# Supporting Information for the article: Dynamics of life expectancy and lifespan equality 

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## Section A. Time series Analysis

A. 1 Finding the order of integration. Formally, a stochastic process is stationary if, for every collection of time indices $1 \leq t_{1} \leq \cdots \leq t_{m}$, the joint distribution ( $x_{t_{1}}, x_{t_{2}}, \ldots, x_{t_{m}}$ ) is the same as the joint distribution of $\left(x_{t_{1+h}}, x_{t_{2+h}}, \ldots, x_{t_{m+h}}\right)$ (1). In loose terms, the properties of the underlying process do not change over time. In contrast, a stochastic process is integrated (i.e. nonstationary) of order $d$, denoted $I(d)$, if it needs to be differenced $d$ times to achieve stationarity (2). When $d=1$, the process has a unit root. If two variables (e.g. life expectancy and lifespan equality) are integrated and possess a unit root, then shocks are accumulated over time with growing variance, which makes statistical inference problematic (3, 4).

There are multiple tests to investigate the existence of a unit root for a stochastic process and its order of integration. Here, we used the Kwiatkowski-Phillips-Schmidt-Shin test (KPSS) (5), which tests the null hypothesis of stationarity, and the augmented Dickey-Fuller test (ADF) (6), which tests whether the process exhibits a unit root by using a parametric auto-regressive structure to capture serial correlation. The AIC information criteria is used to determine the number of lags needed. The combination of KPSS and ADF tests is common because the KPSS test reverses the null hypothesis. In addition, to make our results more robust towards structural breaks, we performed the Zivot-Andrews (7) test on the first differences of the series for all indicators used and results yielded similar conclusions.

Stochastic properties of life expectancy and lifespan equality. We analyzed the stochastic properties of $e_{o}$ and $h$ over time to determine whether they are stationary processes. In case of non-stationarity, we also find the order of integration. We performed the Kwiatkowski-Phillips-Schmidt-Shin test (KPSS) (5) for $e_{o}$ and $h$ and the augmented Dickey-Fuller test (ADF) (6) in their levels and first differences, respectively. ${ }^{1}$ Using the $95 \%$ critical values, the null hypothesis of stationarity can be rejected in $94.9 \%$ of the cases for life expectancy, and $93.9 \%$ for lifespan equality $h$ (Fig. A1). ${ }^{2}$ Moreover, at the same level, the null-hypothesis of a unit-root in their first differences is rejected $97 \%$ for $e_{o}$ and almost $97 \%$ for $h$ (see Fig. A1). ${ }^{3}$ These analyses suggest that the variables are non-stationary processes and achieve stationarity after differencing once for both females and males. For the remainder of the study, we treat both variables as integrated of order one, $I(1)$. We additionally did a consistency check in the seven countries in which KPSS and ADF showed a disagreement. These countries were Chile, South Korea, Croatia, Greece, Israel, Luxembourg and Slovenia. The first three were not considered in the cointegration analysis due to small sample size (less than 20 observations) as stated in the main manuscript. For the subsequent ADF test in the first differences, after not considering those countries that showed a disagreement, the hypothesis of non-stationarity is rejected in $100 \%$ of the cases. Therefore, we believe there is enough evidence to treat the time series as $I(1)$.

[^0]Fig. A1. KPSS and ADF tests for $e_{o}, h, g$ and $v$. Calculations based on data from HMD (8)

b) ADF test on first differences

A. 2 Cointegration. The concept of cointegration was developed to avoid misleading interpretations regarding the relationship between two integrated variables (9). It refers to a model that can adjust for stochastic trends to produce stationary residuals to permit detection of stable long-run relationships among integrated variables.

Formally, two cointegrated variables can be expressed using a 2-dimensional vector autoregressive model in its equilibrium correction (VECM) form, defined as

$$
\Delta z_{t}=\Gamma \Delta z_{t-1}+\alpha \beta^{\prime} z_{t-1}+\mu+\varepsilon_{t} .
$$

Operator $\Delta$ denoted the first difference; $z_{t}$ is a $2 \times 1$ vector of stochastic variables ( $e_{o}$ and lifespan equality in our case) at time $t ; \Gamma$ contains the cumulative long-run impacts; $\alpha$ and $\beta$ are two $2 \times 1$ vectors of full rank, $\mu$ is a vector of constants; and $\varepsilon_{t}$ is a vector of normally, independently, and identically distributed errors with zero means and constant variances. We specify the model with an unrestricted constant in the cointegration space and dummy variables in contexts where life expectancy experienced historical shocks, such as World Wars and epidemics (see Table S2). This is known as a case 4 model (3) and deals correctly with the deterministic components of the observed behavior of life expectancy and lifespan equality (10-12). An advantage of this framework is that, under cointegration, parameter estimates are super consistent, which makes them robust to omitted variables and endogeneity related issues (4).

We use the optimal lag length based on the Akaike Information Criterion (AIC). We applied four information criteria to 45 countries' time series (13-16). In all of them, the most selected lags are one and two as optimal. We decided to choose AIC in our time series analysis based on the accuracy of the method (17) to avoid over-parametrization and to have consistency across countries and sexes.

There are two test statistics for cointegration, the lambda-max and the trace statistic tests (18). The first one tests the null hypothesis of $r$ cointegrating vectors against the specific alternative of $r+1$ cointegrating vector(s). The trace statistic tests the null hypothesis of no cointegrating vector against a general alternative of one or more cointegrating vectors $(r>1)$. We performed both tests to each country for females and males separately. In addition, as a robustness check we used the BIC criterion to select the optimal lag length and the results were very similar. Using BIC, results yielded $90 \%$ of evidence of cointegration with the trace statistic test. Using the lambda max test, results showed $92.2 \%$ of cointegration.

Subsequently, after finding $r$ cointegrating relationships, we estimated the VECM representation of each system. That is, we ran an ordinary least squares (OLS) regression on the lagged differenced variables and the error correction term derived from the previous step. All the analyses were carried out using the programming language R (19).

Moreover, $84 \%$ of the cases exhibit a negative error correction coefficient, which is significant in more than $60 \%$ of cases. These findings confirm that a long-run co-movement exists between life expectancy and lifespan equality. It is worth mentioning that we did not find clear patterns or jumps in the speed of adjustment and in the cointegrating vector, other than similar directions for most countries. As a robustness check we implemented the Toda and Yamamoto approach (20). In most populations under study, $79 \%$ at $p<.05$, we were able to detect Granger causality in at least one direction, but without a consistent direction. The results are that in 14 ( $21 \%$ ) and 9 ( $15 \%$ ) countries at $p<.05$ and $p<.1$ respectively, the null-hypothesis of noGranger causality was not rejected in any direction between life expectancy and lifespan equality h. These countries include Greece, Ireland, Luxembourg and the Maori population of New

Zealand, populations that have been highlighted by the Human Mortality Database (HMD) as problematic in terms of data quality causing artificial fluctuations which could affect the results. Ireland, Northern Ireland, and the UK in general might be also affected by pre-census information which has also been highlighted as an issue in the HMD documentation. It should be emphasized that our main interest is on detecting cointegration rather than on interpreting the coefficients or finding causality between life expectancy and lifespan equality.
A. 3 Exceptions for long-run evidence. Some populations did not show evidence of a long-run relationship: females in Lithuania, Belarus and New Zealand, and males in Lithuania, Belarus and Belgium. This could be a result of data quality and short time series. For instance, data quality is an issue in Eastern European countries from, such as Lithuania and Belarus, especially during the Soviet Period (21). New Zealand has been subject to compositional effects due to the changes in proportions of the Maori and European populations (21). Belgium was only considered for the years after 1950 because earlier gaps in the data series (21).

## Section B. Time derivatives of $\boldsymbol{\vartheta}$ and $\boldsymbol{g}$

Let $\vartheta=\ell(x, t)^{2} d x$, where $\ell(x, t)$ is the probability of surviving from birth to age $x$ at time $t .^{4}$ Applying the chain rule, its derivative with respect to time is

$$
\dot{\vartheta}=\int_{0}^{\infty} 2 \ell(x) \dot{\ell}(x) \mathrm{d} x .
$$

Using that $\dot{\ell}(x)=-\ell(x) \int_{0}^{x} \dot{\mu}(a) \mathrm{d} a$, where $\mu(a)$ is the force of mortality at age $a$, and reversing the order of integration, we get

$$
\begin{align*}
& \dot{\vartheta}=-2 \int_{0}^{\infty} \ell(x)^{2} \int_{0}^{x} \dot{\mu}(a) \mathrm{d} a \mathrm{~d} x=-2 \int_{0}^{\infty} \dot{\mu}(a) \int_{a}^{\infty} \ell(x)^{2} \mathrm{~d} x \mathrm{~d} a \\
& =2 \int_{0}^{\infty} \rho(a) \mu(a) \ell(a) e(a) \frac{\int_{a}^{\infty} \ell(x)^{2} \mathrm{~d} x}{\int_{a}^{\infty} \ell(a) \mathrm{d} x} \mathrm{~d} a \\
& =\int_{0}^{\infty} \rho(a) w(a) 2 \ell(a) \bar{\ell}(a) \mathrm{d} a . \tag{B1}
\end{align*}
$$

Function $e(a)$ is the remaining life expectancy at age $a, \rho(a)$ are the rates of mortality improvement, $w(a)=\mu(a) \ell(a) e(a)$, and

$$
\bar{\ell}(x)=\frac{1}{\ell(x)} \int_{x}^{\infty} c(a) \ell(a) \mathrm{d} a
$$

where $c(a)$ denotes the lifetable age composition.
From Eq. $\mathbf{2}$ and $\mathbf{8}$ in the main manuscript, the time derivative of the life expectancy at birth is

$$
\begin{equation*}
\dot{e}_{o}=\int_{0}^{\infty} w(x) \rho(x) \mathrm{d} x \tag{B2}
\end{equation*}
$$

whereas the time derivative of $g=-\ln G$ cab be expressed as

$$
\begin{equation*}
\dot{g}=\frac{\vartheta}{e_{o}-\vartheta}\left(\frac{\dot{\vartheta}}{\vartheta}-\frac{\dot{e}_{o}}{e_{o}}\right) . \tag{B3}
\end{equation*}
$$

Replacing Eq. B1 and B2 in Eq. B3 yields

[^1]\[

$$
\begin{aligned}
& \dot{g}=\frac{\vartheta}{e_{o}-\vartheta}\left(\frac{1}{\vartheta} \int_{0}^{\infty} \rho(x) w(x) 2 \ell(x) \bar{\ell}(x) \mathrm{d} x-\frac{1}{e_{o}} \int_{0}^{\infty} \rho(x) w(x) \mathrm{d} x\right) \\
& =\int_{0}^{\infty} \rho(x) w(x) \frac{\vartheta}{e_{o}-\vartheta}\left(\frac{2 \ell(x) \bar{\ell}(x)}{\vartheta}-\frac{1}{e_{o}}\right) \mathrm{d} x \\
& =\int_{0}^{\infty} \rho(x) w(x) \frac{\vartheta}{e_{o}-\vartheta}\left(\frac{2 \ell(x) \bar{\ell}(x)}{\vartheta}-\frac{1}{e_{o}}\right) \mathrm{d} x \\
& =\int_{0}^{\infty} \rho(x) w(x)\left(\frac{2 \ell(x) \bar{\ell}(x)-\bar{\ell}}{e_{o}-\vartheta}\right) \mathrm{d} x \\
& =\int_{0}^{\infty} \rho(x) w(x) W_{g}(x) \mathrm{d} x
\end{aligned}
$$
\]

which provides an expression for $\dot{g}$ that proves Eq. 9 in the main manuscript.

## Section C. Time derivatives of $\boldsymbol{\sigma}$ and $\boldsymbol{h}$

Let

$$
\sigma=\sqrt{\int_{0}^{\infty}\left(x-e_{o}\right)^{2} \ell(x) \mu(x) \mathrm{d} x}
$$

be the standard deviation of the lifetable age-at-death distribution. Following Gillespie et al (22), the time derivative of the variance of the age-at-death distribution is

$$
\dot{\sigma^{2}}=2 \int_{0}^{\infty} \rho(x) w(x) \frac{\int_{x}^{\infty} \ell(a)\left(a-e_{o}\right) \mathrm{d} a}{e(x) \ell(x)} \mathrm{d} x
$$

Applying the chain rule, the time derivative of $\sigma$ is given by

$$
\begin{equation*}
\dot{\sigma}=\frac{\dot{\sigma}^{2}}{2 \sigma}=\frac{1}{\sigma} \int_{0}^{\infty} \rho(x) w(x) \frac{\int_{x}^{\infty} \ell(a)\left(a-e_{o}\right) \mathrm{d} a}{e(x) \ell(x)} \mathrm{d} x . \tag{C1}
\end{equation*}
$$

On the other hand, from Eq. 11 in the main manuscript, the time derivative of $v=-\ln C V$ can be expressed as

$$
\begin{equation*}
\dot{v}=\frac{\dot{e}_{o}}{e_{o}}-\frac{\dot{\sigma}}{\sigma} . \tag{C2}
\end{equation*}
$$

Replacing Eq. B2 and C1 in Eq. C2, we get

$$
\begin{aligned}
& \dot{v}=\frac{1}{e_{o}} \int_{0}^{\infty} \rho(x) w(x) \mathrm{dx}-\frac{1}{\sigma} \int_{0}^{\infty} \rho(x) w(x) \frac{\int_{x}^{\infty} \ell(a)\left(a-e_{o}\right) \mathrm{d} a}{\sigma e(x) \ell(x)} \mathrm{d} x \\
& =\int_{0}^{\infty} \rho(x) w(x)\left(\frac{1}{e_{o}}-\frac{1}{\sigma^{2}} \int_{x}^{\infty} \frac{\int_{x}^{\infty} \ell(a)\left(a-e_{o}\right) \mathrm{d} a}{\int_{x}^{\infty} \ell(a) \mathrm{d} a}\right) \mathrm{d} x \\
& =\int_{0}^{\infty} \rho(x) w(x)\left(\frac{1}{e_{o}}-\frac{1}{\sigma^{2}} C V(x)\right) \mathrm{d} x \\
& =\int_{0}^{\infty} \rho(x) w(x) W_{v}(x) \mathrm{d} x .
\end{aligned}
$$

which provides an expression $\dot{v}$ that proves Eq. 12 in the main manuscript.

## Additional figures and tables

Fig. S1. Association between changes in life expectancy and changes in lifespan equality (10-year lag), $g$ and $\boldsymbol{v}$. Calculations based on data from HMD (8).

Periods
1900-1921
$1921-1959$
1960 onwards






Fig. S2. Johansen Trace and Lambda Max Cointegrations tests for $e_{o}$ with $h, g$ and $v$. Calculations based on data from HMD (8). Since we are interested in whether $e_{o}$ and $h$ have a long-run relationship, we focus on the rejection of the null hypothesis of no long-run equilibrium. Each point indicates deviation from the $5 \%$ significance critical value. Positive values indicate a long-run relationship, while negative show no evidence to reject the null hypothesis of no long-run relationship.


Fig. S3. Standardized weights for the changes in lifespan equality $w(x) W(x)$ for $g$ and $v$.

B) Weights from age 5

C) Weights for lifespan equality $\mathrm{v}, \mathrm{w}(\mathrm{x}) \mathrm{W}_{\mathrm{v}}(\mathrm{x})$
D) Weights from age 5

 $-0.02-$

| $-0.02-$ | ó | 25 | 50 | it |
| :--- | :--- | :--- | :--- | :--- |

Fig. S4. Association between changes in life expectancy and lifespan equality ( $g$ and $v$ ) below and above the threshold age. Dotted lines show the directions of the relationship below and above the threshold age. Calculations based on data from HMD (8).


Fig. S5. Threshold age trajectories for $h, g$ and $v$. Calculations based on data from HMD (8).


Threshold age for g


Threshold age for $v$


Table S1. Countries and regions in the Human Mortality Database (8) used in our study, ranked by life expectancy by sex for the latest year available.

| Females <br> No | Country | Initial | Final | $e_{o}$ | $h$ | $g$ | $v$ | $a^{h}$ | $a^{g}$ | $a^{v}$ |
| :--- | :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Japan | 1947 | 2017 | 87.31 | 2.16 | 2.64 | 1.93 | 87.25 | 84.54 | 82.21 |
| 2 | Spain | 1908 | 2016 | 85.79 | 2.17 | 2.65 | 1.94 | 85.55 | 82.96 | 80.73 |
| 3 | Korea | 2003 | 2016 | 85.46 | 2.15 | 2.65 | 1.93 | 84.86 | 82.28 | 80.43 |
| 4 | France (civilian) | 1900 | 2017 | 85.28 | 2.1 | 2.55 | 1.84 | 85.93 | 83.17 | 80.34 |
| 5 | France | 1900 | 2017 | 85.28 | 2.1 | 2.55 | 1.84 | 85.93 | 83.17 | 80.34 |
| 6 | Switzerland | 1900 | 2016 | 85.24 | 2.16 | 2.62 | 1.9 | 85.32 | 82.63 | 80.32 |
| 7 | Italy | 1900 | 2014 | 85.15 | 2.14 | 2.62 | 1.93 | 84.89 | 82.15 | 79.97 |
| 8 | Australia | 1921 | 2016 | 84.93 | 2.11 | 2.57 | 1.87 | 85.1 | 82.25 | 79.83 |
| 9 | Luxembourg | 1960 | 2014 | 84.49 | 2.09 | 2.57 | 1.87 | 84.34 | 81.35 | 79.21 |
| 10 | Israel | 1983 | 2016 | 84.24 | 2.11 | 2.59 | 1.89 | 83.92 | 81.05 | 79.03 |
| 11 | Finland | 1900 | 2015 | 84.19 | 2.14 | 2.6 | 1.91 | 84.18 | 81.41 | 79.04 |
| 12 | Canada | 1921 | 2016 | 84.18 | 1.99 | 2.46 | 1.77 | 84.69 | 81.34 | 78.81 |
| 13 | Sweden | 1900 | 2017 | 84.12 | 2.13 | 2.6 | 1.91 | 84.03 | 81.13 | 78.89 |
| 14 | Norway | 1900 | 2014 | 84.1 | 2.11 | 2.59 | 1.91 | 84.00 | 81.07 | 78.76 |
| 15 | Portugal | 1940 | 2015 | 84.03 | 2.16 | 2.63 | 1.92 | 84.00 | 81.38 | 79.11 |
| 16 | Austria | 1947 | 2017 | 83.89 | 2.14 | 2.6 | 1.9 | 84.06 | 81.31 | 78.83 |
| 17 | Iceland | 1900 | 2016 | 83.88 | 2.13 | 2.61 | 1.96 | 83.51 | 80.34 | 78.23 |
| 18 | Slovenia | 1983 | 2017 | 83.68 | 2.12 | 2.59 | 1.9 | 83.64 | 80.89 | 78.51 |
| 19 | Greece | 1981 | 2013 | 83.67 | 2.11 | 2.6 | 1.89 | 83.14 | 80.6 | 78.6 |
| 20 | New Zealand | 1948 | 2013 | 83.43 | 2.04 | 2.5 | 1.79 | 83.88 | 80.84 | 78.31 |
| 21 | East Germany | 1956 | 2017 | 83.32 | 2.1 | 2.56 | 1.87 | 83.5 | 80.67 | 78.16 |
| 22 | Germany | 1990 | 2017 | 83.29 | 2.1 | 2.56 | 1.86 | 83.56 | 80.69 | 78.14 |
| 23 | West Germany | 1956 | 2017 | 83.28 | 2.1 | 2.55 | 1.86 | 83.57 | 80.7 | 78.14 |
| 24 | Ireland | 1950 | 2014 | 83.21 | 2.05 | 2.54 | 1.84 | 82.95 | 80.03 | 77.89 |


| 25 | Belgium | 1919 | 2015 | 83.15 | 2.07 | 2.52 | 1.83 | 83.65 | 80.69 | 78.02 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 26 | Netherlands | 1900 | 2016 | 83.12 | 2.08 | 2.54 | 1.85 | 83.48 | 80.53 | 77.95 |
| 27 | New Zealand (Non Maori) | 1901 | 2008 | 83.06 | 2.06 | 2.52 | 1.81 | 83.31 | 80.32 | 77.98 |
| 28 | England and Wales (Civilian) | 1900 | 2016 | 83.04 | 2.03 | 2.5 | 1.81 | 83.25 | 80.13 | 77.71 |
| 29 | England and Wales (non-Civilian) | 1900 | 2016 | 83.04 | 2.03 | 2.5 | 1.81 | 83.25 | 80.13 | 77.71 |
| 30 | U.K. | 1922 | 2016 | 82.84 | 2.02 | 2.49 | 1.8 | 83.07 | 79.91 | 77.49 |
| 31 | Taiwan | 1970 | 2014 | 82.82 | 1.99 | 2.47 | 1.78 | 82.75 | 79.53 | 77.40 |
| 32 | Denmark | 1900 | 2016 | 82.79 | 2.03 | 2.51 | 1.84 | 82.7 | 79.47 | 77.21 |
| 33 | Northern Ireland | 1922 | 2016 | 82.3 | 1.99 | 2.46 | 1.76 | 82.58 | 79.37 | 77.00 |
| 34 | Estonia | 1959 | 2017 | 82.29 | 2.03 | 2.5 | 1.83 | 82.4 | 79.4 | 76.96 |
| 35 | Czech Republic | 1950 | 2017 | 81.86 | 2.09 | 2.55 | 1.88 | 81.85 | 78.86 | 76.53 |
| 36 | Poland | 1958 | 2016 | 81.72 | 1.99 | 2.45 | 1.78 | 82.02 | 78.72 | 76.22 |
| 37 | U.S.A. | 1933 | 2017 | 81.39 | 1.86 | 2.31 | 1.64 | 82.45 | 78.67 | 75.96 |
| 38 | Chile | 1992 | 2008 | 81.13 | 1.92 | 2.39 | 1.69 | 81.34 | 78.07 | 75.96 |
| 39 | Scotland | 1900 | 2016 | 81.06 | 1.95 | 2.41 | 1.74 | 81.46 | 78.03 | 75.57 |
| 40 | Croatia | 2002 | 2017 | 80.86 | 2.08 | 2.55 | 1.85 | 80.72 | 77.9 | 75.74 |
| 41 | Slovakia | 1950 | 2017 | 80.57 | 2.01 | 2.48 | 1.79 | 80.56 | 77.59 | 75.32 |
| 42 | Lithuania | 1959 | 2017 | 80.37 | 1.98 | 2.42 | 1.74 | 81.19 | 78.00 | 75.14 |
| 43 | Latvia | 1959 | 2017 | 79.53 | 1.93 | 2.37 | 1.7 | 80.51 | 77.02 | 74.22 |
| 44 | Hungary | 1950 | 2017 | 79.27 | 1.94 | 2.41 | 1.76 | 79.37 | 75.99 | 73.6 |
| 45 | Belarus | 1959 | 2016 | 79.02 | 1.95 | 2.41 | 1.74 | 79.2 | 76.00 | 73.62 |
| 46 | Bulgaria | 1947 | 2010 | 77.25 | 1.93 | 2.36 | 1.64 | 78.13 | 75.09 | 72.66 |
| 47 | Russia | 1959 | 2014 | 76.48 | 1.79 | 2.21 | 1.54 | 78.07 | 74.41 | 71.48 |
| 48 | Ukraine | 1959 | 2013 | 76.2 | 1.82 | 2.27 | 1.59 | 77.11 | 73.58 | 71.14 |
| 49 | New Zealand (Maori) | 1948 | 2008 | 75.64 | 1.72 | 2.21 | 1.57 | 75.95 | 71.02 | 69.34 |


| Males |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No | Country | Initial | Final | $e_{o}$ | $h$ | $g$ | $v$ | $a^{h}$ | $a^{g}$ | $a^{v}$ |
| 50 | Switzerland | 1900 | 2016 | 81.53 | 2.03 | 2.49 | 1.81 | 81.9 | 78.75 | 76.26 |
| 51 | Japan | 1947 | 2017 | 81.13 | 1.97 | 2.44 | 1.78 | 81.36 | 77.95 | 75.52 |
| 52 | Australia | 1921 | 2016 | 80.83 | 1.94 | 2.39 | 1.71 | 81.5 | 78.17 | 75.55 |
| 53 | Sweden | 1900 | 2017 | 80.73 | 2.03 | 2.47 | 1.79 | 81.2 | 78.07 | 75.56 |
| 54 | Israel | 1983 | 2016 | 80.65 | 1.91 | 2.39 | 1.73 | 80.62 | 77.05 | 74.93 |
| 55 | Italy | 1900 | 2014 | 80.55 | 1.99 | 2.46 | 1.78 | 80.67 | 77.38 | 75.08 |
| 56 | Iceland | 1900 | 2016 | 80.3 | 2.04 | 2.48 | 1.79 | 80.76 | 77.78 | 75.25 |
| 57 | Spain | 1908 | 2016 | 80.27 | 1.94 | 2.41 | 1.76 | 80.51 | 76.9 | 74.52 |
| 58 | Canada | 1921 | 2016 | 80.05 | 1.88 | 2.32 | 1.65 | 81.04 | 77.32 | 74.68 |
| 59 | Norway | 1900 | 2014 | 80.03 | 1.99 | 2.44 | 1.76 | 80.36 | 77.05 | 74.69 |
| 60 | Netherlands | 1900 | 2016 | 79.88 | 2.00 | 2.46 | 1.78 | 79.99 | 76.75 | 74.53 |
| 61 | New Zealand | 1948 | 2013 | 79.8 | 1.91 | 2.36 | 1.67 | 80.41 | 77.07 | 74.63 |
| 62 | France (civilian) | 1900 | 2017 | 79.44 | 1.88 | 2.31 | 1.65 | 80.91 | 77.03 | 73.92 |
| 63 | France | 1900 | 2017 | 79.44 | 1.88 | 2.31 | 1.65 | 80.91 | 77.03 | 73.92 |
| 64 | England and Wales (Civilian) | 1900 | 2016 | 79.42 | 1.91 | 2.36 | 1.69 | 80.01 | 76.44 | 73.99 |
| 65 | England and Wales (non-Civilian) | 1900 | 2016 | 79.42 | 1.91 | 2.36 | 1.69 | 80.01 | 76.44 | 73.99 |
| 66 | Korea | 2003 | 2016 | 79.34 | 1.92 | 2.4 | 1.73 | 79.37 | 76.09 | 73.84 |
| 67 | Austria | 1947 | 2017 | 79.28 | 1.94 | 2.39 | 1.73 | 79.85 | 76.14 | 73.68 |
| 68 | U.K. | 1922 | 2016 | 79.18 | 1.9 | 2.35 | 1.68 | 79.82 | 76.22 | 73.74 |
| 69 | Ireland | 1950 | 2014 | 79.16 | 1.91 | 2.38 | 1.69 | 79.43 | 76.16 | 73.84 |
| 70 | Luxembourg | 1960 | 2014 | 79.13 | 1.96 | 2.4 | 1.74 | 79.78 | 76.06 | 73.54 |
| 71 | New Zealand (Non Maori) | 1901 | 2008 | 79.05 | 1.89 | 2.35 | 1.66 | 79.44 | 76.21 | 74.00 |
| 72 | Denmark | 1900 | 2016 | 78.95 | 1.93 | 2.39 | 1.73 | 79.39 | 75.78 | 73.34 |
| 73 | West Germany | 1956 | 2017 | 78.81 | 1.93 | 2.39 | 1.73 | 79.35 | 75.62 | 73.19 |
| 74 | Northern Ireland | 1922 | 2016 | 78.76 | 1.88 | 2.32 | 1.65 | 79.6 | 75.96 | 73.42 |
| 75 | Finland | 1900 | 2015 | 78.62 | 1.9 | 2.35 | 1.71 | 79.34 | 75.54 | 72.94 |
| 76 | Belgium | 1919 | 2015 | 78.57 | 1.91 | 2.35 | 1.70 | 79.23 | 75.47 | 72.99 |
| 77 | Germany | 1990 | 2017 | 78.56 | 1.92 | 2.37 | 1.72 | 79.15 | 75.35 | 72.87 |
| 78 | Greece | 1981 | 2013 | 78.45 | 1.82 | 2.3 | 1.65 | 78.89 | 75.11 | 72.65 |
| 79 | Slovenia | 1983 | 2017 | 78.08 | 1.92 | 2.38 | 1.74 | 78.29 | 74.56 | 72.23 |


| 80 | Portugal | 1940 | 2015 | 78.02 | 1.89 | 2.33 | 1.68 | 79.11 | 75.39 | 72.55 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 81 | East Germany | 1956 | 2017 | 77.66 | 1.87 | 2.32 | 1.69 | 78.38 | 74.29 | 71.73 |
| 82 | Scotland | 1900 | 2016 | 76.93 | 1.8 | 2.25 | 1.6 | 77.99 | 74.01 | 71.35 |
| 83 | Taiwan | 1970 | 2014 | 76.54 | 1.72 | 2.19 | 1.55 | 77.4 | 73.26 | 70.74 |
| 84 | U.S.A. | 1933 | 2017 | 76.35 | 1.68 | 2.12 | 1.47 | 78.31 | 73.82 | 70.94 |
| 85 | Czech Republic | 1950 | 2017 | 76.01 | 1.85 | 2.31 | 1.67 | 76.21 | 72.01 | 70.08 |
| 86 | Chile | 1992 | 2008 | 75.33 | 1.72 | 2.18 | 1.51 | 76.25 | 72.36 | 70.14 |
| 87 | Croatia | 2002 | 2017 | 74.87 | 1.82 | 2.28 | 1.64 | 75.15 | 71.00 | 69.05 |
| 88 | Poland | 1958 | 2016 | 73.84 | 1.68 | 2.15 | 1.53 | 74.25 | 69.32 | 67.53 |
| 89 | Slovakia | 1950 | 2017 | 73.76 | 1.73 | 2.21 | 1.57 | 73.84 | 69.44 | 67.72 |
| 90 | Estonia | 1959 | 2017 | 73.66 | 1.71 | 2.18 | 1.56 | 73.82 | 69.01 | 67.34 |
| 91 | Hungary | 1950 | 2017 | 72.56 | 1.71 | 2.21 | 1.6 | 71.95 | 67.11 | 65.94 |
| 92 | New Zealand (Maori) | 1948 | 2008 | 72.51 | 1.49 | 2.07 | 1.44 | 72.1 | 67.55 | 66.19 |
| 93 | Lithuania | 1959 | 2017 | 70.7 | 1.61 | 2.07 | 1.46 | 71.37 | 66.14 | 64.39 |
| 94 | Bulgaria | 1947 | 2010 | 70.31 | 1.65 | 2.09 | 1.44 | 71.48 | 66.76 | 64.91 |
| 95 | Latvia | 1959 | 2017 | 69.8 | 1.58 | 2.04 | 1.44 | 70.3 | 6.1 | 63.5 |
| 96 | Belarus | 1959 | 2016 | 68.96 | 1.6 | 2.09 | 1.49 | 68.73 | 63.91 | 62.57 |
| 97 | Ukraine | 1959 | 2013 | 66.31 | 1.47 | 1.94 | 1.33 | 67.16 | 61.84 | 60.34 |
| 98 | Russia | 1959 | 2014 | 65.26 | 1.38 | 1.85 | 1.26 | 66.95 | 61.08 | 59.38 |

Table S2. Years in which dummy variables were added. The dummy variables account for the following historic events: 1918 the Spanish flu, 1914-1919 World War 1, 1940-1945 World War II (1939 included for Spain to account for the Spanish civil war) and 1992-1995 dissolution of the Soviet Union.

| Country | Data years | Dummy Years |
| :---: | :---: | :---: |
| Australia | 1921-2016 | None |
| Austria | 1947-2017 | None |
| Belgium | 1920-2015 | 1940-1945 |
| Bulgaria | 1947-2010 | None |
| Belarus | 1959-2016 | 1992-1995 |
| Canada | 1921-2016 | None |
| Czech Republic | 1950-2017 | None |
| Denmark | 1900-2016 | 1918 and 1940-1945 |
| Estonia | 1959-2017 | 1992-1995 |
| Finland | 1900-2015 | 1918 and 1940-1945 |
| France | 1900-2017 | 1914-1919 and 1940-1945 |
| Germany | 1990-2017 | None |
| Greece | 1981-2013 | None |
| Hungary | 1950-2017 | 1992-1995 |
| Iceland | 1900-2016 | None |
| Ireland | 1950-2014 | None |
| Israel | 1983-2016 | None |
| Italy | 1900-2014 | 1914-1919 and 1940-1945 |
| East Germany | 1956-2017 | 1992-1995 |
| West Germany | 1956-2017 | None |
| Japan | 1947-2017 | None |
| Latvia | 1959-2017 | 1992-1995 |
| Lithuania | 1959-2017 | 1992-1995 |
| Luxembourg | 1960-2014 | None |
| Netherlands | 1900-2016 | 1918 and 1940-1945 |
| New Zealand Maori | 1948-2008 | 1918 |
| New Zealand Non-Maori | 1948-2008 | 1918 |
| Norway | 1900-2014 | 1918 and 1940-1945 |
| Northern Ireland | 1922-2016 | 1940-1945 |
| Poland | 1958-2016 | 1992-1995 |
| Portugal | 1940-2015 | None |
| Russia | 1959-2014 | 1992-1995 |
| Scotland | 1900-2016 | None |
| Slovakia | 1950-2017 | None |
| Slovenia | 1983-2017 | None |
| Spain | 1908-2016 | 1918 and 1939-1945 |
| Sweden | 1900-2017 | 1918 |
| Switzerland | 1900-2016 | 1918 |
| Taiwan | 1970-2014 | None |
| Ukraine | 1959-2013 | 1992-1995 |
| UK | 1922-2016 | 1940-1945 |

## References

1. J. M. Wooldridge, Introductory econometrics: A modern approach (Nelson Education, 2015).
2. J. M. Wooldridge, Estimation and inference for dependent processes. Handbook of econometrics 4, 2639-2738 (1994).
3. K. Juselius, The cointegrated VAR model: methodology and applications (Oxford University Press, 2006).
4. B. Pfaff, Analysis of integrated and cointegrated time series with $R$ (Springer Science \& Business Media, 2008).
5. D. Kwiatkowski, P. C. Phillips, P. Schmidt, Y. Shin, Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? Journal of Econometrics 54, 159-178 (1992).
6. D. A. Dickey, W. A. Fuller, Distribution of the estimators for autoregressive time series with a unit root. Journal of the American Statistical Association 74, 427-431 (1979).
7. E. Zivot, D. W. K. Andrews, Further evidence on the great crash, the oil-price shock, and the unit-root hypothesis. Journal of business \& economic statistics 20, 25-44 (2002).
8. University of California Berkeley and Max Planck Institute for Demographic Research (Germany) (2019) Human Mortality Database (https://www.mortality.org/ ).
9. C. W. Granger, Some properties of time series data and their use in econometric model specification. Journal of Econometrics 16, 121-130 (1981).
10. F. W. Ahking, Model mis-specification and Johansen's co-integration analysis: an application to the US money demand. Journal of Macroeconomics 24, 51-66 (2002).
11. J. A. Doornik, Approximations to the asymptotic distributions of cointegration tests. Journal of Economic Surveys 12, 573-593 (1998).
12. M. Osterwald-Lenum, A note with quantiles of the asymptotic distribution of the maximum likelihood cointegration rank test statistics1. Oxford Bulletin of Economics and statistics 54, 461-472 (1992).
13. H. Akaike, Likelihood of a model and information criteria. Journal of econometrics 16, 3-14 (1981).
14. E. J. Hannan, B. G. Quinn, The determination of the order of an autoregression. Journal of the Royal Statistical Society. Series B (Methodological), 190-195 (1979).
15. H. Lütkepohl, New introduction to multiple time series analysis (Springer Science \& Business Media, 2005).
16. G. Schwarz, others, Estimating the dimension of a model. The annals of statistics 6, 461-464 (1978).
17. V. Ivanov, L. Kilian, A practitioner's guide to lag order selection for VAR impulse response analysis. Studies in Nonlinear Dynamics and Econometrics 9, 1-34 (2005).
18. S. r. Johansen, The role of the constant and linear terms in cointegration analysis of nonstationary variables. Econometric reviews 13, 205-229 (1994).
19. R. Ihaka, R. Gentleman, R: a language for data analysis and graphics. Journal of computational and graphical statistics 5, 299-314 (1996).
20. H. Y. Toda, T. Yamamoto, Statistical inference in vector autoregressions with possibly integrated processes. Journal of econometrics 66, 225-250 (1995).
21. B. U. Human Mortality Database. University of California, and Max Planck Institute for Demographic Research (Germany), Human Mortality Database.
22. D. O. Gillespie, M. V. Trotter, S. D. Tuljapurkar, Divergence in age patterns of mortality change drives international divergence in lifespan inequality. Demography 51, 1003-1017 (2014).

[^0]:    ${ }^{1}$ We also performed tests against higher orders of integration, but could not reject the hypothesis that the variables were integrated at a lower level.
    ${ }^{2}$ For $g=-\ln G$ the proportion is $91.8 \%$, and for $v=-\ln C V$ is $91.8 \%$ (see Fig. A1).
    ${ }^{3}$ The corresponding values for $g$ and $v$ are 94.9 and $95.9 \%$, respectively (see Fig. A1).

[^1]:    ${ }^{4}$ For simplicity, variable $t$ will be omitted as an argument in the following.

