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# Minimal single linear functional observers for discrete-time linear systems

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**Abstract**—In this paper, the direct approach which was introduced for the first time by [Rotella and Zambettakis, 2011] in designing minimal functional state observers is extended to deal with discrete-time systems. One of the benefits of this approach is that it does not require solving the Sylvester matrix equation that appears in other observer design procedures. Both stable observers and arbitrary fixed poles observers problems are considered for minimality. A numerical example and simulation results explain the effectiveness and the benefits of the proposed algorithm.

## I. INTRODUCTION

In observer design theory, it does not usually required to observe all the states of the system so only a few number of functions of the states can be estimated. Ever since Luenberger has introduced the concept of functional observer in 1966 [Luenberger, 1966], this field of study has been a subject of intensive researches (see [O'Reilly, 1983], [Aldeen and Trinh, 1999], [Darouach, 2000], [Trinh and Fernando, 2011]). This scheme is more general than reduced order Luenberger observers and it has been applied in several applications like observer-based feedback control, fault detection and system monitoring [Trinh *et al.*, 2013], [Patton and Chen, 1997], [Hou and Muller, 1994]. Because of their ability to reduce the cost and complexity of control systems, there have been considerable attention drawn to functional observers. However, only little attention has been paid toward discrete-time systems.

In literature, we distinguish three design procedures to reconstruct a set of linear functions: the parametric approach ([Trinh *et al.*, 2006], [Trinh *et al.*, 2008]), the conventional approach ([Aldeen and Trinh, 1999], [Darouach, 2000], [Duan, 1992]), and the direct approach [Rotella and Zambettakis, 2011], [Rotella and Zambettakis, 2016].

The main problem faced in designing functional observers is finding a method to solve Sylvester equation that appears in the observer design approach ([Trinh *et al.*, 2006], [Trinh and Fernando, 2011]). These interconnected equations usually have an infinite number of solutions and each method in the solving tries to optimize some criteria and satisfy robustness and performance.

The direct procedure for designing linear state functional observers was first presented in [Rotella and Zambettakis, 2011] to estimate a single function of the states of a Linear Time-Invariant (LTI) system. This scheme is the only available one extended to a Linear Time-Varying (LTV) systems in [Rotella and Zambettakis, 2013]. An extension to designing minimal multi-functional observers is proposed in the paper of [Rotella and Zambettakis, 2016], [Mohajerpoor *et al.*, 2015].

Comparing to the other observer design approaches, the direct scheme is the only method that does not require solving the Sylvester equation to find the observer parameters, it's based on linear algebraic operations in a state space setting.

In the present paper, the direct approach is extended to reconstruct single linear functional of the states for large-scale linear discrete-time systems in such way that the minimum possible order of the observer is achieved.

The paper is structured as follows. The problem is illustrated in Section II, and some preliminary definitions, theorems, and remarks are given. An extension of the direct approach in designing a minimal single linear functional observers to the discrete-time framework is proposed in Section III. In Section IV, the design procedure illustrated with the same numerical example provided in [Rotella and Zambettakis, 2011], and simulation results are reported. The paper is concluded in Section V.

## II. PRELIMINARIES

Lets consider the following discrete-time LTI system:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \quad (1)$$

where  $x(k) \in \mathbb{R}^n$ ,  $u(k) \in \mathbb{R}^p$  and  $y(k) \in \mathbb{R}^m$  are the state vector, the control input vector and the measurement output vector, respectively. Matrices  $A \in \mathbb{R}^n \times \mathbb{R}^n$ ,  $B \in \mathbb{R}^n \times \mathbb{R}^p$  and  $C \in \mathbb{R}^m \times \mathbb{R}^n$  are known constant.

Let  $v(k)$  be a vector that is required to be reconstructed (or estimated), such that:

$$v(k) = Lx(k) \quad (2)$$

and  $L \in \mathbb{R}^{(l \times n)}$  a constant full row rank known matrix.

To reconstruct the state function,  $v(k)$ , the following Luenberger observer structure of order  $q$ ,  $q \leq (n-p)$ , is proposed

$$\begin{aligned} z(k+1) &= Fz(k) + Gu(k) + Hy(k) \\ w(k) &= Pz(k) + Vy(k) \end{aligned} \quad (3)$$

where  $z(k)$  is the  $q$ -dimensional state vector and  $w(k) \in \mathbb{R}^l$  is the estimate of  $v(k)$ .

The observer matrices  $F \in \mathbb{R}^{(q \times q)}$ ,  $G \in \mathbb{R}^{(q \times p)}$ ,  $H \in \mathbb{R}^{(q \times m)}$ ,  $P \in \mathbb{R}^{(l \times q)}$  and  $V \in \mathbb{R}^{(l \times m)}$  are determined such that:

$$\lim_{k \rightarrow \infty} (v(k) - w(k)) = 0 \quad (4)$$

This asymptotic tracking is ensured if  $F$  is a Schur matrix. Namely, if all the eigenvalues of  $F$  are inside an unit open circle.

Following [Fortman and Williamson, 1972], the linear functional observer (3) exists if and only if there exists a  $(q \times n)$  matrix  $T$  such that:

$$G = TB \quad (5)$$

$$TA - FT = HC \quad (6)$$

$$L = PT + VC \quad (7)$$

$$F \text{ is a Schur matrix.} \quad (8)$$

Note that  $L$  can always be chosen to be a controller gain that stabilizes the closed-loop system matrix  $(A + BL)$ , then the linear functional observer (3) would provide an estimate of the corresponding control signal to be directly feedback into the system. On the other hand, the designer can always chose  $L$  to represent any desired partial set of the state vector that needs to be estimated.

In Fig 1 the observer structure design is displayed.

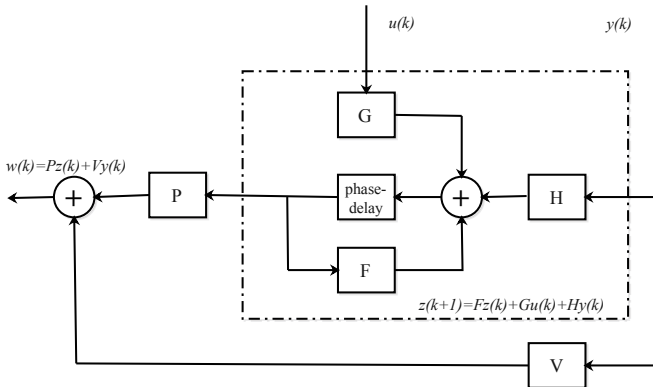


Fig. 1. Linear functional observer structure design.

Recently, the interesting notion of functional observability of the triplet  $(A; C; L)$  defined in [Fernando *et al.*, 2010],

[Jennings *et al.*, 2011] was introduced to tackle the problem of designing a minimal order functional observer.

**Definition 1.** [Fernando *et al.*, 2010], [Jennings *et al.*, 2011] The triple  $(A, C, L)$  is functional observable if and only if

$$\text{rank} \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \\ L \\ LA \\ \vdots \\ LA^{n-1} \end{pmatrix} = \text{rank} \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{pmatrix} \quad (9)$$

**Theorem 1.** [Fernando *et al.*, 2010], [Jennings *et al.*, 2011] There exists an asymptotic functional observer with structure (3) for the system (1) if the triple  $(A, C, L)$  is functional observable.

Clearly when the condition (9) is satisfied, there are matrices  $L_0, \dots, L_{n-1}$ , such that:

$$L = \sum_{i=0}^{n-1} L_i CA^i \quad (10)$$

therefore

$$\text{rank} \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \\ L \end{pmatrix} = \text{rank} \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{pmatrix} \quad (11)$$

Conversely, suppose that  $L$  can be written as (10). Thus, it is easy to prove by induction that, for all  $k \in \{0, \dots, n-1\}$ , there exist  $L_{k,i}$  matrices such that:

$$LA^k = \sum_{i=0}^{n-1} L_{k,i} CA^i$$

These relationships lead to (9). So, we can claim the triplet  $(A; C; L)$  is functionally observable if and only if (11) is fulfilled.

Functional observability is a sufficient but not necessary criteria for the existence of an asymptotic observer. Nevertheless, functional detectability defined in [Fernando *et al.*, 2010] is a necessary and sufficient condition to investigate the existence of a solution to the observer design problem.

An alternative to characterize the functional observability for linear systems has been developed in [Rotella and Zambettakis, 2015]. A necessary and sufficient condition for the existence of a stable multi-functional observer of a time-invariant linear system is defined. This criterion to test functional observability of a triplet  $(A; C; L)$  leads to a constructive procedure of functional observer neither based on the use of canonical state space forms nor on the solution of a Sylvester equation.

### III. DESIGN OF MINIMAL SINGLE LINEAR FUNCTIONAL OBSERVERS IN DISCRETE-TIME FRAMEWORK

Let us define  $q$  as the smallest integer such that:

$$\text{rank}(\Sigma_q) = \text{rank} \left( \begin{bmatrix} \Sigma_q \\ LA^q \end{bmatrix} \right) \quad (12)$$

with:

$$\Sigma_q = \begin{bmatrix} C \\ L \\ CA \\ LA \\ \vdots \\ CA^{q-1} \\ LA^{q-1} \\ CA^q \end{bmatrix} \quad (13)$$

After  $q$  phase-advanced of  $v(k) = Lx(k)$ , we obtain:

$$v(k+q) = LA^q x(k) + \sum_{i=0}^{q-1} LA^i Bu(k+q-1-i) \quad (14)$$

From (12), there exist  $\Gamma_i$  for  $i = 0$  to  $q$  and  $\Lambda_i$  for  $i = 0$  to  $q-1$  such that:

$$LA^q = \sum_{i=0}^q \Gamma_i CA^i + \sum_{i=0}^{q-1} \Lambda_i LA^i \quad (15)$$

Thus (14) can be written as:

$$v(k+q) = \sum_{i=0}^q \Gamma_i CA^i x(k) + \sum_{i=0}^{q-1} \Lambda_i LA^i x(k) + \sum_{i=0}^{q-1} LA^i Bu(k+q-1-i) \quad (16)$$

To eliminate the state  $x(k)$  we have the equalities :

$$\begin{aligned} Lx(k) &= v(k) \\ LAx(k) &= v(k+1) - LBu(k) \\ &\vdots \\ LA^{(q-1)}x(k) &= v(k+q-1) - \sum_{i=0}^{q-2} LA^i Bu(k+q-2-i) \\ Cx(k) &= y(k) \\ CAx(k) &= y(k+1) - CBu(k) \\ CA^2x(k) &= y(k+2) - CABu(k) - CBu(k+1) \\ &\vdots \\ CA^q x(k) &= y(k+q) - \sum_{i=0}^{q-1} CA^i Bu(k+q-1-i) \end{aligned}$$

It yields:

$$\begin{aligned} v(k+q) &= \sum_{i=0}^{q-1} LA^i Bu(k+q-1-i) \\ &+ \Gamma_0 y(k) + \sum_{i=1}^q \Gamma_i \left[ y(k+i) - \sum_{j=0}^{i-1} CA^j Bu(k+i-1-j) \right] \\ &+ \Lambda_0 v(k) + \sum_{i=1}^{q-1} \Lambda_i \left[ v(k+i) - \sum_{j=0}^{i-1} LA^j Bu(k+i-1-j) \right] \end{aligned}$$

$$= \sum_{i=0}^q \Gamma_i y(k+i) + \sum_{i=0}^{q-1} \Lambda_i v(k+i) + \sum_{i=0}^{q-1} \Phi_i u(k+i) \quad (17)$$

where for  $i = 0$  to  $q-2$

$$\Phi_i = \left[ LA^{q-1-i} - \sum_{j=i+1}^q \Gamma_j CA^{j-i-1} - \sum_{j=i+1}^{q-1} \Lambda_j LA^{j-i-1} \right] B \quad (18)$$

and:

$$\Phi_{q-1} = [L - \Gamma_q C] B \quad (19)$$

The input-output recurrence equation (17) can be realized as the  $q$ -order state space observable system:

$$\begin{aligned} z(k+1) &= Fz(k) + Gu(k) + Hy(k) \\ w(k) &= Pz(k) + Vy(k) \end{aligned} \quad (20)$$

where

$$F = \begin{bmatrix} 0 & & & \Lambda_0 \\ 1 & \ddots & & \Lambda_1 \\ & \ddots & 0 & \vdots \\ & & 1 & \Lambda_{q-1} \end{bmatrix}, G = \begin{bmatrix} \phi_0 \\ \phi_1 \\ \vdots \\ \phi_{q-1} \end{bmatrix}$$

$$H = \begin{bmatrix} \Gamma_0 + \Lambda_0 \Gamma_q \\ \Gamma_1 + \Lambda_1 \Gamma_q \\ \vdots \\ \Gamma_{q-1} + \Lambda_{q-1} \Gamma_q \end{bmatrix}, P = [0 \quad \dots \quad 0 \quad 1]$$

and  $V = \Gamma_q$ .

### IV. NUMERICAL EXAMPLE

In order to illustrate the proposed design approach, let us consider the same example in [Trinh *et al.*, 2006] and [Rotella and Zambettakis, 2011] with the continuous time LTI system given by:

$$\begin{aligned} \dot{x}(t) &= A_c x(t) + B_c u(t) \\ y(t) &= C_c x(t) \end{aligned} \quad (21)$$

with

$$A_c = \begin{bmatrix} -1 & 0 & 0 & 1 & -2 \\ 0 & -5 & 3 & 4 & 0 \\ 1 & 1 & -8 & 3 & 0 \\ -4 & 0 & 2 & -6 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}, B_c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (22)$$

$$C_c = [1 \quad 0 \quad 0 \quad 0 \quad 0]$$

and the single functional defined by:

$$L = [1 \quad 14 \quad 42 \quad 79 \quad 2] \quad (23)$$

Discretization of linear state space models (22) with the sampling time  $T = 0.1s$  leads to write in the discrete-time LTI system (1) with:

$$A = \begin{bmatrix} 0.8907 & 0.0002 & 0.0057 & 0.0637 & -0.18 \\ -0.0475 & 0.6156 & 0.1809 & 0.2569 & 0.0033 \\ 0.027 & 0.0532 & 0.4722 & 0.164 & -0.0045 \\ -0.2794 & 0.0053 & 0.1003 & 0.5531 & 0.0304 \\ -0.0152 & 0.0002 & 0.0062 & 0.0714 & 0.9059 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.0093 \\ 0.0001 \\ -0.0002 \\ 0.0011 \\ 0.0952 \end{bmatrix}, C = [ 1 \ 0 \ 0 \ 0 \ 0 ]$$

The following steps illustrate the procedure for designing the minimum observer.

Testifying condition (12), we get  $\text{rank}(\Sigma_2) = 5$  and  $\text{rank} \begin{bmatrix} \Sigma_2 \\ LA^2 \end{bmatrix} = 5$ . It is found that  $q = 2$ .

As:

$$LA^2\Sigma_2^{-1} = \begin{bmatrix} -1.1417 & -0.4325 & 14.3742 & 1.3226 & -22.148 \end{bmatrix}$$

we deduce  $\Lambda_0 = -0.4325$  and  $\Lambda_1 = 1.3226$ , leading to:

$$F = \begin{bmatrix} 0 & -0.4325 \\ 1 & 1.3226 \end{bmatrix}.$$

The eigenvalues of  $F$  are  $\{0.5919, 0.7307\}$ . Then  $F$  is a Schur matrix which means that a minimal second-order observer can be designed.

#### A. Design of the minimal second-order observer

From  $LA^2\Sigma_2^{-1}$  we get  $\Gamma_0 = -1.1417$ ,  $\Gamma_1 = 14.3742$  and  $\Gamma_2 = -22.148$ . Thus, from (20) we get:

$$G = \begin{bmatrix} -0.1534 \\ 0.0535 \end{bmatrix}, H = \begin{bmatrix} 8.4367 \\ -14.9178 \end{bmatrix},$$

$$P = [ 0 \ 1 ], V = -22.1480.$$

For an initial condition  $v(0) - w(0) = 200$ , the simulation results are given in Fig. 2.

It is clear that the estimated function has asymptotically converged to its true value. Moreover, the observer is designed with an arbitrary asymptotic convergence speed due to the eigenvalues of  $F$ . To increase performance and obtaining a fastest convergence rate, we have to augment the order of the observer as follows.

#### B. Design of the minimal third-order observer

A minimal third-order observer with partially fixed poles can be designed using the following procedure.

From:

$$CA^3 = [ \Gamma_{20} \ \Lambda_{20} \ \Gamma_{21} \ \Lambda_{21} \ \Gamma_{22} ] \Sigma_2,$$

$$LA^2 = [ \Pi_0 \ \Delta_0 \ \Pi_1 \ \Delta_1 \ \Pi_2 ] \Sigma_2,$$

we deduce:

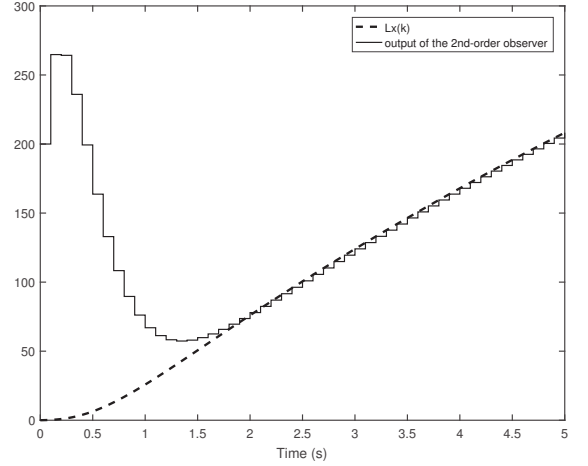


Fig. 2. Simulation results for the implementation of the second-order functional observer

$$\begin{bmatrix} \Gamma_{20} \\ \Lambda_{20} \\ \Gamma_{21} \\ \Lambda_{21} \\ \Gamma_{22} \end{bmatrix} = \begin{bmatrix} 0.2812 \\ -0.0008 \\ -1.3818 \\ 0.0010 \\ 2.1149 \end{bmatrix} \text{ and } \begin{bmatrix} \Pi_0 \\ \Delta_0 \\ \Pi_1 \\ \Delta_1 \\ \Pi_2 \end{bmatrix} = \begin{bmatrix} -1.1417 \\ -0.4325 \\ 14.3742 \\ 1.3226 \\ -22.1480 \end{bmatrix}$$

On the one hand, we have:

$$\begin{aligned} LA^3 &= LA^2A = \Pi_0CA + \Delta_0LA + \Pi_1CA^2 + \Delta LA^2 \\ &\quad + \Pi_2CA^3, \\ &= \Pi_0CA + \Delta_0LA + \Pi_1CA^2 \\ &\quad + \Delta_1 [ \Pi_0 \ \Delta_0 \ \Pi_1 \ \Delta_1 \ \Pi_2 ] \Sigma_2 \\ &\quad + \Pi_2 [ \Gamma_{20} \ \Lambda_{20} \ \Gamma_{21} \ \Lambda_{21} \ \Gamma_{22} ] \Sigma_2, \\ &= \begin{bmatrix} \Delta_1\Pi_0 + \Pi_2\Gamma_{20} \\ \Delta_0\Delta_1 + \Pi_2\Lambda_{20} \\ \Pi_0 + \Delta_1\Pi_1 + \Pi_2\Gamma_{21} \\ \Delta_0 + \Delta_1\Delta_1 + \Pi_2\Lambda_{21} \\ \Pi_1 + \Delta_1\Pi_2 + \Pi_2\Gamma_{22} \end{bmatrix}^T \Sigma_2 \end{aligned}$$

and on the other hand, as:

$$LA^3 = [ \Gamma_0 \ \Lambda_0 \ \Gamma_1 \ \Lambda_1 \ \Gamma_2 \ \Lambda_2 \ \Gamma_3 ] \Sigma_3$$

and  $\Sigma_3$  is not invertible, we are then led to:

$$\begin{aligned} LA^3 &= \Gamma_0C + \Lambda_0L + \Gamma_1CA + \Lambda_1LA + \Gamma_2CA^2 \\ &\quad + \Lambda_2LA^2 + \Gamma_3CA^3, \\ &= \Gamma_0C + \Lambda_0L + \Gamma_1CA + \Lambda_1LA + \Gamma_2CA^2 \\ &\quad + \Lambda_2 [ \Pi_0 \ \Delta_0 \ \Pi_1 \ \Delta_1 \ \Pi_2 ] \Sigma_2 \\ &\quad + \Gamma_3 [ \Gamma_{20} \ \Lambda_{20} \ \Gamma_{21} \ \Lambda_{21} \ \Gamma_{22} ] \Sigma_2, \\ &= \begin{bmatrix} \Gamma_0 + \Lambda_2\Pi_0 + \Gamma_3\Gamma_{20} \\ \Lambda_0 + \Lambda_2\Delta_0 + \Gamma_3\Lambda_{20} \\ \Gamma_1 + \Lambda_2\Pi_1 + \Gamma_3\Gamma_{21} \\ \Lambda_1 + \Lambda_2\Delta_1 + \Gamma_3\Lambda_{21} \\ \Gamma_2 + \Lambda_2\Pi_2 + \Gamma_3\Gamma_{22} \end{bmatrix}^T \Sigma_2 \end{aligned}$$

where  $\Lambda_2$  and  $\Gamma_3$  are two design parameters. It yields

$$\begin{aligned} \Gamma_0 &= \Delta_1\Pi_0 + \Pi_2\Gamma_{20} - \Lambda_2\Pi_0 - \Gamma_3\Gamma_{20}, \\ \Gamma_1 &= \Pi_0 + \Delta_1\Pi_1 + \Pi_2\Gamma_{21} - \Lambda_2\Pi_1 - \Gamma_3\Gamma_{21}, \\ \Gamma_2 &= \Pi_1 + \Delta_1\Pi_2 + \Pi_2\Gamma_{22} - \Lambda_2\Pi_2 - \Gamma_3\Gamma_{22}, \end{aligned}$$

and:

$$\begin{aligned} \Lambda_0 &= \Delta_0\Delta_1 + \Pi_2\Lambda_{20} - \Lambda_2\Delta_0 - \Gamma_3\Lambda_{20}, \\ \Lambda_1 &= \Delta_0 + \Delta_1\Delta_1 + \Pi_2\Lambda_{21} - \Lambda_2\Delta_1 - \Gamma_3\Lambda_{21}. \end{aligned}$$

When  $\Lambda_2$  and  $\Gamma_3$  are chosen, these five parameters are known and we can design the third-order observer (20).

The poles of the matrix  $F$  are the roots of the characteristic polynomial  $p_F(\lambda) = \lambda^3 - \Lambda_2\lambda^2 - \Lambda_1\lambda - \Lambda_0$  which depends on the parameters  $\Gamma_3$  and  $\Lambda_2$ .

We choose these poles  $\{-0.0370 \pm 0.1698i, 0.4346\}$ . It yields  $p_F(\lambda) = \lambda^3 + 0.7437\lambda^2 + 0.0020\lambda + 209.9125$ , we get  $\Lambda_0 = 209.9125$ ,  $\Lambda_1 = 0.0020$  and  $\Lambda_2 = 0.7437$ . These equalities are consistent and yield  $\Gamma_3 = 325.6895$ . For these values we get the Luenberger observer defined by

$$G = \begin{bmatrix} -1.0443 \\ -0.3931 \\ 3.2993 \end{bmatrix}, F = \begin{bmatrix} 0 & 0 & 0.0131 \\ 1 & 0 & 0.0020 \\ 0 & 1 & 0.7437 \end{bmatrix},$$

$$H = \begin{bmatrix} -94.1830 \\ 488.4553 \\ -491.8594 \end{bmatrix}, P = [ 0 \quad 0 \quad 1 ], V = 325.6895$$

For the same initial condition, the simulation results of the third-order observer is given in Fig. 3.

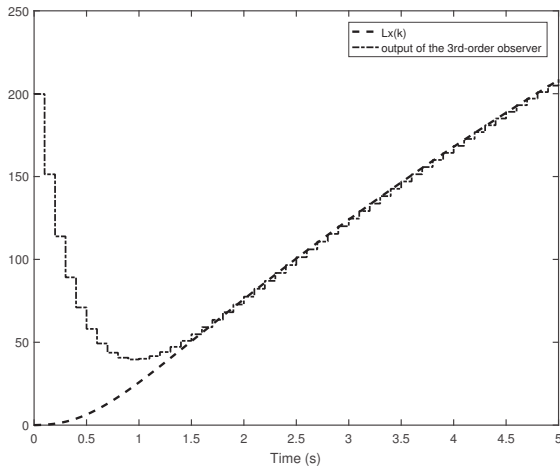


Fig. 3. Simulation results for the implementation of the third-order functional observer

It is clear that all the estimated functions converge asymptotically to their true values.

Moreover, the estimated value with the third-order observer has the fastest convergence rate than the one with the second-order observer (Fig. 3), which was expected due to the selected observer's eigenvalues.

## V. CONCLUSION

A minimal single linear functional observers design for discrete-time LTI systems using the direct approach has been addressed. The observer is designed so that an asymptotic functional observer can be obtained with arbitrary convergence speed. A numerical example and simulation illustrated the effectiveness of the proposed approach. In this example, we have points out that we can fix the observation error at any desired rate.

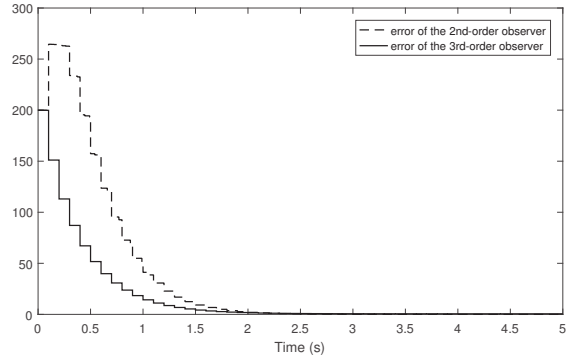


Fig. 4. Estimation errors of the second and the third-order functional observer

Future works will concern the extension of the direct approach to unknown-input functional observers with minimum possible order.

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