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Heterogeneous Design for Feeder Services of Trunk Transit System

Yue Su and Wenbo Fan

Abstract— This paper extends the uniform design of a trunk-feeder system, e.g., rail-bus system, over a grid network in the literature to the heterogeneous design, where the spacings of feeder bus lines and stops are allowed to vary spatially to better serve the demand. The trunk lines are evenly deployed over the study area with the line spacing to be optimized. Employing the method of continuum approximation, we develop a joint design model to minimize the generalized system cost as a sum of transit patrons' cost, agency cost, and emission cost. The proposed model is applied on two types of trunk systems (i.e., rail and Bus Rapid Transit, BRT) in small- and large-sized city scenarios. The results indicate that: (i) the proposed model saves 31% of the system cost as compared with the uniform-designed feeder system; (ii) large-sized cities (e.g., New York City) prefer faster transit system as the trunk transit mode (i.e., rail), while small-sized cities welcome more economical trunk transit mode (i.e., BRT); and (iii) considering emission cost into the optimization model will lead to 9.43% reduction in actual emission cost.

Keywords—trunk-feeder system, heterogeneous design, continuum approximation, emission

I. INTRODUCTION

To fight against the increasing vehicular congestion and environmental problems, many cities in the world resort to the high-capacity public transport systems (e.g., rail or metro) as the urban trunk transit. Due to the expensive cost of building and operating, these trunk transit systems are often limited in network scale, and thus need feeder buses to extend the demand coverage area by picking up and delivering passengers to and from rail stations^{[1][2]}.

Not surprisingly, the trunk-feeder system has been extensively studied in literature. The most relevant works to this study dates back to Wirashinghe et al.^[3] They adopted the method of Continuum Approximation (CA) in formulating parsimonious models for the optimal design of a rail-bus corridor. The design variables such as rail station spacing and feeder bus line spacing are formulated as continuous functions of locations in the corridor. Considering spatially uneven demand, they optimized the rail station spacing and feeder bus line spacing functions as well as the service headways (as scalar variables). Then, ref.^[4] extended to take the rail line length into the decision variables to be determined. A major extension to the trunk-feeder corridor structure was made in Sivakumaran et al.^[5], who established a trunk-feeder system over a grid network and demonstrated their advantage over traditional single-

mode transit network. Most recently, their network was enhanced by ref.^[6] with the consideration of feeder buses' emission cost (as a constraint into the optimization). These two works both assumed a uniform demand over the study domain to simplify the modelling, and all the design variables (i.e., the line spacing and station spacing, service headways) of rail and bus systems are reduced to scalar variables. The corresponding minimization problems become very easy to be solved. It is demonstrated in ref.^[7] that even under the uniform demand assumption, the feeder services need a spatially heterogeneous design in best fit with the demand that perform a many-to-one pattern and accumulate as approaching the trunk stations.

In light of above, this paper explicitly models the heterogeneous design for the feeder bus services of trunk transit over a grid network. The demand is still assumed to follow a uniform distribution to obtain parsimonious models. Emission cost is also taken into consideration to enhance the environmental awareness.

The remainder of this paper is organized as follows: next section formulates the system costs and the optimization problem of the trunk-feeder system by employing CA approach. The analytical analyses of the decision variables/functions are also presented. Section three applies the proposed model to various scenarios with respect to city sizes, demand levels, and trunk transit modes. Insightful findings are obtained by comparing the results of the proposed model with that of the traditional uniform design, and with the case that ignores emission cost. The last section draws conclusions.

II. METHODOLOGY

Consider a square city of size $R \times R$ (km²), we present a trunk-feeder system over a grid network, as illustrated in Fig. 1a. The trunk lines and stations are evenly spaced by $S_t = 2S$ (km), and a square service area of size $S \times S$ (km²) is designed for a feeder system. Without loss of generality, feeder buses are operated to run and visit stops along vertical lines to pickup patrons, and upon reaching the horizontal trunk line they run without stopping to the trunk station, as shown by Fig. 1b; and vice versa for the process of delivering patrons from the trunk station. In the proposed model, the layout of feeder-bus system is allowed to be spatially varying. Let $S_f(x)$ denote the bus line spacing in the neighborhood of the cross-section at x , and $B(x, y)$ be the bus stop spacing in the neighborhood area of (x, y) . To facilitate modeling, a few assumptions are made as follows:

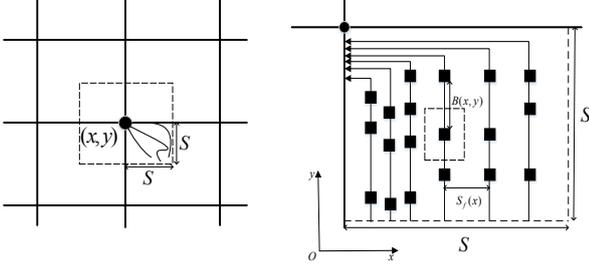
1. The demand is uniformly distributed over the study area with density q (passengers/km²/h).
2. To accomplish their trips, all passengers take feeder buses to access and egress from trunk stations at both trip ends, and ride trunk transit in the middle of trips.

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Yue Su is with Department of Transportation and Logistic, Southwest Jiaotong University, Chengdu 610031, China;

Wenbo Fan is with Department of Transportation and Logistic, Southwest Jiaotong University, Chengdu 610031, China (Corresponding author e-mail: wbfan@swjtu.edu.cn).

3. Passengers choose the nearest stop to board or alight feeder bus. They arrive at stops randomly without pre-trip scheduling.



(a) Trunk system layout (b) Feeder system layout
Fig. 1. Layout of trunk-feeder system

The generalized system cost, GC , is the weighted sum of user cost, C_u , operator cost, C_o , and emission cost, C_p , expressed as below:

$$GC = C_u + C_o + C_p \quad (1)$$

where C_u contains three cost items: the access/egress time to/from transit stops (C_a), the waiting time at origin and transfer stops (C_w), in-vehicle time that on-board passengers overcome in both trunk and feeder lines (C_v). Thus, C_u is expressed by:

$$C_u = C_a + C_w + C_v \quad (2)$$

The operator cost C_o is composed by the lines infrastructure cost (C_l); the stops infrastructure cost (C_s); the vehicle-km related cost (C_{vk}); the vehicle-time related cost (C_{vh}).

$$C_o = C_l + C_s + C_{vk} + C_{vh} \quad (3)$$

Emission cost C_p mainly entails: the emission cost while dwelling at stops (C_p^1); the emission cost at cruising speed (C_p^2).

$$C_p = C_p^1 + C_p^2 \quad (4)$$

Each cost item in (2-4) will be derived by employing CA method for trunk and feeder systems, respectively.

A. User Cost

We first summarize the expressions of C_a , C_w , C_v as (5-7), and then derive them below.

$$C_a = 0 + \frac{2R^2}{S^2} \cdot \int_{y=0}^S \int_{x=0}^S q \cdot \frac{S_f(x)+B(x,y)}{4v_a} dx dy \quad (5)$$

$$C_w = qR^2H + \frac{2R^2}{S^2} \cdot \int_{x=0}^S qS \cdot \left(\frac{h(x)}{2} + t_{f-t} \right) dx \quad (6)$$

$$C_v = qR^2 \left[\frac{2R}{3V} + \frac{2R}{3S_t} \cdot \left(\tau_0 + \frac{qS_t^2 H \tau_1}{2} \right) \right] + C_{vf} \quad (7)$$

In the Right Hand Side (RHS) of (5), item $\frac{R^2}{S^2}$ suggests the total number of feeder service zones in the city area. Notice that the catchment area of a bus stop at (x, y) is a rectangle of $S_f(x) \times B(x, y)$, as shown by the dotted rectangle in Fig. 1b.

Thus the average accessing and egressing walk distance to bus stops can be estimated by $\frac{S_f(x)+B(x,y)}{2}$. Dividing by the average walking speed, v_a (km/h), yields the average access and egress time per patron.

The two items in RHS of (6) are waiting time at trunk stations and feeder bus stops, respectively. It is underscored that the average waiting time per boarding or transferring is the half of headway, i.e., H for trunk service and $h(x)$ for feeder lines at cross-section x . Additionally, (6) accounts for the transfer delay, t_{f-t} , e.g., walking time and inconvenience penalty.

The first RHS item of (7) is the in-vehicle time on trunk lines, of which V (km/h) is the cruising speed of trunk vehicle. The detailed derivation and explanation can be found in Sivakumaran et al. [5] and omitted here for sake of simplification. The second item C_{vf} denotes the in-vehicle travel time on feeder lines, which is formulated by:

$$C_{vf} = C_{vf}^t + C_{vf}^l + C_{vf}^d \quad (8)$$

The first time item C_{vf}^t represents the total in-vehicle time experienced by on-board passengers to overcome a unit distance in a cruising bus, and can be estimated by:

$$C_{vf}^t = 2qS^2 \cdot \frac{S}{v_f} \cdot \frac{R^2}{S^2} \quad (9)$$

It is noted in (9) that under uniform demand, the average travel distance per patron in feeder system is S ; $\frac{S}{v_f}$ yields the average travel time; and the total demand is $2qS^2$ per feeder service zone (including outbound and inbound demand).

The second time item C_{vf}^l is the additional time lost at stops due to acceleration and deceleration, which is formulated by:

$$C_{vf}^l = \frac{2R^2}{S^2} \cdot q\tau_0 \cdot \int_{y=0}^S \int_{x=0}^S \frac{y}{B(x,y)} dx dy \quad (10)$$

where τ_0 is the acceleration and deceleration time at stop (in unit of hour). The formulation logic of (10) is that: The accumulated onboard flow of each bus line passing (x, y) is $q \cdot S_f(x) \cdot y$. For the area of $dx \times dy$, there are $\frac{dx}{S_f(x)}$ lines and $\frac{dy}{B(x,y)}$ stops along each line. Thus, $q \cdot S_f(x) \cdot y \cdot \frac{dx}{S_f(x)} \cdot \frac{dy}{B(x,y)} \cdot \tau_0$ yields the time lost at stops in the domain by $dx \times dy$, which is the result inside the integrals of (10).

The last item C_{vf}^d is the time lost related to bus dwell time at stops due to passenger boarding. The expression is:

$$C_{vf}^d = \frac{2R^2}{S^2} \cdot \frac{q^2 \tau_1 S^2}{2} \int_{x=0}^S S_f(x) h(x) dx \quad (11)$$

To understand (11), consider: A slide $[x, x + dx]$, there are $\frac{dx}{S_f(x)}$ number of feeder bus lines, and $\frac{1}{h(x)}$ number of buses traveling on each line. For each bus traveling along a vertical line at x , the total boarding demand is $q \cdot S \cdot S_f(x) \cdot h(x)$. As the demand is uniformly distributed along the vertical line, the total boarding delay is thus $\frac{\tau_1}{2} \cdot [q \cdot S \cdot S_f(x) \cdot h(x)]^2$,

where τ_1 is the boarding time per patron. Integrating the delay of all buses along all lines over the entire service zones gives us (11).

B. Operator Cost

The cost components, C_l , C_s , C_{vk} , C_{vh} are summarized in (12-15), which convert them into unit of time by dividing patrons' value of time (VOT), μ .

$$C_l = \frac{\pi_l}{\mu} \cdot R^2 \cdot \frac{1}{S_t} + \frac{R^2}{S^2} \cdot \frac{\pi_{lf}}{\mu} \cdot \left(\int_{x=0}^S \frac{2(x+S)}{S_f(x)} dx \right) \quad (12)$$

$$C_s = \frac{\pi_s}{\mu} \cdot R^2 \cdot \frac{1}{S_t^2} + \frac{R^2}{S^2} \cdot \frac{\pi_{sf}}{\mu} \cdot \int_{x=0}^S \int_{y=0}^S \frac{1}{S_f(x)B(x,y)} dx dy \quad (13)$$

$$C_{vk} = \frac{\pi_v}{\mu} \cdot R^2 \cdot \frac{2}{S_t H} + \frac{R^2}{S^2} \cdot \frac{\pi_{vf}}{\mu} \cdot \left(\int_{x=0}^S \frac{2(x+S)}{S_f(x)h(x)} dx \right) \quad (14)$$

$$C_{vh} = \frac{\pi_m}{\mu} \cdot \left[\frac{2R^2}{S_t H V} + \frac{R^2}{S_t^2} \cdot \left(\tau_0 + \frac{qS_t^2 H \tau_1}{2} \right) \right] + C_{vh}^f \quad (15)$$

where the first RHS items of (12-15) are agency costs of trunk system, of which the detailed derivations can be found in Sivakumaran et al. [5]. Parameters, π_l , π_s , π_{vk} , π_{vh} are the unit costs related to trunk system's line infrastructure, station infrastructure, vehicle-km traveled and vehicle-hour traveled per operation hour, respectively.

The second RHS items of (12-15) are related to feeder system. They are straightforward and left for readers to verify. And π_{lf} , π_{sf} , π_{vf} , π_{mf} are the corresponding unit costs of feeder bus system. The expression of C_{vh}^f , i.e., vehicle-hour related cost of feeder buses, is given by:

$$C_{vh}^f = \frac{R^2}{S^2} \cdot \frac{\pi_{mf}}{\mu} \cdot \left(\int_{x=0}^S \frac{2(x+S)}{S_f(x)h(x)v_f} dx + q\tau_1 S^2 + \int_{x=0}^S \int_{y=0}^S \frac{2\tau_0}{S_f(x)B(x,y)h(x)} dx dy \right) \quad (16)$$

Where the first term in the parenthesis is the total bus hours when buses are operating at cruising speed. The second term is the total bus hours accounting for passenger boarding delay. The last term is the bus hours due to acceleration and deceleration delay.

C. Emission Cost

To enhance the environmental awareness, we further consider the emission cost generated by three major pollutants: HC, CO, and NO_x. The total emission cost of trunk-feeder system C_p entails two parts: the emission cost at stops/stations, C_p^1 ; the emission cost between stops/stations, C_p^2 . Each cost item is formulated as the sum of costs generated by feeder system and the cost by trunk system as shown in (17-18):

$$C_p^1 = C_{pf}^1 + C_{pt}^1 \quad (17)$$

where C_{pf}^1 , C_{pt}^1 are the emission costs at stops/stations of feeder/trunk system, respectively;

$$C_p^2 = C_{pf}^2 + C_{pt}^2 \quad (18)$$

where C_{pf}^2 , C_{pt}^2 are the emission costs between stops/stations of feeder/trunk system, respectively;

The emission cost that feeder buses emitted at stops is:

$$C_{pf}^1 = \sum_{n \in \{NO_x, HC, CO\}} \theta_{p,n} \cdot [P_f^{ab} + P_f^d] \cdot \frac{R^2}{S^2} \cdot \frac{1}{\mu} \quad (19)$$

where $\theta_{p,n}$ is the unit vehicle-related damage cost of pollutant n (\$/ton)^[8]; P_f^{ab} and P_f^d are the hourly emission volume generated from by stopping and dwelling at stops. They are estimated by:

$$P_f^{ab} = \int_{x=0}^S \int_{y=0}^S \left(e_n^a \cdot \frac{v_f}{a_v} + e_n^b \cdot \frac{v_f}{b_v} \right) \cdot \frac{1}{B(x,y)} \cdot \frac{1}{S_f(x)} \cdot \frac{1}{h(x)} dx dy;$$

$$P_f^d = \int_{x=0}^S e_n^d \cdot \frac{q^2 \tau_1 S^2}{2} \cdot S_f(x) \cdot h(x) dx;$$

$$n \in \{NO_x, HC, CO\}$$

where e_n^a , e_n^b , e_n^d are the emission rates (ton/hour) for pollutant n , when buses are accelerating, braking, and dwelling, respectively; a_v , b_v are the average acceleration rate and deceleration rates (km/h²).

The emission cost generated by cruising feeder buses C_{pf}^2 is:

$$C_{pf}^2 = \sum_{n \in \{NO_x, HC, CO\}} \left[\int_{x=0}^S e_n^c \cdot \frac{2(S+x)}{S_f(x)h(x)v_f} dx \right] \cdot \theta_{p,n} \cdot \frac{1}{\mu} \cdot \frac{R^2}{S^2} \quad (20)$$

Where e_n^c is the emission rates (ton/hour) for pollutant n when buses are cruising.

Similarly, for trunk system, the emission costs due to vehicle stopping and cruising are:

$$C_{pt}^1 = \sum_{n \in \{NO_x, HC, CO\}} P_t \cdot \theta_{p,n} \cdot \frac{R^2}{S_t^2} \cdot \frac{1}{\mu} \cdot \frac{1}{H} \quad (21)$$

$$C_{pt}^2 = \sum_{n \in \{NO_x, HC, CO\}} \tilde{e}_n^c \cdot \frac{2R^2}{S_t H V} \cdot \theta_{p,n} \cdot \frac{1}{\mu} \quad (22)$$

where $P_t = \left(\tilde{e}_n^a \cdot \frac{V}{a_t} + \tilde{e}_n^b \cdot \frac{V}{b_t} \right) + \tilde{e}_n^d \cdot \frac{q\tau_1 S_t^2 H}{2}$, indicates the volume of emission emitted per vehicle per station. a_t , b_t are trunk vehicles' average acceleration and deceleration rates, respectively. \tilde{e}_n^a , \tilde{e}_n^b , \tilde{e}_n^d are the corresponding emission rates for trunk vehicles.

D. Optimization Model

Thus, the optimization problem is formulated as minimizing the total system cost with five decision variables/functions: $S_f(x)$, $h(x)$, $B(x,y)$, S , H :

$$\min_{H, h(x), S, S_f(x), B(x,y)} GC \quad (23a)$$

subject to:

$$\begin{cases} h_{min} \leq h(x) \leq \frac{K_f}{qS_f(x)S} \\ H_{min} \leq H \leq \frac{6K_t}{qS_t R} \end{cases} \quad (23b)$$

$$H, h(x), S, S_f(x), B(x,y) > 0 \quad (23c)$$

where constraints (23b) guarantee vehicle loads never exceed feeder bus and trunk vehicle capacities.

E. Analytical analysis and solution method

The first-order conditions with respect to feeder system design variables/functions, $B(x, y), S_f(x), h(x)$ can be obtained as follows:

$$B(x, y)^* = \sqrt{\frac{2v_a}{q} \left[2q\tau_0 y + \frac{\pi_{sf}}{\mu S_f(x)} + \frac{\pi_{mf} \cdot 2\tau_0}{\mu S_f(x) h(x)} \right] + \frac{f_{10}(x)}{S_f(x) h(x)}} \quad (24)$$

$$\text{where } f_{10}(x) = \sum_{n=HC, CO, NOx} \frac{\left(e_n^a \frac{v_f}{a_v} + e_n^b \frac{v_f}{b_v} \right) \theta_{p,n}}{\mu};$$

$$S_f(x)^* = \sqrt{\frac{f_{11}(x) + \frac{f_{12}(x)}{h(x)} + \frac{f_{10}(x)}{h(x)} \int_0^S \frac{1}{B(x, y)} dy}{\frac{q}{2v_a} + q^2 h(x) \tau_1 S^2 + f_{13}(x) h(x)}} \quad (25)$$

where:

$$f_{11}(x) = \frac{\pi_{lf}}{\mu} \cdot 2(x+S) + \frac{\pi_{vf}}{\mu} \cdot \frac{2(x+S)}{h(x)} + \frac{\pi_{mf}}{\mu} \cdot \frac{2(x+S)}{h(x) v_f} + \left(\frac{\pi_{sf}}{\mu} + \frac{2\pi_{mf} \tau_0}{\mu h(x)} \right) \int_0^S \frac{1}{B(x, y)} dy;$$

$$f_{12}(x) = e_n^c \cdot \frac{2(S+x) \theta_{p,n}}{v_f \mu};$$

$$f_{13}(x) = e_n^d \cdot \frac{\tau_1 q^2 S^2 \theta_{p,n}}{2\mu};$$

$$h(x)^* = \sqrt{\frac{f_{14}(x) + \frac{f_{12}(x)}{S_f(x)} + \frac{f_{10}(x)}{S_f(x)} \int_0^S \frac{1}{B(x, y)} dy}{qS + q^2 \tau_1 S^2 S_f(x) + f_{13}(x) S_f(x)}} \quad (26)$$

where:

$$f_{14}(x) = \frac{\pi_{vf}}{\mu} \cdot \frac{2(x+S)}{S_f(x)} + \frac{\pi_{mf}}{\mu} \cdot \frac{2(x+S)}{S_f(x) v_f} + \frac{2\pi_{mf} \tau_0}{\mu S_f(x)} \int_0^S \frac{1}{B(x, y)} dy$$

Based on the above results, a bi-level iteration process is proposed. First, with random initial values solve the trunk system design variables, S, H , by the off-shelf fmincon algorithm in Matlab. Then, plug in the iteratively solve $B(x, y), S_f(x), h(x)$ until the convergence is reached.

III. NUMERICAL STUDY

A. Input Data

We suppose that the demand density is 250 passengers/km²/h (similar to San Francisco). The city area is a 10×10km² grid network (representing small size cities). In this case, the value of time is set to be 20\$/h. We investigate two major trunk transit modes: BRT and rail. The cost and operating parameters are taken from the previous study [9], and are summarized as shown in Table 1:

TABLE I. COST AND SYSTEM PARAMETERS FRO DIFFERENT TRANSIT MODES

| Cost parameters for different transit modes | | | | |
|---|-------------------|---------------------|---------------------|------------------------|
| Transit Modes | π_1 (\$/km/h) | π_s (\$/stop/h) | π_v (\$/veh/km) | π_m (\$/veh/h) |
| Bus | 6+0.2 μ | 0.42+0.014 μ | 0.59 | 2.66+3 μ |
| BRT | 162+5.4 μ | 4.2+0.14 μ | 0.66 | 3.81+4 μ |
| Rail | 594+19.8 μ | 294+9.8 μ | 2.20 | 101+5 μ |
| System parameters for different transit modes | | | | |
| Transit Modes | τ (s) | v (km/h) | K (pax/veh) | H _{min} (min) |
| Bus | 30 | 25 | 80 | 1 |

| Cost parameters for different transit modes | | | | |
|---|----|----|------|-----|
| BRT | 30 | 40 | 160 | 1 |
| Rail | 45 | 60 | 3000 | 1.5 |

We assume that all the feeder buses are 12m Compressed Natural Gas (CNG) bus. The emission standard for CNG bus is of China National IV standard. The emission rates of CNG buses at different driving regimes are given in Table 2. The data is adopted from previous study [10]. In this research, the emission generated by rail system is neglected compared with other transit modes:

TABLE II. EMISSION RATES OF POLLUTANTS AT DIFFERENT DRIVING REGIMES

| 12m China National IV Bus | | | | |
|---------------------------|-----------|--------------|--------------|-------------------|
| Pollutants | Idling | Acceleration | Deceleration | Constant velocity |
| NO _x (ton/h) | 1.296E-05 | 5.472E-05 | 2.556E-05 | 4.140E-05 |
| HC(ton/h) | 4.320E-06 | 1.224E-05 | 7.560E-06 | 9.720E-06 |
| CO (ton/h) | 7.596E-05 | 1.703E-04 | 1.091E-04 | 1.307E-04 |

B. The effects of heterogeneous feeder service design

We first compare the rail corridor with and without heterogeneous feeder service design. Table 3 summarizes the results including the optimal design and cost metrics.

TABLE III. RAIL-BUS SYSTEM CHARACTERISTICS FOR DIFFERENT FEEDER DESIGN PATTERNS

| System metrics | Uniform feeder design (rail-bus) | Heterogeneous feeder design (rail-bus) |
|-------------------------------|----------------------------------|--|
| S_f , km | 0.26 | $S_f(x) \in [0.32, 0.39]$ |
| Average B , km | 0.37 | $\bar{B}(x, y) = 0.43$ |
| h , min | 4.72 | $h(x) \in [5.9, 6.7]$ |
| S_t , km | 2.50 | 2.60 |
| H , min | 2.75 | 2.47 |
| avC_{ut}/avC_{uf} , min/pax | 16.75/25.79 | 9.82/28.36 |
| avC_{ot}/avC_{of} , min/pax | 22.70/5.58 | 6.66/4.03 |
| $avGC_t/avGC_f$, min/pax | 39.44/31.37 | 16.48/32.39 |
| $avGC$, min/pax | 70.81 | 48.87 |

As can be seen in Table 3, the average system cost is decreased by 31% (70.81min v.s. 48.87min) due to the heterogeneous design, which verifies the effectiveness of proposed model. A huge saving is found in the agency cost of the trunk system, with a decrease of 71%. Compared to the uniform design, the average stop spacing of feeder bus increases by 16% in the heterogeneous design. The bus line density function in a feeder service zone is presented in Fig. 2, where the circles represent the optimal locations of lines and dash lines are the boundaries of each line converge. The feeder line locations in this case are [0.325, 0.67, 1.033], in unit of km.

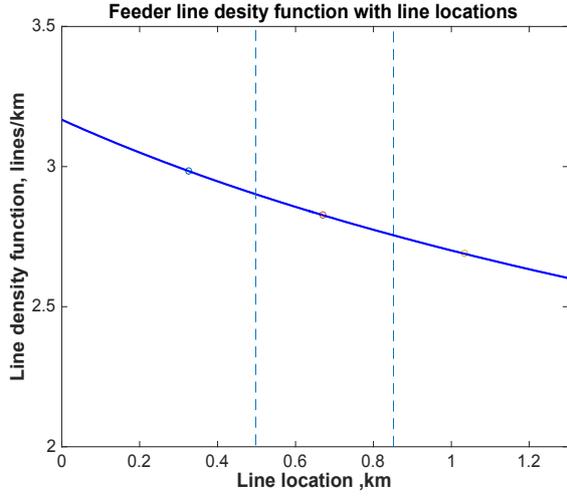
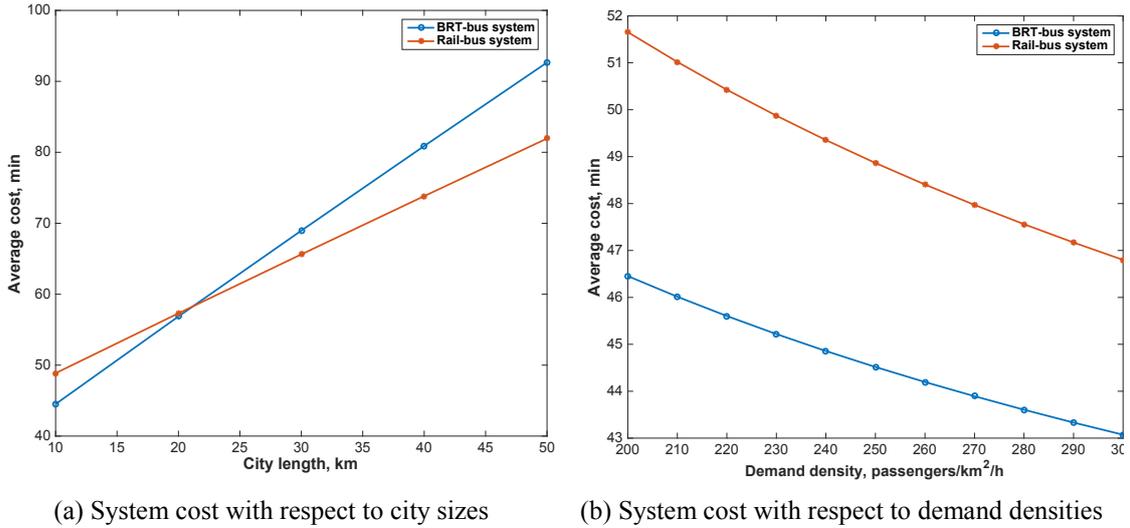


Fig.2. Rail-bus system feeder line optimal locations

We further consider scenarios with BRT as the trunk transit mode, and a large-sized city of 40×40 (km²) (representing the case of New York City). Table 4 summarizes the optimal system design and cost metrics under different scenarios. As seen in Table 4, the BRT-bus system is more competitive in the small-sized cities with lower system cost; whereas the rail-bus system triumphs in large-sized cities. This is reasonable because larger cities favors the trunk technology with higher operating speed and transport capacity.

| Small-sized cities: 10×10 km ² , 250 passengers/km ² /h | | |
|--|---------------------------|---------------------------|
| System metrics | BRT-bus system | Rail-bus system |
| S_f , km | $S_f(x) \in [0.23, 0.28]$ | $S_f(x) \in [0.32, 0.39]$ |
| Average B , km | $\bar{B}(x, y) = 0.37$ | $\bar{B}(x, y) = 0.43$ |
| h , min | $h(x) \in [7.3, 8.3]$ | $h(x) \in [5.9, 6.7]$ |
| S_t , km | 1.47 | 2.60 |
| H , min | 1.8 | 2.47 |
| avC_{ut}/avC_{uf} , min/pax | 13.77/22.18 | 9.82/28.36 |
| avC_{ot}/avC_{of} , min/pax | 3.75/4.81 | 6.66/4.03 |
| $avGC_t/avGC_f$, min/pax | 17.52/26.99 | 16.48/32.39 |
| $avGC$, min/pax | 44.52 | 48.87 |
| Large-sized cities: 40×40 km ² , 250 passengers/km ² /h | | |
| System metrics | BRT-bus system | Rail-bus system |
| S_f , km | $S_f(x) \in [0.29, 0.35]$ | $S_f(x) \in [0.36, 0.44]$ |
| Average B , km | $\bar{B}(x, y) = 0.37$ | $\bar{B}(x, y) = 0.415$ |
| h , min | $h(x) \in [6.3, 7.2]$ | $h(x) \in [5.3, 6.0]$ |
| S_t , km | 2.2 | 3.32 |
| H , min | 0.85 | 2.19 |
| avC_{ut}/avC_{uf} , min/pax | 48.22/24.79 | 33.79/31.03 |
| avC_{ot}/avC_{of} , min/pax | 3.64/4.24 | 5.21/3.79 |
| $avGC_t/avGC_f$, min/pax | 51.86/29.03 | 39.01/34.82 |
| $avGC$, min/pax | 80.89 | 73.83 |

TABLE IV. OPTIMAL RESULTS FOR DIFFERENT TRUNK-FEEDER SYSTEM IN DIFFERENT SCENARIOS



(a) System cost with respect to city sizes

(b) System cost with respect to demand densities

Fig.3. Trunk transit technology selection

To shed insights on the effects of key factors that influence the choice of trunk transit technologies, we conduct sensitivity analysis with respect to various city sizes and demand densities, as shown in Fig. 3. It is observed in Fig. 3(a) that the cost of rail-bus system approximately equals to

that of BRT-bus system in a 20×20 km² city area (with demand density equals to 250 passengers/km²/h). Beyond that, rail-bus system becomes more cost-effective. BRT-bus system appears to be always more preferable than rail-bus system in small sized cities with various levels of demand

density, as long as the vehicle capacity constraint is not binding. See Fig. 3(b).

The optimal designs of feeder services under the above scenarios are depicted in Fig. 4. It is found that the feeder lines of rail-bus system are always sparser than that of BRT-bus system, while feeder bus headway is shorter. It's due to the larger line spacing and higher operator cost of rail system.

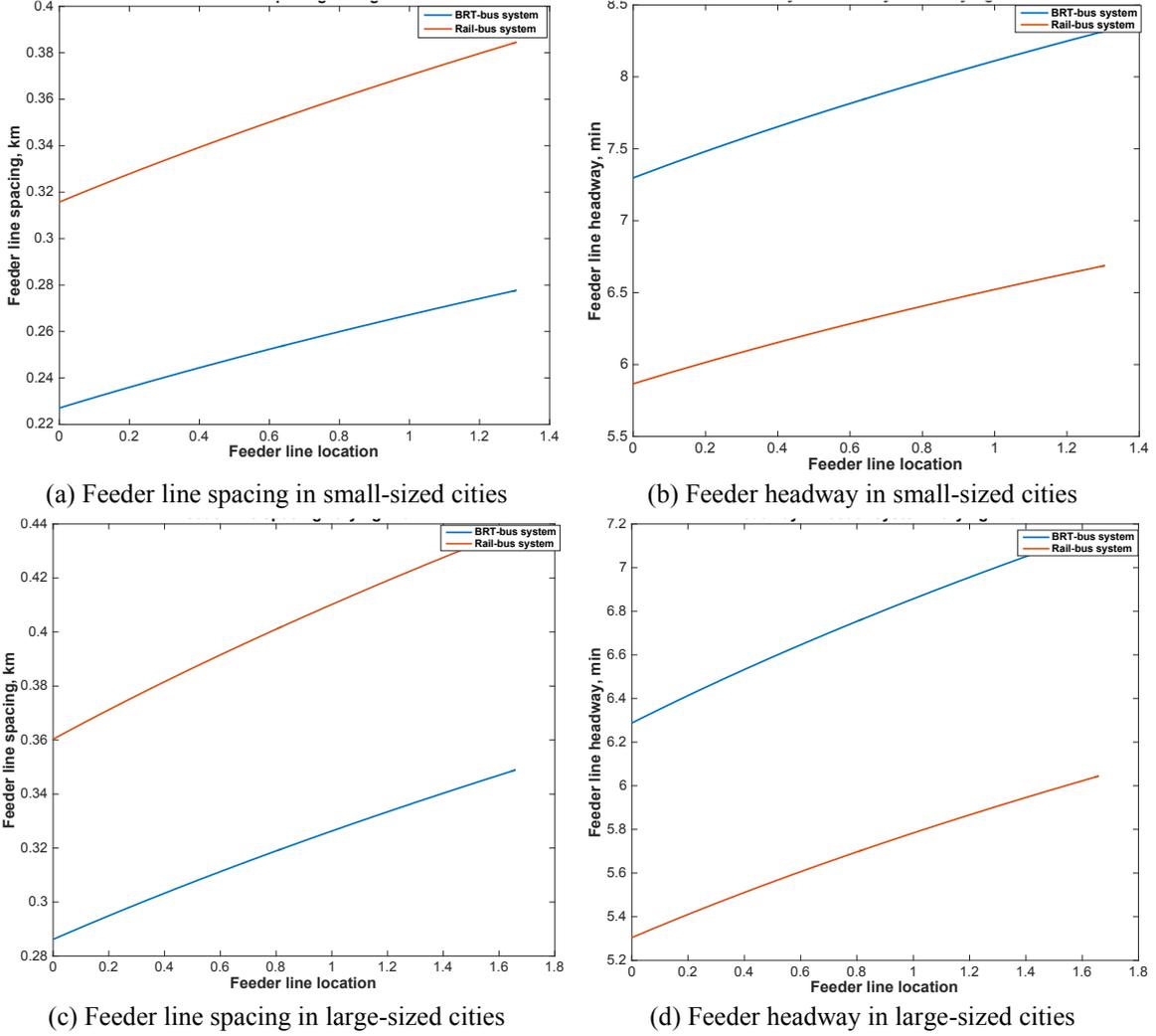


Fig.4. Design parameters of two trunk-feeder systems in different scenarios.

C. The effects of emission cost on system design

Lastly, we compare the optimal design of rail-bus corridor with and without considering emission cost. The results are summarized in Table 5. As seen, with the environmental awareness, the feeder line spacing and service headway increase while the rail lines spacing shrinks. Integrating emission cost into the objective function of the proposed design model achieves a reduction of 9.43% in actual emission cost.

TABLE V. OPTIMAL RESULTS OF RAIL-BUS SYSTEM WITH OR WITHOUT EMISSION CONSIDERATION

| System metrics | Without accounting for emission | With accounting for emission |
|----------------|---------------------------------|------------------------------|
| S_f , km | $S_f(x) \in [0.32, 0.39]$ | $S_f(x) \in [0.34, 0.42]$ |

Besides, we find that both the spacing and service headway of feeder lines become larger as the distance to trunk station increases. This is because the non-stop line-haul length and the infrastructure cost of feeder lines increases as their locations being away from the trunk station. Thus to optimize the system cost, it's more profitable to have larger-spaced feeder lines and fewer dispatched buses in the distant area of trunk stations.

| System metrics | Without accounting for emission | With accounting for emission |
|-------------------------------|---------------------------------|------------------------------|
| Average B , km | $\bar{B}(x, y) = 0.43$ | $\bar{B}(x, y) = 0.43$ |
| h , min | $h(x) \in [5.9, 6.7]$ | $h(x) \in [6.3, 7.4]$ |
| S_t , km | 2.60 | 2.58 |
| H , min | 2.47 | 2.49 |
| avC_{ut}/avC_{uf} , min/pax | 9.82/28.36 | 9.85/29.16 |
| avC_{ot}/avC_{of} , min/pax | 6.66/4.03 | 6.73/3.62 |
| avC_{pt}/avC_{pf} , min/pax | 0/(0.58)* | 0/0.53 |
| $avGC_t/avGC_f$, min/pax | 16.48/32.39 | 16.58/33.31 |
| $avGC$, min/pax | 48.87 | 49.89 |

*In parenthesis is the actual emission cost that is not accounted in the objective function of the scenario.

CONCLUSIONS AND EXTENSIONS

In this research, we establish a joint optimization model for trunk-feeder system that allows feeder bus line spacing and stop spacing vary spatially in fit with the demand. To enhance the environmental awareness, we further consider the emission cost into the objective function that also minimize patrons' and transit agency's cost. Solution method is developed based on analytical analysis of the decision variables/functions. In numerical studies, we consider various scenarios with respect to city sizes, demand levels, and trunk transit modes. Main findings include that: (i) the heterogeneous design may lead to 31% saving in the system cost as compared to traditional uniform feeder design; (ii) rail-bus system better fit large-sized cities, while BRT-bus system is more preferable in small-sized cities; and (iii) the optimally designed feeder bus services may also achieve up to 9.43% emission cost reduction. The results also shed lights to the deployment of bus lines in feeder service zones, e.g., the principle of being larger spaced as the distance to the trunk station increases.

For future extension, we plan to consider the heterogeneous design in trunk network as well, and account for spatially non-uniform demand in respect to more realistic demand patterns.

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