## Simulating SAR images clutter with a sum of scatterers

Flora Weissgerber, Nicolas Trouvé
ONERA
Département Electromagnétisme et Radar Unité Simulation, Environment et Modélisation

26/09/2019


## Plan

1. Introduction

- What do we want to simulate
- How do we want to simulate it
- How do we validate the simulation


## 2. Diversity from the modulus distribution

3. Diversity from the location
4. Diversity from altitude
5. Conclusion

## Example 1



Salon de Provence, SETHI, ONERA

- Fields with oriented structure
- Small woods and trees


## Example 2



Salon de Provence, SETHI, ONERA

- Orchards
- Grass
- Altitude effects


## Parameters we want to retrieve

- Mean Power or mean $\sigma_{0}$


Toulon, TerraSAR-X, LAN3628

- Distribution of power and phase


- Texture or pattern $\rightarrow$ pixel combined 2D-distribution


## Coherent sum of scatterers

$N_{s}$ scatterers defined by :

- A position ( $x, y, z$ )
- A complexe amplitude $r e^{i \varphi}$
- A phase due to their distance to the sensor

A sensor :

- Height $H=514 \mathrm{~km}$
- Incidence angle $\theta=34$

■ $f=9.65 \mathrm{GHz}, \lambda=0.031 \mathrm{~m}$

- Orientation angle $\theta_{s}$


$$
\mathrm{P}=\rho e^{i \phi}=\frac{1}{\sqrt{N_{s}}} \sum_{n=1}^{N_{s}} r_{n} e^{i \varphi_{n}}
$$

## Impact of the orientation $\theta_{s}$

## 90 degrees



## 126 degrees



## 180 degrees



Changing the orientation angle changes the wavefront direction

## Fully developped Speckle

A sum of a large amount of scatterers in a resolution cell which:
■ amplitude $r$ and phase $\varphi$ are statistically independent
■ phase $\varphi$ is equi-distributed on an $2 \pi$ interval.
result is a complex circular Gaussian distribution (Goodman, 1976).

$$
\mathrm{P}=\rho e^{i \phi} \sim \frac{1}{\pi 2 \sigma_{r}^{2}} e^{\frac{\mathrm{pP} *}{2 \sigma_{r}^{2}}}
$$

$\sigma_{r}$ is the second moment of the scatterers modulus distributions.

■ Pixel phase is uniformly distributed on a $2 \pi$-interval
■ Pixel modulus is Rayleigh-distributed

## Quality indicators

In the case of fully developped speckle:

| Indicators | Formula | Value |  |
| :---: | :--- | :--- | :--- |
| $c_{v}$ | $\frac{\sigma}{\mu}$ | $\sqrt{\frac{4-\pi}{\pi}}$ |  |
|  |  | $\approx 0.522723$ |  |
| $\tilde{\kappa}_{2}$ | $\tilde{m}_{2}-\tilde{m}_{1}^{2}$ | $\frac{1}{4} \Psi(1,1)$ | (Nicolas, 2002) |
|  | $\tilde{\kappa}_{3} 0.411234$ |  |  |
| $R$ | $\tilde{m}_{3}-3 \tilde{m}_{1} \tilde{m}_{2}+2 \tilde{m}_{1}^{3}$ | $\frac{1}{8} \psi(2,1)$ | (Nicolas, 2002) |
|  | $E\left[e^{i \phi}\right]$ | 0 |  |
| $R$ |  | (Mardia \& Jupp, 1999) |  |

## Question

How many scatterers do we need to get fully developped speckle depending on :

- Modulus distribution
- Position of the scatterers

Diversity from the modulus distribution

## Plan

1. Introduction

- What do we want to simulate
- How do we want to simulate it
- How do we validate the simulation

2. Diversity from the modulus distribution
3. Diversity from the location
4. Diversity from altitude
5. Conclusion

## Set up of the experiment

- Phase $\phi$ uniformly distributed on a $2 \pi$ interval

■ $x$ and $y$ uniformly distributed in the resolution cell

- $z=0$
- Modulus $\rho$
(1) Constant, with a value of $\sqrt{2} \sigma_{r}$
(2) Gaussian, following $\left|\mathcal{N}\left(0, \sqrt{2} \sigma_{r}\right)\right|$
(3) Uniform, on a $\left[0, \sqrt{6} \sigma_{r}\right]$ interval

■ Increasing the number of scatterers $N_{s}$

## Constant modulus



- Phase is uniformly distributed on the $2 \pi$-interval
- For $N_{s}$ larger than 7, the modulus has a Rayleigh distribution


## Gaussian modulus



- Phase is uniformly distributed on the $2 \pi$-interval
- For $N_{s}$ larger than 4, the modulus has a Rayleigh distribution


## Uniform modulus



- Phase is uniformly distributed on the $2 \pi$-interval
- For $N_{s}$ larger than 3, the modulus has a Rayleigh distribution
$\rightarrow$ When the scatterers phase is uniformly distributed on the $2 \pi$-interval, diversity on the scatterers modulus decrease the number of scatterers needed in a resolution cell

Diversity from the location

## Plan

## 1. Introduction

- What do we want to simulate
- How do we want to simulate it
- How do we validate the simulation

2. Diversity from the modulus distribution
3. Diversity from the location
4. Diversity from altitude
5. Conclusion

## Diversity from the location

## Set up of the experiment

■ Phase $\phi$ of the scatterers is equal to 0

- Modulus $\rho$ of the scatterers is constant
- $z=0$
- $x$ and $y$ :
(1) Uniformly distributed in the resolution cell
(2) Distributed on a sparse grid
- Increasing the number of scatterers $N_{s}$
- Changing the orientation $\theta_{s}$ of the wavefront toward the resolution cell


## Uniformly distributed $x$ and $y$




■ Result similar to the phase randomly distributed on a $2 \pi$-interval.
■ There is $N_{\varphi}=36$ phase cycles projected onto the resolution cell
$\rightarrow$ The scatterers sample uniformly the wavefront

Diversity from the location

## Target distributed on a grid

- The grid is sampling the wavefront
- The orientation angle $\theta_{s}$ influences the sampling

Full grid: the number of nodes is $N_{s}$

- Their is no randomness

Sparse grid: $N_{s}=9,90$ nodes on the grid


$$
\theta_{s}=270
$$

All the scatterers have the same phase

$$
\theta_{s}=126
$$




Diversity from altitude Plan

1. Introduction

- What do we vant to simulate
- How do we want to simulate it
- How do we validate the simulation

2. Diversity from the modulus distribution
3. Diversity from the location
4. Diversity from altitude
5. Conclusion

## Set up of the experiment

- Phase $\phi$ of the scatterers is equal to 0
- Modulus $\rho$ of the scatterers is constant
- $x$ and $y$ are distributed on a regular full grid
- z Uniformly distributed between $\left[-z_{\max }, z_{\max }\right]$
- Increasing the number of scatterers $N_{s}$
- Changing the orientation $\theta_{s}$ of the wavefront toward the resolution cell


## Random scatterers height

At constant location, $z \in[-2,2] c m$ to get $\varphi \in[-\pi, \pi]$.
On a regular grid, the position $(x, y)$ also sample the wavefront. $z \in[-1,1] \mathrm{cm}$ can result in a fully developped speckle for some grid orientation.




$$
\theta_{s}=126, N_{s}=9
$$

## Seamless heightmap

Set up :

- Modulus $\rho$ given by a $\sigma_{0}$ curve (given the local incidence)
- Height $z$ given by a seamless heightmap
- $x$ and $y$ resolution is 25 cm
- Scatterers summed in the SAR grid

$$
\sigma_{0} \text { curve "forage crop" }
$$


$\theta$

Diversity from altitude

## The seamless heightmap



Normal


Diversity from altitude The seamless heightmap

Amplitude


Number of scatterers


Diversity from altitude

## The seamless heightmap

Amplitude histogram


Phase histogram


## Plan

1. Introduction- What do we want to simulate- How do we want to simulate it- How do we validate the simulation
2. Diversity from the modulus distribution
3. Diversity from the location
4. Diversity from altitude
5. Conclusion

## Diversity of the phase is key

- For a phase uniformly distributed on a $2 \pi$-interval, very few scatterers can lead to a fully developped speckle
- Increasing the variability of the modulus decreases the number of scatterers needed
- The uniform distribution of the phase can arise :
- From a random $(x, y)$ position
- From a random altitude $z$
© If the phase of the scatterers is only due to their distance to the sensor, a ( $x, y$ ) position given by a grid can lead to a unique value for the phase
© Introducing an angle $\theta_{s}$ between the sensor trajectory and the vector of the grid can help to retrieve phase diversity
- A textured heightmap is a mean to lead to a textured ground clutter


## Bibliography

Cochin, C., Le Hellard, D., Aubineau, F., \& Gosselin, P. (2008). \{MOCEM:\} An 'all in one tool' to simulate \{SAR\} image. In 7th european conference on synthetic aperture radar.
Farina, a., Russo, a., \& Studer, F. (1986). Coherent radar detection in log-normal clutter. IEE Proceedings F Communications, Radar and Signal Processing, $133(1)$, 39. doi: 10.1049/ip-f-1.1986.0009
Goodman, J. W. (1976). Some fundamental properties of speckle. Journal of the Optical Society of America, 66(11), 1145-1150.
Hammer, H., \& Schulz, K. (2009). Coherent simulation of SAR images.
In Image and signal processing for remote sensing xv. Retrieved from http://proceedings.spiedigitallibrary.org/ proceeding.aspx?doi=10.1117/12.830380 doi: 10.1117/12.830380

Mardia, K. V., \& Jupp, P. E. (1999). Directional Statistics. Wiley.

## Bibliography II

Nicolas, J.-M. (2002). Introduction aux statistiques de deuxième espèce: applications des logs-moments et des logs- cumulants à l'analyse des lois d'images radar. Traitement du Signal, 19(3), 139-167.
Oliver, C. J. (1987). On the simulation of coherent clutter textures with arbitrary spectra. Inverse Problems, 3(3), 463-475. doi: 10.1088/0266-5611/3/3/014

