotted as a function of the axial wavenumber in Fig. 13(a)
 306 for the jets at $T_j = 2T_\infty$ and in Fig. 13(b) for $T_j = 4T_\infty$. They are normalized by M_a^3 assuming, as previously, that the
 307 magnitude of the pressure fluctuations increases with the third power of the jet velocity. For the two temperatures
 308 considered, the pressure decay in the high-wavenumber range is slower for jetT2Mj2 and jetT4Mj2, resulting in more
 309 high-frequency components than for jetT2Ma2 and jetT4Ma2 with lower velocities. This is due to the presence of
 310 more high-intensity, steepened waves in the sound field of jetT2Mj2 and jetT4Mj2 than near the other jets at $M_a = 2$,
 311 as observed in previous experimental studies [24, 48]. Finally, it can be noted that the pressure levels differ in the
 312 low-wavenumber range, for $k_z r_0 \leq 1$. These differences may be due to the fact that at $r = 8r_0$, the pressure field is likely
 313 to include hydrodynamic components which are not expected to follow the M_a^3 velocity scaling [49].

314 C. Convection speeds

315 For the jets at $M_j = 2$, the rise of the jet speed u_j from $2a_\infty$ to $4a_\infty$ mainly explains the higher amplitude of the
 316 steepened waves radiated by the jets at a higher temperature. In that case, the convection speed of the large-scale

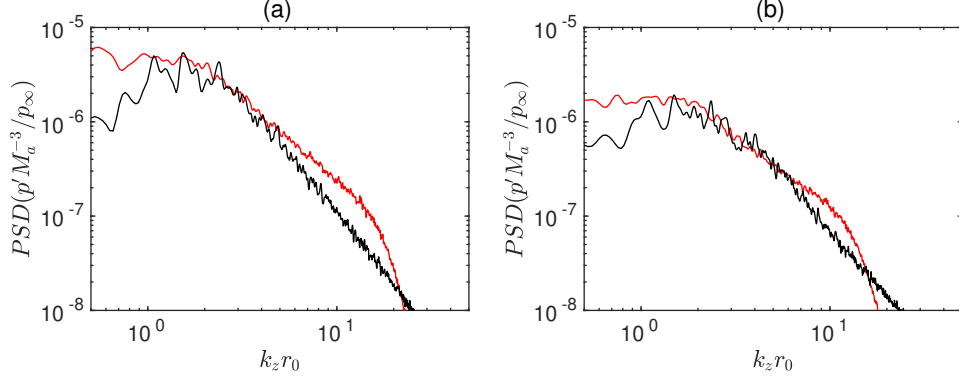


Fig. 13 Pressure spectra at $r = 8r_0$ and $t = t_c + 7r_0/a_\infty$ normalized by M_a^3 for (a) — jetT2Mj2, — jetT2Ma2, and (b) — jetT4Mj2, — jetT4Ma2.

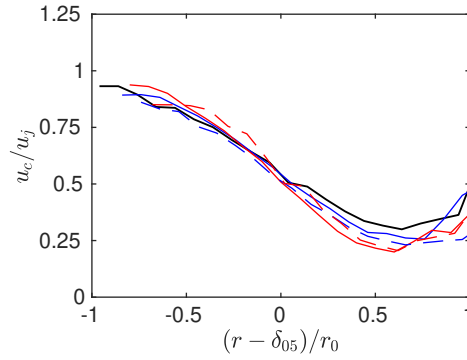


Fig. 14 Radial profiles of the convection speeds at $t = t_c$ normalized by u_j for — jetT1Mj2, — jetT2Mj2, — jetT4Mj2, - - - jetT2Ma2 and - - - jetT4Ma2.

317 coherent structures drastically increases, causing the formation of more intense and more skewed Mach waves, as
 318 observed in the simulations of Buchta & Freund [18] for isothermal mixing layers. In order to understand the reason for
 319 the slight reduction with temperature of the pressure skewness observed for the jets at $M_a = 2$, which have identical
 320 initial velocities, it seems natural to investigate the convection speeds in their shear-layers. To this end, the latter are first
 321 estimated from the space-time correlations of axial velocity fluctuations. The radial variations of the convection speeds
 322 obtained at the core-closing time are plotted in Fig. 14(a) as a function of $(r - \delta_{05})/r_0$, where δ_{05} is the jet half-width.
 323 Despite the differences in temperature, velocity and Mach number, the profiles collapse reasonably well. Since the jet
 324 half-width decreases with temperature, as shown in Fig. 7(a), this indicates that the radial profiles of convection velocity
 325 are shifted toward the jet axis when temperature is raised. Thus, the convection speed at a given radial location is lower
 326 at a lower temperature as in previous studies of spatially-developing hot supersonic jets [6, 50, 51]. This decrease could
 327 explain the reduction with temperature of the pressure skewness for the jets at constant speed. However, to confirm
 328 this hypothesis, it is necessary to determine the convection speed of the turbulent structures that actually produce the
 329 steepened Mach waves.

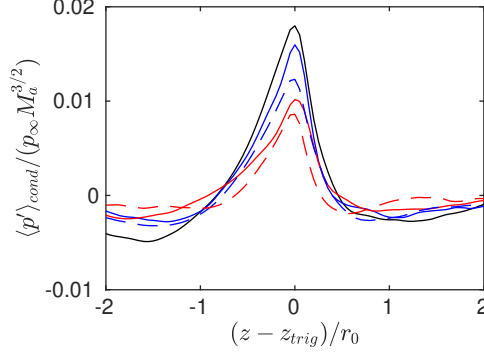


Fig. 15 Conditional pressure waveforms at $r = 4.5r_0$ and $t = t_{\text{trig}} = t_c + 2r_0/a_\infty$ ——— jetT1Mj2, ——— jetT2Mj2, ——— jetT4Mj2, - - - jetT2Ma2 and - - - jetT4Ma2.

D. Conditional averages

Flow features directly linked to noise generation can be obtained by performing correlations between the flow and sound fields, as previously done in studies of temporal [31, 52] and spatial [19, 53] jets. The convection speed can be determined by tracking a specific flow pattern [31, 53] or, in the case of Mach wave radiation, by estimating a mean inclination angle of the wavefronts, yielding an average convection speed using relation (1). This approach will be followed hereafter for the present jets. In practice, the process of steepened waves generation is extracted using conditional averages. To this end, the pressure fluctuations recorded at a distance $r_{\text{trig}} = 4.5r_0$ from the jet axis are first binned in intervals of length $3r_0$. The pressure maxima of each subintervals are then determined, and those whose values are lower than twice the standard deviation are discarded. In a second step, the flow and sound field are synchronized according to the axial and azimuthal coordinates z_{trig} and θ_{trig} of the detected maxima. Finally, they are ensemble averaged in order to highlight the flow features that are correlated with the trigger pressure peaks. For all jets, the conditional averaging procedure is applied at time $t_{\text{trig}} = t_c + 2r_0/a_\infty$, close to the peak of pressure levels at $r = r_{\text{trig}}$. In order to save memory storage, it is not applied to the full three-dimensional fields, but to a simplified reconstruction involving only the first five azimuthal Fourier modes. This simplification is motivated by previous observations [22, 54] that the sound waves radiated by highly supersonic jets are strongly correlated in the azimuthal direction.

The conditional pressure signals obtained at r_{trig} and at time t_{trig} for the five jets are plotted in Fig. 15. They are computed by synchronizing approximately 150 events, and are normalized by $p_\infty M_a^3$. The signals obtained for the different jets are very similar, and all peak for $z = z_{\text{trig}}$, as expected given the spatial synchronization. The distinctive features of the sharp pressure waves depicted in the signals of Fig. 10 are recovered. Indeed, a fast compression and a gradual expansion are visible for $z \geq z_{\text{trig}}$ and $z \leq z_{\text{trig}}$, respectively. The pressure peaks are also weaker for the hot jets than for the isothermal jet, following the decrease with temperature of the normalized pressure levels in Fig. 8(b).

The conditional averages are also evaluated at times $t < t_{\text{trig}}$ by synchronizing the flow and sound fields according to the pressure maxima obtained at $t = t_{\text{trig}}$, which provides a sequential description of the wave generation process.

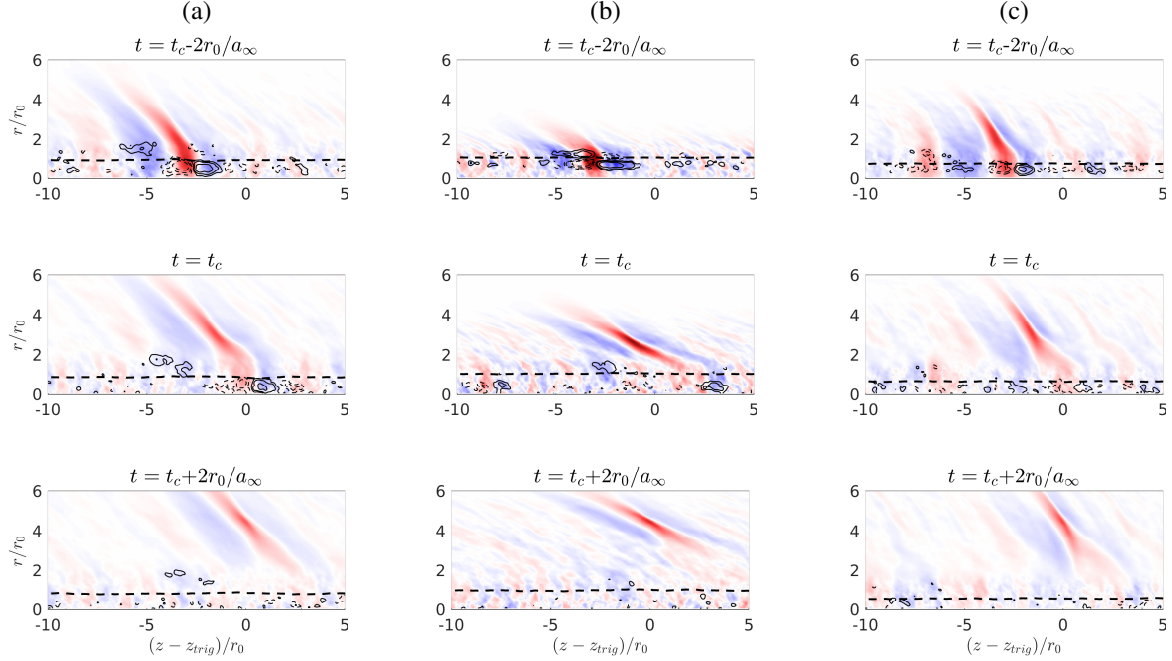


Fig. 16 Conditional pressure and vorticity for (a) jetT1Mj2, (b) jetT4Mj2, and (c) jetT4Ma2; ——— positive and - - - negative vorticity contours, and - - - sonic line. Color scales between (a,b) $\pm 12,000$ Pa and (c) $\pm 5,000$ Pa, from blue to red.

353 The resulting conditional pressure and vorticity fields are shown in Fig. 16 for jetT1Mj2 as well as for the two hottest
354 jets jetT4Mj2 and jetT4Ma2 at $t_c - 2r_0/a_\infty$, t_c and $t_{\text{trig}} = t_c + 2r_0/a_\infty$. Positive contours of vorticity for the values
355 $0.24u_j/r_0$, $0.48u_j/r_0$ and $0.96u_j/r_0$ are represented as black solid lines, and negative contours for the values $-0.96u_j/r_0$,
356 $-0.48u_j/r_0$ and $-0.24u_j/r_0$ are represented as black dashed lines. Some features common to the five jets are discernible.
357 In all cases, an inclined, elongated wavefront emerges from the jet flow at $t_c - 2r_0/a_\infty$. It consists of a band of strong
358 positive levels surrounded by two strips of weaker negative levels. At the lower tip of the wave, an excess of conditional
359 vorticity associated with a pressure deficit is visible inside the jet flow. This zone of positive vorticity has an elliptical
360 shape, and it is located below the sonic line, in the inner core of the jets. Slightly upstream of this vorticity spot, a region
361 of negative vorticity and positive pressure is also apparent. At t_c , the conditional pressure wave moves outward from the
362 jet flow, and sharp pressure gradients are noticeable at its edge. Inside the jet, the vorticity excess is found downstream
363 from its initial position. Finally, at $t_c + 2r_0/a_\infty$, the pressure wave leaves the flow and propagates downstream, while the
364 vortical structure disappears. These features indicate that the steepened waves observed in the near field constitute a
365 particular case of Mach wave radiation. The vorticity spot can indeed be viewed as the trace of large-scale coherent
366 structures convected by the flow. When their convection speed is higher than the ambient sound speed, they act upon the
367 surrounding medium as a moving wavy-wall, leading to the formation of Mach waves, which propagate in the direction
368 of the angle α given by equation (1). For flow perturbations of moderate amplitude, the formation and the propagation

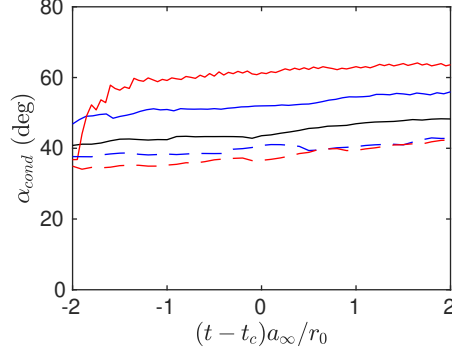


Fig. 17 Time variations of the conditional wavefront orientation angle at the location of minimum dilatation for — jetT1Mj2, — jetT2Mj2, — jetT4Mj2, - - - jetT2Ma2 and - - - jetT4Ma2.

369 of these waves are usually described as linear phenomena, which constitutes the main hypothesis in the classical models
 370 of Mach wave radiation [55–57]. However, when nonlinear effects are taken into account, as in the study of Buchta &
 371 Freund [35], who simulated the nonlinear development of a linear instability wave, weak shocks possessing the main
 372 features of crackle can be formed in the immediate vicinity of the jet flow. Finally, the present conditional averages
 373 are very close to those presented in Appendix, which are obtained by applying the procedure described above to a
 374 simulation of a spatially-developing isothermal jet at Mach 2 and at a diameter-based Reynolds number of 12,500. This
 375 supports that the mechanisms responsible for the formation of steepened waves in temporal flows do not significantly
 376 differ from those in spatially-developing jets.

377 Despite the similarities of the conditional flow and sound fields, the inclination angle of the waves is largest in
 378 Fig. 16 for jetT4Mj2 and lowest for jetT4Ma2. More quantitatively, the inclination angle α_{cond} of the conditional
 379 wavefronts is evaluated by computing the angle of the pressure gradient relatively to the jet direction following

$$\alpha_{\text{cond}} = \tan^{-1} \left(\frac{\partial p / \partial r}{\partial p / \partial z} \right). \quad (7)$$

380 The angle α_{cond} computed for the different jets at the location of minimum dilatation, where the pressure gradient
 381 is strongest, is plotted in Fig. 17 as a function of $(t - t_c)a_{\infty}/r_0$. For $t \geq t_c - r_0/a_{\infty}$, the angle α_{cond} does not change
 382 much over time. It increases with temperature for $M_j = 2$ but decreases for $M_a = 2$, which is consistent with the
 383 trends obtained in the snapshots of Fig. 9. A convection speed directly associated with noise generation is then defined
 384 from the angle α_{cond} using relation (1). This convection speed, time averaged between $t_c - r_0/a_{\infty}$ and $t_c + 2r_0/a_{\infty}$,
 385 ranges from $u_c = 1.3a_{\infty}$ for jetT4Ma2 up to $u_c = 2.1a_{\infty}$ for jetT4Mj2. As previously, the effects of temperature can be
 386 highlighted by considering the ratio between the convection speed and the jet velocity u_j . This ratio is equal to 0.72 for
 387 jetT1Mj2, 0.61 for jetT2Mj2, 0.54 for jetT4Mj2, while it is of 0.66 for jetT2Ma2 and 0.64 for jetT4Ma2. Therefore, the
 388 effects of temperature is to diminish the ratio between the convection speed and the jet velocity.

389 Finally, the influence of the convection speed for the generation of steepened waves is displayed in Fig. 18, where

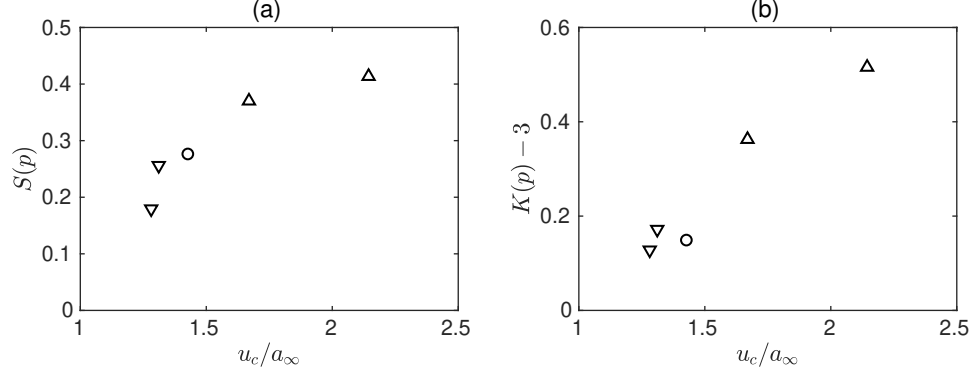


Fig. 18 Time-averaged pressure (a) skewness and (b) kurtosis factors as a function of the convection velocity inferred from the conditional averages for \circ jetT1Mj2 and for the hot jets at $\Delta M_j = 2$ and $\nabla u_j = 2a_\infty$.

390 the pressure skewness and kurtosis factors at $r = 8r_0$, time-averaged from $t_c - 5r_0/a_\infty$ to $t_c + 10r_0/a_\infty$, are represented
 391 as a function of the convection speed obtained above. In Fig. 18(a), the skewness factor continuously increases with the
 392 convection speed, indicating that the acoustic waves radiated by the jets are more skewed as the turbulent structures
 393 inside the jet convect more rapidly. In Fig. 18(b), the kurtosis factor obtained for the jets at $M_a = 2$ is close to 3.1, but
 394 drastically increases with the convection speed for $M_j = 2$, which means that strong deviations are also more likely to
 395 occur at a higher convection speed. Therefore, the present results strongly suggest that the steepened aspect of the waves
 396 radiated by jets at different temperatures is essentially governed by the convection speed of the turbulent structures at
 397 the origin of Mach waves.

398 IV. Conclusion

399 In the present study, the effects of temperature on the formation of steepened acoustic waves near high-speed
 400 supersonic jets are investigated by performing numerical simulations of temporal jets at static temperatures equal to 1, 2
 401 and 4 times that of the ambient medium. One isothermal and four hot jets are considered, which allows us to investigate
 402 separately the effects of a temperature rise at a constant Mach number, as is done in most experimental studies, or at a
 403 constant jet speed. When the Mach number is constant, the acoustic waves radiated by the jets are stronger at a higher
 404 temperature, and their steepened aspect is more pronounced, which is reflected in the rise of the pressure skewness and
 405 kurtosis factors. On the contrary, when the temperature is raised at a constant speed, the pressure levels and skewness
 406 factor diminish while the kurtosis is not significantly affected. The two different trends obtained at a constant Mach
 407 number or at a constant jet speed are explained by considering the variations with temperature of the convection speed
 408 estimated from the autocorrelation functions of the velocity fluctuations. When the temperature is raised, the radial
 409 profiles of convection speed are shifted toward the jet axis leading, for a given radial location, to lower values of the
 410 ratio between the convection speed and the jet velocity. The importance of the convection speed in the formation of
 411 steepened waves is highlighted by performing conditional averages, which shows a direct link between the formation of

412 those waves and the supersonic convection of large-scale structures. These averages also allow us to determine, for each
413 jet, a convection speed from the inclination angle of the conditional waves. This convection speed, which is directly
414 related to noise generation, decreases with temperature when normalized by the jet velocity. Among the various effects
415 of increasing temperature on the present flow, which include changes in the Mach number, the density or the thrust of
416 the jets, the decrease of the normalized convection speed appears to be one of the most important temperature effects
417 related to the formation of steepened waves in the jet near field. For a temperature rise at a constant speed, it causes a
418 reduction of the convection velocity, leading to the formation of less steep Mach waves at higher temperature. When the
419 Mach number is constant, this reduction is however compensated by the rise of the jet velocity, so that the convection
420 speed increases with temperature, causing the formation of more intense steepened waves.

421 Appendix

422 In this Appendix, the conditional averaging procedure described in section III.D is applied to the flow and sound
423 fields of a spatially-developing Mach 2 isothermal round jet at a diameter-based Reynolds number of 12,500 in order to
424 verify that the flow features revealed for the present temporally-developing jets are also observed in spatially-developing
425 jets. This jet is computed using the same numerical methods as those presented in section II.B using a grid resolution
426 comparable to that of the present temporal simulations. The jet exhausts at ideally-expanded conditions from a
427 straight-pipe nozzle in which a Blasius laminar boundary layer profile of momentum thickness $\delta_\theta = 0.018r_0$ is imposed.
428 At each time step, slight velocity fluctuations [58] are added inside the nozzle in order to favor the transition of the shear
429 layers from a laminar to a turbulent state. The amplitude of these perturbation has been tuned such that the turbulence
430 rates at the nozzle exit are of 3%. Therefore, the shear layers of the jet are initially in a weakly disturbed state, so that the
431 exhaust conditions are very close to those of jetT1Mj2. More details on the simulation are available in reference [19]. A
432 snapshot of the pressure fluctuations and vorticity norm obtained are represented in Fig. 19. Notably, straight, inclined
433 Mach waves are visible near the jet flow and display sharp variations indicating the presence of steepened waves.

434 The conditional averaging procedure applied to the spatial jet differs from the one applied to the temporal jets since
435 the shock events are now detected in the temporal pressure signals acquired at a particular position, represented as a black
436 cross in Fig. 19. The radial coordinate r_{trig} of this position is equal to $4.5r_0$, as for the conditional averages computed in
437 section 19, and its axial location z_{trig} is equal to $17.5r_0$ and corresponds to that of peak pressure fluctuations at $r = r_{trig}$.
438 The conditional averages are computed by synchronizing the temporal signals according to the pressure maxima. To
439 this end, the signal acquired at (r_{trig}, z_{trig}) is splitted in intervals of length $3r_0/u_j$ and, for each interval, the time t_{trig}
440 and azimuthal location θ_{trig} of the pressure maxima that are higher than twice the standard deviation constitute the
441 reference according to which the signals are synchronized. Then, as in section III.D, the synchronized flow and sound
442 fields are ensemble-averaged at times $t \leq t_{trig}$ in order to describe the events leading to the formation of one generic,
443 steepened wave. Finally, as for the averages in section III.D, the procedure is applied to a reconstruction of the flow and

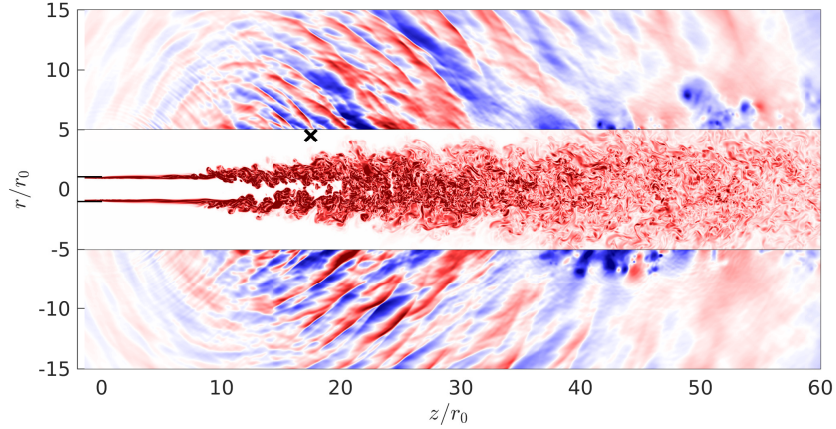


Fig. 19 Snapshots of pressure fluctuations and vorticity norm for a Mach 2 isothermal jet at $Re_D = 12,500$. The scales range from -4,000 to 4,000 Pa for the pressure and from 0 up to $3u_j/r_0$ for the vorticity.

444 sound field involving the first five azimuthal Fourier modes and are computed using approximately 150 events.

445 The conditionally-averaged pressure fluctuations are represented in Fig. 20 at $t_{trig} - 6r_0/a_\infty$, $t_{trig} - 3r_0/a_\infty$ and
 446 t_{trig} , along with the conditional vorticity and the location of the sonic line, defined as $\langle u_z \rangle = u_j$. The features revealed
 447 in the averaged fields are very similar to those described in Fig. 16(a) for jetT1Mj2. In particular, the pressure waves
 448 are connected to a large-scale vorticity excess located below the sonic line and convected downstream by the jet flow.
 449 This supports that the mechanisms leading to the formation of steepened waves in temporally-developing jets do not
 450 significantly differ from those in spatially-developing ones.

Acknowledgement

451 This work was granted access to the HPC resources of FLMSN (Fédération Lyonnaise de Modélisation et Sciences
 452 Numériques), partner of EQUIPEX EQUIP@MESO, and of the resources of IDRIS (Institut du Développement et
 453 des Ressources en Informatique Scientifique) under the allocation 2018-2a0204 made by GENCI (Grand Equipement
 454 National de Calcul Intensif). It was performed within the framework of the Labex CeLyA of Université de Lyon,
 455 within the programme ‘Investissements d’Avenir’ (ANR-10-LABX-0060/ANR-16-IDEX-0005) operated by the French
 456 National Research Agency (ANR).
 457

References

- 458
 459 [1] Tanna, H. K., Dean, P. D., and Fisher, M. J., “Influence of temperature on shock-free supersonic jet noise,” *Journal of Sound
 460 and Vibration*, Vol. 39, No. 4, 1975, pp. 429–460. doi:https://doi.org/10.1016/S0022-460X(75)80026-5.
 461 [2] Tanna, H. K., “An experimental study of jet noise. Part 1 : turbulent mixing noise,” *Journal of Sound and Vibration*, Vol. 50,
 462 No. 3, 1977, pp. 429–444. doi:https://doi.org/10.1016/0022-460X(77)90493-X.

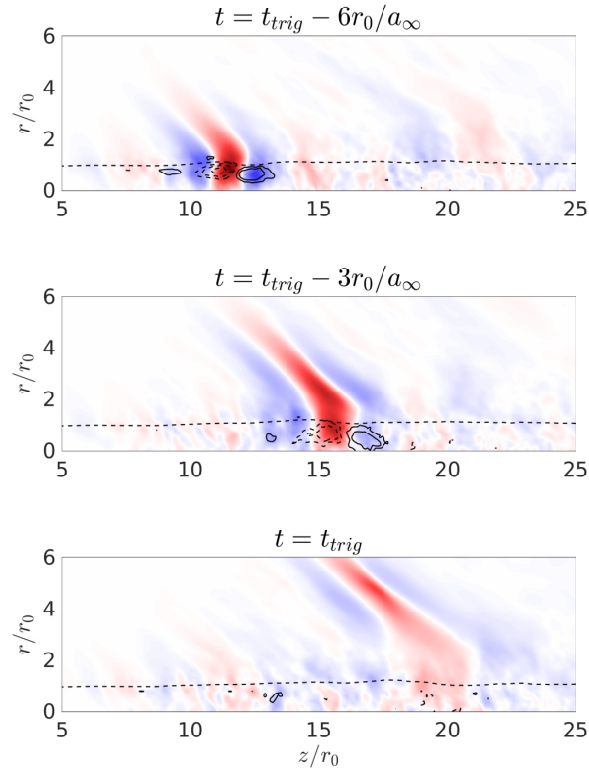


Fig. 20 Conditional pressure and vorticity fields for a Mach 2 spatially developing jet. See Fig. 16 for legend.

- 463 [3] Seiner, J., Ponton, M. K., Jansen, B., and Lagen, N., “The effects of temperature on supersonic jet noise emission,” *In 14th*
 464 *DGLR/AIAA aeroacoustics conference*, Vol. 1, 1992, pp. 295–307.
- 465 [4] Krothapalli, A., Venkatakrishnan, L., and Lourenco, L., “Crackle: A dominant component of supersonic jet mixing noise,”
 466 *AIAA paper 2000-2024*, 2000. doi:<https://doi.org/10.2514/6.2000-2024>.
- 467 [5] Viswanathan, K., “Aeroacoustics of hot jets,” *Journal of Fluid Mechanics*, Vol. 516, 2002, pp. 39–82.
 468 doi:<https://doi.org/10.1017/S0022112004000151>.
- 469 [6] Liu, J., Corrigan, A., Kailasanath, K., and Gutmark, E., “Effects of temperature on noise generation in supersonic jets,” *AIAA*
 470 *paper 2016-2937*, 2016. doi:<https://doi.org/10.2514/6.2016-2937>.
- 471 [7] Kearney-Fischer, M., Kim, J. H., and Samimy, M., “A study of Mach wave radiation using active control,” *Journal of Fluid*
 472 *Mechanics*, Vol. 681, 2011, pp. 261–292. doi:<https://doi.org/10.1017/jfm.2011.196>.
- 473 [8] Greska, B., Krothapalli, A., Horne, W. C., and Burnside, N., “A near-field study of high temperature supersonic jets,” *AIAA*
 474 *paper 2008-3026*, 2008. doi:<https://doi.org/10.2514/6.2008-3026>.
- 475 [9] Mora, P., Heeb, N., Kastner, J., Gutmark, E. J., and Kailasanath, K., “Impact of heat on the pressure skewness and kurtosis in
 476 supersonic jets,” *AIAA Journal*, Vol. 52, No. 4, 2014. doi:<https://doi.org/10.2514/1.J052612>.

- 477 [10] Panda, J., “Experimental investigation of turbulent density fluctuations and noise generation from heated jets,” *Journal of Fluid*
478 *Mechanics*, Vol. 591, 2007, pp. 73–96. doi:<https://doi.org/10.1017/S0022112007007173>.
- 479 [11] Ffowcs Williams, J. E., Simson, J., and Virchis, V. J., “‘Crackle’: an annoying component of jet noise,” *Journal of Fluid*
480 *Mechanics*, Vol. 71, 1975, pp. 251–271. doi:<https://doi.org/10.1017/S0022112075002558>.
- 481 [12] Gee, K. L., Sparrow, V. W., James, M. M., Downing, J. M., Hobbs, C. M., Gabrielson, T. B., and Atchley, A. A., “The role of
482 nonlinear effects in the propagation of noise from high-power jet aircraft,” *Journal of the Acoustical Society of America*, Vol.
483 123, No. 6, 2008, pp. 4082–4093. doi:<https://doi.org/10.1121/1.2903871>.
- 484 [13] Petitjean, B., and McLaughlin, D. K., “Experiments on the nonlinear propagation of noise from supersonic jets,” *AIAA paper*
485 *2003-3127*, 2003. doi:<https://doi.org/10.2514/6.2003-3127>.
- 486 [14] de Cacqueray, N., and Bogey, C., “Noise of an overexpanded Mach 3.3 jet: non-linear propagation effects and correlations
487 with flow,” *International Journal of Aeroacoustics*, Vol. 13, No. 7-8, 2014, pp. 607–632. doi:[https://doi.org/10.1260/1475-](https://doi.org/10.1260/1475-472X.13.7-8.607)
488 [472X.13.7-8.607](https://doi.org/10.1260/1475-472X.13.7-8.607).
- 489 [15] Papamoschou, D., “Evidence of shocklets in a counterflow supersonic shear layer,” *Physics of Fluids*, Vol. 7, No. 2, 1995, pp.
490 233–235. doi:<https://doi.org/10.1063/1.868621>.
- 491 [16] Fiévet, R., Tinney, C. E., Baars, W. J., and Hamilton, M. F., “Coalescence in the sound field of a laboratory-scale supersonic jet,”
492 *AIAA Journal*, Vol. 54, No. 1, 2016, pp. 254–265. doi:<https://doi.org/10.2514/1.J054252>.
- 493 [17] Nichols, J. W., Lele, S. K., Ham, F. E., Martens, S., and Spyropoulos, J. T., “Crackle noise in heated supersonic jets,” *Journal of*
494 *Engineering for Gas Turbines and Power*, Vol. 135, No. 5, 2013. doi:<http://dx.doi.org/10.1115/1.4007867>.
- 495 [18] Buchta, D. A., and Freund, J. B., “The near-field pressure radiated by planar high-speed free-shear-flow turbulence,” *Journal of*
496 *Fluid Mechanics*, Vol. 832, 2017, p. 383–408. doi:[10.1017/jfm.2017.671](https://doi.org/10.1017/jfm.2017.671).
- 497 [19] Pineau, P., and Bogey, C., “Study of the generation of shock waves by high-speed jets using conditional averaging,” *AIAA paper*
498 *2018-3305*, 2018. doi:<https://doi.org/10.2514/6.2018-3305>.
- 499 [20] Baars, W., Tinney, C. E., and Hamilton, M. F., “On cumulative nonlinear acoustic waveform distortions from high-speed jets,”
500 *Journal of Fluid Mechanics*, Vol. 749, 2014, pp. 331–366. doi:<https://doi.org/10.1017/jfm.2014.228>.
- 501 [21] Murray, N. E., and Lyons, G. W., “On the convection velocity of source events related to supersonic jet crackle,” *Journal of*
502 *Fluid Mechanics*, Vol. 793, 2016, pp. 477–503. doi:<https://doi.org/10.1017/jfm.2016.127>.
- 503 [22] Pineau, P., and Bogey, C., “Steepened Mach waves near supersonic jets: study of azimuthal structure and generation process
504 using conditional averages” *Journal of Fluid Mechanics* (accepted), 2019.
- 505 [23] Nichols, J. W., Lele, S. K., and Spyropoulos, J. T., “The source of crackle noise in heated supersonic jets,” *AIAA paper*
506 *2013-2197*, 2013. doi:<https://doi.org/10.2514/6.2013-2197>.

- 507 [24] Petitjean, B., Viswanathan, K., and McLaughlin, D., “Acoustic pressure waveforms measured in high speed jet
508 noise experiencing nonlinear propagation,” *International Journal of Aeroacoustics*, Vol. 5, No. 2, 2006, pp. 193–215.
509 doi:<https://doi.org/10.1260/2F147547206777629835>.
- 510 [25] Bogey, C. and Sabatini, R., “Effects of nozzle-exit boundary-layer profile on the initial shear-layer instability, flow field and
511 noise of subsonic jets,” *Journal of Fluid Mechanics*, Vol. 876, 2019, pp. 288–325. doi:<https://doi.org/10.1017/jfm.2019.546>.
- 512 [26] Brès, G., Jordan, P., Jaunet, V., Le Rallic, M., Cavalieri, A. V. G., Towne, A., Lele, S. K., Colonius, T. and Schmid, O.
513 “Importance of the nozzle-exit boundary-layer state in subsonic turbulent jets,” *Journal of Fluid Mechanics*, Vol. 851, 2018, pp.
514 83–124. doi:<https://doi.org/10.1017/jfm.2018.476>.
- 515 [27] Bogey, C., Marsden, O. and Bailly, C., “Influence of initial turbulence level on the flow and sound fields of a sub-
516 sonic jet at a diameter-based Reynolds number of 10^5 ,” *Journal of Fluid Mechanics*, Vol. 701, 2012, pp. 352–385.
517 doi:<https://doi.org/10.1017/jfm.2012.162>.
- 518 [28] Nonomura, T., Nakano, H., Ozawa, Y., Terakado, D., Yamamoto, M., Fujii, K., and Oyama, A., “Large eddy simulation of acoustic
519 waves generated from a hot supersonic jet,” *Shock waves*, 2019, pp. 1–22. doi:<https://doi.org/10.1007/s00193-019-00895-2>.
- 520 [29] Bogey, C. and Pineau, P., “Potential-core closing of temporally-developing round jets: Mach number scaling and conditional
521 averaging of flow and sound fields,” *Physical Review of Fluids*, submitted in revised form, 2019.
- 522 [30] Zaman, K. B. M. Q., “Effect of initial condition on subsonic jet noise,” *AIAA Journal*, Vol. 23, No. 9, 1985, pp. 1370–1373.
523 doi:<https://doi.org/10.2514/3.9094>.
- 524 [31] Bogey, C., “On noise generation in low Reynolds number temporal round jets at a Mach number of 0.9,” *Journal of Fluid
525 Mechanics*, Vol. 859, 2019, pp. 1022–1056. doi:<https://doi.org/10.1017/jfm.2018.864>.
- 526 [32] Papamoschou, D., and Roshko, A., “The compressible turbulent shear layer : an experimental study,” *Journal of Fluid
527 Mechanics*, Vol. 197, 1988, pp. 453–477. doi:<https://doi.org/10.1017/S0022112088003325>.
- 528 [33] Anderson, J. D., “Modern Compressible Flows : With Historical Perspective”, McGraw-Hill, 2nd edition, 1990, isbn
529 0-07-100665-6.
- 530 [34] Liu, J., Corrigan, A., Kailasanath, K., and Taylor, B., “Impact of the specific heat ratio on noise generation in a high-temperature
531 supersonic jet,” *AIAA paper 2016-2125*, 2016. doi:<https://doi.org/10.2514/6.2016-2125>.
- 532 [35] Buchta, D. A., and Freund, J. B., “Intense sound radiation by high-speed flow: Turbulence structure, gas properties, and
533 near-field gas dynamics,” *Physical Review Fluids*, Vol. 4(4), 2019, p.044605. doi:10.1103/PhysRevFluids.4.044605.
- 534 [36] Joseph, J. G., Tinney, C. E. and Murray, N., “Ideal gas effects in aeroacoustics,” *AIAA paper 2016-0688*, 2016.
535 doi:<https://doi.org/10.2514/6.2017-0688>.
- 536 [37] Bogey, C., and Bailly, C., “A family of low dispersive and low dissipative explicit schemes for noise computation,” *Journal of
537 Computational Physics*, Vol. 194, No. 1, 2004, pp. 194–214. doi:<https://doi.org/10.1016/j.jcp.2003.09.003>.

- 538 [38] Mohseni, K., and Colonius, T., “Numerical treatment of polar coordinate singularities,” *Journal of Computational Physics*, Vol.
539 157, No. 10, 2002, pp. 3593–3600. doi:<https://doi.org/10.1006/jcph.1999.6382>.
- 540 [39] Bogey, C., Marsden, O., and Bailly, C., “Finite differences for coarse azimuthal discretization and for reduction of effective
541 resolution near origin of cylindrical flow equations,” *Journal of Computational Physics*, Vol. 230, 2011, pp. 1134–1146.
542 doi:<https://doi.org/10.1016/j.jcp.2010.10.031>.
- 543 [40] Bogey, C., de Cacqueray, N., and Bailly, C., “A shock-capturing methodology based on adaptative spatial filter-
544 ing for high-order non-linear computations,” *Journal of Computational Physics*, Vol. 228, 2009, pp. 1447–1465.
545 doi:<https://doi.org/10.1016/j.jcp.2008.10.042>.
- 546 [41] Tam, C. K. W., and Dong, Z., “Radiation and outflow boundary conditions for direct computation of acoustic and
547 flow disturbances in a nonuniform mean flow,” *Journal of Computational Acoustics*, Vol. 4, No. 2, 1996, pp. 175–201.
548 doi:<https://doi.org/10.1142/S0218396X96000040>.
- 549 [42] Freund, J. B., Lele, S. K., and Moin, P., “Compressibility effects in a turbulent annular mixing layer. Part 1. Turbulence and
550 growth rate,” *Journal of Fluid Mechanics*, Vol. 421, 2000, pp. 229–267. doi:<https://doi.org/10.1017/S0022112000001622>.
- 551 [43] Bogey, C., and Bailly, C., “Influence of nozzle-exit boundary-layer conditions on the flow and acoustic fields of initially laminar
552 jets,” *Journal of Fluid Mechanics*, Vol. 663, 2010, pp. 507–538. doi:<https://doi.org/10.1017/S0022112010003605>.
- 553 [44] Lu, G., and Lele, S. K., “On the density ratio effect on the growth rate of a compressible mixing layer,” *Physics of Fluids*,
554 Vol. 6, 1994, pp. 1073–1075. doi:<https://doi.org/10.1063/1.868340>.
- 555 [45] Pantano, C., and Sarkar, S., “A study of compressibility effects in the high-speed turbulent shear layer using direct simulation,”
556 *Journal of Fluid Mechanics*, Vol. 451, 2002, pp. 29–371. doi:<https://doi.org/10.1017/S0022112001006978>.
- 557 [46] Lau, J. C., “Effects of exit Mach number and temperature on mean-flow and turbulence characteristics in round jets,” *Journal of*
558 *Fluid Mechanics*, Vol. 105, 1981, pp. 193–218. doi:<https://doi.org/10.1017/S0022112081003170>.
- 559 [47] Ffowcs Williams, J. E., “The noise from turbulence convected at high speed,” *Philosophical Transactions of the Royal Society*
560 *of London*, Vol. 255, No. 1061, 1963, pp. 469–503. doi:<https://doi.org/10.1098/rsta.1963.0010>.
- 561 [48] Schlinker, R., Liljenberg, S., Polak, D., Post, K., and Chipman, C., “Supersonic jet noise source characteristics and propagation:
562 engine and model scale,” *AIAA paper 2007-3623*, 2007. doi:<https://doi.org/10.2514/6.2007-3623>.
- 563 [49] Arndt, R. E. A., Long, D. F., and Glauser, M. N., “The Proper Orthogonal Decomposition of pressure fluctuations surrounding
564 a turbulent jet,” *Journal of Fluid Mechanics*, Vol. 340, 1997, pp. 1–33. doi:<https://doi.org/10.1017/S0022112097005089>.
- 565 [50] Ecker, T., Lowe, K. T., and Ng, W. F., “Eddy convection in developing heated supersonic jets,” *AIAA Journal*, Vol. 53, No. 11,
566 2015, pp. 3305–3315. doi:<https://doi.org/10.2514/1.J053946>.
- 567 [51] Gojon, R., Baier, F., Gutmark, E., and Mihaescu, M., “Temperature effects on the aerodynamic and acoustic fields of a
568 rectangular supersonic jet,” *AIAA paper 2017-0002*, 2017.

- 569 [52] Pineau, P., and Bogey, C., “Numerical study of the sound fields of temporally-developing supersonic round jets,” *AIAA paper*
570 *2017-3209*, 2017. doi:<https://doi.org/10.2514/6.2017-3209>.
- 571 [53] Bogey, C., “Two-dimensional features of correlations in the flow and near pressure fields of Mach number 0.9 jets,” *AIAA paper*
572 *2019-0806*, 2019, pp. 1–14. doi:<https://doi.org/10.2514/6.2019-0806>.
- 573 [54] de Cacqueray, N., Bogey, C., and Bailly, C., “Investigation of a High-Mach-Number Overexpanded Jet Using Large-Eddy
574 Simulation,” *AIAA Journal*, Vol. 49, No. 10, 2011, pp. 2171–2182. doi:<https://doi.org/10.2514/1.J050952>.
- 575 [55] Sinha, A., Rodriguez, D., Brès, G., and Colonius, T., “Wavepacket models for supersonic jet noise,” *Journal of Fluid Mechanics*,
576 Vol. 742, 2014, pp. 71–95. doi:<https://doi.org/10.1017/jfm.2013.660>.
- 577 [56] Tam, C. K. W., and Burton, D. E., “Sound generated by instability waves of supersonic flows. Part 2. Axisymmetric jets,”
578 *Journal of Fluid Mechanics*, Vol. 138, 1984, p. 273–295. doi:<https://doi.org/10.1017/S0022112084000124>.
- 579 [57] Tam, C. K. W., and Chen, P., “Turbulent mixing noise from supersonic jets,” *AIAA Journal*, Vol. 32, No. 9, 1994, pp. 1774–1780.
580 doi:<https://doi.org/10.2514/3.12173>.
- 581 [58] Bogey, C. and Marsden, O. and Bailly, C., “Large-eddy simulation of the flow and acoustic fields of a Reynolds
582 number 10^5 subsonic jet with tripped exit boundary layers,” *Physics of Fluids*, Vol. 23, 035104, 2011, pp. 1–20.
583 doi:<https://doi.org/10.1063/1.3555634>.