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Technological catching-up and structural convergence among US industries

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Introduction

National productivity changes arise from two origins. First, a technological catching-up effect related to the fact that less productive industries put in extra effort to grow faster than the leading sectors. Second, a convergence mechanism due to homogenization of production processes among industries do occur over time. This structural convergence is connected to an output/input deepening or expending effect and diminishing returns of the production technology. Structural efficiency is mainly driven by marginal factor productivity and price harmonization among industries within the same economy. Over the last decades, import competition has been a driver of productivity gains. For both technical and structural components, the price cost margin (PCM) convergence leads to price harmonization among industries in developed countries. Most of previous studies about productivity gains simply highlight technical efficiency at the individual level, but pay less or no attention to a structural effect at a more aggregated level.

This paper attempts to analyze the efficiency convergence process within a group of 63 industries which cover the whole U.S. economy over the period of 1987–2016. We intend to decompose the growth in productivity at the macro level by splitting overall efficiency evolution into two components: technological catching-up and structural convergence. The technical effect measures an efficiency gap between the evaluated sector and its benchmark located on the production frontier. Its reduction over time discloses a technological catching-up process: the inefficient industry has reached the benchmark progressively. The structural effect measures the differences in the input and output mixes among industries impacting productivity ratios at the macro level. Over time, a decrease of this effect means that input and output combinations are becoming more homogenous among the different industries which contribute to improve the productivity level for the global economy.

Two contributions are developed in this paper. First we derive technological catching-up and structural convergence processes through non parametric production technology estimations. Second, we correlate the input and output price evolutions with the changes of technical and structural components measured in the first step.

Traditionally, technological catching-up analysis is based on TFP comparisons that present several caveats. First, they need to define a technological leader a priori (generally the US) instead of letting data choose the benchmark to reach. Second, the technology estimation requires a particular functional form (Cobb–Douglas, CES, Translog...). Third, the constant returns to scale assumption does not take into account size heterogeneity across production plans and may bias TFP indexes and the underlying catching-up process. To avoid these three drawbacks, a non-parametric framework can be considered as an alternative to the usual approaches. Based on an axiomatic definition of

production sets (Koopmans, 1951; Debreu 1951; Shephard 1953, 1970), and directional distance function (Chambers et al, 1996), the analysis of catching-up mechanisms can be re-examined without imposing any functional form on the production frontier, or proposing any hypothesis for the market structure. In addition, an arbitrary country does not have to be chosen as the world leader and allow for eventual technical and/or allocative inefficiencies for economies.

Using such a non-parametric programming framework, our study analyzes both structural convergence and technological catching-up among 63 North American industries over the period 1987-2016. We first separate efficiency gaps into two components: a technology effect taking into account industry size heterogeneity by relaxing the constant return to scale assumption and a structural element which highlights the impacts of an input-output deepening or expanding effect on technological transfer over time. The convergence processes on each of them are analyzed. Secondly, we perform a panel data regression to explain the input and output price evolutions by the changes in technical efficiencies and input-output mixes.

This paper is organized as follows. In the next section we present a brief literature review on catching-up and convergence processes and their link to production technology estimations and efficiency measurements. In section 2, we use a directional distance function to define the production frontier and evoke the measures of technical and structural effects which may impact the convergence process within a set of units. Section 3 presents data and results; the latter are discussed in section 4. The last section presents the main conclusions.

1. Efficiency convergence process: a brief literature review

The two concepts of technological catching-up and structural convergence developed in this paper take their source in the traditional convergence literature. According to Solow, two simultaneous processes support income convergence between countries: a capital deepening effect and a technological transfer/diffusion related to Total Factor Productivity (TFP) gaps. Through the initial Solow's Model, the neoclassical standard theory devoted most attention to the first process. Assuming an exogenous and non-costly technological progress, technology adoption issues were not explicitly taken into account. For Solow (1994) this restrictive hypothesis was necessary at that initial step of progress of the growth theory. Later, the identical production technologies assumption was rejected by authors such as Jorgenson (1995) or Durlauf and Johnson (1995). In the same vein, a less drastic approach adopted by Abramovitz (1986) considers a common available technology among economies which may diverge in their capability to join and use it. As a result the concept of "social capabilities" was introduced to explain different productivity levels between countries and

concern in cross-country TFP gaps has become a major issue to investigate economic growth (Islam, 2003).

As empirical measure of technology can be linked to TFP estimations, the concept of TFP-convergence investigates whether production plans of industries are capable of catching-up in terms of achieving highest observed TFP levels. Most of the empirical studies of TFP convergence have focused on international comparison of TFP and have shown that differences in technology are related to gaps in TFP levels. For example, Dowrick and Nguyen (1989) analyzed TFP-convergence by carrying out a regression analysis

of the productivity growth rates on the initial TFP levels of fifteen OECD nations.. Substantial signs of TFP catching-up among developed nations were established. However, their restraining hypothesis of a single capital-output ratio for all countries is a major limitation.. Wolff (1991) developed a TFP catching-up equation including a capital/labor ratio growth rate for the G-7 countries in order to study the relationship between technical change and capital deepening. He found a positive influence of capital accumulation on TFP catching-up. Later, Dougherty and Jorgenson (1997) revealed a process of sigma-convergence of TFP levels among the G-7 countries through a significant reduction of their coefficients of variation over time.

TFP growth due to the interaction between technology adoption and capital accumulation was mainly studied for East Asian economies during the nineties. Several authors (Young, 1992, 1994, 1995; Kim and Lau 1994) found that TFP growth did not play a major role in the expansion of their economies. As a result, Krugman (1994) deduced that East Asian economic development mainly resulted from factor accumulation. Nevertheless, Collins et al. (1996) and Klenow and Rodriguez (1997) showed that TFP played more substantial role in the growth of some East Asian economies such as Singapore.

While a huge body of literature has been devoted to productivity convergence at country level, the sources of aggregate productivity changes at the industry level remain largely unstudied. Through data on sectors, Bernard and Jones (1995) analyzed the sources of aggregate labor productivity convergence among the U.S. states over the period 1963-1989. They estimated the individual sector's contribution to aggregate convergence. Their main result is that productivity growth in the manufacturing industries explained the main part of private non-farm productivity growth. Focusing on productivity changes by sector from 1963-1989, Barro and Sala-i-Martin (1991) pointed out that convergence was happening in all industries, although this process has been more significant in manufacturing than in other types of industries. They also established a change in macro-convergence after the early 70s mainly due to price changes in oil industry. More recently, Cardarelli and Lusinyan (2015) studied the aggregate US TFP slowdown using TFP estimators

across U.S. states over the last two decades. They revealed that the deceleration of TFP growth was quite common among the states and not correlated to the presence of IT- producing or IT- using industries. Gaps in productive efficiency across U.S. states are mainly explained by differences in rates of investments in education and R&D.

Boussemart and al. (2017) estimated productivity gains and their distribution among inputs and outputs for 63 American industries over the period 1987-2012. The study showed that TFP of US industries increased at an average trend of 0.8% and highlighted that employees and firms' profitability were the winners while purchasers of good and services and providers of fixed capital were the losers in the distribution of productivity gains. Beyond these global results, TFP growth rates have been significantly different between the 63 industries over the last 26 years. Clearly, the computer and electronic products industry had the highest level of TFP growth (7.48%) followed by other sectors such as support activities for mining and wholesale trade (2.32% and 2.09%) while the oil and gas extraction industry registered the lowest performance (-1.22%).

Kumar and Russell (2002) estimated productivity indexes with a non-parametric method and investigated the catching-up hypothesis across 57 poor and rich nations. More precisely, they decomposed variations of the cross-country distribution of labor productivity into dissimilarities in levels of technology and technical changes over time. They showed that much of income convergence was due to technology transfer or to alignment in capital/labor intensities. They demonstrated a clear evidence of technological catching-up, as most of the countries have moved closer to the production frontier; non-neutral technological progress and a dominant role of capital deepening occurred compared to technological catching-up inducing both growth and income divergence between countries. A Data Envelopment Analysis (DEA) approach was also used by Christopoulos (2007) to study the effect of human capital and international trade (i.e.: economic globalization) on efficiency within a group of 83 developed and less developed nations. He confirmed that more openness significantly improves countries' productive performances while human capital does not impact the efficiency to a great degree. Nonetheless, a constant returns to scale hypothesis still characterized the underlying technology.

Relaxing this restrictive constant returns to scale assumption for the technology, Färe et al. (1994) decomposed productivity growth into technical progress effect and an efficiency change component that was referred to as catching-up process for 17 OECD countries over the period 1979-1988. Additionally, the catching-up component was split into two terms: a pure technical efficiency change and a scale efficiency change. Their results showed that Japan obtained the highest TFP growth rate.

Boussemart et al. (2006) examined the catching-up hypothesis at the industry level across the main OECD countries. Using panel data econometric models involving technological gap indicators were developed. A nonparametric distance function estimation revealed that there is statistical evidence of a catching-up process at the industry level.

2. Analyzing convergence process with directional distance functions

We aim at estimating both a technological catching-up effect between observed production plans of industries and their maximum achievable levels of TFP, and a convergence process of input-output ratios among industries. A technological catching-up process reveals the ability to fit the current technology and a structural convergence process considers the diversity across industries regarding their respective input or output intensity evolutions. This latter can be related to an input/output deepening or expanding effect.

In the followings paragraphs, the concepts of technological catching-up and structural convergence are defined. Moreover, methodological tools to measure these effects are developed.

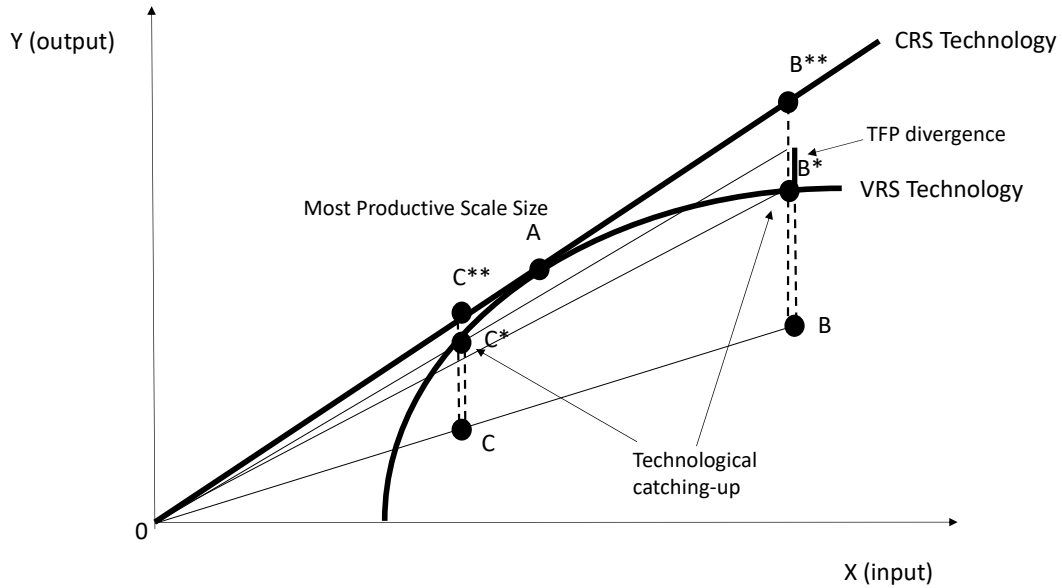
2.1 Definition and measure of a technological catching-up process

A technological catching-up process happens when less efficient industries tend to catch up more efficient ones over time by moving toward the production frontier. Thus, one can observe a convergence process to the efficient frontier if the technical inefficiency level is decreasing over time. Less efficient industries can progressively adopt technological innovations, managerial procedures, or organizational capabilities from the most productive ones.

Traditionally, in the literature, technological adoption is viewed as comparison of TFP levels across sectors or countries and testing an inverse link between TFP growth rates and their original levels. Convergence process occurs if industries with the smallest TFP levels display the highest growth rates. The assumption of constant returns to scale (CRS) is necessary since the best production plan, set as a benchmark for all sectors, has the maximal observed productivity. Nevertheless, if the CRS assumption is not fulfilled and increasing and/or decreasing returns to scale (variable returns to scale, VRS) appear to be more appropriate, the maximal feasible productivity level may not correspond to the maximal observed productivity level and should be estimated for each sector relative to its own size. Indeed, this size is constrained by the industry's scale of operations which can be considered as quasi-fixed in the short-run. In fact, if a CRS technology is retained while a VRS is more appropriate for the data, the analysis of technological transfer can lead to substantial bias. One can observe a divergence in productivity levels among industries when they achieve the production frontier and contribute to a technological catching-up process as it is shown in Figure 1.

Let us consider 3 sectors A, B and C which use one input (X) to produce one output (Y) under variable return to scale (Figure 1). One can observe that sectors B and C characterized by a similar productivity level are inefficient while industry A is efficient and has the most productive scale size (mpss). If we suppose that sector A is a benchmark for all industries, we implicitly assume a CRS technology. That is, if sectors B and C could achieve B^{**} and C^{**} , TFP convergence will occur as all industries will reach the same maximal productivity level. However, if the true VRS technology holds, sectors B and C will only be able to achieve B^* and C^* for which productivity divergence is observed. Since B and C will never be able to achieve B^{**} and C^{**} , one can draw a conclusion about divergence of productivity levels between the industries even if they have reached their respective maximal feasible productivity levels located on the VRS production frontier. In that case, their technical inefficiencies have decreased over time denoting a clear technological catching-up process.

Figure 1. Maximal observed productivity level under CRS assumption versus maximal feasible productivity level under VRS assumption



In order to measure technical inefficiency, we develop an activity analysis model assuming that all industries face the same VRS technology in the sense that they are able to produce a common gross output from similar resources such as fixed capital, labor and intermediate inputs:

In a more general way, let us consider a vector of inputs $\mathbf{x} \in \mathbb{R}_+^N$ and a vector of outputs $\mathbf{y} \in \mathbb{R}_+^M$ for an observed industry or DMU (decision making unit). At time t , the technology can be simply defined by the production set which includes all the feasible production plans:

$$T_{VRS}^t = \{(\mathbf{x}^t, \mathbf{y}^t) : \mathbf{x}^t \text{ can produce } \mathbf{y}^t\} \quad (1)$$

To better structure and clarify the definition of T_{VRS}^t , we consider two assumptions on the production possibility set: free disposability of inputs and outputs and convexity. Now from a sample of K observed DMUs, we achieve an operational definition of T_{VRS}^t as:

$$T_{VRS}^t = \left\{ (\mathbf{x}^t, \mathbf{y}^t) : \mathbf{x}^t \in R_+^N, \mathbf{y}^t \in R_+^M, \sum_{k=1}^K \mu_k y_k^{m,t} \geq y^{m,t}, m=1, \dots, M, \right. \\ \left. \sum_{k=1}^K \mu_k x_k^{n,t} \leq x_k^{n,t}, n=1, \dots, N, \sum_{k=1}^K \mu_k = 1, \mu_k \geq 0, k=1, \dots, K \right\}. \quad (2)$$

We measure the gaps between any DMU and the technology frontier at time t using a directional distance function. An axiomatic approach to technical efficiency measurement with directional distance function is developed in Briec et al. (2011). In our context, the directional distance function is defined as:

$$\bar{D}_{T^t}(\mathbf{x}^t, \mathbf{y}^t; \mathbf{g}_x^t, \mathbf{g}_y^t) = \sup_{\lambda^t} \left\{ \lambda^t \in \mathfrak{R}_+ : (\mathbf{x}^t + \lambda^t \cdot \mathbf{g}_x^t, \mathbf{y}^t + \lambda^t \cdot \mathbf{g}_y^t) \in T_{VRS}^t \right\}, \quad (3)$$

where $\mathbf{g}^t = (\mathbf{g}_x^t; \mathbf{g}_y^t) \in (-R_+^N; R_+^M)$ characterizes the direction of the projection onto the annual production frontier. In our analysis we define $\mathbf{g}_x^t = 0$ and $\mathbf{g}_y^t = \sum_{k=1}^K \mathbf{y}_k^t$. Therefore, the technical inefficiency for any evaluated DMU “ a ” can be estimated with a linear program (LP1) presented in Appendix 1.

Considering the group of K industries or DMUs called “AGREG”, all individual technical inefficiencies can be summed up to obtain the technical inefficiency score at the aggregate level:

$$I_{AGREG}^{TECH,t} = \sum_{a=1}^K \bar{D}_{T_{VRS}^t}(\mathbf{x}_a^t, \mathbf{y}_a^t; \mathbf{g}_x^t, \mathbf{g}_y^t). \quad (4)$$

Thus, a decrease with time of $I_{AGREG}^{TECH,t}$ will denote a general catching-up process to the maximal feasible productivity levels for the majority of industries.

2.2 Definition and measure of a structural convergence process

If we consider a multi outputs-inputs technology, diversity in input and output allocations among industries can cause structural inefficiency (Ferrier et al. 2010). As we can observe at Figure 2, efficient sectors A and B which produce the same output level but with different input mixes, create technical inefficiency at the aggregate level. Related structural effects in the output and input-output spaces are displayed in Figures 3 and 4. Thus, differences in relative input and output endowments between the two technically efficient industries induce such a structural inefficiency.

Under perfect competition, a given input-output price vector leads industries to adopt identical input-output allocations. As a result, less resistance in the reallocation process reduces misallocation of resources and improves aggregate productivity. Consequently, in the spirit of Debreu's (1951) concerning coefficients of resource utilization, market allocation inefficiency can be revealed through the structural inefficiency. Thus, the decrease of this component over time is correlated to aggregate productivity growth at the group level since sectors homogenize their input-output allocations gradually disclosing a structural convergence process.

Figure 2. Structural inefficiency and convergence in the input space

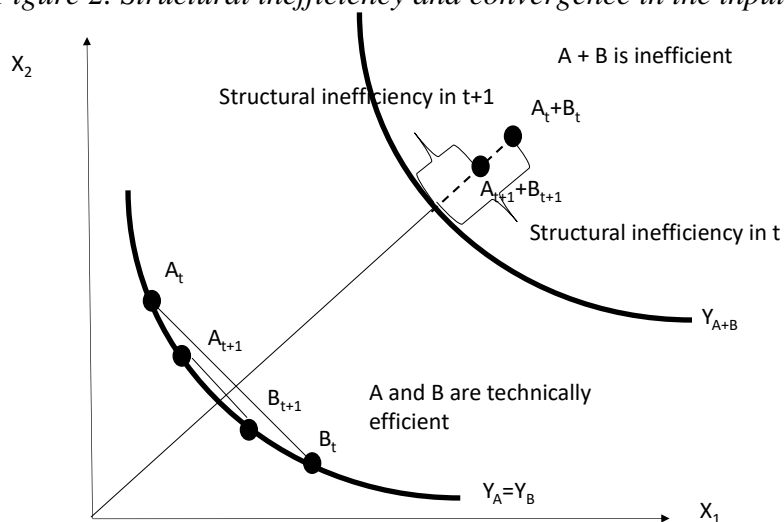


Figure 3. Structural inefficiency and convergence in the output space

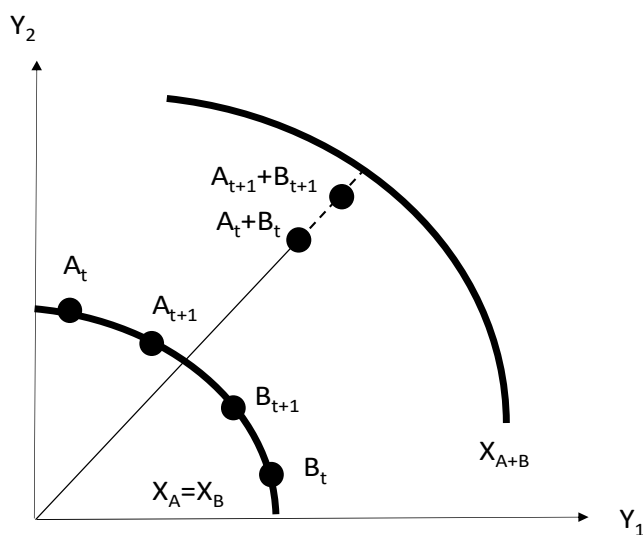
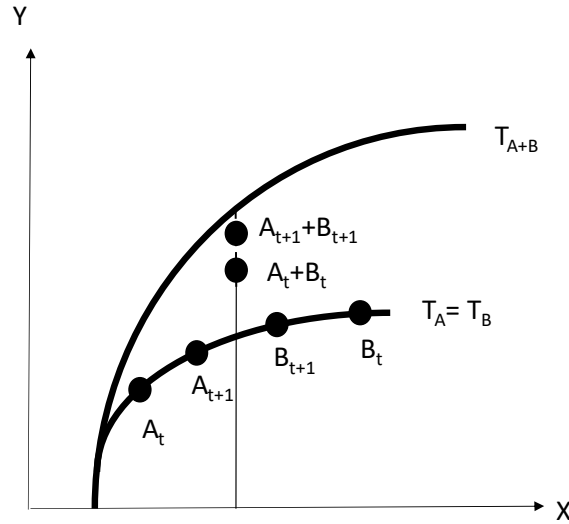


Figure 4. Structural inefficiency and convergence in the input-output space



We intend to estimate structural inefficiency scores for all industries at the group and individual levels. To obtain the inefficiency scores at individual level, we first estimate structural inefficiency at the group level. As previously discussed, we consider K industries or DMUs which constitute the total group AGREG and we suppose, in a formal way, that the group technology is the sum of the K DMUs technologies:

$$T^{AGREG,t} = \sum_{k=1}^K T^k. \quad (5)$$

Li and Ng (1995) proved that under convexity assumption the VRS aggregate technology $T_{VRS}^{AGREG,t}$ is equal to K times the individual technology:

$$T_{VRS}^{AGREG,t} = \sum_{k=1}^K T_{VRS}^k = K \times T_{VRS}^t. \quad (6)$$

We estimate the overall inefficiency as the technical inefficiency for AGREG with the linear program (LP2) presented in appendix 1.

This linear program allows us to identify overall inefficiency which measures the technical inefficiency of the aggregated production plan merging the K DMUs.

$$I_{AGREG}^{OVERALL,t} = \bar{D}_{T_{VRS}^{AGREG,t}} \left(\sum_{k=1}^K x_k^t, \sum_{k=1}^K y_k^t; \mathbf{g}_x^t, \mathbf{g}_y^t \right) \quad (7)$$

The difference between the overall component estimated by LP2 and the sum of individual technical scores given by LP1 defines the structural inefficiency coming from the heterogeneity in relative input/output allocations among the K DMUs. Thus, structural inefficiency is defined at the group level:

$$I_{AGREG}^{STRUC,t} = I_{AGREG}^{OVERALL,t} - I_{AGREG}^{TECH,t} = \vec{D}_{TVRS}^{AGREG,t} \left(\sum_{k=1}^K x_k^t, \sum_{k=1}^K y_k^t; g_x^t, g_y^t \right) - \sum_{a=1}^K \vec{D}_{TVRS}^a (x_a^t, y_a^t; g_x^t, g_y^t) \quad (8)$$

As a result, if structural inefficiency decreases over time, we observe an input/output mix convergence process among the K DMUs.

While technical inefficiency can be retrieved directly at individual level through LP1, structural inefficiency is computed as a part of the overall inefficiency for the whole group. Nevertheless, we can allocate the overall inefficiency across DMUs by using the shadow prices derived in LP2 (Briec and al., 2003) in order to deduce the individual structural inefficiency:

$$I_{AGREG}^{OVERALL,t} = \sum_{a=1}^K I_a^{OVERALL,t} \Rightarrow I_a^{STRUC,t} = I_a^{OVERALL,t} - I_a^{TECH,t}$$

(9)

$$\text{and } I_{AGREG}^{STRUC,t} = \sum_{a=1}^K I_a^{STRUC,t}$$

3. Data and results

In order to analyze the technological catching-up and structural convergence processes among the industries, the previous models are now applied to a dataset on 63 different sectors covering the whole US economy over the period 1987-2016. In a second step, a panel data analysis is performed to explore the link between the changes in both technological catching-up and structural inefficiencies and the distribution of productivity gains among the different inputs and output.

3.1. Data

Data was collected from the Bureau of Economic Analysis (BEA) website (<http://www.bea.gov/>). For each industry, we have the current values (expressed in current U.S. dollars) and the quantity indexes (base year 100=2009) of their gross output net of taxes on production-less subsidies, intermediate inputs, labor (compensation of employees), and consumption of fixed capital (equipment, structures, and intellectual property products). The quantity indexes of taxes and subsidies on production are directly linked to the quantity output indexes. The labor quantity is estimated in a full-time equivalent employee. The volume of capital consumption (the sum of equipment, structure, and intellectual property products) is calculated by the cost depreciation at a constant price. Thus, for each sector, we can compute both the value and the volume for each variable stated. Finally, the underlying technology is defined as a production function of one output (gross output) which depends on 3 inputs (intermediate inputs, labor and fixed capital).

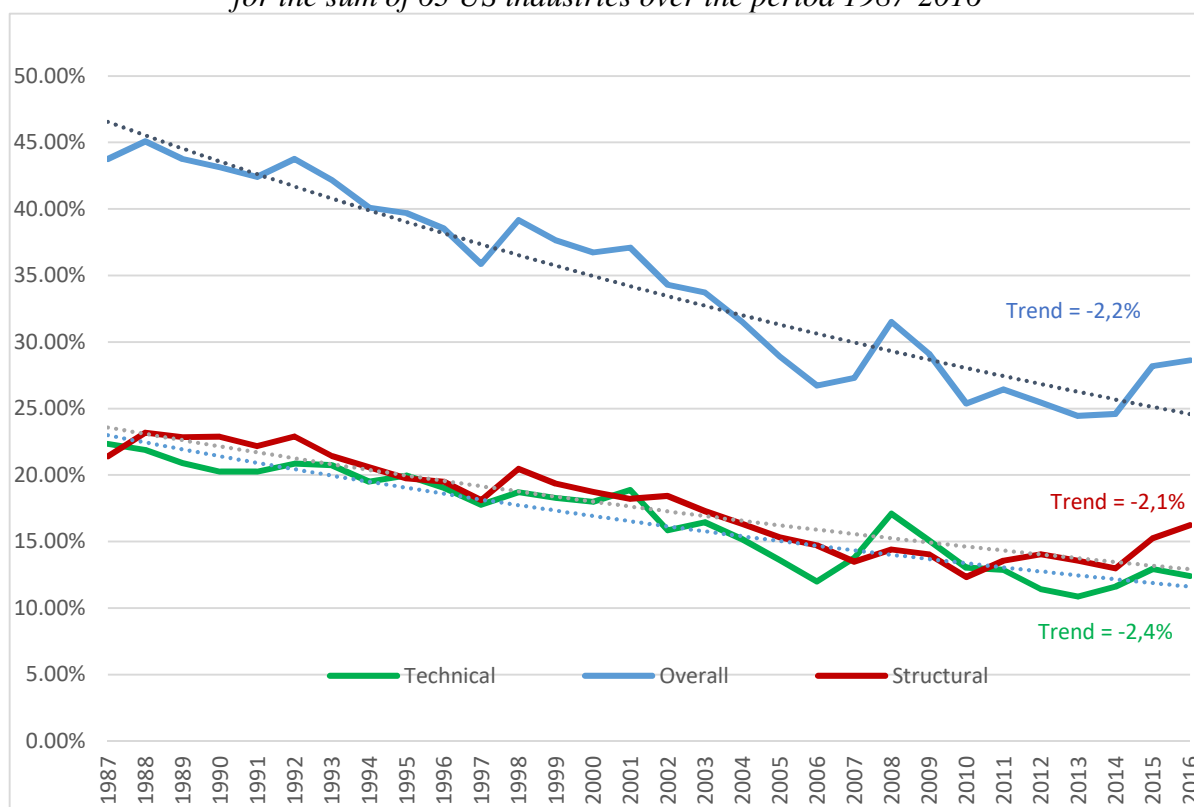
3.2. Technological catching-up and structural convergence processes among US industries

Production frontiers are estimated annually over the period 1987-2016. In a first step, we compute technical inefficiency scores at industry level using a directional distance function. The direction is defined as the sum of gross outputs of all industries. For each evaluated industry, efficiency scores reveal potential growth computed in terms of percentages of the total US gross output. Based on this common direction, the individual efficiency scores can be directly aggregated.. In a second step, we estimate overall inefficiency for all industries at the aggregate level. Finally, for each sector, the structural component is deduced through the difference between individual overall and technical inefficiencies. As the production frontier is year-specific, the number of efficient industries can change over time; although some of them are always located on the production frontier. The seven stable efficient sectors over the whole period 1987-2016 are: legal services; funds, trusts, and other financial vehicles; real estate; construction; state and local governments, petroleum and coal products: food, beverages, and tobacco products.

The respective evolutions of technical, structural and overall inefficiencies for the sum of 63 industries are presented in Figure 5. All three types of inefficiencies demonstrate convergence processes over the thirty-year period at the macro level even if a break in these trends is observed in the last few years. The technical and structural inefficiency dynamics follow a rather similar pattern with average annual decrease rate of respectively 2.4% and 2.1%. As a result, the overall inefficiency is decreasing over time with a trend of -2.2%.

Figure 5. Evolution of the overall, technical and structural inefficiencies

for the sum of 63 US industries over the period 1987-2016



Individual industries contribute to the technological catching-up and structural convergence processes differently. For instance, compared to computer systems design, chemical products, hospitals and nursing facilities, textile, electrical equipment industries and food services face significant and regular technical inefficiency decreases. Concerning the homogenization of input-output mixes, the individual effects seem more irregular. Examples of industries with the most important convergence rates of technological catching-up effect and homogenization of input-output mixes over the period considered are given in Figures 6 and 8 respectively. Examples of industries without technological catching-up effect and input-output mixes homogenization are presented respectfully in Figures 7 and 9.

Figure 6. Examples of industries with significant technological catching-up effects

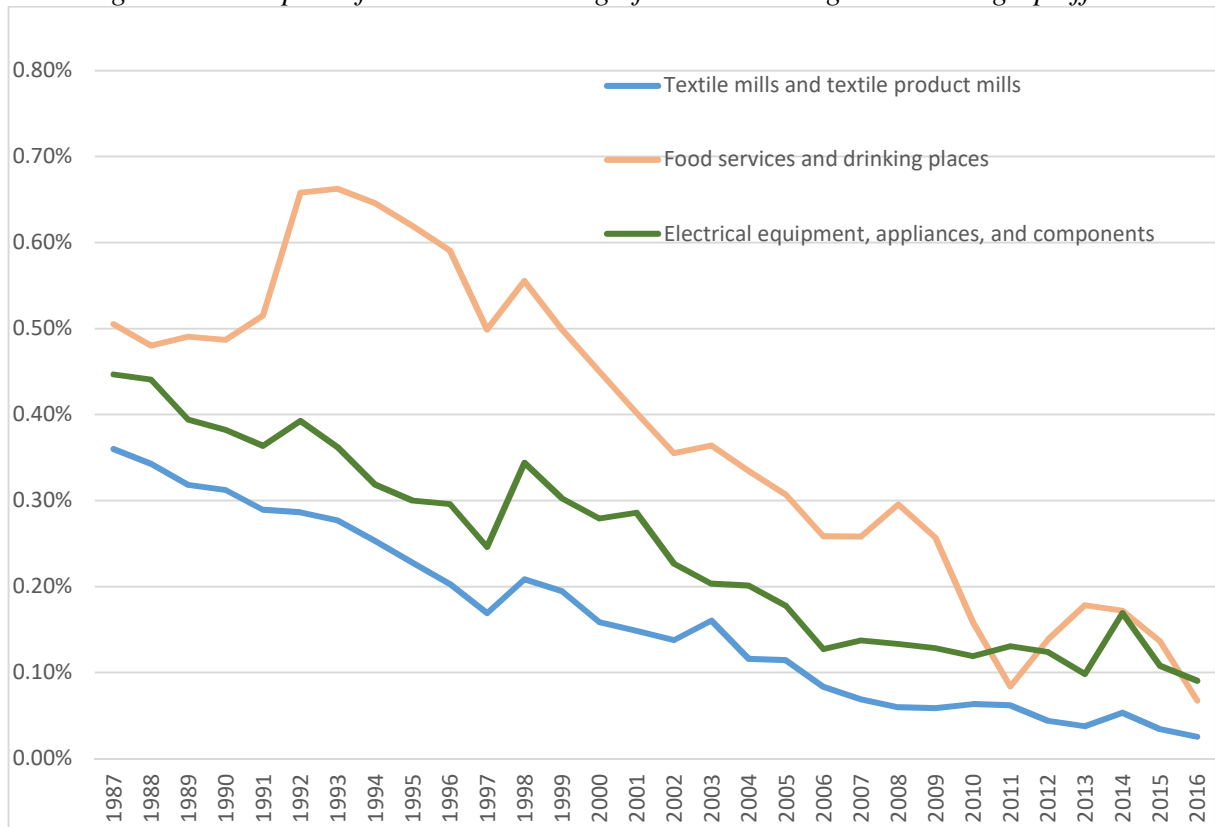


Figure 7. Examples of industries without technological catching-up effects

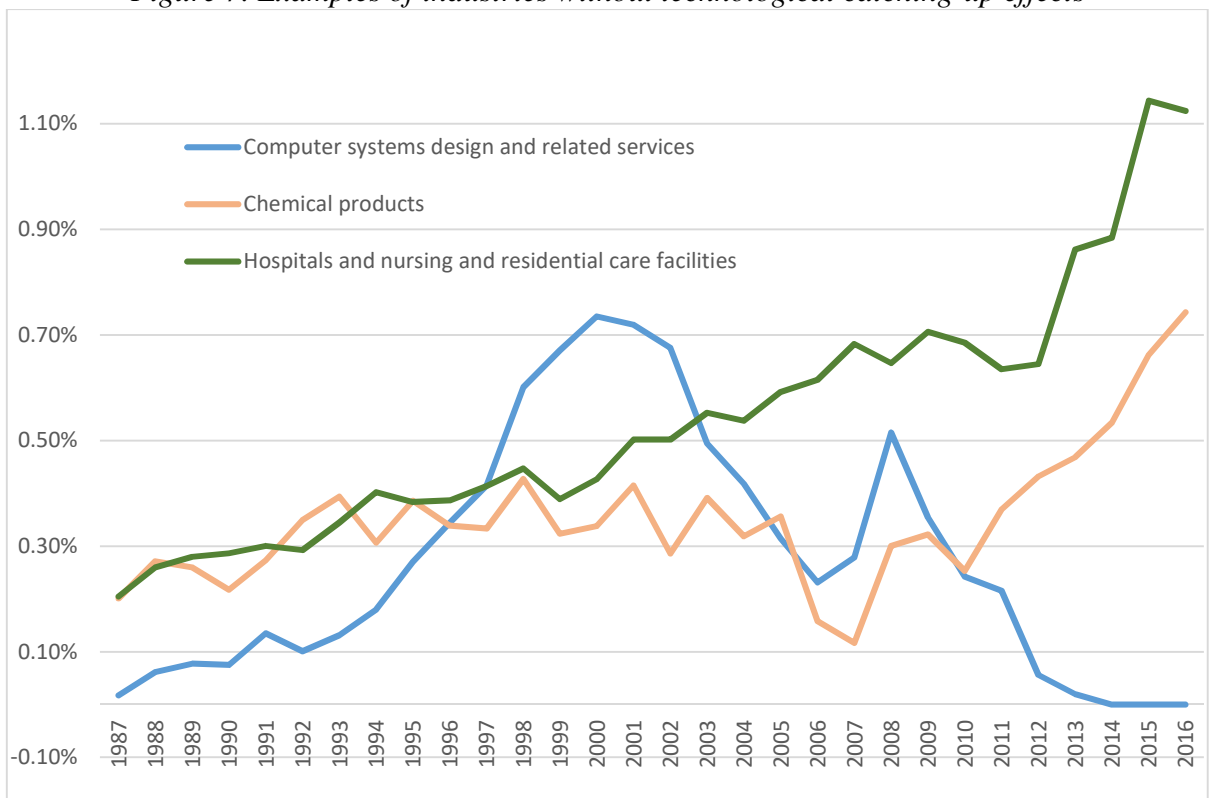


Figure 8. Examples of industries with homogenization of input-output mixes

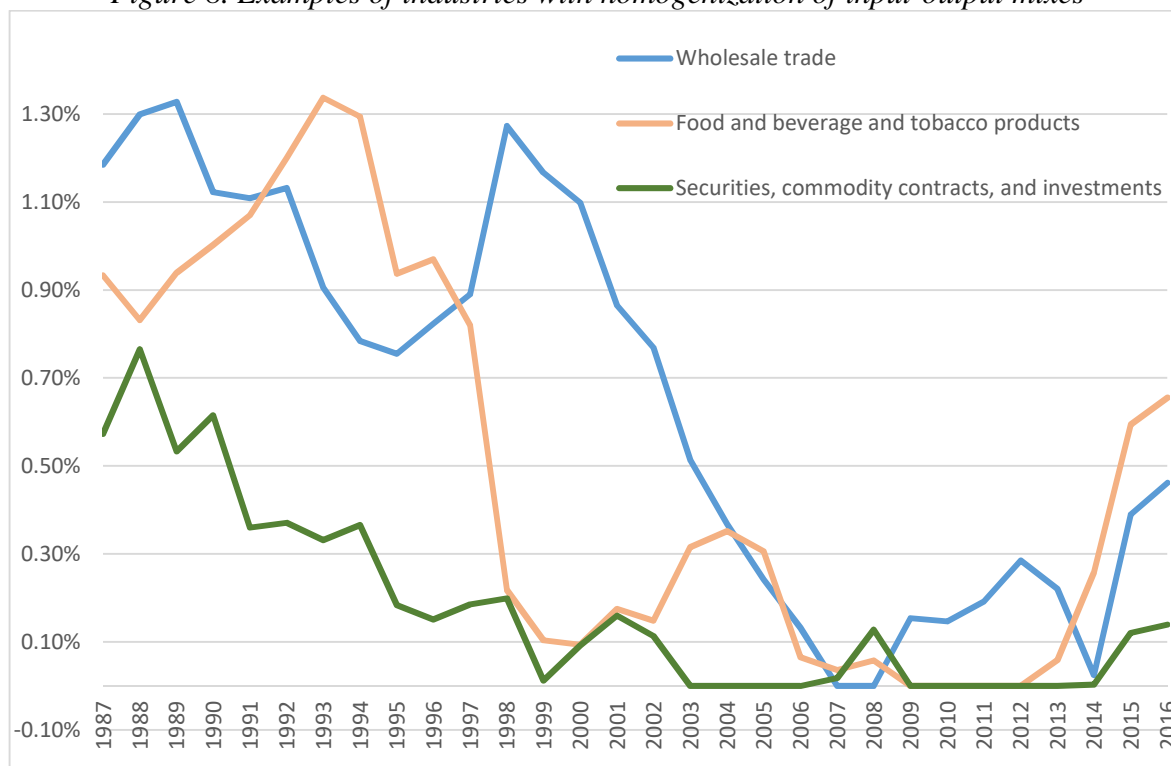
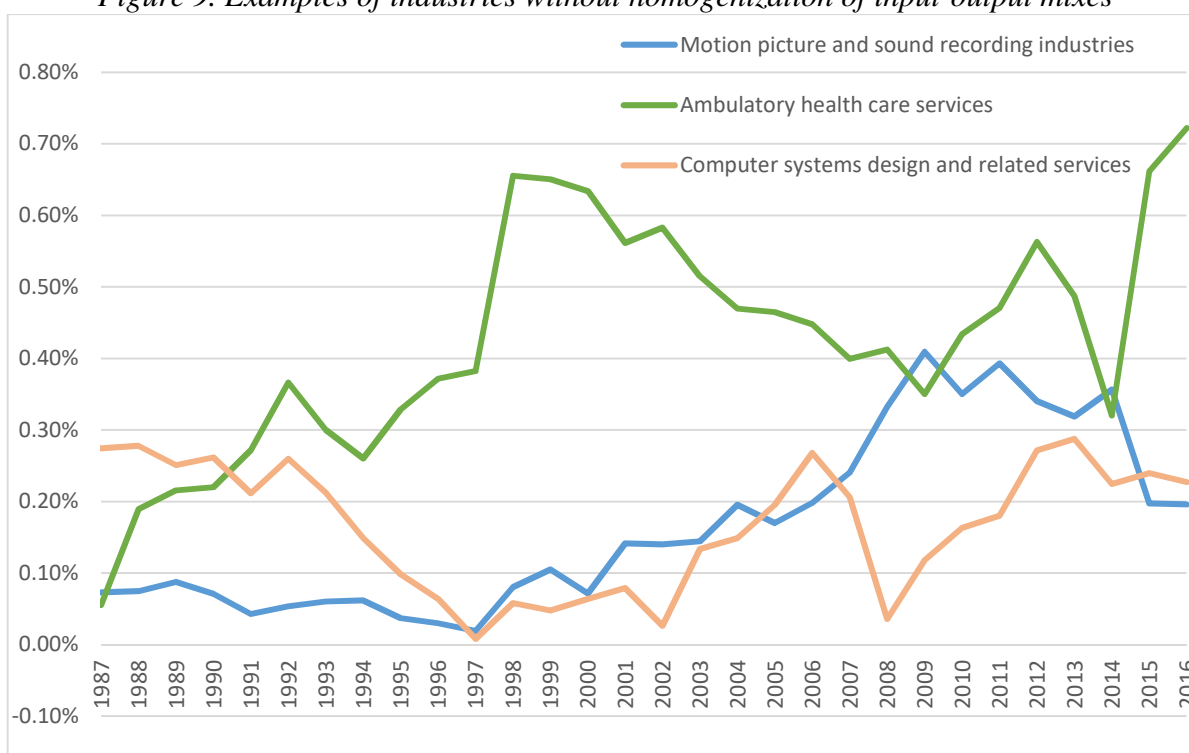


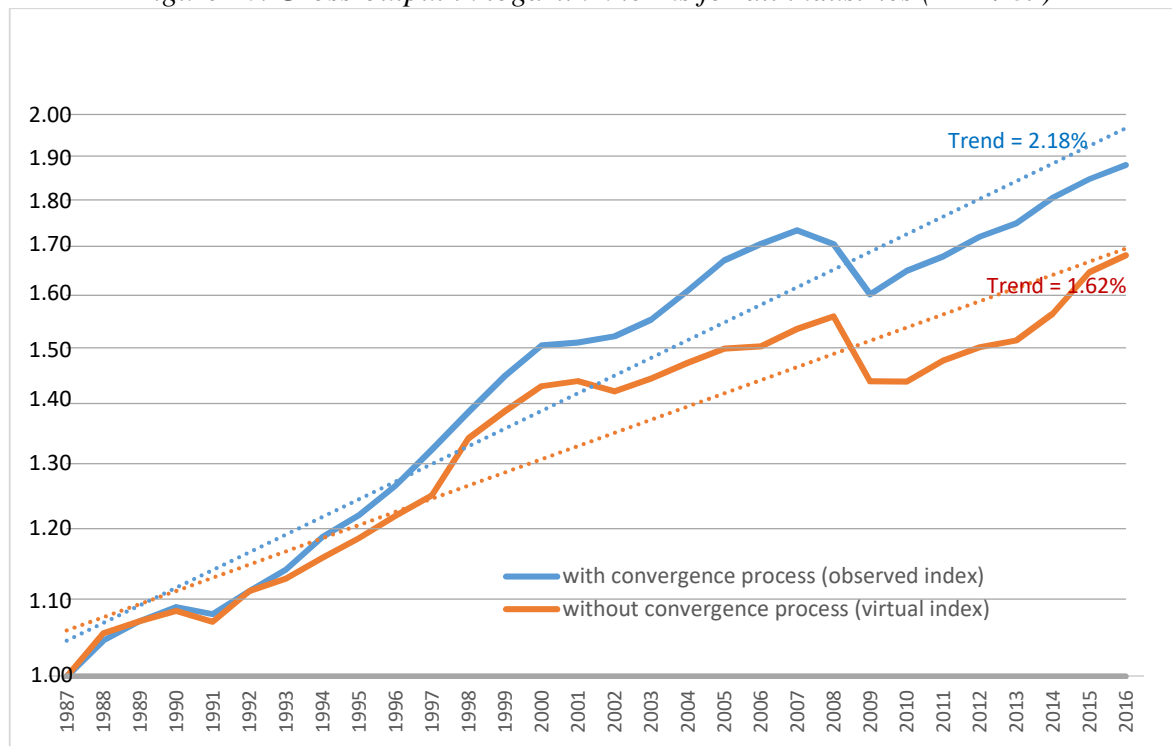
Figure 9. Examples of industries without homogenization of input-output mixes



Furthermore, the impacts of overall convergence process (technical + structural effects) on the US output growth can be estimated. By correcting all observed output levels with their respective

annual inefficiency scores, one can compute the virtual output growth along the production frontier. Considering that technical and structural inefficiency reductions over time denote additional productivity gains for the US economy, the difference of the observed and the previous virtual growth rates gives an estimation of the impact of overall convergence process on the US growth which is around 0.56 percentage-points (2.18% - 1.62%) implying that convergence accounts for 26% of measured growth (i.e., 0.56/2.18). These virtual and observed output changes are displayed in Figure 10.

Figure 10. Gross-output in logarithm terms for all industries ($I = 1987$)



3.3. The linkage between stakeholder's price advantages and convergence processes

Technological catching-up and structural convergences imply productivity gains at the industry and macro levels. These gains should be transferred to the different stakeholders (namely purchasers of goods and services, intermediate input suppliers, employees, fixed capital suppliers and profitability). These productivity transfers may be related or not to the input/output price evolutions reflecting financial advantages or disadvantages that the stakeholders have benefitted or suffered over time. In this perspective, we investigate the long-run relationship between real output/input price evolutions and technical/structural efficiency scores at the industry level for the panel of 63 US industries over the period 1987-2016.

a) Definition of price advantages

Considering that a value change of any input n between two periods t and $t+1$ can be split into Bennet price and quantity effects as:

$$\Delta(w^n x^n) = \left[\frac{1}{2} (x_t^n + x_{t+1}^n) \Delta w^n \right] + \left[\frac{1}{2} (w_{t+1}^n + w_t^n) \Delta x^n \right] \quad (10)$$

with x_t^n = quantity of input n at period t and w_t^n = price of input n at period t .

On the right hand side, the first bracket measures a price effect called the price advantage of the stakeholder with input n . The price advantage or remuneration change over the two periods for any stakeholder is equal to the difference between the quantity weighted changes in its input price. If $\Delta w^n > 0$ (price increasing between the two periods), the price advantage gives a positive remuneration change to the stakeholder. The second bracket measures a quantity effect of the related input. As a result, the combination of the price and quantity effects allows the retrieval of the change in value over the two periods.

In a similar way, the change in value of any output m between periods t and $t+1$ is decomposed into Bennet price and a quantity effects as:

$$\Delta(p^m y^m) = \left[\frac{1}{2} (y_t^m + y_{t+1}^m) \Delta p^m \right] + \left[\frac{1}{2} (p_{t+1}^m + p_t^m) \Delta y^m \right] \quad (11)$$

with y_t^m = quantity of output m at period t and p_t^m = price of output m at period t .

From equation (11), the price advantage related to the stakeholder or purchaser m is defined by

$-\left[\frac{1}{2} (y_t^m + y_{t+1}^m) \Delta p^m \right]$ where the negative sign indicates a positive price advantage in case of a selling price decrease over the two periods.

b) Price advantages and TFP growth

Adopting a productivity accounting approach outlined by Grifell-Tatjé and Lovell (2015), we estimate productivity gains and their distribution among inputs and output for the US industries. In this perspective the traditional surplus accounting approach, initially developed by CERC¹ (1980), is performed in order to compute the respective stakeholders' price advantages related to the corresponding input and output price changes. These price advantages allow us to determine which stakeholders benefit or do not benefit from productivity gains over time.

¹ Centre d'Etudes des Revenus et des Coûts

According to the surplus approach, the TFP growth rate is just equal to the aggregation of price advantage ratios defined as the percentages between price advantages and the total output value. These results are derived in Appendix 2.

$$\frac{DTFP}{TFP} = \frac{-\sum_{m=1}^M \frac{\dot{a}^m}{a^m} Dp^m y^m + \sum_{n=1}^{N+1} \frac{\dot{a}^n}{a^n} Dw^n x^n}{\sum_{m=1}^M \frac{\dot{a}^m}{a^m} p^m y^m} \quad (12)$$

c) The model linking price advantages and technical and structural inefficiency scores

For each industry and each year, available data enable the establishment of the following balanced production account:

Gross output value = Intermediate inputs + Compensation of employees + Depreciation of capital + Net operating surplus

In this accounting identity, the net operating surplus can be equated to a cost including dividends, interest costs or managers' remunerations before tax that remunerates a virtual input x^{N+1} defined as the managerial input. This specific cost gauges the capacity of an industry to achieve a financial surplus after covering the costs of intermediate consumptions and primary inputs (labor and fixed capital). Therefore, one can associate 5 different stakeholders: purchasers, suppliers of intermediate inputs, employees, suppliers of fixed capital and managers who are remunerated through the net operating surplus.

In this context, our model combines six equations. The first five equations are related to output/input price advantages. The sixth one refers to the previous equation (12) by linking the distribution of TFP gains between the five stakeholders. Consequently, the model can be described through the following simultaneous equations:

- one equation related to the gross output:

$$\left(\frac{\Delta py}{py} \right)_a^{t+1/t} = \alpha \Delta I_a^{TECH,t+1/t} + \beta \Delta I_a^{STRUC,t+1/t} + d_a + f^t + \varepsilon_a^t \quad (13)$$

- four equations related to the inputs including the profitability:

$$\left(\frac{\Delta w^n x^n}{py} \right)_a^{t+1/t} = \alpha^n \Delta I_a^{TECH,t+1/t} + \beta^n \Delta I_a^{STRUC,t+1/t} + d_a^n + f^{n,t} + \varepsilon_a^{n,t} \quad (14)$$

- one equation linking the TFP growth rate to the different price advantages

$$\frac{\frac{\Delta DTFP}{TFP} \frac{\dot{\phi}^{t+1/t}}{\phi_a^{t+1/t}}}{\frac{\dot{\phi}^t}{\phi_a^t}} = \frac{\frac{\Delta py}{py} \frac{\dot{\phi}^{t+1/t}}{\phi_a^{t+1/t}}}{\frac{\dot{\phi}^t}{\phi_a^t}} + \sum_{n=1}^4 \frac{\frac{\Delta Dw^n x^n}{py} \frac{\dot{\phi}^{t+1/t}}{\phi_a^{t+1/t}}}{\frac{\dot{\phi}^t}{\phi_a^t}} \quad (15)$$

with $a = 1, 2, \dots, 63$ industries, $t = 1987, 1988, \dots, 2016$, $n = 1, 2, 3, 4$ inputs.

Technological catching-up and structural convergence processes generate productivity gains. As a result, if the coefficients $\alpha, \alpha', \beta, \beta'$ are negative, the productivity gains (derived from inefficiency decreases) exert positive effects on the stakeholders' price advantages.

For estimation purpose, we face a system of five equations linked together through the last TFP identity. As a result, cross-equations constraints must be imposed. Indeed from equations 13,14 and

15, error terms should respect: $\varepsilon_a^t + \sum_{n=1}^4 \varepsilon_a^{n,t} = 0$. This justifies estimating them simultaneously

through the seemingly unrelated regression (SUR) procedure, proposed by Zellner (1962). SUR procedure consists of two steps. First, we estimate the set of equations with the constraint imposed on error terms, using a diagonal covariance matrix of the disturbances across equations. Second, the SUR estimator uses the results of step 1 to form a consistent estimate of the covariance matrix of the disturbances. Then the model is re-estimated by using the later as a weighting matrix. Zellner (1962) showed that these estimates are consistent and asymptotically normal. Parameter estimates and asymptotic standard errors can be used for the usual inferences. The SUR procedure is considered asymptotically more efficient than the standard OLS estimates of single equation regressions. It is adapted to the estimation of our 6-equations model. The econometric results are presented in Table 1.

Table 1. SUR procedure results

Equation	Stakeholders	Coef.	t-stat	R²	DW
<i>Output</i>					
	<i>Purchasers</i>				
Tech. catching-up		-19.22	-12.27	0.21	1.74
Convergence process		-4.44	-3.81		
<i>Intermediate inputs</i>					
	<i>Suppliers</i>				
Tech. catching-up		3.10	3.61	0.25	1.88
Convergence process		2.24	3.51		
<i>Labor</i>					
	<i>Employees</i>				
Tech. catching-up		-0.23	-0.58	0.18	2.18
Convergence process		-0.20	-0.66		
<i>Fixed Capital</i>					
	<i>Capital providers</i>				
Tech. catching-up		-0.14	-1.05	0.14	1.00
Convergence process		0.08	0.82		
<i>Profitability</i>					
	<i>Managers</i>				
Tech. catching-up		-10.66	-8.44	0.82	2.17
Convergence process		-3.175	-3.37		

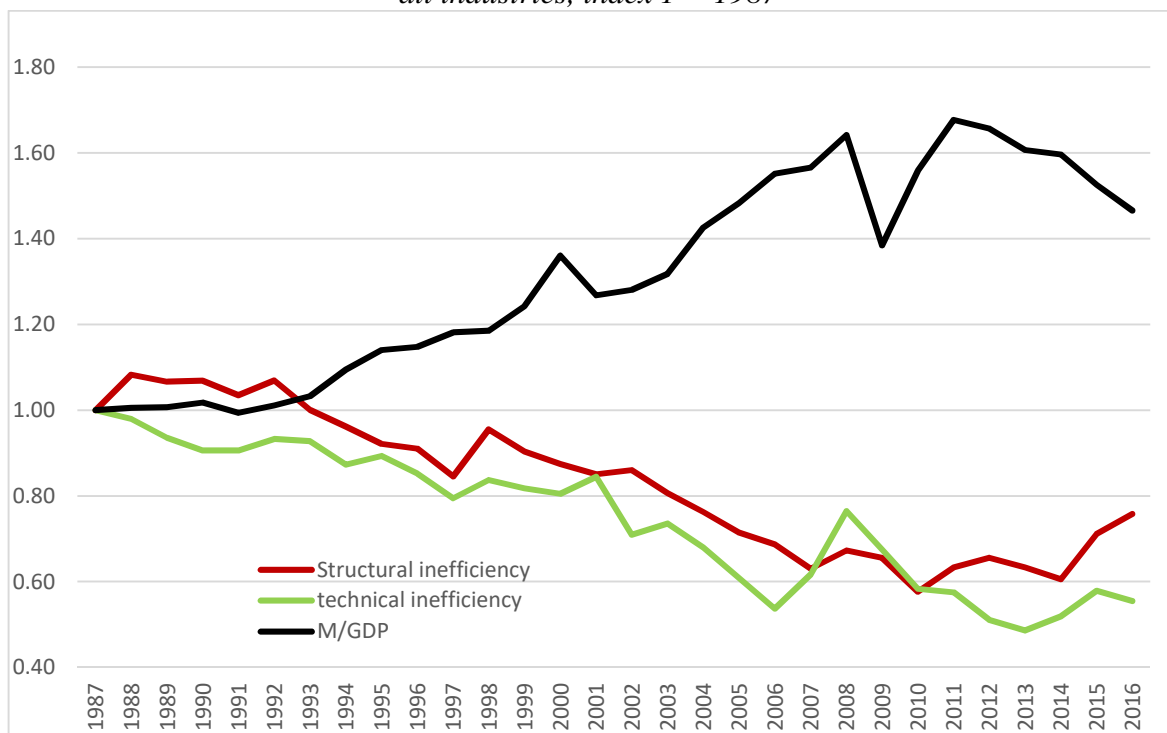
The results show that both technical and structural efficiency scores have positive effects on purchasers' price advantages and profitability; the coefficients have negative effects on supplier's

price advantages. Labor and fixed capital stakeholders seem not to be dependent on structural and technical efficiency scores. We also notice that compared to the structural component, technical inefficiency scores have higher impact on buyers' price advantages and profitability.

4. Discussion

While the two types of efficiencies are different by nature, our results clearly show that convergence is observed for both technical and structural efficiencies. This reveals that technology transfer and reallocation process among sectors generate significant productivity gains at the country level. The import of goods and services due to the liberalization of international trade over the last decades is surely one of the main drivers of technological catching-up and price harmonization. Figure 11 shows a strong negative correlation between the evolution of the two inefficiencies and the ratio of import of goods and services to GDP at the US macro-economy level ($R=-0.926$ for technical inefficiency and $R=-0.959$ for structural inefficiency).

Figure 11. Technical and structural inefficiencies versus ratio of imports over GDP all industries, index 1 = 1987



One can observe that from 1987 to 2011, the higher the import competition, measured as M/GDP, the greater the incentives for inefficient industries to catch-up the production frontier by eliminating technical inefficiencies and improving their productivity levels.

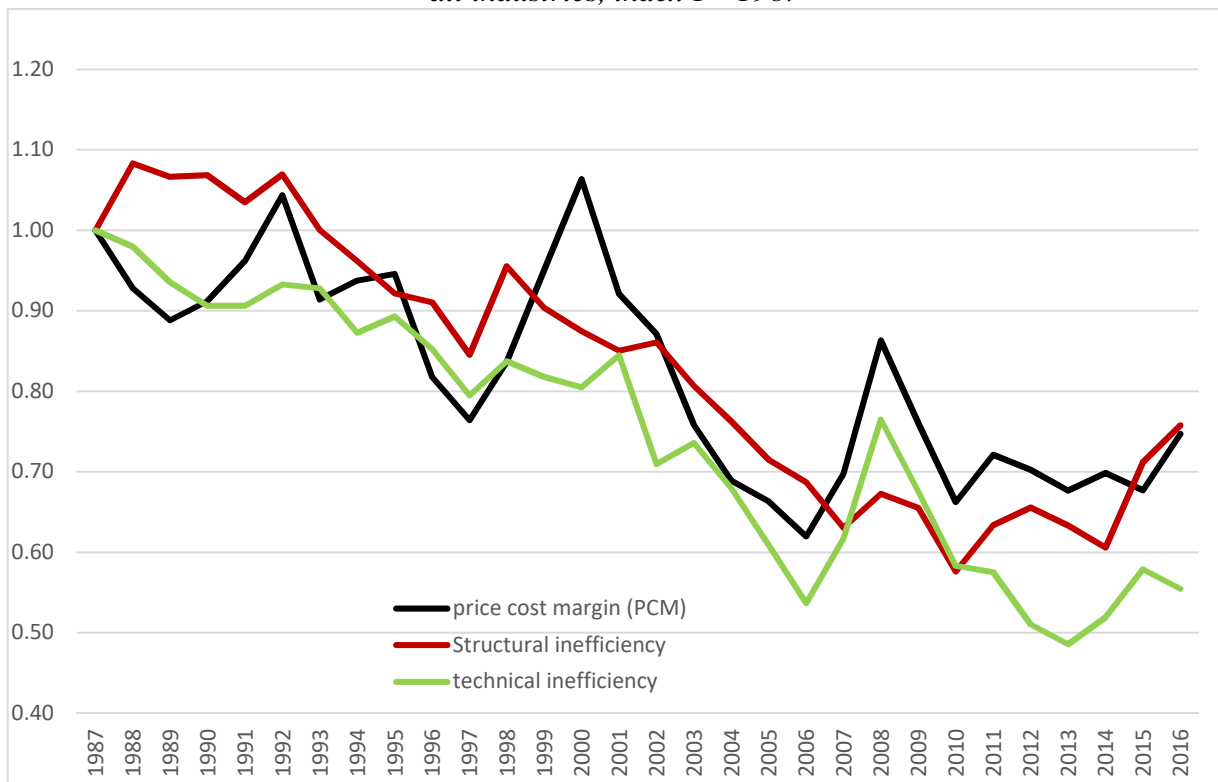
At the same time, competitive pressure results in price and cost harmonization through profitability rate convergence among industries. To analyze this phenomenon, a price cost margin indicator is

$$\text{first computed as } PCM_a^t = \left(\frac{py - \sum_{n=1}^N w^n x^n}{py} \right)_a^t$$

with $a = 1, 2, \dots, 63$ industries, $t = 1987, 1988, \dots, 2016$, $n = 1, 2, 3, 4$ inputs.

The yearly coefficients of variation between industries are calculated and plotted against the two inefficiency components (series are computed as time change indexes, base 1 = 1987). The strong correlation between PCM and structural inefficiency ($R=0.81$) demonstrates the impact of import competition on price harmonization leading to technological and structural adjustments via marginal input productivity convergence.

Figure 12. Technical and structural inefficiencies versus Price cost Margin (PCM) all industries, index 1 = 1987



After 2011, the import pressure on the US economy reduced leading to a reversal of technical efficiency improvements (cf. Figure 11). International trade data compiled by the World Bank show

that the annual rate of growth in imports by USA from 1987 (\$508.714 billion) to 2011 (\$2.686 trillion) was 17.5 percent compared to 0.31 percent between 2011 and 2016 (\$2.736 trillion).²

Simultaneously, the price cost margin convergence has stopped initiating an interruption of homogenization of production processes among industries revealed by an increasing structural inefficiency over the five last years (cf. Figure 12).

Finally our econometric results, displayed in Table 1, show that, over the last 30 years, industries with significant inefficiency decreases have profited purchasers and firms through lower output prices or higher profitability rates. On the contrary, relative compensations of labor and fixed capital do not seem to be impacted by inefficiency decreases over time. Indeed, price changes for labor and fixed capital are mainly determined by their specific national market structure and their own availability in accordance with the macroeconomic business cycle.

5. Conclusion

This paper evaluates two types of inefficiencies: technical inefficiency between an industry and its benchmark on the production frontier and structural inefficiency seen as heterogeneity between input-output mixes among sectors. We define these two inefficiencies at individual and group levels. Finally, we link these two inefficiency measures to the stakeholders' price advantages.

This analysis was applied to US industries from 1987-2016. The results clearly show that convergence is observed for both technical and structural efficiencies even if a break in these trends is observed in the last few years. This reveals that, over time, technology transfer and the reallocation processes among sectors generate significant productivity gains at the country level. We estimate the impact of these convergence processes on the US economy at around 0.56 percentage points of additional growth that represents an increase of 26%. The liberalization of international trade and the resulting increase in imports are probably the main sources of technological catching-up and price harmonization among the US industries. Over the past three decades, trade liberalization has resulted in continuous increase in imports and US trade deficits, contributing to decreases in technical and structural inefficiencies. Trade liberalization or economic globalization predicts these findings.

The panel data analysis performed for 63 US industries over the study period demonstrates positive influence of the two convergence processes on final demand prices and profitability and negative influence on suppliers' prices. The purchasers and managers get significant advantage from efficiency gains which occur in their specific industries; this is not the case for the suppliers.

² Source(<https://data.worldbank.org/indicator/NE.IMP.GNFS.CD?locations=US>).

Finally, no link can be established between technological catching-up process and input-output mixes homogenization and employees or capital providers' price advantages. For these last three stakeholders, we conclude that their remuneration changes do not benefit from their respective sectoral efficiency gains. Finally, we argue that the competitive pressure on the US industries from international trade affects positively consumers' purchasing power and profitability of corporate businesses while suppliers and employees are the forgotten ones in the distribution of productivity gains.

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Appendix 1

Linear program for technical efficiency estimation

$$\begin{aligned}
 I_a^{TECH,t} &= \bar{D}_{T_{VRS}^t}(\mathbf{x}_a^t, \mathbf{y}_a^t; \mathbf{g}_x^t, \mathbf{g}_y^t), \forall a \in \{1, 2, \dots, K\} \\
 \bar{D}_{T_{VRS}^t}(\mathbf{x}_a^t, \mathbf{y}_a^t; \mathbf{g}_x^t, \mathbf{g}_y^t) &= \max_{\mu, \lambda_a^t} \lambda_a^t \\
 s.t. \quad \sum_{k=1}^K \mu_k y_k^{m,t} &\geq y_a^{m,t} + \lambda_a^t \mathbf{g}_y^t \quad \forall m = 1, \dots, M \\
 \sum_{k=1}^K \mu_k x_k^{n,t} &\leq x_a^{n,t} + \lambda_a^t \mathbf{g}_x^t \quad \forall n = 1, \dots, N \\
 \sum_{k=1}^K \mu_k &= 1 \\
 \lambda_a^t &\geq 0 \\
 \mu_k &\geq 0 \quad \forall k = 1, \dots, K
 \end{aligned} \tag{LP1}$$

Linear program for overall efficiency estimation

$$\begin{aligned}
 \bar{D}_{T_{VRS}^{AGREG,t}}\left(\sum_{k=1}^K \mathbf{x}_k^t, \sum_{k=1}^K \mathbf{y}_k^t; \mathbf{g}_x^t, \mathbf{g}_y^t\right) &= \max_{\mu, \lambda_{AGREG}^t} \lambda_{AGREG}^t \\
 s.t. \quad K \sum_{k=1}^K \mu_k y_k^{m,t} &\geq \sum_{k=1}^K y_k^{m,t} + \lambda_{AGREG}^t \mathbf{g}_y^t \quad \forall m = 1, \dots, M \\
 K \sum_{k=1}^K \mu_k x_k^{n,t} &\leq \sum_{k=1}^K x_k^{n,t} + \lambda_{AGREG}^t \mathbf{g}_x^t \quad \forall n = 1, \dots, N \\
 K \sum_{k=1}^K \mu_k &= K \Leftrightarrow \sum_{k=1}^K \mu_k = 1 \\
 \lambda_{AGREG}^t &\geq 0 \\
 \mu_k &\geq 0 \quad \forall k = 1, \dots, K
 \end{aligned} \tag{LP2}$$

Appendix 2

We establish that the TFP growth rate is equivalent to the aggregation of price advantage ratios defined as the percentages between price advantages and the total output value (equation A6).

Starting from the definition of the profit which is a residual term between total output value and total cost:

$$P = \sum_{m=1}^M \overset{\circ}{a} p^m y^m - \sum_{n=1}^N \overset{\circ}{a} w^n x^n \quad (A1)$$

The following accounting identity holds for any particular industry in any market condition:

$$\sum_{m=1}^M \overset{\circ}{a} p^m y^m = \sum_{n=1}^N \overset{\circ}{a} w^n x^n + P \quad (A2)$$

In this accounting identity, the profit can be considered as a cost that remunerates a virtual input x^{N+1} defined as the managerial input: $P = w^{N+1} x^{N+1}$. By replacing the profit value by this specific managerial cost, (A2) is rewritten as:

$$\sum_{m=1}^M \overset{\circ}{a} p^m y^m = \sum_{n=1}^{N+1} \overset{\circ}{a} w^n x^n \quad (A3)$$

Given the previous equation, changes in output and input values between periods t and $t+1$ can be measured in terms of changes in quantity and price components. Denoting that $p_{t+1}^m = (p_t^m + Dp^m)$, $y_{t+1}^m = (y_t^m + Dy^m)$, $w_{t+1}^n = (w_t^n + Dw^n)$ and $x_{t+1}^n = (x_t^n + Dx^n)$, the time variation of equation (A3) leads to equation (A4), considering a Bennet approach resulting from the average of a Laspeyres and a Paasche approaches.

$$\underbrace{\sum_{m=1}^M \frac{(p_t^m + p_{t+1}^m)}{2} Dy^m - \sum_{n=1}^{N+1} \frac{(w_t^n + w_{t+1}^n)}{2} Dx^n}_{PS} = - \underbrace{\sum_{m=1}^M Dp^m \frac{(y_t^m + y_{t+1}^m)}{2} + \sum_{n=1}^{N+1} Dw^n \frac{(x_t^n + x_{t+1}^n)}{2}}_{PA} \quad (A4)$$

In equation (A4), the left hand side characterizes a productivity surplus (PS) defined as the difference between price weighted changes in output and input quantities while the right hand side aggregates the different stakeholders' price advantages (PA). Such price variations result in reallocations among stakeholders that are constrained by the productivity surplus level. More precisely, equation (A4) ensures that the total amount of remuneration changes shared among the different agents (PA) including the managerial input cannot surpass the total productivity growth (PS).

Through equation (A4), *PS* estimates productivity gains expressed in level terms (i.e. in dollars) which can also be directly linked to the usual measure of TFP changes defined in terms of relative

growth rates (%). As a result TFP changes can be computed as the weighted output variations not explained by weighted input changes:

$$\frac{DTFP}{TFP} = \overset{M}{\underset{m=1}{\overset{\circ}{\mathbf{a}}}} a_m \frac{Dy^m}{y^m} - \overset{N+1}{\underset{n=1}{\overset{\circ}{\mathbf{a}}}} b_n \frac{Dx^n}{x^n} \quad (\text{A5})$$

where α represents the vector of M output shares in total revenue and β the vector of N input shares in total cost.

Substituting $a_m \frac{Dy^m}{y^m}$ by $\frac{p^m D y^m}{\overset{M}{\underset{m=1}{\overset{\circ}{\mathbf{a}}}} p^m y^m}$ and $b_n \frac{Dx^n}{x^n}$ by $\frac{w^n D x^n}{\overset{N+1}{\underset{n=1}{\overset{\circ}{\mathbf{a}}}} w^n x^n}$ and seeing that the total revenue

equals the total cost ($\overset{M}{\underset{m=1}{\overset{\circ}{\mathbf{a}}}} p^m y^m = \overset{N+1}{\underset{n=1}{\overset{\circ}{\mathbf{a}}}} w^n x^n$), TFP growth rate can be measured as:

$$\frac{DTFP}{TFP} = \frac{\overset{M}{\underset{m=1}{\overset{\circ}{\mathbf{a}}}} p^m D y^m - \overset{N+1}{\underset{n=1}{\overset{\circ}{\mathbf{a}}}} w^n D x^n}{\overset{M}{\underset{m=1}{\overset{\circ}{\mathbf{a}}}} p^m y^m} \quad (\text{A6})$$

As a result, the TFP growth rate is just equal to the productivity surplus rate defined by the ratio between PS and the total output value. Additionally, an interesting link between TFP growth rate and price advantage changes can be proven. From the equality $PS = PA$, TFP growth rate is equivalent to the aggregation of price advantage ratios defined as the percentages between price advantages and the total output value):

$$\frac{DTFP}{TFP} = \frac{PS}{\overset{M}{\underset{m=1}{\overset{\circ}{\mathbf{a}}}} p^m y^m} = \frac{PA}{\overset{M}{\underset{m=1}{\overset{\circ}{\mathbf{a}}}} p^m y^m} = \frac{- \overset{M}{\underset{m=1}{\overset{\circ}{\mathbf{a}}}} D p^m y^m + \overset{N+1}{\underset{n=1}{\overset{\circ}{\mathbf{a}}}} D w^n x^n}{\overset{M}{\underset{m=1}{\overset{\circ}{\mathbf{a}}}} p^m y^m} \quad (\text{A7})$$