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FIXED POINTS IN MULTIPLE EQUILIBRIA PROBABILITY-MIGRATION MODELS

M. Garçon, J. Garnier, A. Omrane

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Abstract. We analyze a probability-migration model in which the probability of migration depends on human capital (education essentially). In this model, the human capital can converge to two possible values (fixed points), a low or high value. The aim of this paper is to analyze how belief mechanisms can lead to the selection of a particular fixed point.
In particular, we prove that for any belief mechanism, there exists a critical value H such that the result can always be expressed by an assertion of the form: if initially, the human capital is smaller than the critical value H, then the human capital will converge to the low fixed point, while if the initial human capital is larger than H, then the human capital will converge to the high fixed point value. The fixed points do not depend on the belief mechanism, while the critical value strongly depends on it.

Introduction. Migration problems in developing countries are an old subject of study. The article deals with the probability-migration problem which depends on the economic growth and where we distinguish between the high-skilled and low-skilled emigration creating a 'brain-drain' problem as shown by several authors as by Mountford [15], or by Miyagiwa [14] who has developed a

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theoretical model to analyze human capital formation (education) for both host and source countries. Miyagiwa showed that a 'brain drain' will impact upon the availability of intermediate-skilled workers in the source country. Conversely, Stark et al. [17] (see also Stark and Wang [18]) argued that migration raises the return on human capital that will in turn raise the average level of human capital in the source country. For the general theory of probability migration, one can see the details in Bhagwati and Rodriguez [2]. Some models were also developed by Beine et al. [1], Docquier et al. [8], Card and Krueger [3], Chen [4][5], and the recent works by De la Croix and Docquier [7], or by Mountford and Rapoport [16], and the references therein.

In this article, we investigate the impact of migration on economic growth through the role of human capital from the source country point of view. We generalize our work by Garçon et al. [12] where we studied a simpler case.

We consider the case where migration is dependent of human capital (high education essentially). We use the model by Chen [4] and Vidal [19] who proposed a model where the probability of migration is endogenous; this model naturally depends on human capital. As we will see in detail, people living in a source country with higher average human capital are traditionally more incited to emigrate to a foreign country than those living in a source country with lower average human capital.

As in Chen [4], we consider the case where the probability of migration can take only two values: the low (respectively high) value is taken when the human capital is smaller (respectively larger) than a threshold level. Moreover, the convergence follows the two possible scenarios:

— The first scenario is when the probability of migration depends on *prior* human capital, which is the case when the human capital is the one inherited from the parents or equivalently the one of the agents before their education period; we will call it in this paper the *traditional* case. In this scenario, the threshold level affects the economic behavior in the long run. More precisely, if the human capital threshold is sufficiently low (respectively high), then the economy converges to a high (respectively low) steady state level. However, if the human capital threshold is at the median level, club convergence may occur. Moreover, it depends on the initial condition. We will give a review of the results in Section 3.

— In the second scenario, the probability of migration depends on *current* human capital (at the end of the agents education period); we will call it the *anticipative* case, and we will discuss it carefully in Section 4, which contains our main results. Here, the dynamic transition of the economy is determined by perceptions of the future and we have indetermination in the sense that agents may choose

among two possible solutions. The choice between the two solutions depends on a common belief. A belief in higher probability of migration in the future provides an incentive for agents to invest more in their education, thereby raising their accumulation of human capital, which in turn lead to a higher probability of migration. Through a careful analysis of several belief mechanisms, we come to the conclusion that there always exist a critical value for the initial human capital condition below which the human capital will converge to a low fixed point, and above which the human capital will converge to the high fixed point. The main point is that this critical value strongly depends on the belief mechanism.

The paper is organized as follows: In Sections 2 and 3 we recall the most important results in the traditional scenario and we add some remarks. Section 4 is devoted to the anticipative scenario. Here, it is shown that migration can be a source of indeterminacy. We introduce three different belief mechanisms: the optimistic, pessimistic and conservative mechanisms, where we explain in detail that indeterminacy emphasizes the role of beliefs: people with similar backgrounds may well follow different equilibrium paths simply because they have different beliefs about their future probability of migration.

2. Position of the problem. In a small open economy characterized by an infinite horizon, we consider a no-growth overlapping generations model, where agents live for two periods N_t and N_{t+1}. In each period a new generation is born, agents born in period t are endowed with parental human capital h_t, and are supposed to allocate their time between gaining education e_t and engaging in leisure $1 - e_t$ in the first period of life. In the second period, agents can migrate to a foreign country (country B) with probability $p_{t+1} \in [0, 1]$ or remain into the home country (country A) with probability $1 - p_{t+1}$. During this second period of life, agents spend their time working to earn income for consumption.

Moreover, if w_A and w_B represent the respective real wage per unit of human capital in countries A and B, the earnings of agents depend on their level of human capital h_{t+1}, through the formula $w_A h_{t+1}^{\theta_A}$ (respectively $w_B h_{t+1}^{\theta_B}$). That is, the expected utility function, which is identical for all agents is defined for $\beta > 0$ by:

$$u_t = \ln(1 - e_t) + \beta \left[(1 - p_{t+1}) \ln(w_A h_{t+1}^{\theta_A}) + p_{t+1} \ln(w_B h_{t+1}^{\theta_B}) \right]. \tag{1}$$

From period t to period $t + 1$ the human capital evolves following the relation

$$h_{t+1} = A e_t^{\gamma} h_t^{\delta}, \qquad \gamma, \delta \in (0, 1). \tag{2}$$

We distinguish two migration processes: the traditional process of migration in which the probability of migration of the young adults p_{t+1} is determined by the human capital of the parents, $p_{t+1} = \mathcal{P}(h_t)$, and the anticipative process in which the probability of migration of the young adults p_{t+1} is determined by the human capital of those adults at the end of their first period h_{t+1}, that is $p_{t+1} = \mathcal{P}(h_{t+1})$. As we will see, indeterminacy can occur in this anticipative situation, since the time spent in education e_t, and therefore the human capital of the young adults at the end of their first period h_{t+1}, then depend on the probability of migration p_{t+1}.

The variation of the utility function u_t with respect to education is given by

$$\frac{\partial u_t}{\partial e_t} = \frac{-1}{1-e_t} + \frac{\gamma\beta}{e_t}\left[\theta_A + (\theta_B - \theta_A)p_{t+1}\right], \tag{3}$$

and the optimal decision e_t^* for agents staying in country A, which is reached at $(\partial u_t)/(\partial e_t) = 0$, is given by

$$e_t^* = \frac{\gamma\beta\left[\theta_A + (\theta_B - \theta_A)p_{t+1}\right]}{1 + \gamma\beta\left[\theta_A + (\theta_B - \theta_A)p_{t+1}\right]}. \tag{4}$$

We deduce that

$$\frac{\partial e_t^*}{\partial p_{t+1}} = \frac{\gamma\beta(\theta_B - \theta_A)}{\Big(1 + \gamma\beta\left[\theta_A + (\theta_B - \theta_A)p_{t+1}\right]\Big)^2}. \tag{5}$$

Remark 1. *Note that $e_t^{*\prime} > 0$ iff $\theta_B w_B > \theta_A w_A$ which means that an increase in p_{t+1} will give an incentive for agents to invest more in their education, and in contrary if $\theta_B w_B \leq \theta_A w_A$, the agents are more incite to keep in their own country.*

The human capital (2) evolves following the equation

$$h_{t+1} = A\{e_t^*\}^\gamma h_t^\delta = A\left\{\frac{\gamma\beta\left[\theta_A + (\theta_B - \theta_A)p_{t+1}\right]}{1 + \gamma\beta\left[\theta_A + (\theta_B - \theta_A)p_{t+1}\right]}\right\}^\gamma h_t^\delta. \tag{6}$$

3. The traditional model. The probability of migration is assumed to be dependent on average human capital H_t. We suppose that the agents are homogeneous, then the average human capital is equal to the personal human

capital in each period $H_t = h_t$. In this subsection, we consider the traditional model of migration, that is:

$$p_{t+1} = \mathcal{P}(h_t), \tag{7}$$

which means that the probability of migration is dependent on human capital lagged by one period (i.e. the average human capital of the parents). We also suppose that

$$\mathcal{P}(h) = \begin{cases} p_1 & \text{if} \quad h < h^{\#} \\ p_2 & \text{if} \quad h \geq h^{\#} \end{cases} \tag{8}$$

for some probability constants $0 \leq p_1 < p_2 \leq 1$, where $h^{\#}$ is a nominative threshold human capital as in [4]. For $j = 1, 2$, we finally denote by

$$e_j = \frac{\gamma\beta\left[\theta_A + (\theta_B - \theta_A)p_j\right]}{1 + \gamma\beta\left[\theta_A + (\theta_B - \theta_A)p_j\right]}. \tag{9}$$

Notice that we have $e_1 < e_2$.

Proposition 2. *The sequence of human capitals $(h_t)_t$ converge to a fixed point as $t \to \infty$.*
The two possible fixed points are $\bar{h}_1$ and $\bar{h}_2$ (with $\bar{h}_1 < \bar{h}_2$) defined by

$$\bar{h}_j = \left(A\, e_j^{\gamma}\right)^{\frac{1}{1-\delta}}, \quad j = 1, 2. \tag{10}$$

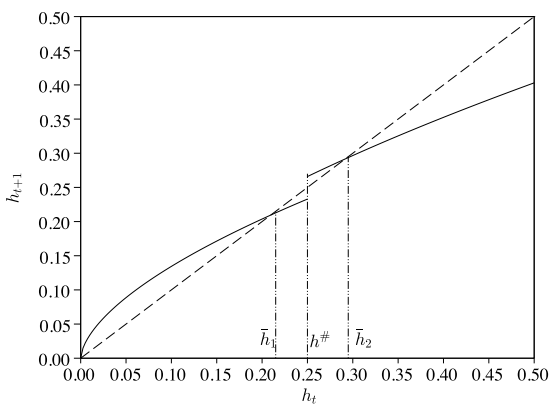

Fig. 1. The traditional case: the solid line plots recursive formula (11). Here $\bar{h}_1 < h^{\#} < \bar{h}_2$.

We have the following:

- *If $\bar{h}_1 > h^{\#}$, then the sequence $(h_t)_t$ converges to $\bar{h}_2$ for every h_0.*
- *If $\bar{h}_2 < h^{\#}$, then the sequence $(h_t)_t$ converges to $\bar{h}_1$ for every h_0.*
- *If $\bar{h}_1 < h^{\#} < \bar{h}_2$, then*
  - *(a) if $h_0 < h^{\#}$, the sequence $(h_t)_t$ converges to $\bar{h}_1$,*
  - *(b) if $h_0 > h^{\#}$, the sequence $(h_t)_t$ converges to $\bar{h}_2$.*

This proposition is an application of standard results on the convergence of sequences defined by a recursive relation of the form $h_{t+1} = \mathcal{H}(h_t)$:

$$h_{t+1} = \begin{cases} Ae_1^{\gamma} h_t^{\delta} & \text{if} \quad h_t < h^{\#}, \\ Ae_2^{\gamma} h_t^{\delta} & \text{if} \quad h_t \geq h^{\#}. \end{cases} \tag{11}$$

With this traditional migration model (i.e when the probability of migration is dependent on the human capital of the parents), the human capital threshold $h^{\#}$ determines the growth of the economy which will converge to one of the two fixed points $\bar{h}_1$ and $\bar{h}_2$ given by (10).

4. The anticipative model. In this section we assume that the probability of migration is dependent on the average human capital in period $t+1$ as in [4]:

$$p_{t+1} = \mathcal{P}(h_{t+1}), \tag{12}$$

with $\mathcal{P}$ defined by (8). Then the recursive relation (11) becomes an implicit relation (h_{t+1} is on the right and on the left of the relation):

$$h_{t+1} \quad = \quad \begin{cases} A\, e_1^{\gamma}\, h_t^{\delta} & \text{if} \quad h_{t+1} < h^{\#}, \\ A\, e_2^{\gamma}\, h_t^{\delta} & \text{if} \quad h_{t+1} \geq h^{\#}. \end{cases} \tag{13}$$

Let us define

$$h_o^{\#} = \left(\frac{h^{\#}}{Ae_2^{\gamma}}\right)^{\frac{1}{\delta}} \quad \text{and} \quad h_p^{\#} = \left(\frac{h^{\#}}{Ae_1^{\gamma}}\right)^{\frac{1}{\delta}}. \tag{14}$$

Observe that we have

$$h_o^{\#} < h^{\#} < h_p^{\#}. \tag{15}$$

Equation (13) is implicit and, given the value h_t, there may be several possible values for h_{t+1}. This shows that the dynamics of human capital depends on households perceptions and beliefs about the future. The following lemma addresses the easy situation in which there is no indeterminacy.

Lemma 3. *Let h_t be the human capital at period t. The human capital h_{t+1} at period $t+1$ must satisfy equation* (13)*. Then we have the following:*
1) *If $h_t < h_o^{\#}$ then there exists a unique possible value $h_{t+1} = Ae_1^{\gamma} h_t^{\delta}$.*
2) *If $h_t > h_p^{\#}$ then there exists a unique possible value $h_{t+1} = Ae_2^{\gamma} h_t^{\delta}$.*

The proof of Lemma 3 is easy and follows from (15) and (13).

From now on in this paper we consider the case in which the following hypothesis is fulfilled by the parameters of our model:

$$e_1^{\gamma(\delta-1)} e_2^{-\gamma\delta} < A\Big(h^{\#}\Big)^{\delta-1} < e_1^{-\gamma\delta} e_2^{\gamma(\delta-1)}. \tag{16}$$

We now discuss useful equivalent formulations of Hypothesis (16) and a convergence result in the following proposition:

Proposition 4. *Hypothesis (16) is fulfilled if and only if*

$$[0, h_o^{\#}) \text{ and } (h_p^{\#}, +\infty) \text{ are stable through the relation (13)} \tag{17}$$

if and only if the two possible fixed points given by (10) satisfy

$$\bar{h}_1 \in [0, h_o^{\#}) \text{ and } \bar{h}_2 \in (h_p^{\#}, +\infty). \tag{18}$$

Moreover, if (16) is satisfied, and if $h_0 > 0$ is the initial human capital, then we have the following assertions:

1. *If $h_0 < h_o^{\#}$ then the resulting human capital sequence $(h_t)_t$ converges to the fixed point $\bar{h}_1$.*
2. *If $h_0 > h_p^{\#}$ then the resulting human capital sequence $(h_t)_t$ converges to the fixed point $\bar{h}_2$.*

The complementary case of (16) is addressed in [11]. Hypothesis (16) ensures the stability of the intervals $[0, h_o^{\#})$ and $(h_p^{\#}, +\infty)$ through the anticipative model (13), and shows that the two fixed points belong to the two different stable regions. Moreover, Proposition 4 states that stability and existence of the two fixed points in the stable regions give convergence of the sequence defined by (13).

The proof of Proposition 4 is given in the Appendix. We need now to address the case in which $h_o^{\#} < h_0 < h_p^{\#}$. In this case indeterminacy occurs, as stated in the following lemma (see Figure 2).

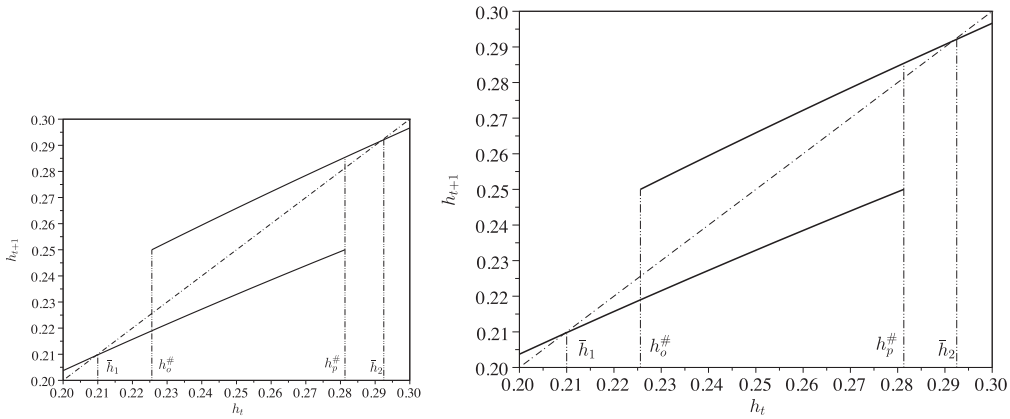

Fig. 2. The anticipative case: the solid line stands for the anticipative formula (13). Indeterminacy (multiple solutions) occurs when $h_t \in \left[h_o^\#, h_p^\#\right]$

Lemma 5. *Let h_t be the human capital at period t. If $h_o^\# < h_t < h_p^\#$, then there exist two different possible values for the solution h_{t+1} of (13):*

$$h_{t+1,j} = A\, e_j^\gamma\, h_t^\delta \qquad \textit{for} \quad j = 1, 2. \tag{19}$$

The lemma shows that it is necessary to give a mechanism to select between the two possible solutions for h_{t+1} in the case in which $h_o^\# < h_t < h_p^\#$. We will address different belief mechanisms in Subsection 4.1 and Subsection 4.2 below.

4.1. Optimistic and pessimistic belief mechanisms. The pessimistic belief mechanism consists in choosing the smallest value for the human capital when there are two possible choices. The optimistic belief mechanism consists in choosing the largest value for the human capital when there are two possible choices. These are the two extremal belief mechanisms. We will consider an intermediate mechanism later in Subsection 4.2.

The following proposition gives the main result in the case of the two belief mechanisms (see Figures 3 and 4).

Proposition 6. *Let h_0 be an initial human capital in the multivalued region i.e $h_o^\# < h_0 < h_p^\#$. We have the following assertions:*

(1) *With the pessimistic belief mechanism, the sequence $(h_t)_t$ converges to the fixed point $\bar{h}_1$ if $h_0 < h_p^\#$, and converges to the fixed point $\bar{h}_2$ if $h_0 > h_p^\#$.*

(2) *With the optimistic belief mechanism, the sequence $(h_t)_t$ converges to the fixed point $\bar{h}_1$ if $h_0 < h_o^\#$, and converges to the fixed point $\bar{h}_2$ if $h_0 > h_o^\#$.*

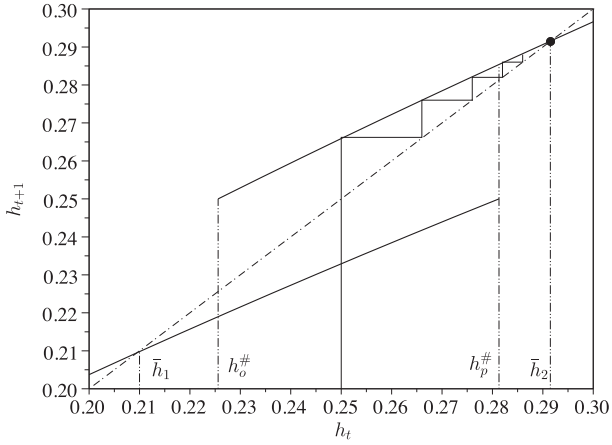

Fig. 3. The optimistic case: convergence to the high human capital (fixed point) $\bar{h}_2$ when $h_0 \in \left[h_o^{\#}, h_p^{\#}\right]$

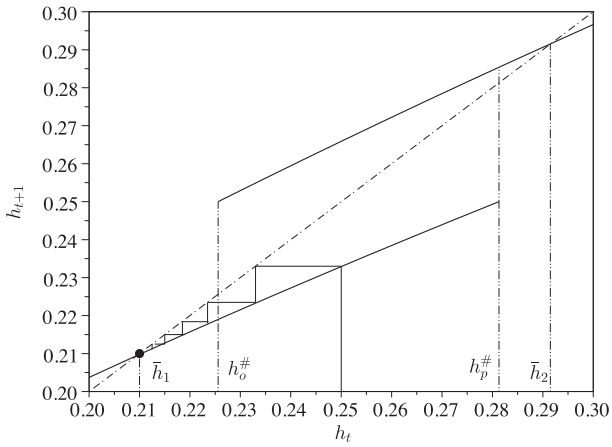

Fig. 4. The pessimistic case: convergence to the low fixed point $\bar{h}_1$ when $h_0 \in \left[h_o^{\#}, h_p^{\#}\right]$

This proposition shows that the threshold value for the initial condition below which the human capital converges to the low fixed point is *smaller* with the optimistic mechanism than with the pessimistic mechanism.

4.2. Conservative belief mechanism. We still use Hypothesis (16). The conservative belief mechanism consists in choosing for the human capital h_{t+1} at period $t+1$ the value that is the closest from h_t when there are two possible choices.

Let us define

$$h_c^{\#} = \left(A\frac{e_1^{\gamma} + e_2^{\gamma}}{2}\right)^{\frac{1}{1-\delta}}. \tag{20}$$

Lemma 7. *Let h_0 be an initial human capital in the multivalued region i.e $h_o^{\#} < h_0 < h_p^{\#}$. We have the two following assertions:*

(a) If $h_0 > h_c^{\#}$, then we have $h_t > h_c^{\#}$ for all t and $h_{t+1} = Ae_2^{\gamma}h_t^{\delta}$ is the solution selected by the conservative mechanism. Moreover, the sequence $(h_t)_t$ converges to the fixed point $\bar{h}_2$.

(b) If $h_0 < h_c^{\#}$, then we have $h_t < h_c^{\#}$ for all t and $h_{t+1} = Ae_1^{\gamma}h_t^{\delta}$ is the solution selected by the conservative mechanism. Moreover, the sequence $(h_t)_t$ converges to the fixed point $\bar{h}_1$.

We recall that the threshold value in the traditional case is $h^{\#}$. In the anticipative case with the optimistic (resp. pessimistic) belief mechanism it is $h_o^{\#}$ (resp. $h_p^{\#}$). The following proposition summarizes the convergence results when the conservative belief mechanism is used:

Proposition 8. *With the conservative belief mechanism, the sequence $(h_t)_t$ converges to the fixed point $\bar{h}_1$ (resp. $\bar{h}_2$) if $h_0 < H$ (resp. $h_0 > H$) where we have:*

(a) $H = h_p^{\#}$ if $h_c^{\#} > h_p^{\#}$,
(b) $H = h_o^{\#}$ if $h_c^{\#} < h_o^{\#}$,
(c) $H = h_c^{\#}$ if $h_o^{\#} < h_c^{\#} < h_p^{\#}$.

In the goal to have a high economy level, we notice that the optimistic anticipation mechanism is the one that gives the smallest critical value H from which we have convergence to the highest fixed point $\bar{h}_2$. Conversely, the pessimistic anticipation mechanism is the one that gives the largest critical value.

5. Conclusion. If the possibility of migration-induced multiple equilibria has been known since Mounford [15], the role of beliefs in defining the threshold value human H such that for initial human capital h_0 smaller (larger) than H, the economy converges to the low (high) steady-state is new.

It is worthwhile to note that, whatever the type of evolution equation for the human capital (traditional or anticipative) and for any belief mechanism, the result can always be expressed by an assertion of the form: if the initial capital h_0 is smaller than a threshold value H, then the human capital will converge to the low fixed point $\bar{h}_1$, while if the initial capital h_0 is larger than a threshold value H, then the human capital will converge to the high fixed point $\bar{h}_2$. The values

of the fixed points $\bar{h}_1$ and $\bar{h}_2$ do not depend on the type of evolution equation and on the belief mechanism. Only the threshold value H depends on the type of evolution equation and on the selection mechanism. In particular we show that beliefs can have a strong impact on the threshold value H.

We have analyzed the impact of international migration on economic growth of a source country in a probabilistic setting through the role of human capital, using the model by Chen [4]. We also established some examples which helped us to understand the impact of beliefs on the probability migration.

The two scenarios introduced above can be used in other type of problems about probability migration: the case where we have to distinguish between public and private schooling, following the ideas by Chen [5], or the case where the low-skilled and high-skilled workers problem has to be considered as in Lien and Wang [13]. Some of the answers can be found in Garçon [11].

6. Appendix – Proofs and complementary results.

6.1. Proof of Proposition 4. We prove the stability of the interval $[0, h_o^{\#})$ in (17). Let h_t be such that $h_t < h_o^{\#}$. This is equivalent to $Ae_2^{\gamma} h_t^{\delta} < h^{\#}$. From Lemma 3 we have $h_{t+1} = Ae_1^{\gamma} h_t^{\delta}$. Using the RHS of Hypothesis (16) we have $A\left(h^{\#}\right)^{\delta-1} < (e_1^{\gamma})^{-\delta}(e_2^{\gamma})^{\delta-1}$ which is equivalent to $Ae_1^{\gamma} < \left(\dfrac{h^{\#}}{Ae_2^{\gamma}}\right)^{\frac{1-\delta}{\delta}}$. This implies:

$$h_{t+1} = Ae_1^{\gamma} h_t^{\delta} < \left(\frac{h^{\#}}{Ae_2^{\gamma}}\right)^{\frac{1-\delta}{\delta}} h_t^{\delta} = \left(\frac{Ae_2^{\gamma} h_t^{\delta}}{h^{\#}}\right) h_o^{\#} < h_o^{\#},$$

which proves the stability of the interval $[0, h_o^{\#})$.
For the stability of the interval $(h_p^{\#}, +\infty)$ we use the same type of arguments: if $h_t > h_p^{\#}$, then we use the LHS of Hypothesis (16), i.e $e_1^{\gamma(\delta-1)} e_2^{-\gamma\delta} < A\left(h^{\#}\right)^{\delta-1}$ to prove that $h_{t+1} > h_p^{\#}$.
Now we prove (18). We have the following equivalence with (16):

$$A\left(h^{\#}\right)^{\delta-1} < (e_1^{\gamma})^{-\delta}(e_2^{\gamma})^{\delta-1} \quad \text{iff} \quad (Ae_1^{\gamma})^{\delta} < \left(\frac{h^{\#}}{Ae_2^{\gamma}}\right)^{1-\delta} \quad \text{iff} \quad \bar{h}_1 < h_o^{\#},$$

and

$$A\left(h^{\#}\right)^{1-\delta} > e_1^{\gamma(\delta-1)} e_2^{-\gamma\delta} \quad \text{iff} \quad (Ae_2^{\gamma})^{\delta} > \left(\frac{h^{\#}}{Ae_1^{\gamma}}\right)^{1-\delta} \quad \text{iff} \quad \bar{h}_2 > h_p^{\#}.$$

We end by proving the convergence result:
Case (1) If $h_0 < h_o^{\#}$, then the above stability shows that $h_t < h_o^{\#}$, which defines

$h_{t+1} = Ae_1^\gamma h_t$ by Lemma 3. Therefore, the sequence $(h_t)_t$ converges to the fixed point $\bar{h}_1$, which is the unique fixed point of the recursive relation $h_{t+1} = Ae_1^\gamma h_t$ in $(0, h_o^\#)$ by (18).
Case (2) The same type of arguments can be used and the result holds true. □

6.2. Proof of Proposition 6. Case (1) If $h_0 < h_p^\#$ then we can show by induction that for all t we have $h_t < h_p^\#$ and $h_{t+1} = Ae_1^\gamma h_t^\delta$. Indeed, if $h_t < h_p^\#$, then the pessimistic mechanism selects $h_{t+1} = Ae_1^\gamma h_t^\delta$ and therefore $h_{t+1} < Ae_1^\gamma (h_p^\#)^\delta = h^\# < h_p^\#$. Note that this argument also shows that $h_t < h^\#$ as soon as $t \geq 1$. Since $\bar{h}_1$ is the unique fixed point of the recursive relation $h_{t+1} = Ae_1^\gamma h_t^\delta$ in $(0, h^\#)$, the sequence $(h_t)_t$ must converge to the fixed point $\bar{h}_1$. From the other side, if $h_0 > h_p^\#$ then by applying Proposition 4 we find that the sequence $(h_t)_t$ converges to the fixed point $\bar{h}_2$.

Case (2) can be addressed in the same way. □

6.3. Proof of Lemma 7. Let the initial condition be such that $h_o^\# < h_0 < h_p^\#$. In Case (a) we have $h_0 > h_c^\#$. By induction we can show that $h_t > h_c^\#$ and $h_{t+1} = Ae_2^\gamma h_t^\delta$ for all t. Indeed, if $h_t > h_c^\#$, then we have two possible solutions by Lemma 5 : $h_{t+1,1} = Ae_1^\gamma h_t^\delta$ and $h_{t+1,2} = Ae_2^\gamma h_t^\delta$. However $h_t > h_c^\#$ is equivalent to $2h_t > A\left(e_1^\gamma + e_2^\gamma\right) h_t^\delta$, and therefore we have $h_t - h_{t+1,1} > h_{t+1,2} - h_t > 0$. Consequently the conservative mechanism selects $h_{t+1,2}$. This point is larger than h_t, which is itself larger than $h_c^\#$, which completes the proof of the induction. Finally, the unique fixed point of the recursive relation $h_{t+1} = Ae_2^\gamma h_t^\delta$ in $(h_c^\#, \infty)$ is $\bar{h}_2$ by (18). Therefore the sequence $(h_t)_t$ must converge to $\bar{h}_2$. We use the same arguments for the proof of Case (b). □

Remark 9. To be complete we can mention that Cases (a) and (b) in Lemma 7 can indeed occur. More exactly, Case (a) occurs if $h_c^\# < h_p^\#$. Case (b) occurs if $h_c^\# > h_o^\#$. The fact that $h_c^\# < h_p^\#$ is equivalent to $A(h^\#)^{\delta-1} < e_1^{\gamma(\delta-1)} \left(\dfrac{e_1^\gamma + e_2^\gamma}{2}\right)^{-\delta}$, which is compatible with Hypothesis (16). The fact that $h_c^\# > h_o^\#$ is equivalent to $A(h^\#)^{\delta-1} > e_2^{\gamma(\delta-1)} \left(\dfrac{e_1^\gamma + e_2^\gamma}{2}\right)^{-\delta}$, which is also compatible with Hypothesis (16).

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