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1 **Evaluating hydrodynamic parameters accounting for**  
2 **water retention hysteresis in a large sand column using**  
3 **surface GPR**

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4 **Abstract.** This study presents laboratory experiments using surface Ground  
5 Penetrating Radar measurements to monitor hysteresis of the water reten-  
6 tion function of a sand. A GPR system monitored the volumetric water con-  
7 tent changes in a large sand column submitted to different hydraulic heads  
8 applied at its bottom during one cycle of drainage followed by imbibition.  
9 The average velocity of the electromagnetic waves measured during drainage  
10 was consistently smaller than the one measured at the same applied head  
11 during imbibition. This was attributed to the hysteresis of the sand water  
12 retention curve. Coupled hydrodynamic and electromagnetic modeling was  
13 used to simulate radargrams whereas hydrodynamic modeling coupled with  
14 1D optical ray path was used to estimate the hydrodynamic parameters of  
15 the sand from GPR reflection two-way travel times. Statistical and uncer-  
16 tainty analysis were performed on numerical and experimental data. The range  
17 of optimized parameters were compared to those obtained with classical lab-  
18 oratory methods such as the hanging water column and the constant head  
19 permeameter.

## 1. Introduction

20 Understanding water status and transport in the vadose zone requires accurate knowl-  
21 edge of the water retention function,  $\theta(h)$ , of the vadose zone materials as well as their  
22 hydraulic conductivity function,  $K(\theta)$ . The first one relates water pressure head,  $h$ , with  
23 volumetric water content,  $\theta$ , while the later makes the link between hydraulic conductivity  
24 and volumetric water content or pressure head. This classical definition of the hydrody-  
25 namic functions assumes that  $h$  is a unique function of  $\theta$ . However, as first pointed out  
26 by *Haines* [1930], this assumption is not valid because of hysteresis phenomena.

27 The basis of hysteresis theory, first observed in magnetism and adsorption on solids,  
28 was set in the works of *Néel* [1942, 1943], *Everett and Whitton* [1952]; *Everett* [1954, 1955]  
29 and *Preisach* [1935]. *Everett* [1955] and *Enderby* [1955] developed in parallel the indepen-  
30 dent domain theory, which was later suggested to be applicable to soil capillary hysteresis  
31 by *Collis-George* [1955]. The assumption behind the independent domain theory of soil-  
32 water hysteresis is that the pore space can be divided into discrete pores, each of which  
33 drains and fills independently of the state of other pores. The first published account  
34 of capillary hysteresis in term of independent domains was performed by *Poulovassilis*  
35 [1962], through experiments on glass bead materials. He observed hysteresis in the water  
36 retention function depending on wetting or drying processes and found good agreement  
37 between measurements and expected values from the independent domain theory. His  
38 results were pursued with success on sands by *Poulovassilis* [1970] and *Talsma* [1970].  
39 However, disagreement between theory and experiments was also reported for glass bead  
40 material [*Topp and Miller*, 1966; *Morow and Harris*, 1965], for sand [*Vachaud and Thony*,

4.1 1971], for sandy loam [Topp, 1969] and for silty loam and clay loam [Topp, 1971]. These  
4.2 studies suggest that the independent domain theory applies only to data obtained during  
4.3 static-equilibrium or steady state flow conditions but fails to depict hysteresis properly in  
4.4 case of unsteady flow [Topp, 1971].

4.5 Following Topp's conclusion, Mualem [1973] improved and developed a new formal-  
4.6 ism based on the similarity hypothesis [Philip, 1964]. This hypothesis is based on the  
4.7 assumption that the pore water distribution function can be described as a product of  
4.8 two independent functions, the pore opening radii distribution and the pore body radii  
4.9 distribution. This new formalism resulted in a succession of papers [Mualem, 1973, 1974;  
5.0 Mualem and Dagan, 1975; Mualem, 1977] where Mualem's formalism of hysteresis is im-  
5.1 proved. These works gave the basis of the currently known Mualem's theory of hysteresis,  
5.2 finally developed and published in Mualem [1984].

5.3 In parallel to the models presented above, qualified as conceptual, empirical models  
5.4 have been proposed based on fitting water retention curves data. They are defined for  
5.5 specific types of soils and do not claim general validity because their derivation is not  
5.6 based on a physical representation of hysteresis. In the present study, we will use the  
5.7 empirical model of Kool and Parker [1987], based on the scanning model of Scott *et al.*  
5.8 [1983], applied to sand.

5.9 Knowledge of the water physical state in soil is required for understanding hydrostatic  
6.0 and hydrodynamic processes in soils. Methods exist for sampling directly the vadose zone,  
6.1 but they can disrupt the system of interest and can rarely, if ever, provide the sampling  
6.2 rate (spatial or temporal) required to fully understand the complex, interrelated processes  
6.3 that operate in this zone.

64 A large amount of classical laboratory methods have been developed to obtain the hy-  
65 draulic properties of soil samples. Depending on soil type and/or the parameters aimed to  
66 be retrieved different techniques were designed such as, the hanging water column [*Dane*  
67 *and Hopmans*, 2002], the evaporation methods [*Wind*, 1966; *Peters and Durner*, 2008],  
68 or the multi-step outflow [*Eching and Hopmans*, 1993; *Eching et al.*, 1994]. The hang-  
69 ing water column method can get the soil-water retention function parameters for the  
70 drainage and wetting phase whereas the multi-step outflow experiment allows to obtain  
71 the hydraulic conductivity parameters as well as the soil-water retention function param-  
72 eters. These classical methods are robust and already proved their efficiency. However  
73 they need quite a long experimental time (several days), are facility-demanding and can  
74 not be run *in situ* on the field.

75 Advances in electronics in the past thirty years have significantly reduced the costs  
76 and improved the acquisition rates of geophysical methods. Their non-invasive nature  
77 makes them highly suitable for studying the vadose zone. Among them, electrical resistiv-  
78 ity [*Zhou et al.*, 2001; *Goyal et al.*, 2006] and electromagnetic [*Sheets and Hendrickx*, 1995;  
79 *Akbar et al.*, 2005] measurements such as Nuclear Magnetic Resonance (NMR) [*Knight*  
80 *et al.*, 2012] and Ground Penetrating Radar (GPR) [*Huisman et al.*, 2003] are the most  
81 commonly used. They provide physical properties, such as electrical conductivity, NMR  
82 relaxation time constants and dielectric permittivity. These parameters are highly sensi-  
83 tive to soil composition, structure, density and water content.

84 The field of hydrogeophysics is developing since decades [*Vereecken et al.*, 2008; *Lesmes*  
85 *and Friedman*, 2005; *Huisman et al.*, 2003; *Hubbard and Rubin*, 2000]. However, few

86 studies dealt with geophysical measurements as data to be fitted to quantify the hysteresis  
87 of water retention and hydraulic conductivity functions.

88 *Knight* [1991] conducted laboratory resistivity measurements on sandstone samples dur-  
89 ing imbibition and drainage. She found that the resistivity measured during imbibition  
90 was consistently less than the one measured at the same saturation during drainage. She  
91 attributed this difference to the presence of electrical conduction at the air/water interface  
92 and concluded on the dependence of the geophysical properties on saturation history. This  
93 first study on the effect of pore-scale fluid distribution on the electrical behavior during  
94 imbibition and drainage was followed by several others [*Knight and Abad*, 1995; *Fleury*  
95 *and Longeron*, 1998; *Moss et al.*, 1999].

96 *Lai et al.* [2006] gave a GPR evidence of hysteresis of soil water functions. They acquired  
97 GPR measurements over soil and asphalt at various degrees of saturation. Their data  
98 exhibited dielectric hysteresis for both soil and asphalt, depending on drying or wetting  
99 processes.

100 More recently *Weihnacht and Boerner* [2014] performed a GPR monitoring of the soil  
101 water content in a high rectangular Plexiglas<sup>®</sup> tank filled with homogeneous sand. They  
102 used transmission radar data, one antenna on each side of the tank, at different depths  
103 to obtain the volumetric water content during a moving water table experiment. The  
104 obtained data showed different sand water retention functions for drainage and imbibition.  
105 Despite the fact that no hydrodynamic modeling was used, the idea of coupling GPR  
106 measurements with Multi-Step Outflow (MSO) experiment was already an advancement.

107 The Institute of Environmental Physics in Heidelberg University recently performed in-  
108 teresting studies [*Dagenbach et al.*, 2013] and [*Klenk et al.*, 2015]. They used the ASSES-

109 GPR test site to monitor the water table during fluctuating water table experiments. Both  
110 studies were inspired by the precursory works of *Tsolfias et al.* [2001] and *Endres et al.*  
111 [2000] interested in capillary fringe observation with GPR. *Dagenbach et al.* [2013] set  
112 a stationary GPR antenna installed at the soil surface and recorded a trace at constant  
113 time step. They studied the experimental capillary fringe reflections during drainage  
114 and imbibition and compared them to those obtained numerically with three different  
115 water retention models, namely *Brooks and Corey* [1964] and the constrained and uncon-  
116 strained *van Genuchten* [1980] models. The study they conducted on the capillary fringe  
117 reflection, amplitude and phase is similar to the one carried by out *Saintenoy and Hop-*  
118 *mans* [2011] and *Diamanti and Redman* [2014]. *Klenk et al.* [2015] performed radargram  
119 profiles at different experimental times and studied the wavelet associated with peculiar  
120 reflections during drainage/relaxation/imbibition cycles. Their comparison between nu-  
121 merical modeling and experiments allowed them to obtain information on the relative  
122 hydraulic properties of the materials.

123 All the studies presented above did not go further than qualitative measurements and  
124 paved the way for hydrodynamic parameter estimation using geophysical methods and  
125 particularly GPR.

126 In the present study we developed an inversion method to obtain the Mualem-van  
127 Genuchten (M-vG) [*Mualem*, 1976; *van Genuchten*, 1980] parameters characterizing sep-  
128 arately the drainage and the imbibition water retention curves. To achieve this goal, we  
129 first set up numerical experiments to test the possibility of characterizing hysteresis from  
130 GPR data acquired with commercial antennae placed at the top of a soil column subjected  
131 to controlled water table variations. We provided an inversion of the synthetic data to



132 obtain the M-vG parameters characterizing independently water retention functions for  
 133 drainage and imbibition.

134 The aim of the study was to develop a method able to obtain parameters as accurate as  
 135 the classical laboratory methods but with a much shorter experimental time. This goal  
 136 has been realized by working under dynamical conditions.

137 Uncertainty analysis was conducted using multiple inversions performed on independent  
 138 noise-added synthetic data. Finally we inverted experimental GPR data acquired in a  
 139 laboratory experiment to retrieve hydraulic properties in drainage or wetting condition of a  
 140 large sand column. The dispersion observed in the data, namely shift in frequency, is taken  
 141 into account to model the observed phenomena. Statistical analysis on the experimental  
 142 data is drawn using bootstrapping method [Efron, 1979]. The results are compared with  
 143 the M-vG parameters estimated from classical laboratory experiments such as hanging  
 144 water column [Dane and Hopmans, 2002] and falling head experiments.

## 2. Background

### 2.1. Soil hydraulic functions

145 The soil hydraulic functions can be described by several mathematical expressions [Ko-  
 146 sugi et al., 2002] requiring a different number of parameters depending on the chosen  
 147 model. For the  $\theta(h)$  relationship, one of the largest groups of models represented in the  
 148 literature is the power function model of the form:

$$(1 - S_e^{-a})^b S_e = \alpha h, \quad (1)$$

149 where  $S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}$  is the effective saturation,  $\theta_s$  is the saturated volumetric water content,  
 150  $\theta_r$  the residual volumetric water content, and  $\alpha (>0)$ ,  $a$  and  $b$ , are fitting parameters.

151 Many models based on this power form have been proposed to fit water retention data,  
 152 such as those presented in *Gardner* [1958], *Brooks and Corey* [1964], *Fredlung and Xing*  
 153 [1994] and *van Genuchten* [1980] among others. Their mathematical equivalence was  
 154 studied by *Fuentes et al.* [1992].

155 In the current study we used the equation from *van Genuchten* [1980],

$$S_e = (1 + (\alpha h)^n)^{-m}, \quad (2)$$

156 where  $\alpha$  is the inverse of the absolute value of the soil matric potential at the inflection  
 157 point of the  $\theta(h)$  curve. The two dimensionless fitting parameters,  $n$  and  $m$ , are linked by

$$m = 1 - \frac{k}{n}, \quad (3)$$

158 with  $n > k$  [*Haverkamp et al.*, 2005; *Leij et al.*, 2005]. For simplicity,  $k$  takes integer  
 159 values which correspond to different models giving closed form analytic expressions for  
 160 the hydraulic conductivity functions. The case  $k = 1$  corresponds to the model of *Mualem*  
 161 [1976], while  $k = 2$  gives the conductivity model of *Burdine* [1953]. Using  $k = 1$ , the  
 162 hydraulic conductivity function is written as

$$K(\theta) = K_s S_e^\lambda \left[ 1 - \left[ 1 - S_e^{\frac{n}{n-1}} \right]^{\frac{n-1}{n}} \right]^2, \quad (4)$$

163 with  $K_s$  the saturated hydraulic conductivity and  $\lambda$  a parameter accounting for pore  
 164 tortuosity. The whole description of water flow in the unsaturated zone, considering no  
 165 hysteresis, with equations (2) and (4) requires the determination of 6 parameters:  $\theta_r$ ,  $\theta_s$ ,  
 166  $n$ ,  $\alpha$ ,  $K_s$  and  $\lambda$ .

167 The choice of *van Genuchten* [1980]'s model for describing the water retention function  
 168 was motivated by the good fitting of our hanging water column data and by hydrody-  
 169 namic modeling reasons since *van Genuchten* [1980]'s model was the only one taking into

170 account hysteresis in the Hydrus1D code that we used for hydrodynamical modeling (see  
 171 section 2.5).

## 2.2. Hysteresis of the water retention function

172 A schematic representation of hysteresis for the soil water retention curve is presented in  
 173 Figure 1. The boundary hysteresis loop consists of the main drying (red) and main wetting  
 174 (green) curves, following the model of *van Genuchten* [1980]. In our study, we will use  
 175 this model because it is implemented in the Hydrus1D code to solve the hydrodynamical  
 176 equations with hysteresis. In this theoretical representation, the main drying curve is  
 177 described by the van Genuchten parameter vector,  $vG^d = \{\theta_r^d, \theta_s^d, n^d, \alpha^d\}$ . The main  
 178 wetting curve is described by  $vG^w = \{\theta_r^w, \theta_s^w, n^w, \alpha^w\}$ . Considering only the hysteresis  
 179 on the water retention function, we are left with a total of eight unknown parameters to  
 180 determine.

On the basis of the work of *Šimuněk et al.* [1999] and *Likos et al.* [2013], we will assume

$$\theta_r^w = \theta_r^d = \theta_r, \quad (5)$$

$$\theta_s^w = \theta_s^d = \theta_s, \quad (6)$$

and

$$n^w = n^d = n. \quad (7)$$

181 The two first conditions imply that changes in the volume of entrapped air during rewet-  
 182 ting due to temperature differences are disregarded [*Hopmans and Dane*, 1986]. The last  
 183 condition imposes that the curvature of the two water retention functions are similar. We  
 184 will see later in section 4.1, that this last assumption is representing well the hysteresis  
 185 cycle measured for the Fontainebleau sand used in our experimental data. Imposing these

186 constraints has the advantage of further reducing the number of model parameters, but  
 187 will obviously result in some loss of flexibility in describing the hysteresis of the water  
 188 retention function, since the main drying and main wetting curves differ only by their  $\alpha$   
 189 values.

### 2.3. Electromagnetic wave velocity as a function of soil dielectric properties

190 Surface GPR consists of two antennae, being electric dipoles, positioned on the surface  
 191 of the medium to be sounded. The transmitting antenna emits short pulses of spherical  
 192 electromagnetic (EM) waves in response to an excitation current source. The receiving  
 193 antenna converts the incoming EM fields to electrical signals. Following the work of *Annan*  
 194 [1999], the velocity of an electromagnetic wave propagating in a non-magnetic soil, with  
 195 low electrical conductivity, is

$$v = \frac{c}{\sqrt{\varepsilon'}}, \quad (8)$$

196 where  $\varepsilon'$  denotes the real part of the relative dielectric permittivity and  $c$  denotes the  
 197 velocity of EM waves in air (0.3 m/ns).

### 2.4. Petrophysical Relationships

198 A petrophysical relationship linking dielectric permittivity to water content is needed to  
 199 interpret GPR measurements. The bulk soil has a frequency-dependent relative dielectric  
 200 permittivity  $\varepsilon_b$ , expressed by a complex number with the loss factor represented by the  
 201 imaginary part, defined as:

$$\varepsilon_b = \varepsilon'_b(f) - i \left[ \varepsilon''_b(f) + \frac{\sigma_b}{2\pi f \varepsilon_0} \right], \quad (9)$$

202 where  $f$  is the frequency,  $\varepsilon'_b(f)$  the real part of soil permittivity,  $\varepsilon''_b(f)$  the imaginary  
 203 part, including the relaxation loss, and  $\sigma_b$  the bulk conductivity leading to conductivity

204 losses.  $\varepsilon_0$  is the permittivity of free space. In the literature, petrophysical models are  
 205 divided into four categories: effective-medium, empirical, semi-empirical and volumetric  
 206 models [Alharthi and Lange, 1987; Knoll, 1996; Shivola, 1999]. We used the volumetric  
 207 CRIM model [Birchak et al., 1974; Roth et al., 1990], where the relative dielectric permit-  
 208 tivity of the porous geological material,  $\varepsilon_b$ , is a function of the material porosity, its state  
 209 of saturation and the respective permittivity of each of its individual constituents. The  
 210 soil we used in our experiments was a Fontainebleau sand. Using the CRIM relationship  
 211 for a tri-phasic medium composed of water, air and silica, we obtain

$$\varepsilon_b^\gamma = \theta \varepsilon_w^\gamma + (1 - \phi) \varepsilon_s^\gamma + (\phi - \theta) \varepsilon_a^\gamma, \quad (10)$$

212 where  $\varepsilon_w$ ,  $\varepsilon_s$  and  $\varepsilon_a$  are respectively the dielectric permittivity of water, silica and air,  $\phi$   
 213 the porosity and  $\theta$  the volumetric water content.  $\gamma$  is an empirical coefficient that depends  
 214 on soil structure, set to 0.5 in this study. The dielectric permittivity of air is real and its  
 215 value is set to be the same as in vacuum, thus its relative permittivity is equal to 1. We  
 216 measured  $\varepsilon_s = 2.23$  using a time-domain reflectometer. We assumed that the solid phase  
 217 was non conductive and therefore its dielectric permittivity was independent of frequency.  
 218 The permittivity of water  $\varepsilon_w$  is a function of frequency and conductivity, and is expressed  
 219 in time domain by Debye's equation [Debye, 1929]:

$$\varepsilon_w = \varepsilon_\infty + \frac{\varepsilon_{st} - \varepsilon_\infty}{1 + i\omega\tau}, \quad (11)$$

220 in which  $\varepsilon_\infty = 4.9$  is the high-frequency limit of the real dielectric permittivity,  $\varepsilon_{st} =$   
 221 80.088 the static value of the real dielectric permittivity at 20°C [Klein and Swift, 1977],  
 222  $\tau$  is the relaxation time of water at 20°C [Stogryn, 1971]. We further assumed that there  
 223 was no loss due to the bound water, or at least it was negligible for the soil used.

## 2.5. Hydrodynamical Modeling

224 In this study we considered one-dimensional vertical soil water flow described by the  
 225 one-dimensional *Richards* [1931]'s equation expressed in terms of water content as

$$\frac{\partial \theta}{\partial t} = \frac{\partial K(\theta)}{\partial z} + \frac{\partial}{\partial z} \left[ K(\theta) \frac{dh}{d\theta} \frac{\partial \theta}{\partial z} \right], \quad (12)$$

226 where  $K(\theta)$ , and  $\theta(h)$  are described by the Mualem-van Genuchten model, presented in  
 227 equations (2) and (4).

228 We used the Hydrus-1D code [*Šimunek et al.*, 2008] to solve this equation, with an  
 229 atmospheric (Neumann) boundary condition at the top. We changed the bottom bound-  
 230 ary conditions with time (through a Dirichlet boundary condition) using the evolution  
 231 of the applied hydraulic head we used in our experiments. The considered media was  
 232 divided into 5 mm thick horizontal layers, thin compared to the shortest wavelength of  
 233 the electromagnetic waves propagating through it.

## 2.6. Electromagnetic Modeling

### 2.6.1. Finite Difference Time Domain

234 Numerous techniques are available for simulating GPR data, such as ray-based methods  
 235 (e.g. *Cai and McMechan* [1995] or *Sethian and Popovici* [1999]), time-domain finite-  
 236 difference full-waveform methods (e.g. *Kunz and Luebbers* [1993] or *Kowalsky et al.*  
 237 [2001]), or the Finite Differences Time Domain (FDTD) method (e.g. *Irving and Knight*  
 238 [2006]). We used the GprMax2D code [*Giannopoulos*, 2005], which uses FDTD modeling  
 239 to solve Maxwell's equations in 2 dimensions. The synthetic radargrams presented below  
 240 have been simulated using GprMax2D.

### 2.6.2. 1D velocity modeling

243 To avoid time-consuming FDTD modeling in our inversion algorithm, we computed the  
244 Two Way Travel Time (TWT) of the wave reflected at the bottom of the tank assuming  
245 a 1D propagation through a stack of 5 mm thick layers. The electromagnetic wave inside  
246 each layer was computed with equation (8) taking the dielectric permittivity value defined  
247 by equations (10) and (11), and the water content distribution modeled by HYDRUS-1D.  
248 We took into account the dispersion of the wave due to the Debye relaxation phenomena,  
249 evaluated for the maximum frequency of the considered reflection. To do so, we selected  
250 a time window where the bottom tank reflection wavelet was included and took the dom-  
251 inant frequency resulting from Instantaneous Fast Fourier Transform (IFFT). However,  
252 this observed dispersion originated from two phenomena, Debye dispersion and variations  
253 of the antenna coupling through time. These two phenomena are difficult to separate  
254 in experimental data. However, we show that this dispersion was included in the TWT  
255 uncertainties in the numerical simulations showed in the next section.

### 3. Numerical experiments

#### 3.1. Set-up

256 We considered a homogeneous cylindrical medium (71 cm height and 60 cm diameter),  
257 with a porosity set to 0.39 and hydrodynamic properties in the range of what we measured  
258 with water hanging column experiments and UMS Ksat System on Fontainebleau sand  
259 (section 4.1). These parameters are summarized as  $p_{in}$  in Table 1. A transmitter (emitting  
260 a Ricker wavelet centered on 1000 MHz) and a receiver were positioned at the sand column  
261 surface. The draft of the apparatus is presented in Figure 2. The tube on the right  
262 indicates the different water table levels set during the experiment. The two triangles  
263 symbolize the GPR transmitter and receiver. A very thin layer of Perfect Electrical

264 Conductor (PEC) was positioned at the bottom of the sand column. At the initial state,  
265 the water level was set at the top of the column, corresponding to  $h_0 = 71$  cm.

266 Different hydraulic heads were applied to the bottom of the cylinder. The hydraulic  
267 heads were chosen in agreement with experimental data, in order to later compare nu-  
268 merical radargrams with experimental ones.

269 Two types of experiment were conducted: i) a multi-step (subscript MS) experi-  
270 ment where we applied successively  $h_1 = 41.5$  cm,  $h_2 = 31.5$  cm,  $h_3 = 22.5$  cm,  
271  $h_4 = 0$  cm,  $h_5 = h_3$ ,  $h_6 = h_2$ ,  $h_7 = h_1$  and  $h_8 = h_0$  at experimental times  $t_{MS} =$   
272 [100; 1600; 2700; 4000; 5800; 6900; 7900; 9200; 10500] seconds, and ii) a one-step (subscript  
273 OS) experiment where the heads applied to the bottom of the cylindrical medium were  
274  $h_0 = 71$  cm,  $h_1 = 0$  cm and  $h_2 = h_0$  at experimental times  $t_{OS} = [700; 6600; 8100]$ .

275 A volumetric water content profile was computed at each experimental time step using  
276 Hydrus-1D with the M-vG parameters  $p_{in}$  presented in Table 1. The hysteresis effect was  
277 fully represented by the parameter  $\alpha$ , taking  $\alpha_w = 2\alpha_d$  according to *Kool and Parker*  
278 [1987] and *Šimunek et al.* [1999]. The two water retention functions (wetting and drying  
279 curves) corresponding to the parameters  $p_{in}$  are shown on Figure 1. Arrows symbolize  
280 the direction of the cycle, first drainage then wetting. The GPR antenna set at the top  
281 was used to monitor the water content of the sand, using the TWT of the reflection on  
282 the PEC surface at the bottom of the tank.

## 3.2. Forward modeling

### 3.2.1. 1D velocity modeling

284 The water content profiles generated at each experimental time step were converted to  
285 permittivity profiles using relations (10). The TWT of the PEC reflection was computed



286 using the 1D velocity modeling. Figure 3 displays the TWTs obtained when varying the  
287 water table levels with the multi-step protocol (a), and the one-step protocol (b).

288 In both cases, we present the TWTs obtained from the simulated water content profile  
289 accounting for hysteresis of the water retention function following the model of *Kool and*  
290 *Parker* [1987] (red circles) and without hysteresis (blue curves). As expected, since the  
291 simulated water content profiles during the drying and wetting cycle will be different for  
292 the same heads applied, the TWTs will be consequently different. The major advantage  
293 of our experiments resides in making measurements during dynamical processes, without  
294 waiting for hydrostatic equilibrium. We see that the TWTs are different from the TWTs  
295 obtained while the sand is wetting (Figure 3-a) for  $h = 31.5$  and  $22.5$  cm). Working under  
296 non-equilibrium allows to decrease the whole experimental time.

### 297 3.2.2. Finite Difference Time Domain

298 The water content distribution outputs from Hydrus-1D were used as inputs for the  
299 GprMax2D code using equations (10) and (11). The simulated GPR monitoring of the  
300 dynamic water level variation is presented in Figure 4-a) for the multi-step and 4-b) for the  
301 one-step case. The reflection on the PEC layer at the base of the sand is arriving between  
302 15 and 22 ns. Because of relaxation losses due to the presence of water, and the dielectric  
303 permittivity gradient due to retention properties of the medium the electromagnetic wave  
304 is traveling through, the frequency spectrum of the electromagnetic wave changes while  
305 it is propagating. The dominant frequency associated with the bottom tank reflection  
306 found between 15 and 22 ns will thus evolve with the applied head. This is represented  
307 in Figure 5, where the maximum of the instantaneous frequency associated with this  
308 reflection is presented for each experimental time step. On this figure, we display the

309 picking of the maximum instantaneous frequency as circles and the TWT of the bottom  
310 tank reflection picked from the radargrams presented in Figure 4 (red line). The drier the  
311 column, the higher the frequency content is, and, similarly, the wetter the column, the  
312 more present are the relaxation effects, the steeper is the dielectric permittivity gradient,  
313 and the smaller is the frequency content of the bottom tank reflection.

### 3.3. Uncertainty analysis

#### 314 3.3.1. Data uncertainty quantification

315 As stated above, the frequency shift observed in the numerical radargrams (Figures 4  
316 and Figure 5) has two sources: Debye relaxation and the permittivity gradient created  
317 by the retention properties of the medium. Our 1D-velocity modeling does not take into  
318 account these sources of signal dispersion and we observe a small discrepancy between  
319 the picking of the reflection presented in the radargram 4-a) and b) and our 1D velocity  
320 algorithm results. These two TWT curves are displayed on Figure 6-a) for the one-step  
321 protocol. The Root Mean Squared Error (RMSE) between these two curves is 0.09 ns.

322 An other source of uncertainty in our data is the one associated with the applied head.  
323 For determining the TWT uncertainties associated with the applied hydraulic head uncer-  
324 tainties, we considered the RMSE between the TWT curve resulting from the hydraulic  
325 head presented in section 3.1 for the one-step experiment, and one other TWT curve  
326 generated with the initial hydraulic head  $h_0$  decreased by 1 *cm*. These TWT curves are  
327 presented in Figure 6-b), where the red curve represents TWT computed with initial  
328 hydraulic head, whereas blue curve represents the initial hydraulic head  $-1$  *cm*. The  
329 RMSE between these curves is 0.11 ns. At last we considered the picking uncertainties by  
330 considering that time picking can not be better than the one time step, which was 0.058

ns for experiments presented in section 4.3. Even if the time-step was much smaller for  
the numerical experiment we chose to take the experimental time step in order to bet-  
ter represent the uncertainty. Taking these three sources of uncertainties in our one-step  
numerical experiments, we estimated our overall uncertainty associated with each TWT  
data point, using a quadratic summation, to  $\sigma_{OS} = 0.15$  ns. The uncertainties for the  
multi-step case was computed with the same procedure to  $\sigma_{MS} = 0.13$  ns.

### 3.4. Inversions on Numerical Data Perturbed by Gaussian Noise

We used the TWT of the bottom tank reflection as presented in Figure 3-a) for the  
multi-step experiment and 3-b) for the one-step case as data to be fitted to get the set  
of M-vG parameter values  $\mathbf{p} = \{\theta_r, \theta_s, \alpha_d, \alpha_w, n, K_s, \lambda\}$ . The optimization procedure was  
the Shuffled Complex Evolution (SCE-UA) algorithm [Duan *et al.*, 1992] minimizing the  
objective function expressed as the RMSE between the modeled and observed TWT of  
the bottom tank reflection to be fitted. This RMSE was given by

$$RMSE = \sqrt{\frac{1}{N} \sum_{j=1}^N [\tau_{xp}(t_j) - \tau_{mod}(t_j, \mathbf{p})]^2}, \quad (13)$$

where  $\tau_{xp}$  is the radargram-measured TWT generated with the hydrodynamic parameters  
 $\mathbf{pin}$  at experimental time  $t_j$ , and  $\tau_{mod}$  is the modeled TWT reflection at time  $t_j$  subject to  
soil hydraulic parameter vector  $\mathbf{p}$ , and  $N$  is the number of points used for comparisons,  
i.e. the number of considered experimental times  $t_j$ .

We sampled the parameters in ranges defined according to the values found in the  
literature. The range for  $\theta_r$  was  $[0.01; 0.1] \text{ cm}^3/\text{cm}^3$ , the one for  $\theta_s$  was  $[0.3; 0.4] \text{ cm}^3/\text{cm}^3$ .  
 $K_s$  interval was  $[0.05; 3] \text{ cm.min}^{-1}$ ,  $n$  was allowed to get values between 2 and 10, and  $\alpha_w$   
and  $\alpha_d$  ranges were both set to  $[0.01; 0.1] \text{ cm}^{-1}$ . We set the  $\lambda$  range to  $[0; 10]$ , considering

351 this parameter as a fitting one. The historical paper of *Mualem* [1976] proposed  $\lambda$  values  
352 in the range of  $[-1; 2.5]$  and conclude on a good fitting for most soils with  $\lambda = 0.5$ . This  
353 value is the most used in the literature. The studies of *Schaap and Leij* [2000]; *Schaap*  
354 *et al.* [2001]; *Schaap and van Genuchten* [2005] were among the first to develop wider  
355 range for  $\lambda$  parameter  $[-1; 1]$ . We then based our  $\lambda$  range on the study of *Wösten et al.*  
356 [1999], on the database of European soils, where  $\lambda$  could reach values higher than unity.

357 For each generated parameter set  $\mathbf{p}$ , we computed the TWT of the bottom tank reflec-  
358 tion using the 1D velocity model. This inversion modeling procedure is presented as a flow  
359 diagram in Figure 7, where the objective function is the RMSE. We used 20 complexes,  
360 50,000 iterations for each loop and a constraint on 30,000 iterations to stop if no better  
361 model was found during the Complex search.

362 Since the inversion algorithm, SCE-UA, is not following the Bayesian rule, the statistical  
363 analysis performed on one single inversion cannot be considered as mathematically correct  
364 (see the works of *Vrugt et al.* [2003] for discussion on this topic). We decided to perform a  
365 statistical analysis on models obtained by independent SCE-UA optimization procedures  
366 carried out on different noise-added generated TWT data. Examples of the perturbed  
367 TWT are presented in Figure 8 as purple points. These data correspond to the original  
368 simulated one, in blue, to which we added small time delays sampled from a Gaussian  
369 distribution centered on the original value, with standard deviation  $\sigma_{MS} = 0.13 \text{ ns}$  for  
370 the multi-step case and  $\sigma_{OS} = 0.15 \text{ ns}$  for the one-step case.

371 We performed 80 inversions on 80 different Gaussian noise-added data in both cases.  
372 In the multi-step case, we selected the models with associated RMSE smaller than or  
373 equal to  $2 \sigma_{MS} = 0.26 \text{ ns}$ . In the one-step case, we selected the models with associated

374 RMSE smaller than or equal to  $2 \sigma_{OS} = 0.30 \text{ ns}$ . Possible double models were taken  
375 out in order to keep the same weight for all models, and not influence the histograms. It  
376 resulted on 968,291 models in the multi-step case and 448,507 models in the one-step case.  
377 The statistical distributions of each parameter of all the selected models are presented in  
378 Figure 9-a) for the multi-step case and Figure 9-b) for the one-step case. The value for  
379 each parameter set as input is represented with the vertical dashed orange bar and can  
380 be found in Table 1.

381 Except for  $\theta_s$  and  $\lambda$  these histograms show that the input parameters are included in  
382 every distribution. The volumetric water content at saturation is over estimated by a  
383 mean value of 0.025 % in both experimental procedures.

384 The histograms presented in the one-step case, on Figure 9-b) seems to have more  
385 difficulty than the multi-step case, to estimate accurately the parameters  $\alpha_d$ , and  $\alpha_w$ .  
386 One possible reason for  $\alpha_w$  is the rewetting phase observed with only 10 data points in  
387 the one-step case, and 40 points in the multi-step case (compare Figure 3-a) and b)).  
388 In addition, inter-correlation between parameters exist, as presented in Figure 10, where  
389 objective function plots are displayed for the multi-step case ( a)-c)-e)-g) ) and one-  
390 step case ( b)-d)-f)-g) ). The objective function plots are different depending on the  
391 experimental case (multi or one-step). This means that the inter-correlation observed  
392 is partially due to the theoretical link between parameters as well as the design of the  
393 experiment.

394 One can see on 10-a)-b)-c)-d) correlations between  $K_s$  and  $\alpha_w$  and  $n$ . Figures 10-g)-h)  
395 show a link between  $\lambda$  and  $n$  depending on the experimental protocol. These objective  
396 function plots show that the regions with lowest RMSE (dark blue areas) always con-

tain the input parameter, except for  $\lambda$ . This is particularly well illustrated on the  
Figure 10-e) where we see that  $\alpha_w = 2\alpha_d$  corresponds to the dark blue region.

The case of  $\lambda$  parameter and its link with parameters require another analysis. On  
Figures 8-a) we displayed 3 TWT curves of the bottom tank reflection for the one-step  
case. The set of parameters taken for the original blue line data is the one set as input  
(Table 1). The two other lines represent the result of an increase or a decrease of the  $n$   
value by 2. Increasing  $n$  results in increasing the TWT variations and vice-versa. Figure 8-  
b) shows as well the effect of an increase or a decrease of the parameter  $\lambda$ . At the opposite  
of  $n$ , increasing  $\lambda$  from 0.5 to 2 results in decreasing the TWT variations. Figures 8-c)  
and d) are the equivalent for the multi-step case. From these figures, we see that different  
couples of  $(n, \lambda)$  will fit as well the noisy data (purple points). A too small  $n$  value  
associated with a too small  $\lambda$  value will result in the same curve than too high values for  
both  $n$  and  $\lambda$ . This is illustrated by the objective function plot Figure 10-g).

In addition, multi-step data are not sensitive to  $\lambda$  variations in the first three steps and  
the last one (Figure 8-d)), whereas most of the one-step data are sensitive to this parameter  
variations. This might explain differences in the two experimental data inversion results.

This numerical analysis showed the robustness of our modeling for inverting most of  
the hydrodynamic parameters as well as some difficulties due to inter-correlations between  
parameters. However, we see that working under dynamical processes, without waiting for  
static equilibrium at each step, did not prevent us to retrieve most of the hydrodynamical  
parameters even if we applied a quite brutal uncertainty analysis. For instance, we applied  
the same uncertainty to every point of the TWT data not considering the influence of the  
frequency shift uncertainties is stronger when the soil is becoming drier. In addition, an

420 uncertainty of 1 cm on the hydraulic head is a pessimistic estimation. To be convinced that  
421 the noise we added was really strong, Table 1 shows the optimized parameters retrieved  
422 on non-noisy data with a RMSE smaller than 0.015 ns.

## 4. Laboratory Measurements

### 4.1. Characteristics of the sand

423 The van Genuchten water retention parameters were obtained by fitting the data ob-  
424 tained with three hanging water column (HWC) experiments performed on Fontainebleau  
425 sand core ( $250 \text{ cm}^3$ ) samples packed at  $1.67 \text{ g.cm}^{-3}$  bulk density. We considered un-  
426 certainties on  $h$  at  $\sigma_h = 1 \text{ cm}$ ,  $\theta = 0.03 \text{ cm}^3.\text{cm}^{-3}$ . We suspect that some evaporation  
427 happened, which gave the low value on  $\theta_r$  parameter. The van Genuchten parameters  
428 obtained are presented in Table 2, under the label “Lab. meas.”.

429 Figure 11 shows the retention curves data acquired from the HWC experiments. They  
430 were fitted using the van Genuchten model. As expected for this type of soil,  $\alpha_w$  was  
431 close to  $2\alpha_d$ . The HWC results confirmed that the  $n$  parameter could be considered as  
432 independent from drying or wetting processes, since the curvature of the water retention  
433 curves are very similar for drying and wetting branches.

434 The saturated hydraulic conductivity was determined on several  $250 \text{ cm}^3$  soil core sam-  
435 ples packed at  $1.6 \text{ g/cm}^3$ , using the UMS Ksat System, under constant head. The range  
436 of values obtained are presented in Table 2. The  $\lambda$  parameter was not determined, but its  
437 value was initially expected to be close to 0.5 [Mualem, 1976] without having any other  
438 measurements.

439 Granulometric determinations on the Fontainebleau sand used for the experiments  
440 showed that 13% of its grains had a diameter larger than 0.25 mm and 98% larger than

441 0.125 mm. Its average particle density was  $2.8 \pm 0.1 \text{ g cm}^{-3}$ . The sand was packed to a  
442 porosity estimated to be 0.39.

## 4.2. Experimental Set-up

443 The experimental apparatus consisted of two cylindrical tanks (80 cm height and 60 cm  
444 diameter), one tank,  $T_s$ , filled with the Fontainebleau sand and the other one,  $T_w$ , filled  
445 with water at a level corresponding to the hydraulic head.  $T_s$  was filled with 321 kg of  
446 sand, compacted manually in order to obtain a bulk density of  $\rho_b = 1.7 \text{ g.cm}^{-3}$ . The sand  
447 layer was put on top of a 5 cm gravel layer in order to drain and wet homogeneously the  
448 sand. To obtain a strong reflection at the bottom of the sand we put between the gravel  
449 layer and the sand a thin aluminum sheet, finely meshed, to let the water go through.  
450 Three pictures of the experimental set-up are presented in Figure 12. Figure 12-a) shows  
451  $T_w$ , where we see the holes used as overflow to maintain the water table during drainage  
452 and where we see the green water supply pipe.  $T_s$  and  $T_w$  bottoms were linked through  
453 four independent plastic tubes with taps (Figure 12-b) ). Figure 12-c) displays  $T_s$  with  
454 the antenna set at the top of the sand, maintained in position by a load. The white thin  
455 layer is a geo-textile enclosing the sand. Piezometers were connected at the bottom of  
456 each tank to indicate the applied head.

## 4.3. Experimental measurements

457 Four experiments were carried out using this experimental setup. For each experiment,  
458 the whole soil profile was initially water saturated in order to be on the main drainage  
459 curve during the first cycle. Both tanks were hydraulically separated by closing the taps  
460 on the linking tubes. The level of water was set to the desired level (for example 42



461 cm) in  $T_w$  using an overflow hole. Then the hydraulic contact between both tanks was  
462 established again, opening the taps. The water drained out of  $T_s$  was evacuated through  
463 the overflow hole in  $T_w$ . This procedure was repeated for all steps during draining and  
464 wetting processes.

465 The radargram measured during one of the multi-step experiment is shown in Fig-  
466 ure 13-a), and during a one-step experiment, in Figure 14-a). The horizontal axis is the  
467 experimental time in seconds, and the vertical axis is the TWT in nanoseconds. During  
468 the drainage and imbibition cycle, the direct wave signal evolves through time, due to  
469 the evolution of the media properties surrounding the antennae, which affects the anten-  
470 nae coupling. The reflection on the aluminum sheet at the base of the sand is arriving  
471 around 28 ns, other reflections are tank side effects. As expected the TWT of this re-  
472 flection evolves with the water table level. In the multi-step experiment, the heads were  
473 changed at [800;1500;2000;2900] seconds for drainage and [4400;5200;5800;6800] seconds  
474 for re-wetting. On Figure 13-b, we display the instantaneous maximum frequency of the  
475 considered reflection. On this figure we display as well the TWT of this reflection, rel-  
476 ative to its initial TWT, when saturated. We see that the behavior of this reflection is  
477 comparable to the one we simulated (Figures 3, 4 and 5). Despite the fact that we used  
478 a Malå Ramac antennae with a nominal frequency of 1.6 GHz, we clearly see that the  
479 center frequency is far from 1.6 GHz, due to antenna coupling, Debye dispersion and the  
480 dielectric permittivity gradient inside the column sand.

481 Graphics presented in Figure 14-a) and b) are the analog of Figure 13-a) and b) for the  
482 one-step experiment. The level of water is brought down to 0 cm from 71 cm (saturated)  
483 and then brought back to full saturation of the large tank. We see that the TWT of

484 the bottom reflection on Figure 13-b) exhibits the same behavior as the simulated one  
485 presented in Figure 3-b).

#### 4.4. Inversion of Experimental Data

##### 486 4.4.1. Frequency shift correction

487 The frequency shift observed in the experimental data (Figure 13-b) and Figure 14-  
488 b)) has to be taken into account in order to correct for the higher frequency picking,  
489 since all our TWT are relative to the saturated case (low frequency). Using the following  
490 procedure, we corrected the TWTs due the frequency variations during the experiment.

491 We considered that the wavelet associated with the bottom tank reflection could be  
492 assimilated as a time derived Ricker wavelet. We then measured the time delay between  
493 the maximum peak arrival of a wavelet computed with the lower frequency of 460 MHz  
494 (observed when the sand column is fully saturated), and the maximum peak arrival of  
495 a wavelet computed with the frequency associated with the experimental time at which  
496 we want to apply a correction. This principle is represented in Figure 15. Figure 15-b)  
497 represents the time correction with experimental time for the multi-step case. As expected,  
498 when the frequency difference between the initial measurement with the current one is  
499 larger the maximum time difference is larger. All numerical TWT were corrected using  
500 this procedure.

##### 501 4.4.2. Data Inversion and Uncertainty Analysis with Bootstrapping

502 Using the same inversion loop as the one presented in the numerical case summarized in  
503 Figure 7, we inverted the experimental TWT data, sampled every 20 second, to obtain the  
504 hydrodynamic parameters of the considered soil. The parameters minimizing the objective  
505 function (Eq. 13) between the simulated and the experimental data are presented in

506 Table 2. We will present the uncertainty analysis only on two experiments, the multi-step  
507 experiment 15-01-26 and the One-step experiment 15-01-27. These two experiments will  
508 be further referred as multi-step and one-step experiment.

509 The uncertainty analysis was based on running multiple SCE-UA inversions on down-  
510 sampled data. This approach is similar to bootstrapping without replacement. For each  
511 experimental case, we chose randomly 50 sets of 80 points. For the multi-step data,  
512 we selected 40 points during the drainage and 40 point during the rewetting. For the  
513 mono-step data, we randomly took 60 points during drainage and 20 points during the  
514 rewetting phase. We performed inversions on each bootstrapped data subset. In each  
515 case, we selected the models with a RMSE value smaller than or equal to the RMSE of  
516 the best fitting model adding 10% to its value. Possible double models were taken out. It  
517 resulted into 231,192 parameter sets for the multi-step case and 238,432 parameter sets for  
518 the one-step case. The distributions of each model parameter are presented as histograms  
519 in Figure 16. These models are also displayed as objective function plots, Figure 17 and  
520 Figure 18, for the multi and one-step case respectively.

521 For each sub-figures (a) to k)) the parameter plane is label on x and y axis. For instance  
522 Figures 17-c) and 18-c) plots in the  $\{\alpha_w, n\}$  plane all the models fitting the bootstrapped  
523 data with a RMSE value smaller than or equal to the one of the model optimizing the  
524 original data sampled at 20 s, plus 10%, for the multi-step and one-step experiment  
525 respectively. The colorbar in Figure 17-l) and Figure 18-l) represents the RMSE between  
526 modeled data and inversed data as computed from Eq. 13. The parameters fitting the  
527 HWC laboratory measurements are displayed as black points.  $K_s$  extreme values are  
528 represented by two black dashed lines as well as  $\lambda = 0.5$ . The optimum parameters (see

529 Table 2) obtained by inverting the original data sampled at 20 s are symbolized by black  
530 triangles.

#### 4.5. Analysis of the Experimental Results

531 A look at the parameter sets obtained for the four different laboratory experiments,  
532 summed up in Table 2, shows differences in results that are hard to explain. We gain  
533 much more information looking at the histograms and the objective function plots issued  
534 from the bootstrapping analysis. The main differences between multi-step and one-step  
535 results concern the parameters  $\lambda$  and  $n$ .

536 The objective function plots including the  $\lambda$  parameter are in Figures 17-a), -d), -g) and  
537 -j) and 18-a), -d), -g) and -j). On Figure 17-a), we see that  $\lambda$  and  $n$  are correlated, with  
538 two regions of minimum RMSE around the points  $(\lambda = 1, n = 3)$  and  $(\lambda = 9, n = 7)$ .  
539 This situation is similar to our previous analysis using Figure 10-g).

540 However, the WHC measurements indicate high value of  $n$ , even higher than the upper  
541 value found after bootstrapped data inversion. The WHC experiment is giving data at  
542 hydrostatic equilibrium. Therefore they do not involve hydraulic conductivity and the  $n$   
543 parameter fitting these data is independent of the  $\lambda$  value. Considering the WHC results,  
544 the couple  $(\lambda = 9, n = 7)$  seems to be more reasonable than the couple  $(\lambda = 1, n = 3)$ .

545 The one-step experiment is exploring directly the whole possible range of water saturation  
546 attainable in our 71 cm high column whereas the multi-step is cutting this range in  
547 four smaller ranges. It might explain why the one-step data fitting can not be achieved  
548 with small  $n$ , and small  $\lambda$  values (Figure 18-a)-d)-g)-j)).

549 If we exclude all the models with small  $n$  and  $\lambda$  values in Figure 17, all optimized  
550 parameters are in similar ranges in both experimental cases except for  $\alpha_w$  (see histograms

551 of Figure 16). Figures 17 and 18 d) to f) show that  $K_s$  is very well defined, even if a small  
552 discrepancy is present, comparing to the measured data (Table 2).

553 Figures 17 and 18 h) show that the factor of two between  $\alpha_d$  and  $\alpha_w$  is more or less  
554 retrieved and no other correlation are found. In addition  $\alpha_d$  seems to have a bi-modal  
555 shape over the two types of experiments, even if the separation between both peaks is quite  
556 small. Anyhow, both  $\alpha_d$  and  $\alpha_w$  are more efficiently retrieved in the one-step experiment  
557 than the multi-step one.

558 Diagrams on Figures 17 and 18 show us that  $\theta_r$ ,  $n$  and  $\lambda$  optimized parameters do not  
559 match the HWC parameters determined in the laboratory. At least two factors must be  
560 considered when comparing GPR-derived parameters and classical laboratory parameters.  
561 First, the parameters obtained in laboratory are made on 250 cm<sup>3</sup> core samples. Since  
562 about 330 kg of Fontainebleau sand were used in the large tank, the compaction was  
563 definitely not as homogeneous as in the case of small 250 cm<sup>3</sup> core samples, despite the  
564 fact that we tried to reach the same bulk density. In addition we are convinced that  
565 evaporation influenced the HWC measurements, which could explain the low  $\theta_r$  values  
566 obtained with these experiments.

567 Second, the hydraulic head range covered by the HWC experiment is different than  
568 the one covered by the large scale experiment. Indeed, looking back at the HWC data  
569 presented in Figure 11, we see that the first drainage curve reaches 140 cm of suction,  
570 whereas in the case of our dynamical experiment we are dealing with at the most 70 cm.  
571 Then it is possible that we are not on the main wetting curve, but rather on an inner  
572 cycle curve. However this effect is taken into account by the hydrodynamical modeling.

## 5. Conclusion

573 The 1D velocity algorithm we used to invert the TWT from GPR data gave precise  
574 and accurate results for the numerical examples and allowed us to obtain most of the  
575 hydrodynamic parameters taking into account hysteresis in the water retention function.

576 Differences between the parameters obtained by our GPR experimental data inversion  
577 and those obtained through classical laboratory experiments (HWC and Ksat) are proba-  
578 bly caused by homogeneity problems in the large tank and evaporation on the HWC soil  
579 samples. The hydraulic conductivity at saturation  $K_s$  is consistently estimated in all our  
580 bootstrapped data inversion into the range  $[0.8;1.2]$  cm.min<sup>-1</sup>. The one-step experimental  
581 GPR data give more precise results than the multi-step experimental GPR data.

582 Values of  $\lambda$  between 6 and 10 and values of  $n$  between 4 and 7 were estimated from our  
583 GPR experiments. Both of these ranges are much higher than usually found in the litera-  
584 ture for classical and/or specially designed laboratory experiments on soil samples [*Likos*  
585 *et al.*, 2013; *Abbasi et al.*, 2012; *Kumar et al.*, 2010]. These methods give dynamic volu-  
586 metric water content data to be fitted using Equation (4), with the constrain on  $\lambda = 0.5$ .  
587 We tend to think that this constrain led to small  $n$  values. The parameter  $\lambda$  is important  
588 to retrieve, because it has a lot of influences on the other parameters to be estimated.

589 In the study presented here, the hysteresis of the water retention function is represented  
590 by the  $\alpha$  parameter only. One of the possible improvement would be to give a higher degree  
591 of freedom by representing the hysteresis effect as well wit the  $\theta_s$ ,  $n$  and  $\lambda$  parameters.

592 Even if this work did not aim to study the efficiency of the empirical model of hysteresis  
593 used for the water retention function (model of *Kool and Parker* [1987]), we tend to think  
594 that other models should be tested. We particularly think that alternative models based

595 on contact angle-dependent hysteresis, such as the one of *Zhou* [2013], are very promising  
596 and may lead to a better physical representation of the hysteresis phenomena. However  
597 in this case the Richards' equation need to be solved by another code than the current  
598 version of Hydrus-1D.

599 Finally, the hydrodynamical parameters optimizing the TWT data would be better  
600 constrained using amplitudes of the reflections, particularly for the  $(n, \lambda)$  couple. That is  
601 why we encourage the development of a full-waveform inversion algorithm.

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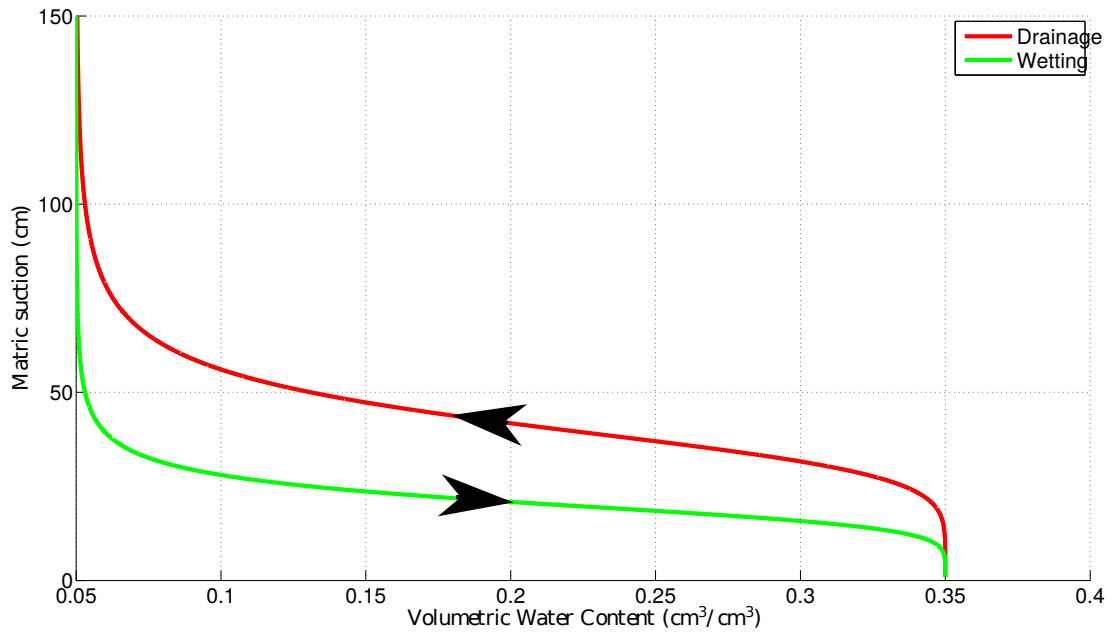


	$\theta_r$ ( $\text{cm}^3.\text{cm}^{-3}$ )	$\theta_s$ ( $\text{cm}^3.\text{cm}^{-3}$ )	$\alpha^d$ ( $\text{cm}^{-1}$ )	$\alpha^w$ ( $\text{cm}^{-1}$ )	$n$	$K_s$ ( $\text{cm}/\text{min}$ )	$\lambda$	RMSE (ns)
$p_{in}$	0.05	0.35	0.025	0.05	6	1	0.5	-
multi-step - heads = [71; 42; 32; 23; 0; 23; 32; 42; 71] <i>cm</i>								
$p_{MS}^{op}$	0.048	0.38	0.024	0.052	5.29	1.08	0.55	0.014
one-step - heads = [71; 0; 71] <i>cm</i>								
$p_{OS}^{op}$	0.056	0.38	0.024	0.053	6.06	1.08	0.54	0.013

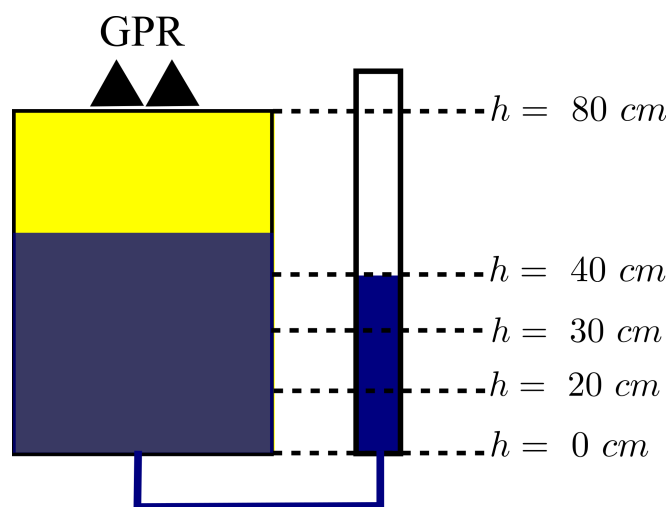
**Table 1.** Hydrodynamical M-vG parameters of the numerical experiments.  $p_{in}$  is the set of parameters used as input to generate the radargrams presented in Figures 4-a) and 4-b). The set of parameters optimized by the SCE-UA global method is  $p_{MS}^{op}$  in the multi-step case and  $p_{OS}^{op}$  in the one-step case.

	$\theta_r$ ( $\text{cm}^3.\text{cm}^{-3}$ )	$\theta_s$ ( $\text{cm}^3.\text{cm}^{-3}$ )	$\alpha^d$ ( $\text{cm}^{-1}$ )	$\alpha^w$ ( $\text{cm}^{-1}$ )	$n$	$K_s^*$ ( $\text{cm}/\text{min}$ )	$\lambda^{**}$	RMSE (ns)
Lab. meas.	$0.03 \pm 0.005$	$0.36 \pm 0.005$	$0.024 \pm 0.002$	$0.043 \pm 0.003$	$8.5 \pm 0.5$	[0.96; 1.5]	0.5	-
multi-step - heads = [71; 41.5; 31.5; 22.5; 0; 22.5; 31.5; 41.5; 71] <i>cm</i>								
15-01-26	0.052	0.37	0.033	0.094	6.6	0.82	10.4	0.14
15-01-20	0.04	0.36	0.023	0.047	4.4	0.54	0.25	0.09
one-step - heads = [71; 0; 71] <i>cm</i>								
15-01-23	0.045	0.40	0.023	0.052	5.52	0.90	4.9	0.149
15-01-27	0.04	0.4	0.028	0.042	5.14	0.88	8.54	0.148

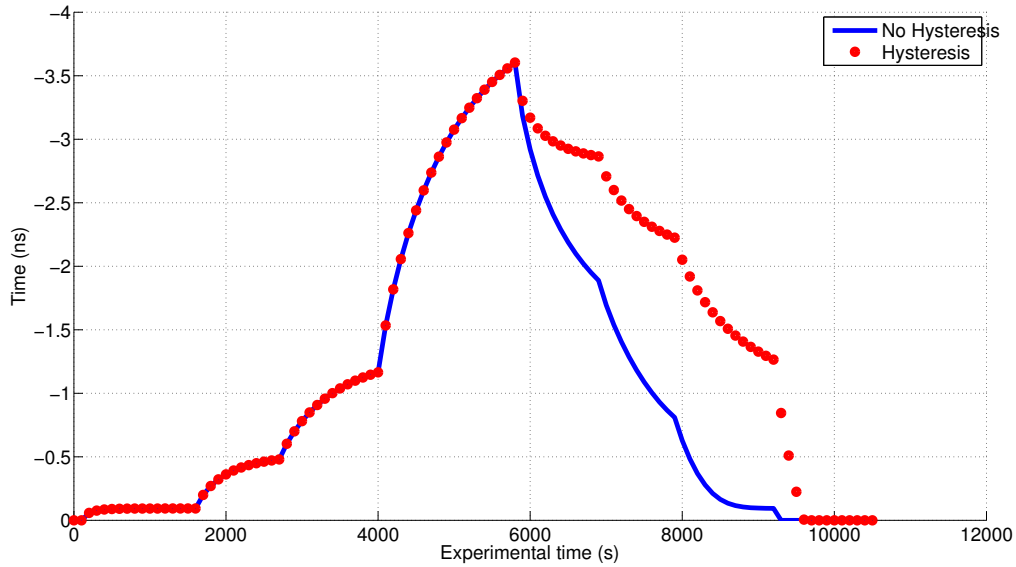
**Table 2.** Hydrodynamical M-vG parameter sets from classical laboratory measurements and optimizing laboratory GPR data monitoring. \* obtained from Ksat. \*\* according to *Mualem* [1976].



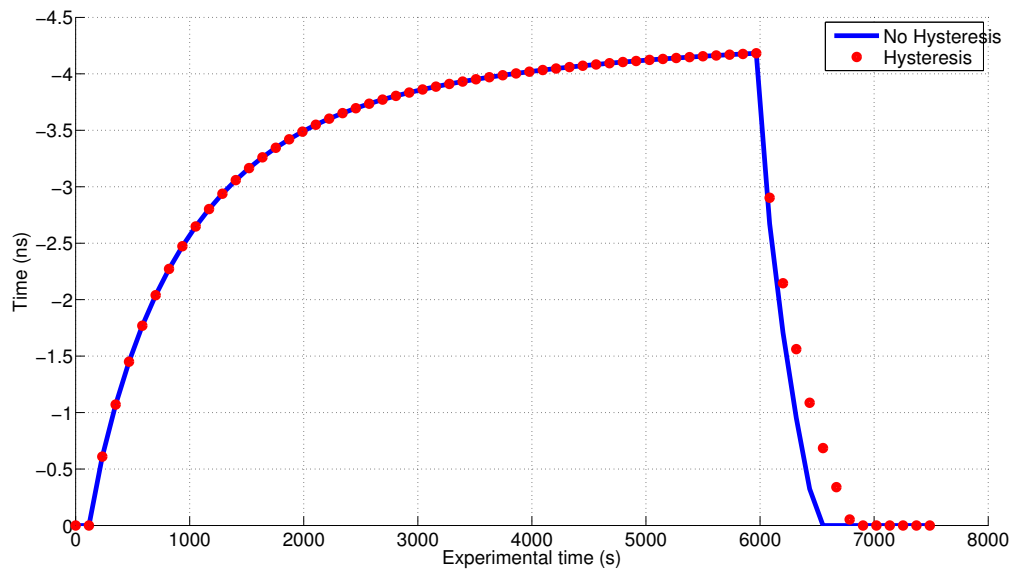
**Figure 1.** The red curve is the main drying curve, the green is the main wetting curve. The black arrow symbolizes the direction of the cycle. The hysteresis is only determined by the  $\alpha$  parameters, with  $\alpha_w = 2\alpha_d$ . See Table 1 for the full parameter set.



**Figure 2.** Draft of the experiment. The tank is 80 cm high and filled with sand. The small diameter reservoir on the right represent the different heads applied at the bottom of the tank. GPR antennae are set on top of the sand reservoir.

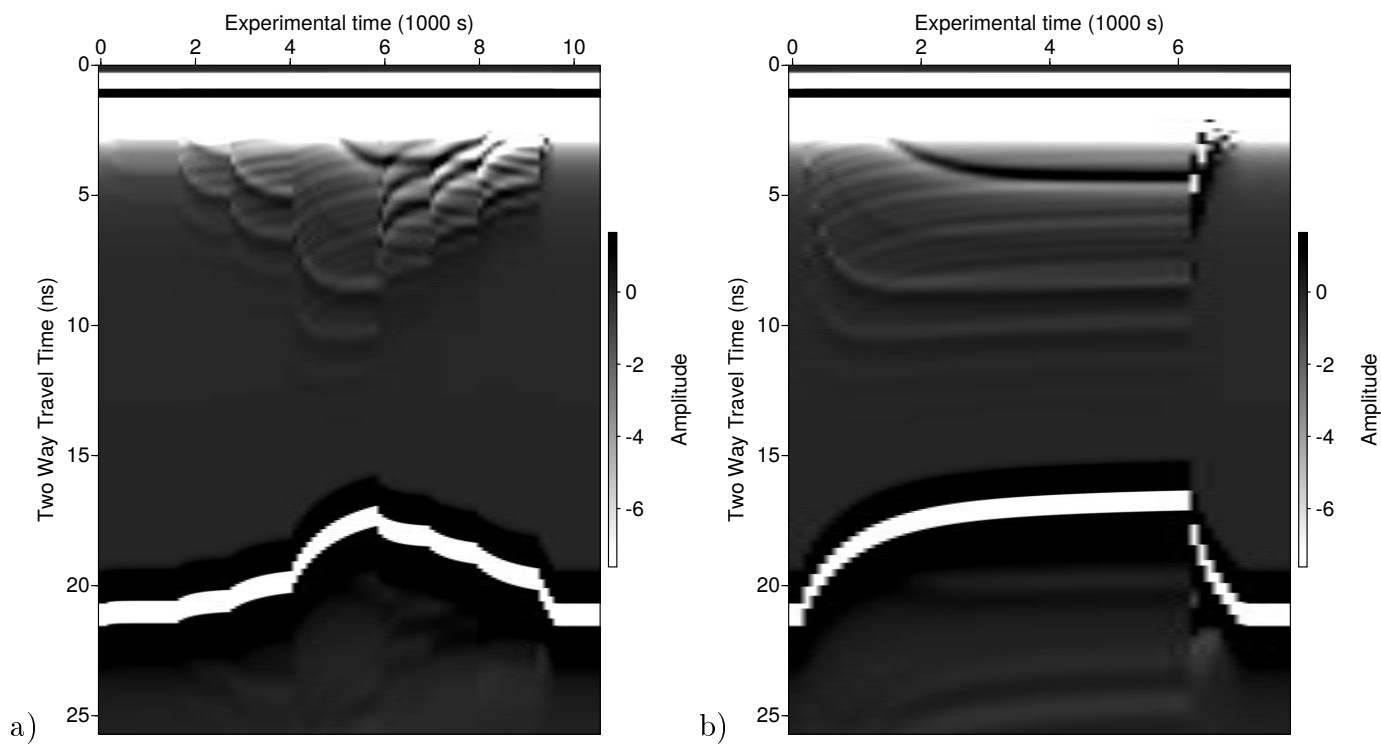


a)

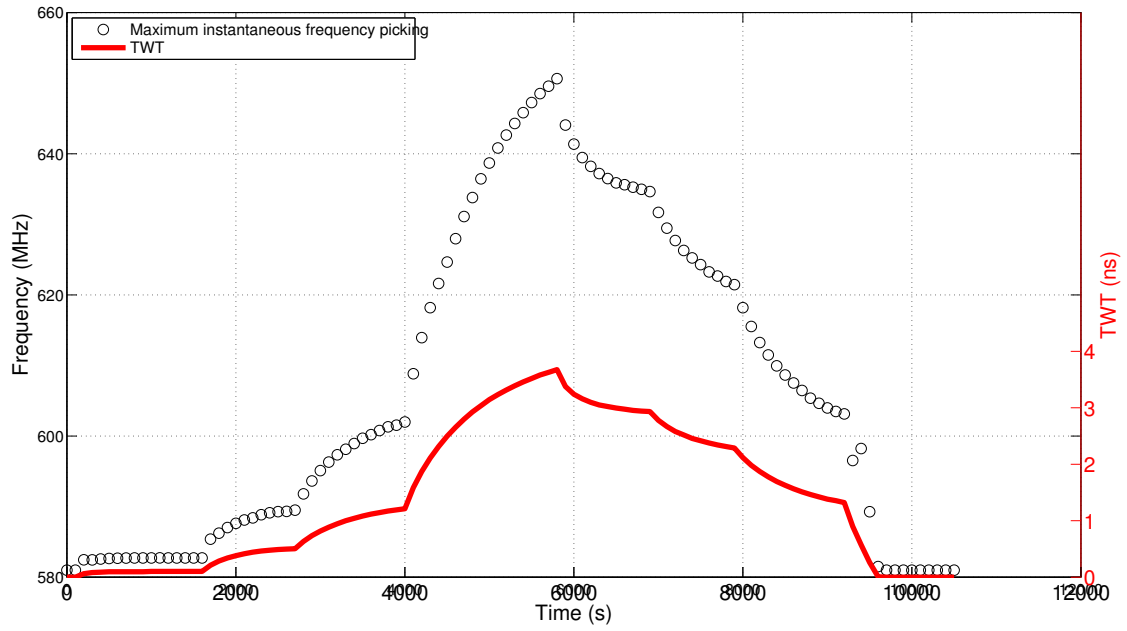


b)

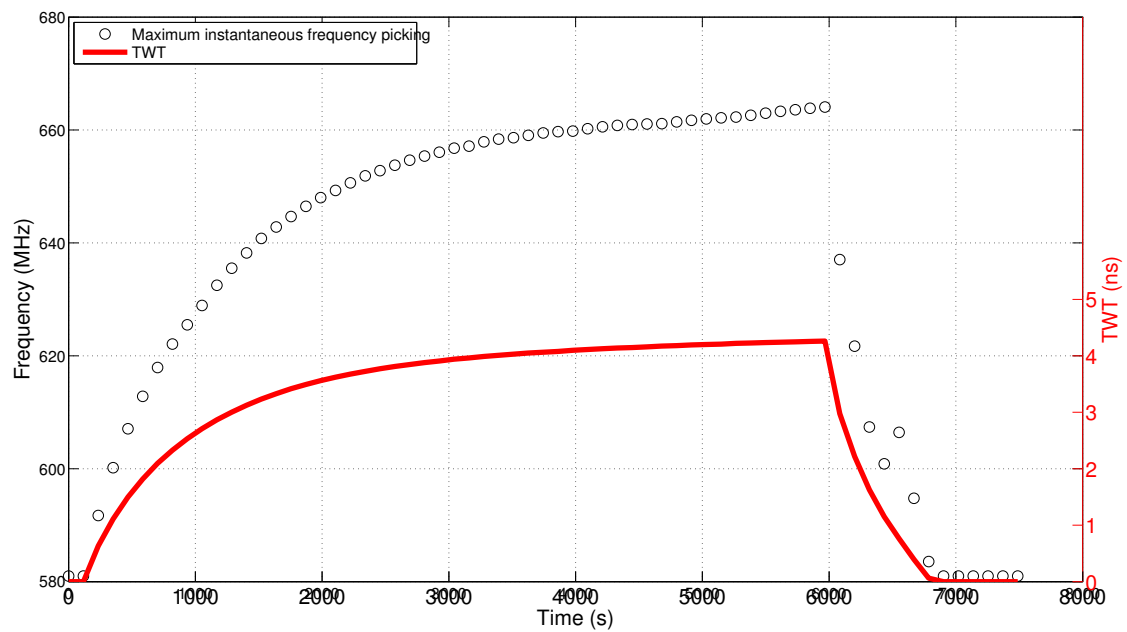
**Figure 3.** TWT of the bottom tank reflection. a): The heads applied at the bottom boundary condition were 71 cm (saturated)-42-32-23-0 cm, then re-wetting following the same steps. b) The heads applied were 71 cm (saturated)-0 cm, then re-wetting to 71 cm. Blue curves represent the computed TWT using a hydrodynamic function without taking into account hysteresis effects, whereas the red circles are modeled using hysteresis. The parameters used for the hydrodynamical modeling are presented in Table 1.



**Figure 4.** Radargrams generated from the simulated water content profiles using GprMax2D: a) multi-step experiment b) one-step experiment. The parameters used as input for the hydrodynamical modeling are presented in Table 1.

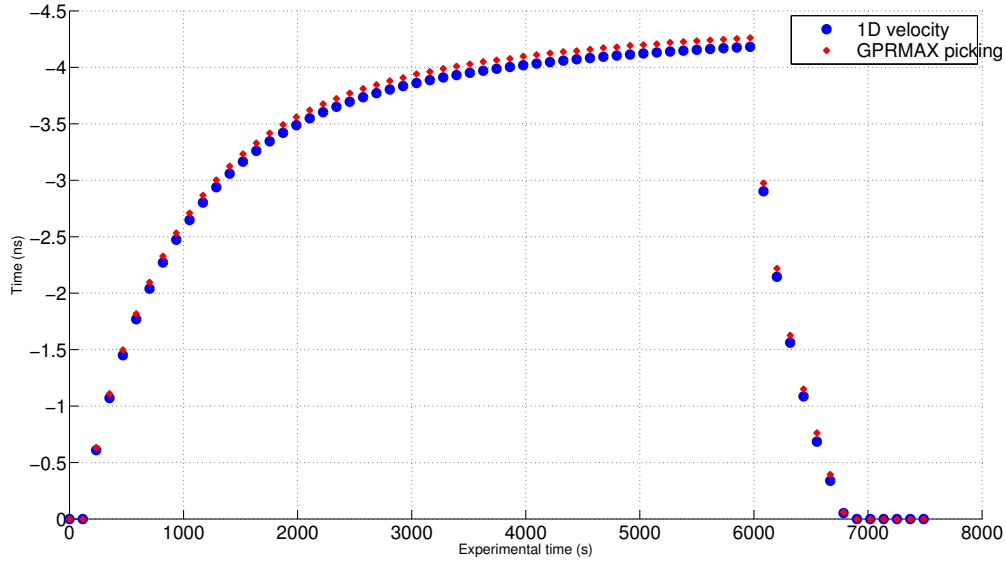


a)

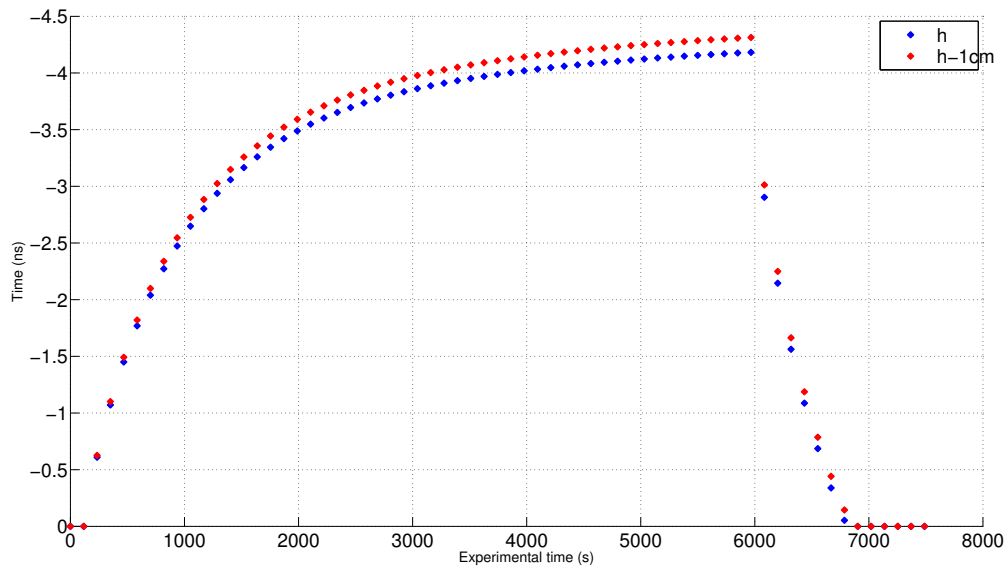


b)

**Figure 5.** Maximum frequency of the bottom tank reflection for a) multi-step and b) one-step experiment. Automatically picked maximum instantaneous frequency from the signal at the bottom of the tank (circles), TWT relative to the initial state (red line).

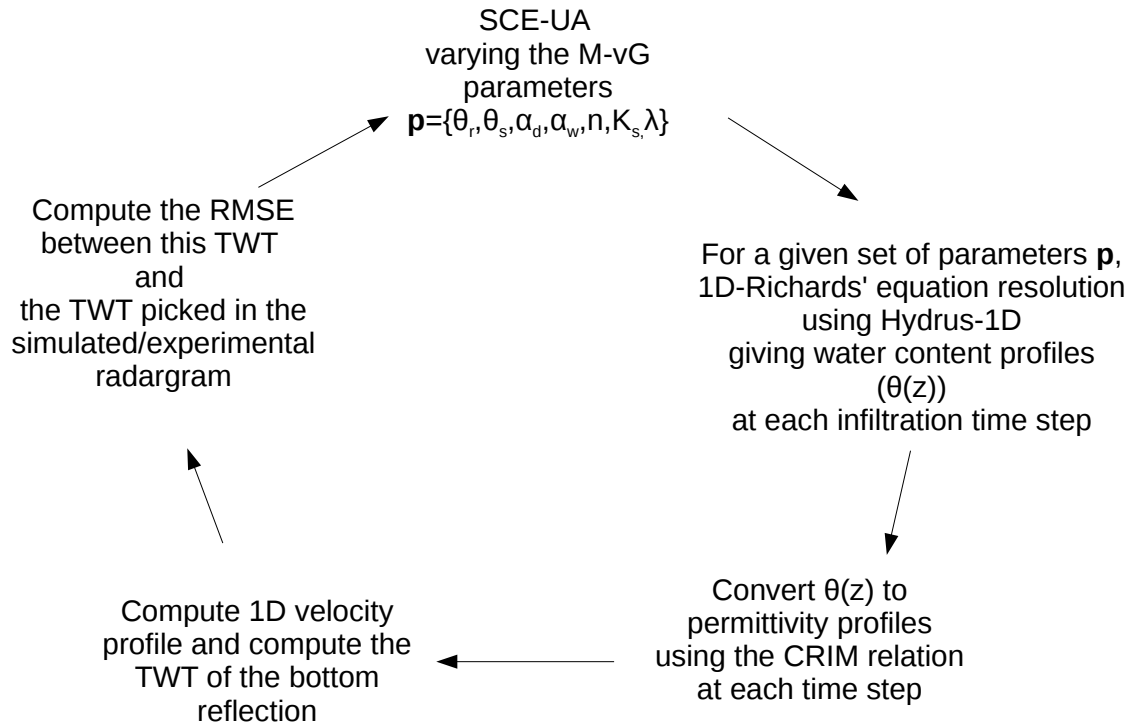


a)



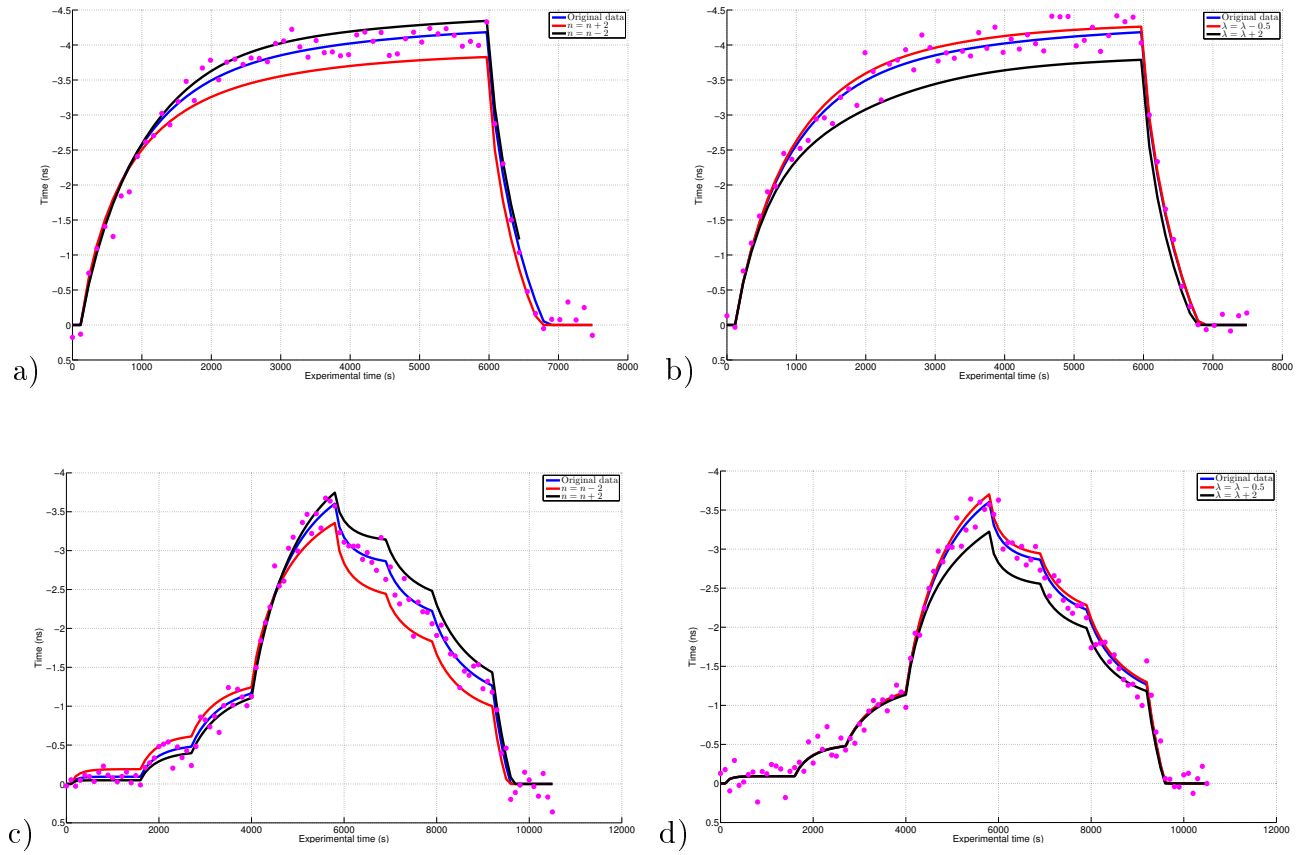
b)

**Figure 6.** TWT from the bottom tank reflection, relative to the initial saturated case, for the one-step case. a) Comparison between the TWT computed using our simple 1D-velocity modeling (blue dots) and the TWT picked in the GPRMAX2D radargram (red dots). These differences (RMSE=0.09 ns) are due to frequency dispersion, taken into account in GPRMAX2D but not in our 1D modeling. b) Comparison between the TWT computed using our simple 1D-velocity with the hydraulic heads presented in Table 1 (blue dots), and with the hydraulic heads decreased by 1 cm (red dots). The RMSE associated with this discrepancy is 0.11 ns.

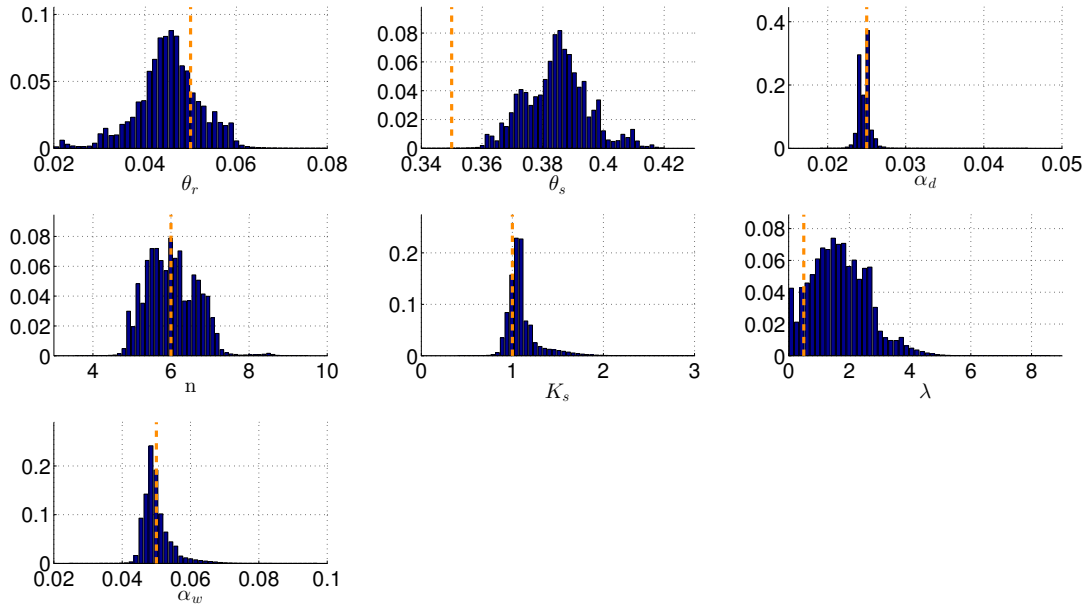


**Figure 7.** Flow diagram of the inversion procedure. “SCE-UA”: Shuffled Complex Algorithm, “M-vG”: Mualem-van Genuchten, “CRIM”: Complex Refractive Index Method, “TWT”: Two Way Travel-time, “RMSE”: Root Mean Squared Error.

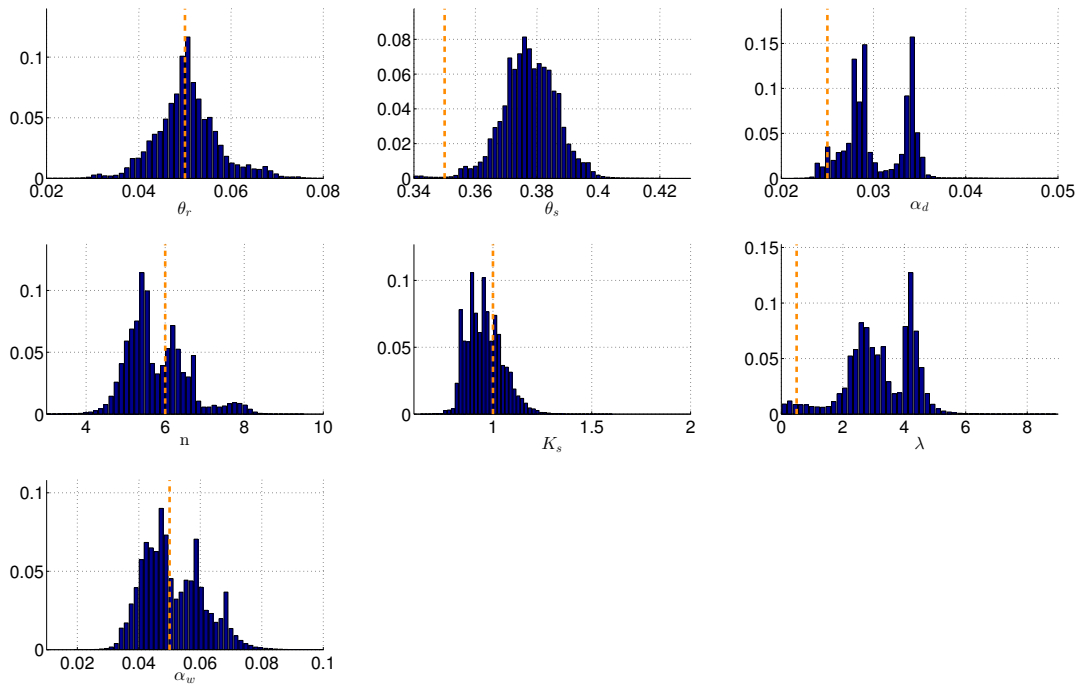




**Figure 8.** TWT with input parameters (Table 1) in blue, red and black curves represent TWT computed with different  $n$  values (one-step in a) and multi-step case in c)) and  $\lambda$  values (one-step in b) and multi-step case in d)). Purple points are the perturbed input data adding noise sampled in a Gaussian distribution with standard deviation 0.13 ns for the multi-step case and 0.15 ns for the one-step case.

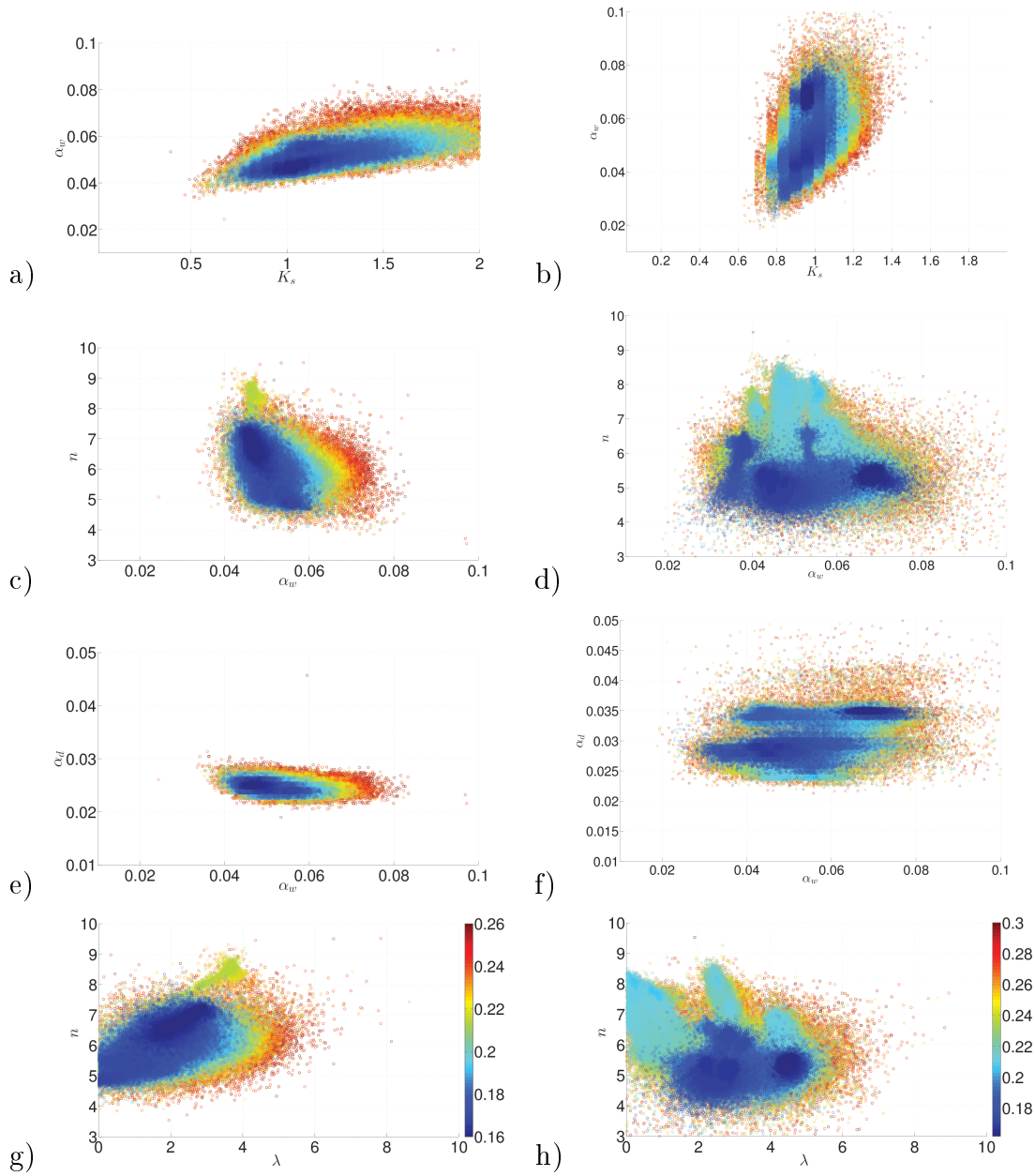


a)

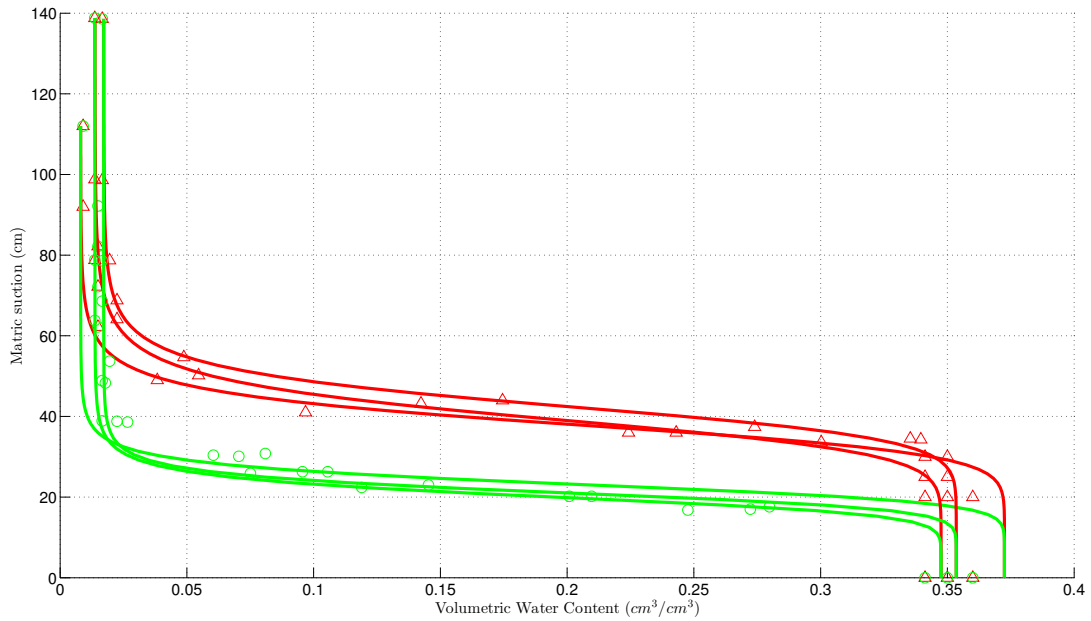


b)

**Figure 9.** Histograms realized on models fitting simulated data, a) in the multi-step case, presented on radargram 4-a), b) in the one-step case, presented on radargram 4-b). For each model the RMSE is smaller than  $2\sigma$  (0.26 ns in the multi-step case and 0.3 ns in the one-step case). The orange vertical lines symbolize the values used as input parameters.



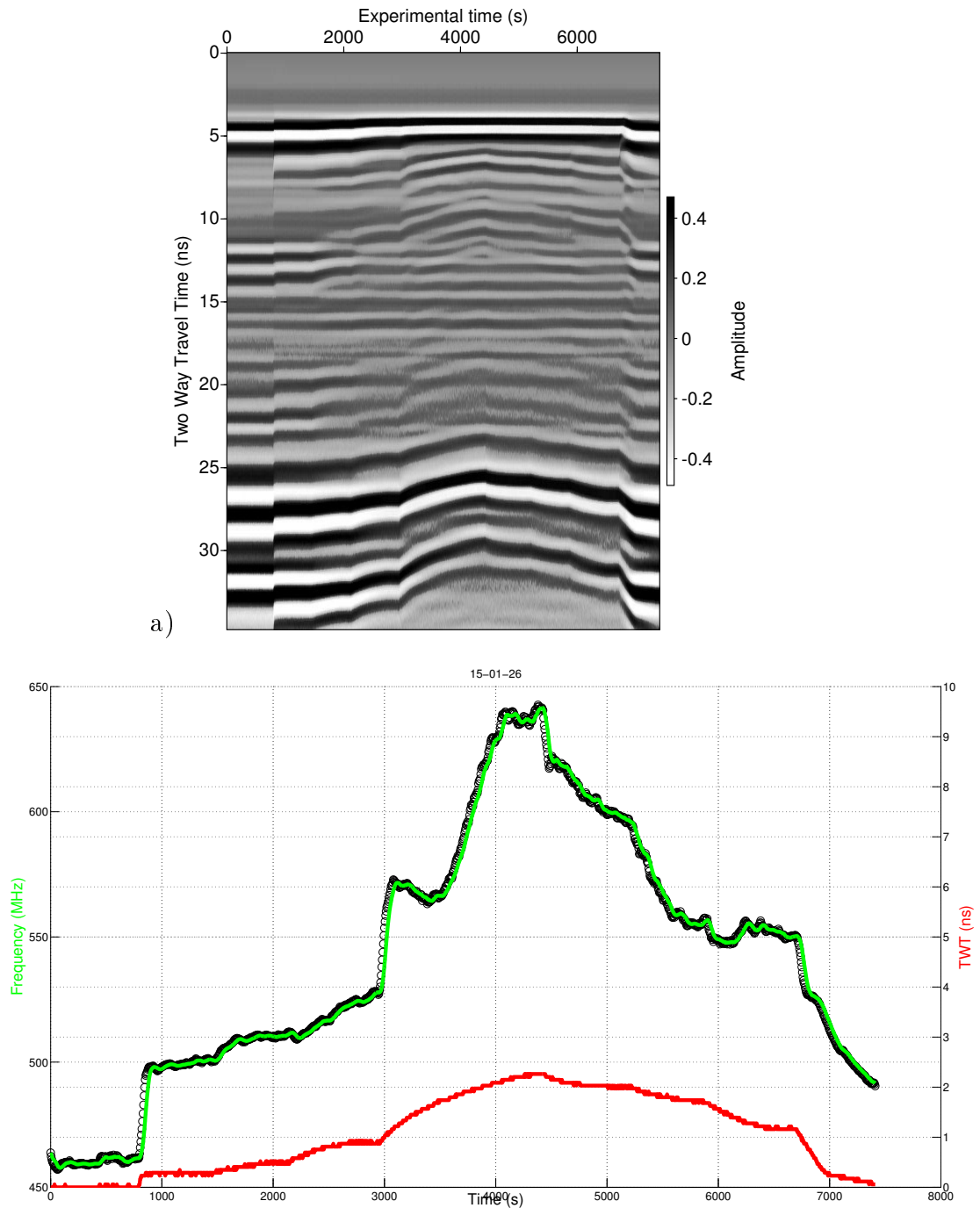
**Figure 10.** Objective function plots realized on noise-added simulated data keeping all parameter sets giving a RMSE value smaller or equal than  $2\sigma$ . Diagrams in the left column (a, c, e and g) were computed for the multi-step case while those in the right column (b, d, f and h) were computed for the one-step case. The color bars displayed in g) and h) are RMSE (ns) and are valid for all objective function plots in each column.



**Figure 11.** Results of water hanging column experiments including a drying and wetting cycle. Samples were prepared with the same bulk density as the sand in the cylindrical tank experiment. Red triangles: drying data; green circles: wetting data. Plain curves represent the model fitting the data with the parameters  $p_{lab}$  in Table 2.

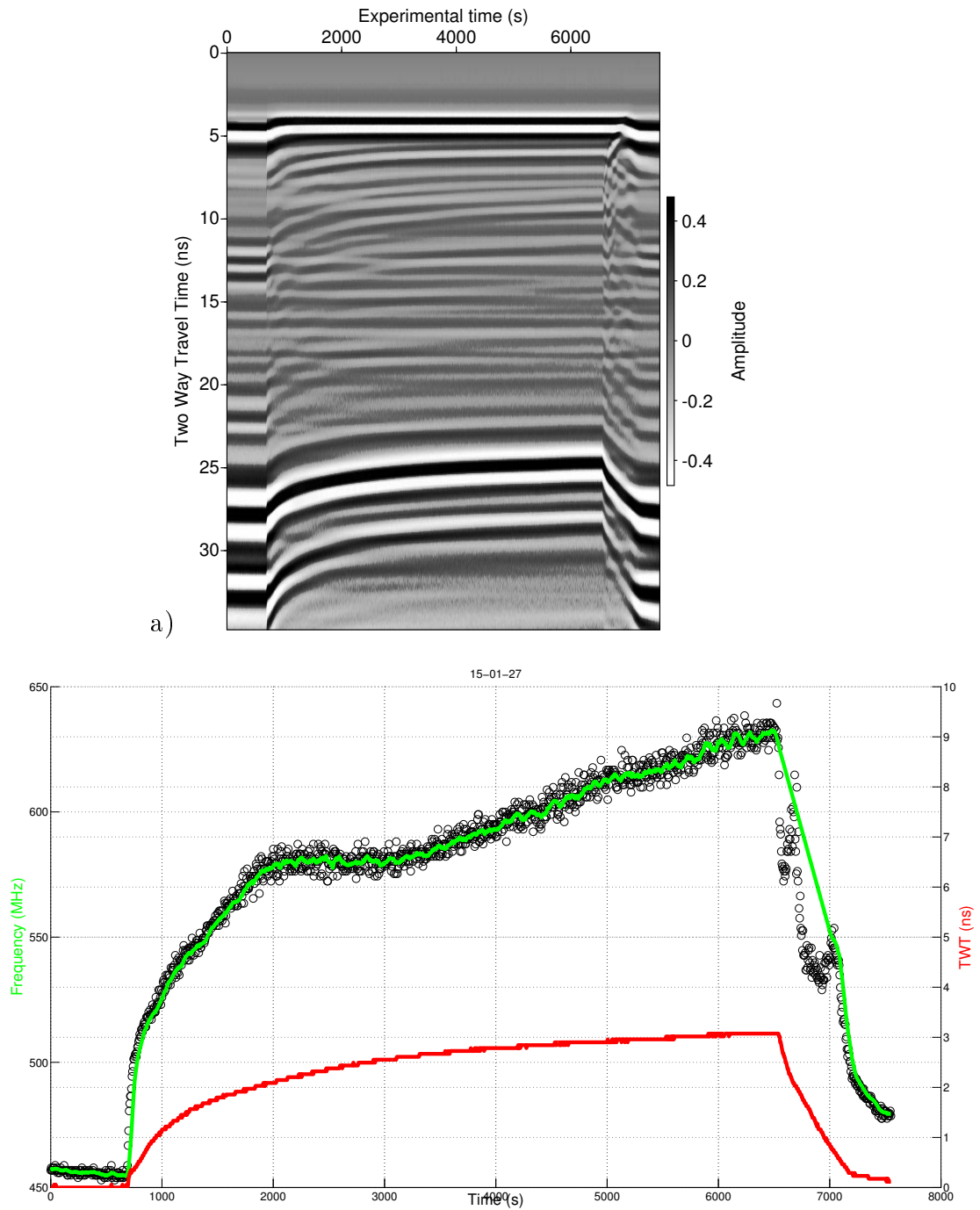


**Figure 12.** Pictures of the experiments. On the left is presented the tank filled with water, with its overflow connections. Middle picture shows the tubing linking both tanks. The picture on the right displays the sand tank with the GPR set at the top with a load on it.



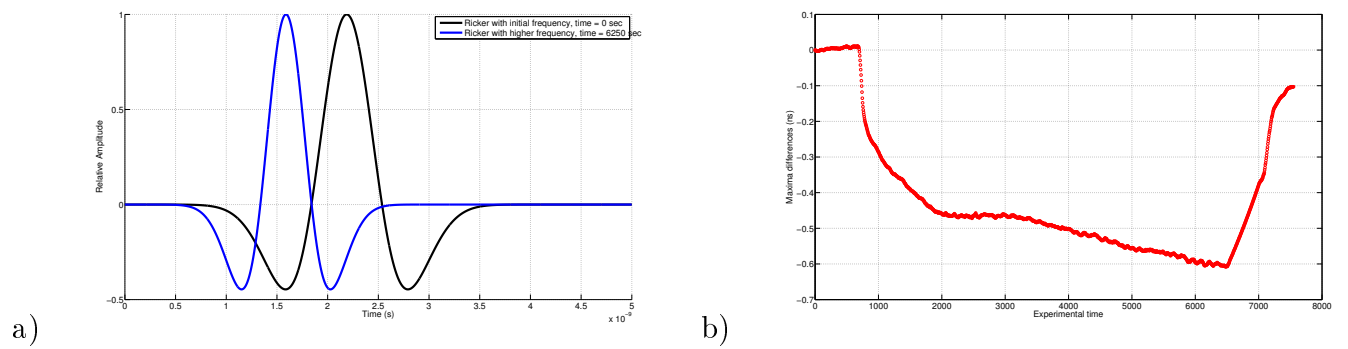
b)

**Figure 13.** a) Radargram acquired along a multi-step experiment. The reflection coming from the bottom of the tank is arriving between 25 and 30 ns. Other reflections are coming from tank edges and the capillary fringe. b) Maximum frequency of the reflection at the bottom of the tank (circles), after moving window averaging (green plain line), TWT relative to its initial value (red line).



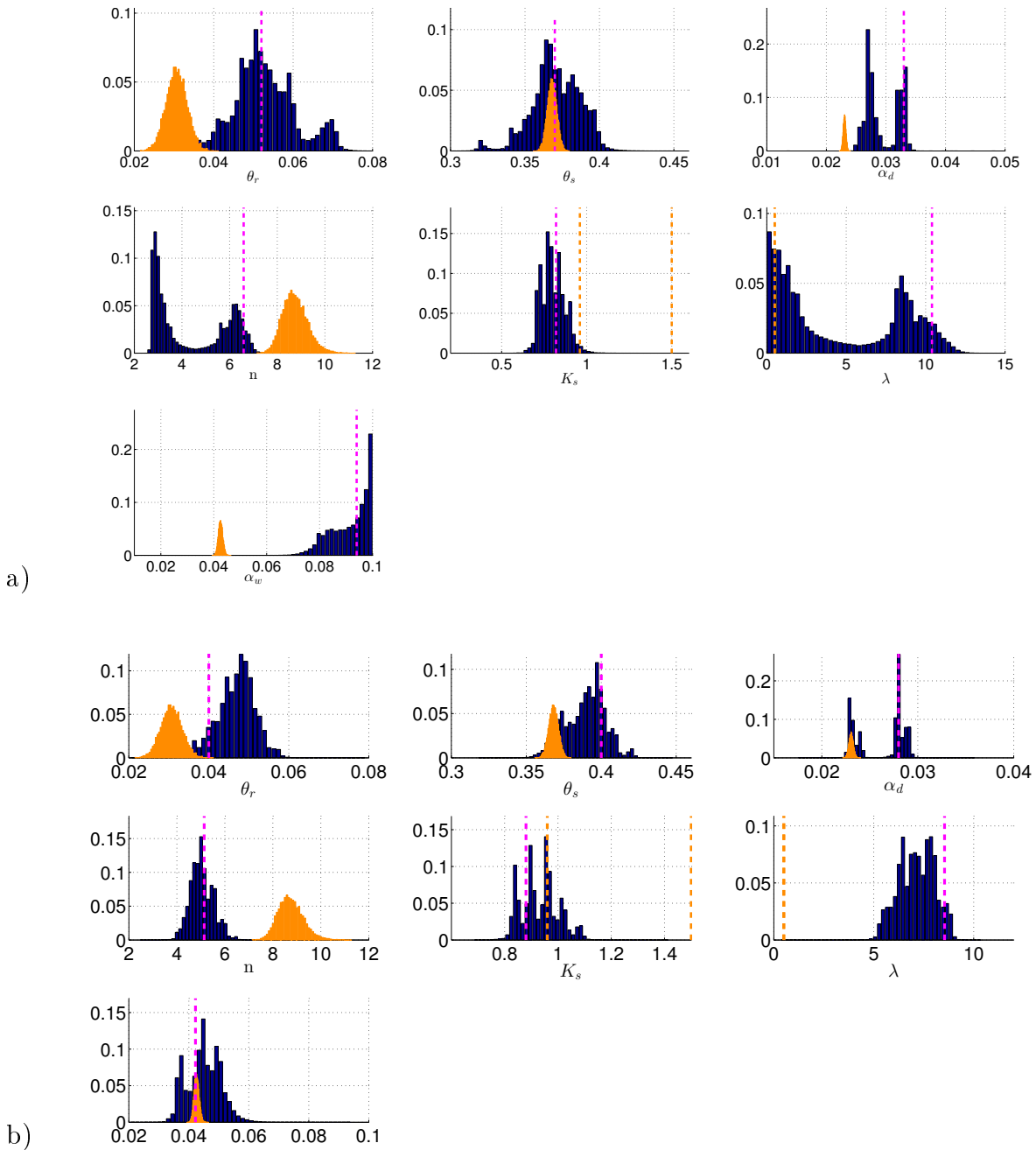
b)

**Figure 14.** a) Radargram acquired along a one-step experiment. The reflection coming from the bottom of the tank is arriving between 25 and 30 ns. Other reflections are coming from tank edges and the capillary fringe. b) Maximum frequency of the reflection at the bottom of the tank (circles), after moving window averaging and manual correction (green plain line), TWT relative to its initial value (red line).

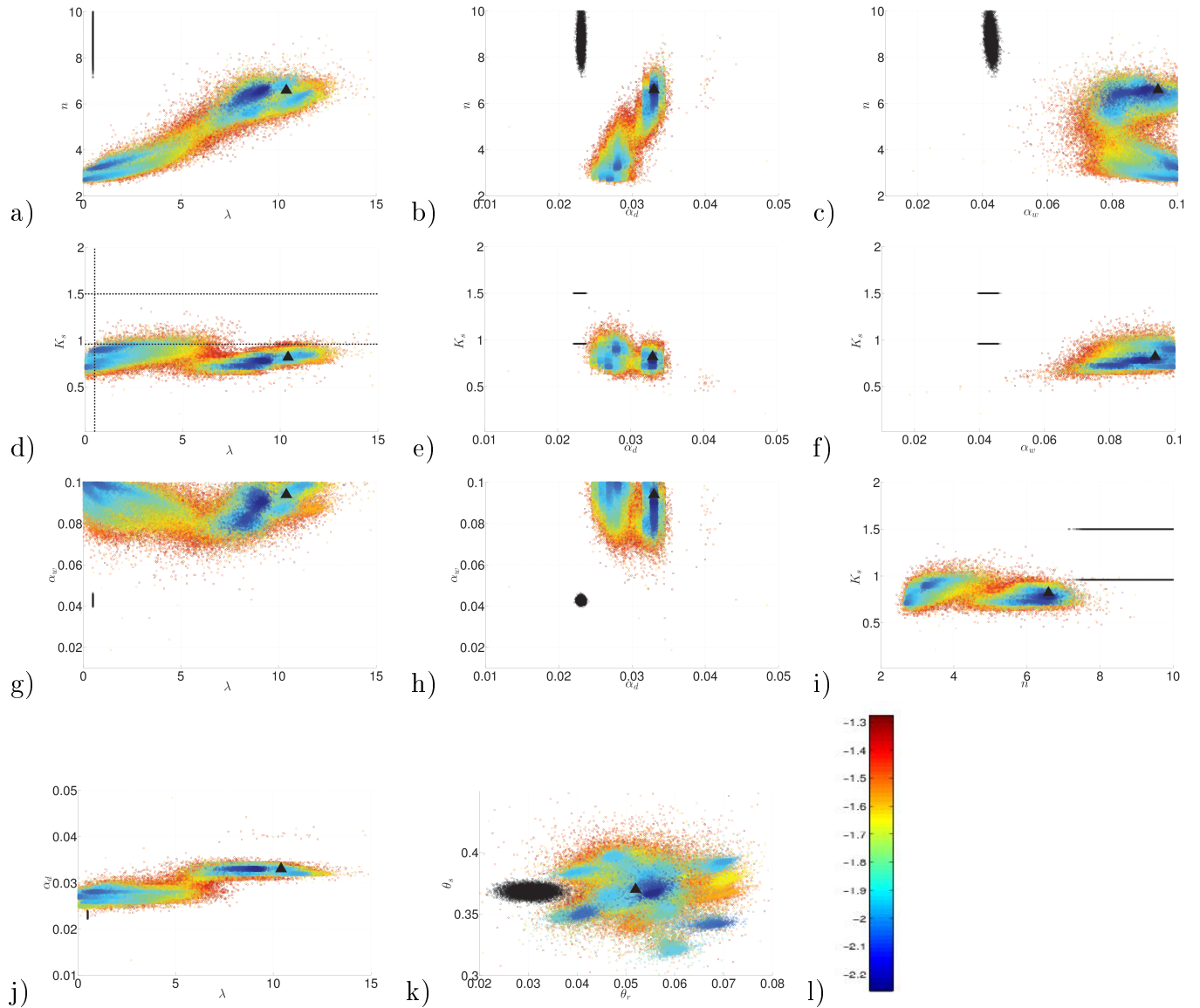


**Figure 15.** a) Time derived Ricker wavelet with two different maximum frequencies, b) computed time delays to be applied to measured TWT of the reflection coming from the bottom of the tank, in the multi-step case.

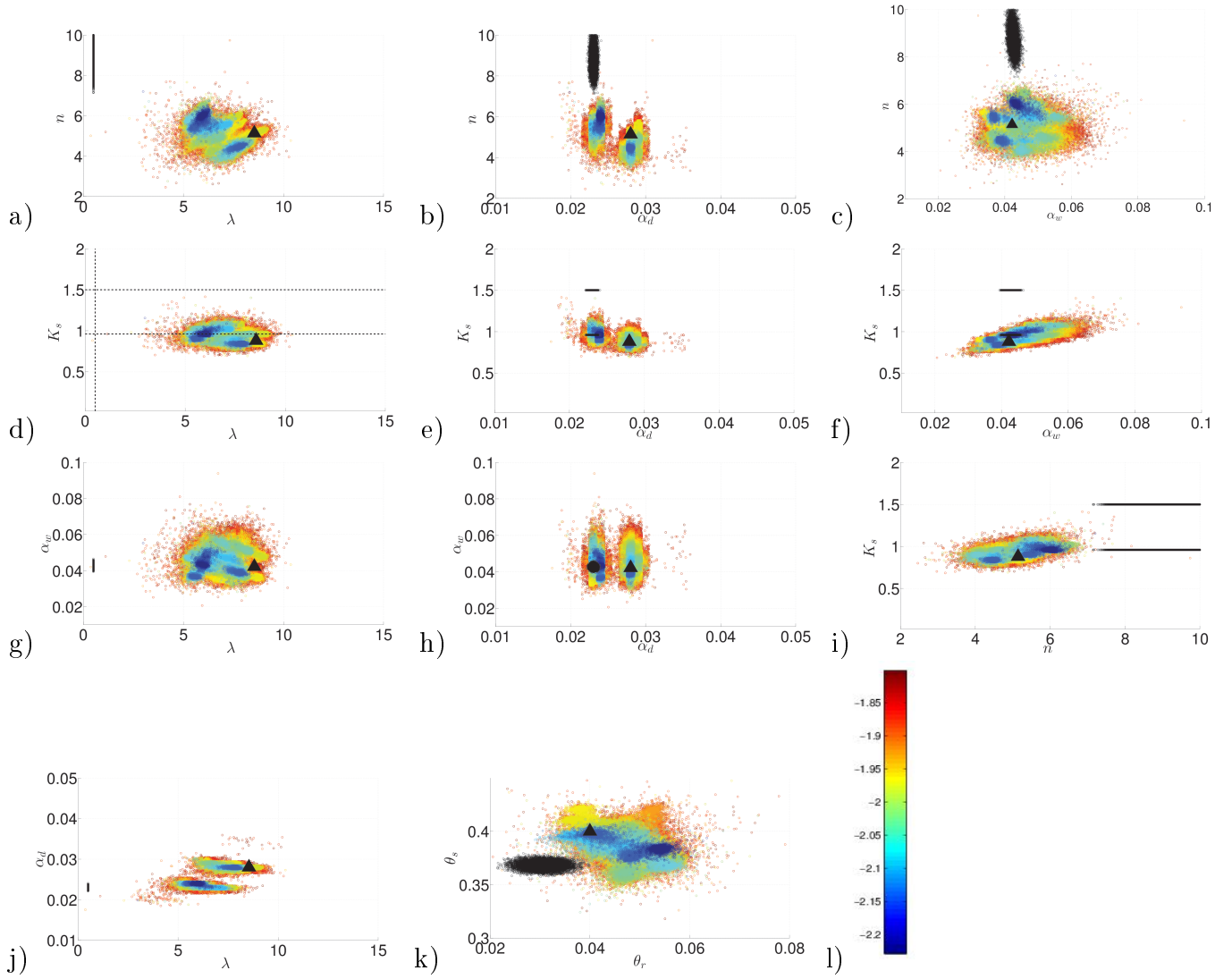




**Figure 16.** Histograms of the models fitting the experimental a) multi-step data with a RMSE smaller than or equal to  $0.154 ns$  and b) one-step data with a RMSE smaller than or equal to  $0.163 ns$ . Orange histograms represent the laboratory data obtain with HWC, orange dashed lines limit the range for  $K_s$  obtain with KSAT measurement and  $\lambda = 0.5$  from *Mualem* [1976]. The pink lines represent the optimized parameters (see Table 2).



**Figure 17.** Objective function plots obtained for a multi-step experiment done with 231,192 models fitting experimental data subsets with a RMSE smaller than or equal to  $\sigma = 0.154$  ns. The colorbar corresponds to RMSE values. Black triangles symbolize the parameter set optimizing the original data set sampled with a 20 s time step (giving the best fit with a RMSE of 0.14 ns). The black dots are the parameters estimated from HWC and KSAT experiment, and the usual  $\lambda$  value found in literature.



**Figure 18.** Objective function plots obtained for a one-step experiment done with 238,432 models fitting experimental data subsets with a RMSE smaller than or equal to  $\sigma = 0.163 ns$ . The colorbar corresponds to RMSE values. Black triangles symbolize the parameter set optimizing the original data set sampled with a 20 s time step (giving the best fit with a RMSE of 0.148 ns). The black dots are the parameters estimated from HWC and KSAT experiment, and the usual  $\lambda$  value found in literature.