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# Analytical modeling of the CNC machine axis motion in high-speed milling with local smoothing

Mohamed Essid<sup>1,2</sup>  · Bassem Gassara<sup>1</sup> · Maher Baili<sup>3</sup> · Moncef Hbaieb<sup>1,2</sup> · Gilles Dessein<sup>3</sup> · Wassila Bouzid Saï<sup>1</sup>

## Abstract

In high-speed milling (HSM) of free-form surfaces, the tool path is mainly characterized by a set of short discontinuous segments in tangency. These discontinuities bring an intense kinematic parameter fluctuation of the computer numerical control (CNC) machine axes. The smoothing of the programmed tool path and the optimal choice of the manufacturing process parameters ensure a high productivity with the required quality of the machined surfaces. Precise estimation of the machining time allows an accurate evaluation of the machined product cost. Thereby, this is achieved by studying the effect of the tool path smoothing and the CNC controller parameters on the axis kinematics and the following errors. This paper focuses on the geometrical modeling of the local smoothing block adopted by Sinumerik CNC. Further, a kinematic model is proposed to simulate the axis motion in linear interpolation mode with local smoothing. The identification of the interpolator reduction effect on the programmed tolerance leads to the identification of the smoothing model adopted by the CNC unit. Then, the axis kinematic behavior is modeled while taking into account the drive parameter axes defined by the manufacturer of the CNC unit. The experimental results showed a smooth variation of the axis feed rate along the smoothing blocks and a good correlation with the proposed models.

**Keywords** Tool path setpoint · Interpolator · Following error · Hermit polynomial · Feed rate

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## Nomenclature

$S_i(u)$	axis position along the smoothing block at point $P_i$
$S_i'(u)$	first parametric derivative of $S_i(u)$
$S_i''(u)$	second parametric derivative of $S_i(u)$
$T_{L_i}$	transition length of $S_i(u)$
$\varepsilon_i$	maximum contour tolerance of $S_i(u)$
$\varepsilon_X$	X-axis tolerance
$\varepsilon_Y$	Y-axis tolerance
$\rho$	programmed contour tolerance
$e_{fj}$	j-axis following error
$\alpha$	weight of two tangential vectors ( $T_i$ and $T_{i+1}$ ) to $S_i(u)$
$\beta_i$	reduction rate of the programmed tolerance bound to $S_i(u)$
$\theta_i$	deviation angle of two successive linear blocks at point $P_i$
$\Delta_i$	bisector of the angle formed by two successive segments of the programmed tool path at point $P_i$
$k_i(u)$	curvature of the smoothing block $S_i(u)$
$j = 1 : 3$	for the X-, Y-, and Z-axes of the machine tool

$T_i(u)$	unit tangential vector to $S_i(u)$
$N_i(u)$	unit normal vector to $S_i(u)$
$V_{M_j}$	maximum velocity of axis $j$
$A_{M_j}$	maximum acceleration of axis $j$
$J_{M_j}$	maximum jerk of axis $j$ in linear interpolation mode
$Jtb_{M_j}$	maximum jerk of axis $j$ specific to the transition block
$R_A$	rate of acceleration allowed in radial acceleration
$R_j$	rate of jerk allowed in radial jerk
$A_{RM_j}$	maximum radial acceleration of axis $j$
$A_{TM_j}$	maximum tangential acceleration of axis $j$
$Jtb_{RM_j}$	maximum radial jerk of axis $j$
$Jtb_{TM_j}$	maximum tangential jerk of axis $j$
$t_{cyl}$	cycle time
$f_s$	sampling factor
$t_s$	sampling time
$u_i(t)$	parametric position of the axes along $S_i(u)$
$\dot{u}_i(t)$	parametric feed rate along $S_i(u)$
$\dot{u}_{iL} _{u=0}$	parametric feed rate limit at the input of $S_i(u)$
$\dot{u}_i _{u=0}$	parametric feed rate at the input of $S_i(u)$
$u_i(t)$	parametric acceleration along $S_i(u)$
$u_i _{u=0}$	parametric acceleration at the input of $S_i(u)$
$u_i(u)$	parametric jerk along $S_i(u)$
$VF_P$	programmed feed rate
$Vf_j(t)$	feed rate of axis $j$ along the tool path
$V(t)$	axis feed rate along $S_i(u)$
$Vf(t)$	feed rate along the tool path
$Vin_{iL}$	limit feed rate at the input of $S_i(u)$
$Vin_i$	feed rate at the input of $S_i(u)$
$Vout_i$	feed rate at the output of $S_i(u)$
$A_i(t)$	axis acceleration along $S_i(u)$
$J_i(t)$	axis jerk along $S_i(u)$

## 1 Introduction

In the field of high-speed sculptured surface machining, productivity, quality, and accuracy criteria of the finished product become more and more exigent. The improvement of these criteria requires a special attention to the manufacturing process.

The workpiece geometry is often defined by parametric splines in computer-aided design (CAD) environment. Then, the computer-aided manufacturing (CAM) system converts the CAD geometry into a polyhedron.

The choice of machining strategy leads to the identification of a set of the polyhedron linear segments describing the tool trajectory. These are transformed into a numerical control (NC) program through the post processor. During the machining phase, the programmed tool path undergoes two essential transformations. The interpolator converts the NC file into

position setpoint, and the drive controller transforms the tool path setpoint into analogic data and allows the control and the regulation of axis motion (Fig. 1).

The tangential discontinuity of tool path, the high dynamics of the computer numerical control (CNC) machine axes, and the offset between the setpoint and the actual axis position are the essential loss roots of time, quality, and precision of the machined surfaces. Tool path smoothing allows the transition from an intense fluctuation of the axis feed rate to a regular variation, hence a gain in machining time and machined surface quality [1, 2].

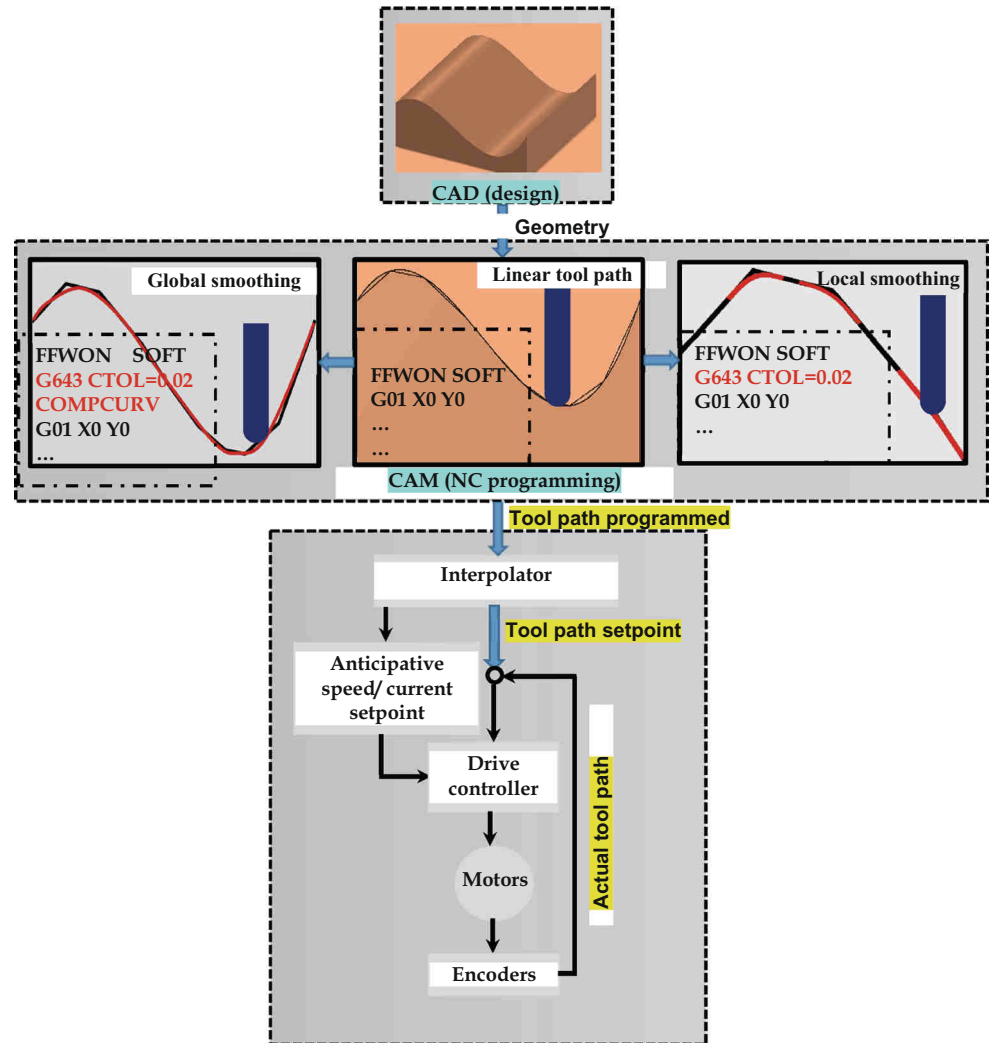
The feed-forward control function (anticipative setpoint speed or current) and the optimal choice of the manufacturing process parameters reduce the offset between the setpoint position and the actual position of the machine axes [1, 2].

For the reason that most of the simulation software parameters of the CNC machine behavior do not take into account the kinematic limits bound to the smoothing operation (Catia, Mastercam, NX CAM), several research activities are conducted to simulate the smoothed tool path and the feed rate ( $Vf(t)$ ). The literature dealing with this subject can be classified into two groups:

- *The real-time machining*: The user defines the model of the smoothing. Mono and double parametric curves of degrees 3, 4, and 5 and those of type Bezier, b-spline, and Nurbs have been proposed as transition blocks inserted at the level of the discontinuities to ensure the continuity in tangency and curvature of the tool trajectory [3–13]. However, this machining mode is still limited at the level of research units.
- *The machining with a CNC machines*: The CNC units have two smoothing modes (Fig. 1): the first one consists of transformation of linear motion into parametric spline motion (global smoothing), and the second mode is to insert a transition element at the junction points of the linear blocks (local smoothing) [1, 14]. In particular, the Sinumerik CNC unit has two local smoothing modes. The smoothing element geometry of the first mode is a circular arc with a programmed radius. The second mode is to insert a spline element which can be defined in the NC program by three different geometric parameters [1]: transition length ( $T_L$ ), axis tolerance ( $\varepsilon_X, \varepsilon_Y$ ), and contour tolerance ( $\rho$ ) (Fig. 2).

Until now, the literature has addressed only two smoothing modes: local smoothing using a circle arc and a spline with axis tolerance programmed as a smoothing block. The CNC machine axis kinematics in local smoothing mode with a circular arc has been widely studied [15–17]. With a constant limit feed rate at the start and at the end of the smoothing blocks along the tool path, Pessoles et al. [18, 19] have

Fig. 1 Manufacturing process in HSM



considered the smoothing element with programmed axis tolerance to a polynomial of degree 5.

At this moment, the previous research works on this topic are based on two hypotheses:

1. H1: The programmed smoothing tolerance is equal to the actual tolerance of the tool path [3–10, 12, 13, 18, 19]. In the servo systems (case of CNC machining), there is always a discrepancy between the setpoint and the actual system status. To this offset between the setpoint tool path and the actual, the effect of the dynamic axes is added: under the effect of the system inertia, the tool tends to move away from the tool path setpoint in the tangential direction. There are always an offset between the setpoint and the actual tool path [20], and this error depends on the axis velocity and the drive controller parameters [1, 2]. Logically, in order to mitigate the kinetic energy effect and to ensure an acceptable machining tolerance, the interpolator must reduce the programmed contour tolerance

( $\rho$ ): the tool path setpoint must be characterized by a lower tolerance ( $\varepsilon$ ) (Fig. 2).

2. H2: The parameters limiting the axis kinematics during the smoothing transition phase are machine parameters expressed in the tangential/normal basis (T, N) bound to the geometry of the programmed smoothing block: each CNC machine is characterized by its radial acceleration and radial jerk [3–8, 15–17, 21, 22]. However, according to Sinumerik [1], the transition phase is characterized by its specific kinematic limits: each axis  $j$  is characterized by its radial jerk ( $Jtb_{RM_j}$ ) and radial acceleration ( $A_{RM_j}$ ) in addition to the parametric synchronization mode.

The objective of this paper is to model the axis kinematic behavior in 3-axis HSM of free-form surfaces with locally smoothing and programmed contour tolerance. The model must consider the effect of the interpolator on the programmed tolerance, the axis synchronization

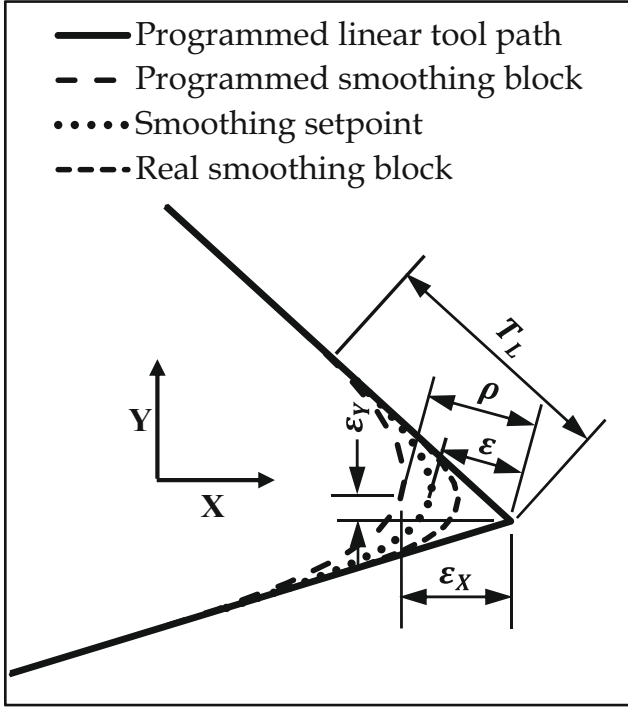


Fig. 2 Geometric parameters of the local smoothing operation

mode, and the axis kinematic parameters defined in the machine coordinate system  $(X, Y, Z)$ .

The first part is devoted to the geometric modeling of the tool path setpoint locally smoothed while taking into account the geometrical boundary conditions, ensuring the continuity in tangency and the maximum contour tolerance of the tool path setpoint ( $\epsilon_i$ ).

Further, based on the geometric model of the smoothing block and the axis kinematic capacity bound to the transition phase between two linear blocks (radial axis acceleration and jerk specific to the transition block), a new analytical approach describing the kinematics and the axis parametric synchronization along the smoothing block will be presented in the second part.

Finally, an experimental reduction rate identification of the contour tolerance programmed ( $\beta_i$ ) and the validation of the developed geometric and kinematic models are presented in the third part.

## 2 Geometric modeling of the smoothing element

The new CNC machine tools are capable to interpolate cubic and quintic parametric curve. Cubic parametric polynomial is used to model the transition block geometry.

The geometric model is approached in the plan  $(X, Y)$  of machine coordinate system (MCS). In continuous-path mode with defined contour tolerance, each axis may have a different rounding path [1]. The geometry of the smoothing element

$(S_i(u))$  can be expressed in MCS as follows:

$$S_i(u) = (X_i(u), Y_i(u)), 0 \leq u \leq 1 \quad (1)$$

Consider a tool path program composed of two discontinuous linear segments in tangency ( $[P_{i-1}P_i]$ ,  $[P_iP_{i+1}]$ ).  $S_i(u)$  is contained in tangent to the two segments ( $[P_{i-1}P_i]$  and  $[P_iP_{i+1}]$ , respectively) at the points  $Q_{i-1}$  and  $Q_{i+1}$  and passing through the point  $Q_i^*$ . The segment  $[P_iQ_i^*]$  represents the maximum contour tolerance of the tool path setpoint ( $\epsilon_i$ ) (Fig. 3).

$S_i(u)$  is a cubic parametric curve with

$$\begin{cases} S_i(u) = A_i u^3 + B_i u^2 + C_i u + D_i \\ S_i(0) = Q_{i-1} \\ S_i(1) = Q_{i+1} \end{cases} \quad (2)$$

where  $Q_{i-1}$  and  $Q_{i+1}$  represent the start and ending points of the rounding contour ( $S_i(u)$ ), respectively, with

$$\begin{cases} Q_{i-1} = (X_{Q_{i-1}}, Y_{Q_{i-1}}) \\ Q_{i+1} = (X_{Q_{i+1}}, Y_{Q_{i+1}}) \end{cases} \quad (3)$$

The adopted approach to model the trajectory with local smoothing consists of the following:

- In a first time, to define the parametric polynomial expressions describing the tool motion along the smoothing element
- In a second time, to identify the coordinates of the start and ending point of the smoothing block ( $S_i(u)$ ) according to the geometric boundary conditions of the smoothing operation

The geometric boundary conditions of the rounding contour limit are presented as hypotheses.

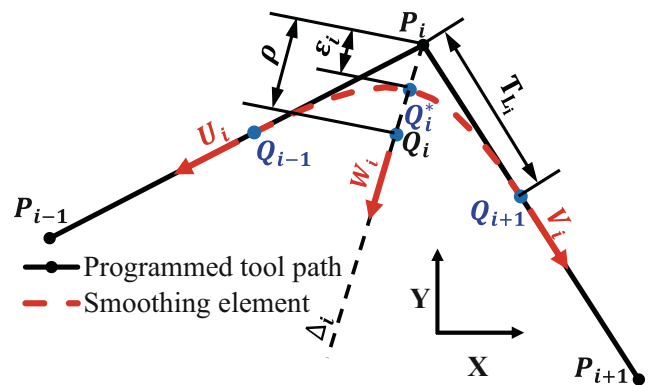


Fig. 3 Geometric modeling of the smoothing element

## 2.1 Hypotheses

The following hypotheses are used:

- i. The first parametric derivative of the smoothing curve ( $S'_i(u)$ ) at two points ( $Q_{i-1}$  and  $Q_{i+1}$ ) is respectively proportional to the two vectors ( $T_i$  and  $T_{i+1}$ ). From geometry, it can be verified that

$$\begin{cases} S'_i(0) = \alpha T_i \\ S'_i(1) = \alpha T_{i+1} \end{cases} \quad (4)$$

where  $T_i$  and  $T_{i+1}$  are the two vectors tangent to the transition block and defined as

$$\begin{cases} T_i = [T_{ix}, T_{iy}]^T = [X_{P_i} - X_{Q_{i-1}}, Y_{P_i} - Y_{Q_{i-1}}]^T \\ T_{i+1} = [T_{i+1x}, T_{i+1y}]^T = [X_{Q_{i+1}} - X_{P_i}, Y_{Q_{i+1}} - Y_{P_i}]^T \end{cases} \quad (5)$$

where  $\alpha$  is the weight of two tangential vectors ( $T_i$  and  $T_{i+1}$ ).

- ii. The smoothing element ( $S_i(u)$ ) is symmetrical with respect to the bisector of the angle formed by the two segments of the tool path ( $\Delta_i$ ); this geometric condition can be expressed as follows:

$$S_i(0.5) = Q_i^* = (X_{Q_i^*}, Y_{Q_i^*}) \quad (6)$$

- iii. The lengths of the two linear tool paths are sufficiently long, in which the CNC unit does not reduce the transition length ( $T_{L_i}$ ), and  $T_{L_i}$  must satisfy the following equation:

$$\begin{cases} T_{L_i} U_i = T_i \\ T_{L_i} V_i = T_{i+1} \end{cases} \quad (7)$$

where  $U_i$  and  $V_i$  are two unit vectors of directions  $T_i$  and  $T_{i+1}$ , respectively. They are calculated as follows:

$$\begin{cases} U_i = \frac{T_i}{\|T_i\|} = [U_{ix}, U_{iy}]^T \\ V_i = \frac{T_{i+1}}{\|T_{i+1}\|} = [V_{ix}, V_{iy}]^T \end{cases} \quad (8)$$

- iv. The interpolator reduces the programmed tolerance with a rate  $\beta_i$

$$\beta_i \rho W_i = [X_{Q_i^*} - X_{P_i}, Y_{Q_i^*} - Y_{P_i}]^T = \varepsilon_i W_i \quad (9)$$

where  $W_i$  is a vector defined as follows:

$$W_i = \frac{-U_i + V_i}{\|-U_i + V_i\|} = [W_{ix}, W_{iy}]^T \quad (10)$$

- v. The segment  $[P_i Q_i]$  represents the programmed tolerance

$$[P_i Q_i] = \rho W_i \quad (11)$$

## 2.2 Solving the equations

According to Eqs. (2), (4), and (5),  $S_i(u)$  can be expressed as a function of the two coordinate points ( $Q_{i-1}$  and  $Q_{i+1}$ ) and of two vectors ( $T_i$  and  $T_{i+1}$ ) as follows:

$$S_i(u) = (u^3, u^2, u, 1) \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Q_{i-1} \\ Q_{i+1} \\ \alpha T_i \\ \alpha T_{i+1} \end{bmatrix} \quad (12)$$

By referring to Eqs. (5), (7), (9), and (12) and using the boundary condition defined in Eq. (6) yields the below expression of  $T_{L_i}$  according to the coordinates of two points ( $Q_i$  and  $P_i$ )

$$\begin{aligned} T_{L_i} &= \frac{8 \beta_i \rho W_{ix}}{4(U_{ix} - V_{ix}) + \alpha (V_{ix} - U_{ix})} \\ &= \frac{8 \beta_i \rho W_{iy}}{4(U_{iy} - V_{iy}) + \alpha (V_{iy} - U_{iy})} \end{aligned} \quad (13)$$

The function ( $S_i(u)$ ) is assimilate to a hermit spline of degree 3, defined by his start and ending points ( $Q_{i-1}$ ,  $Q_i$ ) and the vectors ( $\alpha T_i$ ,  $\alpha T_{i+1}$ ), and the value of  $\alpha$  and  $\beta_i$  will be determined experimentally in the section “[Experimental tests and results.](#)”

## 3 Kinematic modeling

The precise estimation of the machining time and the guarantee of the machined surface quality require the modeling of the axis kinematic behavior according to the manufacturing process parameters.

Local smoothing switches the tool motion from one linear mode to the combined linear-polynomial motion. Each axis motion (linear, polynomial) is characterized by its synchronization mode and axis kinematic capacities [1].

With the exception of the first and the last linear blocks of the tool path smoothed, a smoothing block upstream and another one downstream delimit each linear block. The axis kinematic in linear interpolation mode has been widely studied in the literature [18, 23–25]. The axis feed rate ( $Vf_X, Vf_Y, Vf_Z$ ) is defined by up to 7 phases. The type and the time of each phase depend on the length of linear block, the axis kinematic capacity, the programmed feed rate ( $VF_P$ ), the feed rate at the end of the upstream smoothing block ( $Vout_{i-1}$ ) and the feed rate limit at the start of the downstream smoothing block ( $Vin_{iL}$ ) [23].

The axis kinematic behavior in polynomial interpolation is even less developed. The kinematic models developed in the literature do not take into account the synchronization mode in polynomial interpolation and the axis kinematic capacities specific to the smoothing block. A new analytical approach of the axis feed rate ( $Vf_j(t)$ ) in polynomial interpolation is proposed in this section.

Along the smoothing block, each CNC machine tool axis ( $j$ ) is characterized by its maximum kinematic capacities: maximum feed rate ( $V_{M_j}$ ), maximum acceleration ( $A_{M_j}$ ), and maximum transition block jerk ( $Jtb_{M_j}$ ) [1]. The radial and tangential accelerations of the axis  $j$  ( $(A_{RM_j})$  and  $(A_{TM_j})$ , respectively) are defined according to the  $A_{M_j}$  and the machine parameters: the rate of acceleration allowed in radial acceleration ( $R_A$ ) is expressed by [1]

$$\begin{cases} A_{RM_j} = R_A A_{M_j} \\ A_{TM_j} = (1-R_A) A_{M_j} \end{cases} \quad (14)$$

Likewise, for the radial and tangential jerks of the axis  $j$  ( $(Jtb_{RM_j})$  and  $(Jtb_{TM_j})$ , respectively) are defined according to  $Jtb_{M_j}$  and the machine parameter: the rate of jerk allowed in radial jerk ( $R_J$ ) is given as follows [1]:

$$\begin{cases} Jtb_{RM_j} = R_J Jtb_{M_j} \\ Jtb_{TM_j} = (1-R_J) Jtb_{M_j} \end{cases} \quad (15)$$

In polynomial interpolation mode, the machine axis synchronization is of parametric type [1]. Note  $u_i, \dot{u}_i, \ddot{u}_i$  and  $\ddot{\dot{u}}_i$  are the parametric position, parametric feed rate, parametric acceleration, and parametric jerk along the smoothing block ( $S_i(u)$ ), respectively. At each moment ( $t_m$ ), these parametric variables can be expressed using the Taylor series [22] as follows:

$$\begin{cases} \ddot{\dot{u}}_i(t_m) \\ \ddot{u}_i(t_m) = \ddot{u}_i(t_{m-1}) + t_s \ddot{\dot{u}}_i(t_{m-1}) \\ \dot{u}_i(t_m) = \dot{u}_i(t_{m-1}) + t_s \ddot{u}_i(t_{m-1}) + \frac{1}{2} (t_s)^2 \ddot{\dot{u}}_i(t_{m-1}) \\ u_i(t_m) = u_i(t_{m-1}) + t_s \dot{u}_i(t_{m-1}) + \frac{1}{2} (t_s)^2 \ddot{u}_i(t_{m-1}) + \frac{1}{6} (t_s)^3 \ddot{\dot{u}}_i(t_{m-1}) \end{cases} \quad (16)$$

where  $t_s$  is the CNC sampling time: the CNC unit generates motion instructions at time intervals equal to the sampling times [1].  $t_m$  is the moment of synchronization between the machine axes, and it is expressed as a function of  $t_s$

$$t_m = t_{m-1} + t_s \quad (17)$$

The sampling time ( $t_s$ ) is defined according to machine parameters: the cycle time ( $t_{cyl}$ ) and the sampling factor ( $f_s$ ) as follows [1]:

$$t_s = f_s t_{cyl}, 0 < f_s \leq 1 \quad (18)$$

We denote by the prime and the points, respectively, the derivatives with respect to  $u$  and the time ( $t$ ). Derivatives with respect to time can be transformed into derivatives with respect to  $u$  and derivative with respect to the element of displacement ( $s$ ) through the chain rule

$$\frac{d}{dt} = \frac{d}{du} \dot{u} = \frac{d}{ds} \dot{s} \quad (19)$$

Along the smoothing element ( $S_i(u)$ ) and applying the chain rule (Eq. (19)), the unit tangent and normal vectors ( $T_i$  and  $N_i$ , respectively) and the curvature ( $k_i$ ) can be written as a function of time in the MCS as follows:

$$\begin{cases} T_i = \frac{\frac{dS_i(u)}{dt}}{\left\| \frac{dS_i(u)}{dt} \right\|} = \frac{S_i'(u)}{\sigma_i} = [T_{ix}, T_{iy}, T_{iz}]^T \\ N_i = \frac{\frac{dT_i}{dt}}{\left\| \frac{dT_i}{dt} \right\|} = \frac{S_i''(u) \sigma_i - \sigma_i' S_i'(u)}{|S_i'(u) S_i''(u)|} = [N_{ix}, N_{iy}, N_{iz}]^T \\ k_i = \left\| \frac{dT_i}{ds} \right\| = \frac{S_i'(u) S_i''(u)}{\sigma_i^3} \end{cases} \quad (20)$$

With

$$\sigma_i = \|S_i'(u)\| = \sqrt{(X_i'(u))^2 + (Y_i'(u))^2} \quad (21)$$

The feed rate, acceleration, and jerk of the CNC machine axes ( $V_i(t)$ ,  $A_i(t)$ , and  $J_i(t)$ , respectively) can be expressed in the vector basis ( $T_i, N_i$ ) as follows:

$$\begin{cases} V_i(t) = v_i T_i = [Vf_X \ Vf_Y]^T \\ A_i(t) = \dot{v}_i T_i + v_i^2 k_i N_i \\ J_i(t) = (v_i - v_i^3 k_i^2) T_i + \left( 3 \dot{v}_i v_i k_i + v_i^2 \frac{dk_i}{dt} \right) N_i \end{cases} \quad (22)$$



With

$$\begin{cases} v_i = \sigma_i \dot{u}_i \\ \dot{v}_i = \sigma_i' (\dot{u}_i)^2 + \sigma_i u_i \\ \ddot{v}_i = \sigma_i'' (\dot{u}_i)^3 + 3 \sigma_i' \ddot{u}_i \dot{u}_i + \sigma_i \ddot{u}_i \\ \frac{dk_i}{dt} = \frac{dk_i}{ds} v_i = \frac{S_i' S_i'' - 3 \sigma_i \sigma_i' k_i}{\sigma_i^4} v_i \end{cases} \quad (23)$$

The parametric motion law (Eq. (16)) generated by the NC unit must respect the maximum kinematic capacity of the machine tool axes along the smoothing element. This condition results in the following equation:

$$\begin{cases} v_i T_{ij} \leq V_{M_j} \\ \dot{v}_i T_{ij} \leq A_{TM_j} \\ v_i^2 k_i N_{ij} \leq A_{RM_j} \\ (\dot{v}_i - v_i^3 k_i^2) T_{ij} \leq Jtb_{TM_j} \\ \left( 3 \dot{v}_i v_i k_i + v_i^2 \frac{dk_i}{dt} \right) N_{ij} \leq Jtb_{RM_j} \end{cases} \quad (24)$$

At the junction point of the smoothing element ( $S_i(u)$ ) and the upstream linear block ( $Q_{i-1}$ ), the kinematic axes are defined by parametric feed rate and parametric acceleration ( $\dot{u}_i|_{u=0}, \ddot{u}_i|_{u=0}$ ). These two parameters depend on the speed profile along the linear increment [ $P_{i-1}Q_{i-1}$ ] and the kinematic boundary conditions defined in Eq. (24).

To identify the speed and parametric acceleration at the start point of the smoothing block, the following approach is adopted:

- i. The first step is to identify the parametric feed rate limit ( $\dot{u}_i|_{u=0}$ ) at the input of the smoothing block, assuming that the axis acceleration is null

$$\dot{v}_i|_{u=0} = 0 \quad (25)$$

From Eqs. (22)–(24), the parametric limit speed ( $\dot{u}_i|_{u=0}$ ) at the start point of  $S_i(u)$  can be expressed as a function of the programmed feed rate ( $VF_P$ ) and the kinematic characteristics of the CNC machine axes, as follows:

$$\dot{u}_i|_{u=0} = \min_{j=1,3} \left( \frac{VF_P}{\sigma_i}, \frac{V_{M_j}}{\sigma_i}, \frac{1}{\sigma_i} \sqrt{\frac{A_{RM_j}}{k_i N_{ij}}}, \sqrt{\frac{Jtb_{RM_j}}{\left( \sigma_i^3 \frac{dk_i}{ds} \right) N_{ij}}} \right) \quad (26)$$

The limit feed rate at the input of the smoothing block ( $Vout_{iL}$ ) is equal to the limit speed at the output of the linear block upstream

$$Vout_{iL} = \sigma_i \dot{u}_i|_{u=0} \quad (27)$$

- ii. The second step is to identify the feed rate and the acceleration profile of the axes along the linear segment upstream [ $P_{i-1}Q_{i-1}$ ] (Fig. 3). This step consists to identify the number of the phases, the time of each phase, and the speed at the ending point of the linear block (equal to the feed rate at the input of the smoothing block:  $Vin_i$ ) [23].
- iii. The third step is to identify the parametric feed rate and acceleration at point  $Q_{i-1}$  by referring to Eq. (23) and take into account the boundary condition defined in Eq. (25)

$$\begin{cases} \dot{u}_i|_{u=0} = \frac{Vin_i}{\sigma_i} \\ \ddot{u}_i|_{u=0} = \frac{\left( -\sigma_i' (\dot{u}_i|_{u=0})^2 \right)}{\sigma_i} \end{cases} \quad (28)$$

As in linear interpolation, the ‘‘Bang-Bang’’ law with limited jerk is adopted by CNC machine tools. That is, one of the moving axes uses its maximum jerk. Assume later that the smoothing element is generated with a maximum parametric jerk. Referring to Eqs. (23) and (24), the absolute value of  $u_i$  can be expressed as follows:

$$|u_i| = \min_{j=1,3} \left( \left[ \left| \frac{Jtb_{TM_j}}{T_{ij}} \right| - \left| \sigma_i'' (\dot{u}_i)^3 + 3 \sigma_i' \ddot{u}_i \dot{u}_i - (\sigma_i \dot{u}_i)^3 k_i^2 \right| \right] \frac{1}{\sigma_i} \right) \quad (29)$$

The parametric jerk can take three values

$$\ddot{u}_i(t) = \begin{cases} |\ddot{u}_i| \\ 0 \\ -|\ddot{u}_i| \end{cases} \quad (30)$$

Based on the parametric motion law (Eq. (19)), the kinematic limits related to the capacities of the CNC machine axes (Eq. (26)), and the expression of the parametric jerk (Eq. (29)), an algorithm is proposed which allows to simulate the axis feed rate along the smoothing element ( $S_i(u)$ ).

Algorithm: velocity profile along the smoothing block

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```

Input:   Smoothed bloc geometry, maximum kinematic capacities
Step 1:  Identification of the parametric speed limit:  $\dot{u}_i|_{u=0}$ 
Step 2:  Calculation of the kinematic parameters at start point:  $\dot{u}_i|_{u=0}, \ddot{u}_i|_{u=0}$ 
Step 3:  Axes feed-rate profile
        While  $u < 1$ 
             $t = t + t_s$ ; Calculation of the absolute parametric jerk:  $|\ddot{u}_i|$ 
             $\ddot{u}_i = |\ddot{u}_i|$ ; Calculation of the parametric kinematic parameters:  $(u_i, \dot{u}_i, \ddot{u}_i)$ 
            If Condition checked (Eq.24)
                 $V(t) = \sigma_i u_i T_i$ 
            Else
                 $\ddot{u}_i = 0$ ; Calculation of the parametric kinematic parameters:
                 $(u_i, \dot{u}_i, \ddot{u}_i)$ 
                If Condition checked (Eq.24)
                     $V(t) = \sigma_i u_i T_i$ 
                Else
                     $\ddot{u}_i = -|\ddot{u}_i|$ ; Calculation of the parametric kinematic
                    parameters:  $(u_i, \dot{u}_i, \ddot{u}_i)$ 
                End
            End
        End
    
```

## 4 Experimental tests and results

In order to validate the analytical models of the tool path with local smoothing and the machine axis feed rate along the path, experimental tests were carried out on a high-speed machine (Spinner MVC 850) with a Sinumerik 840D CNC unit.

### 4.1 Identification of $\alpha$ and $\beta_i$

The analytical model validation of the smoothing element requires in a first step an experimental identification of the two parameters ( $\beta_i$  and  $\alpha$ , respectively), the reduction rate of the programmed tolerance, and the weight of two vectors tangent to the smoothing block, for the reason that the change of the axis motion direction increases the effect of the inertia on offset between the real and setpoint tool path. The reduction rate of the programmed contour tolerance may depend on two essential parameters: deviation angle between two successive linear blocks ( $\theta_i$ ) and the changing of the axis motion direction along the smoothing block.

Figure 4 shows the reference programmed tool path of the tool tip, chosen as an identification test case of the two parameters ( $\beta_i$  and  $\alpha$ ).

Table 1 presents the geometry of the tool path programmed and direction of the axis motion.

The test parameters are the following: programmed feed rate ( $VF_P = 3$  m/min) and contour tolerance programmed ( $\rho = 0.1$  mm). Figure 5 shows the programmed trajectory and the experimental results of the setpoint tool path with details at the discontinuity points ( $P_{i+1}, P_{i+2}, P_{i+3}$ ). The coordinates of the points ( $Q_i^*$ ), specific to each block ( $S_i(u)$ ), are experimentally identified:  $Q_i^*$  is the intersection between  $\Delta_i$  and the setpoint tool path. The contour tolerance of the tool path setpoint ( $\varepsilon_i$ ) is equal to the length of the  $[P_i Q_i^*]$  segment.

Thereafter, the reduction rate value ( $\beta_i$ ) of the programmed tolerance ( $\rho$ ) is deduced as follows:

$$\beta_i = \varepsilon_i / \rho \quad (31)$$

The results presented in Fig. 5 show that the last two smoothing blocks (at the two points  $P_{i+2}$  and  $P_{i+3}$ ) have the same smoothing tolerance setpoint ( $\varepsilon_i$ ): ( $\beta_i$ ) does not

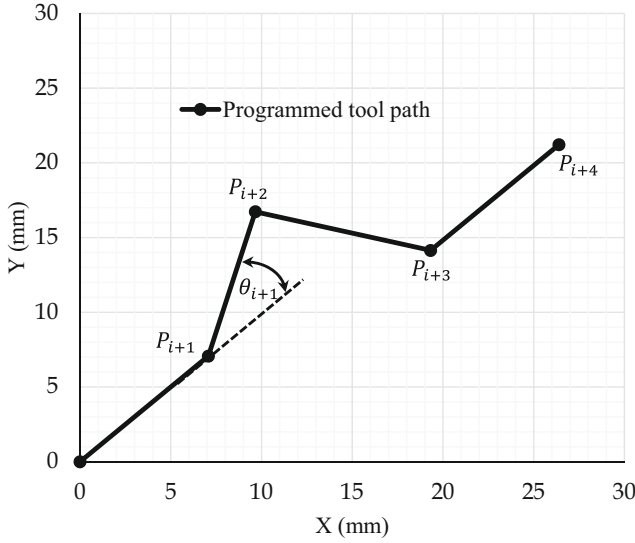


Fig. 4 Programmed linear tool path

depend on the deviation angle between the linear blocks; furthermore, it can take two values: 0.6 if one of the axes changes the motion direction along the smoothing blocks and 0.8 if not.

The second step of geometric validation is the weight value identification of the two tangent vectors defining the smoothing element ( $\alpha$ ). The transition length ( $T_{L_i}$ ) of smoothing element is expressed according to  $\alpha$  (Eq. (13)). The  $\alpha$  parameter is identified by comparing the experimental transition length to the simulated transition length with three values of  $\alpha = (2, 2.5, 3)$  (Fig. 6).

The results presented in Fig. 6 show that for  $\alpha$  equal to two, the simulated transition length is lower than that experimental, although for a value of three, the simulated transition length is higher than the experimental length; 2.5 is the optimal value of  $\alpha$ , which allows simulating the transition length of the local smoothing blocks with programmed contour tolerance.

Table 1 Geometric tool path parameters

Points	Coordinate points		$\theta_i$ (°)	With changing direction
	Axis X	Axis Y		
$P_i$	0	0		
$P_{i+1}$	7.071	7.071	30	No
$P_{i+2}$	9.659	16.73	90	Yes
$P_{i+3}$	19.318	14.142	60	Yes
$P_{i+4}$	26.389	21.213		

## 4.2 Tool path validation

The last step of geometric validation is the application of the analytical model on a spline tool tip path.

The spline geometric test is defined as follows:

$$S(u) = \begin{cases} X(u) = 100u \\ Y(u) = 100(3u^3 - 4u^2 + u) \end{cases}, 0 \leq u \leq 1 \quad (32)$$

The chord error of the CAM software parameter used is 0.5 mm, and the programmed parameters of the test are as follows:  $VF_P = 3$  m/min and  $\rho = 0.1$  mm.

Figure 7 shows the geometric validation results: superposition of the tool path setpoint, simulated without taking the reduction effect of  $\rho(\beta_i = 1)$  and  $\beta_i \neq 1$ .

The three details in Fig. 7 confirm that the proposed geometrical approach taking into account the reduction effect of  $\rho$  makes it possible to simulate the tool path setpoint with a high accuracy.

To justify the identification of  $\beta_i$ , we compare the geometric model with  $\beta_i \neq 1$  and  $\beta_i = 1$ . The details at point  $P_3$  and  $P_8$  show a significant offset between the curvatures of the two geometric models (with  $\beta_i = 1$  and  $\beta_i \neq 1$ ).

Figure 8 shows a comparison of the transition distances of the two geometric models: the simulated transition lengths with  $\beta_i = 1$  are greater than those with  $\beta_i \neq 1$ . This geometric gap leads to an underestimating of the machining time and a great difference between the simulated feed rate profile and the real one. That does not allow to have an accurate estimation of the machining cost and a reliable optimization of the manufacturing parameter process.

## 4.3 Kinematic validation

The simulation of the axis feed rate profile is based on the analytical model developed by Dong et al. [23] in linear interpolation mode and the model presented in this paper to identify the limit feed rate at the output of each linear block and the axis kinematic behavior along the smoothing blocks.

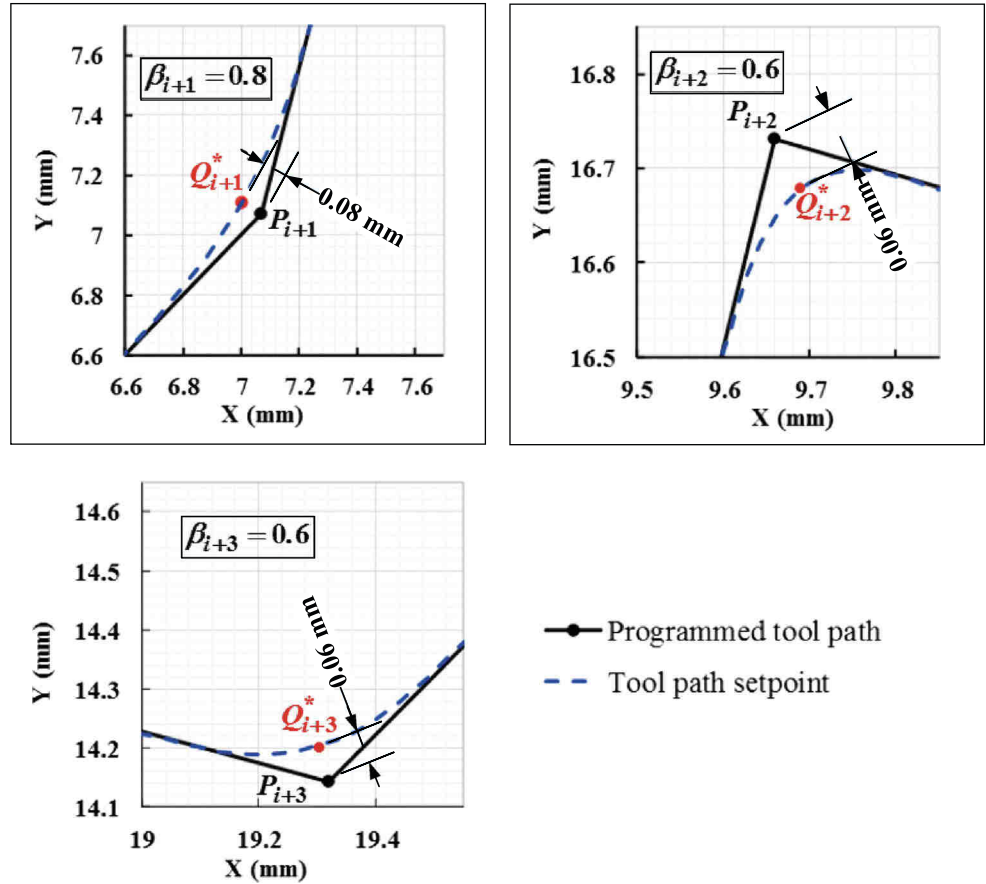
The experimental validation of the proposed kinematic model was carried out using the same tool path and programmed feed rate to validate the geometric model. The validation test was carried out without changing the parameters preset in the NC unit. Except the sampling factor ( $f_s$ ), which is by default set to one, was changed to 0.125 in order to have a more precise sampling time (0.001 s).

The servo control parameters are shown in Table 2.

The experimental profile of the axis feed rate and those simulated (with  $\beta_i = 1$  and  $\beta_i \neq 1$ ) are represented in Figs. 9 and 10.

The validation results of the kinematic model show that taking into account the reduction effect on the contour tolerance makes it possible to simulate the feed rate

Fig. 5  $\beta_i$  identification



variation with high precision. This allowed machining time estimation with an accuracy of 98%: the real total machining time is 3.024 s, and the simulated one (with  $\beta_i \neq 1$ ) is 3.063 s.

The failure to account the tolerance reduction effect leads to a dissimilar simulated velocity profile to the experimental one and an underestimation of the machining time: the machining time estimated with  $\beta_i = 1$  is 2.943 s.

The gap between the experimental results and those of the kinematic model (with  $\beta_i \neq 1$ ) is due to the axes following error ( $e_{f_j}$ ). The effect of inertia, axis velocity, and the change of the axis direction motion on this parameter are shown in Fig. 11. The X-axis is characterized by an offset less than 10  $\mu\text{m}$  along the tool path, although the Y-axis is characterized by two rise values: the first is of the order of 20  $\mu\text{m}$  and the second is equal to 14  $\mu\text{m}$ . These two experimental values are recorded at the two smoothing blocks which are characterized by the change of the Y-axis motion direction.

The machining with constant axis feed rate (null axis acceleration) allows to limit the effect of inertia on the axes following error ( $e_{f_j}$ ). Referring to the experimental results in Figs. 9, 10, and 11, we deduce that the following error is

proportional to the axis acceleration, except in the change of the axis direction.

In order to assess the advantages of the new kinematic model presented in this manuscript, a comparison study between the results obtained by the proposed model and those reported in the literature is carried out. For this study, the model of Gassara et al. [15] was designed to

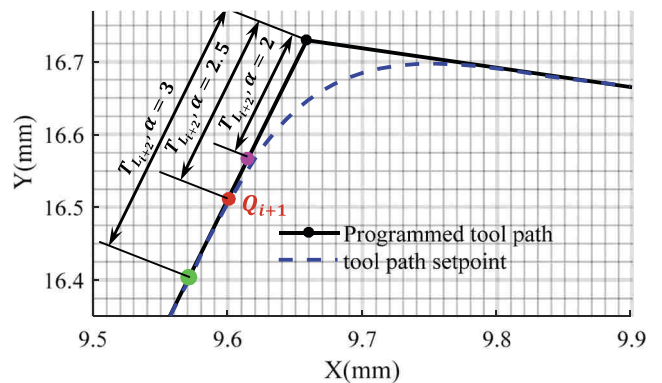


Fig. 6  $\alpha$  identification

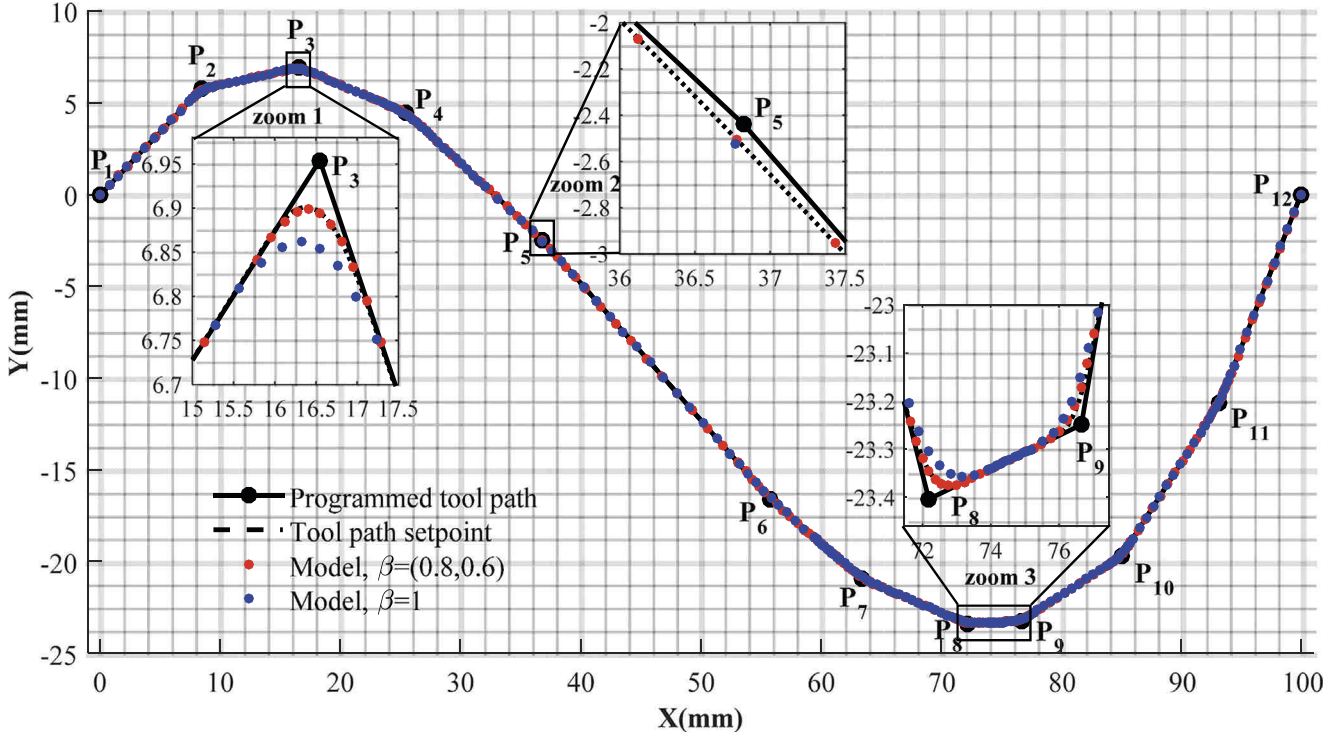


Fig. 7 Geometric model validation

calculate the feed rate variation according to the tool path and kinematic constraints shown respectively in Fig. 7 and Table 2.

The analytical approach of the feed rate proposed by Gassara et al. [15] is based on the following hypotheses:

- H1: The kinematic limits along the smoothing block are machine parameters defined in basis (T, N).
- H2: The polynomial smoothing block can be assumed to a circle arc.

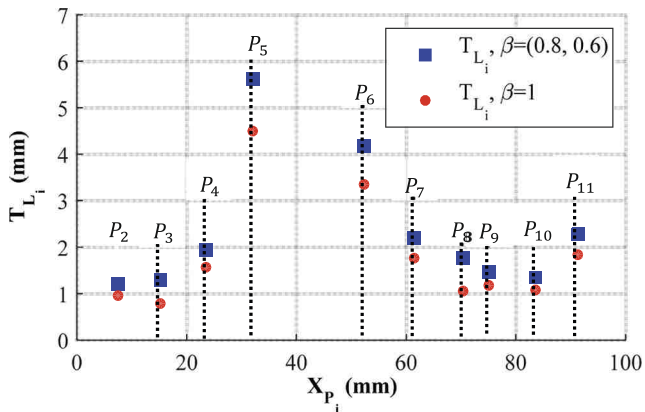


Fig. 8 The reduction effect of  $\rho$  on  $T_{L_i}$

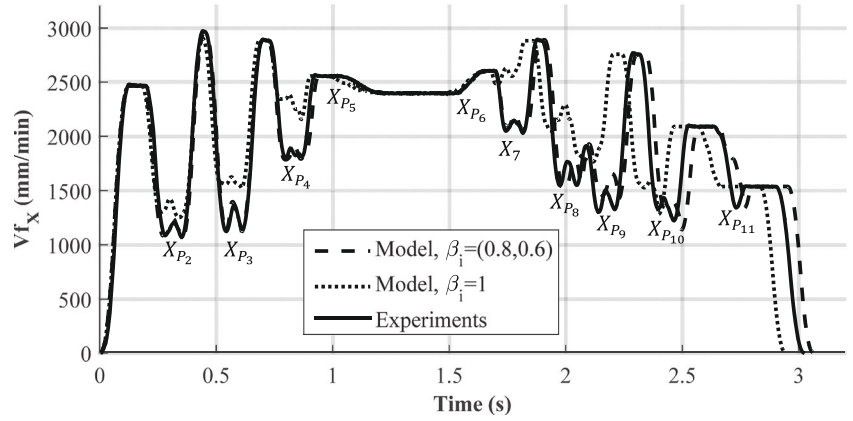
Figure 12 shows a superposition of the proposed model and that of Gassara et al. [15] in addition to the experimental results of the feed rate.

As shown in the detail of the discontinuity in point  $P_4$  (Fig. 12), the main difference between the two results of models depicted in Fig. 12 is the feed rate limit at the start point ( $Q_3$ ) and at the end point ( $Q_5$ ) of the smoothing block and the feed rate variation along the transition phase. This disparity was the results of three reasons. Firstly, for the proposed model, the considered trajectory at the transition is a parametric polynomial that is the same as generated by

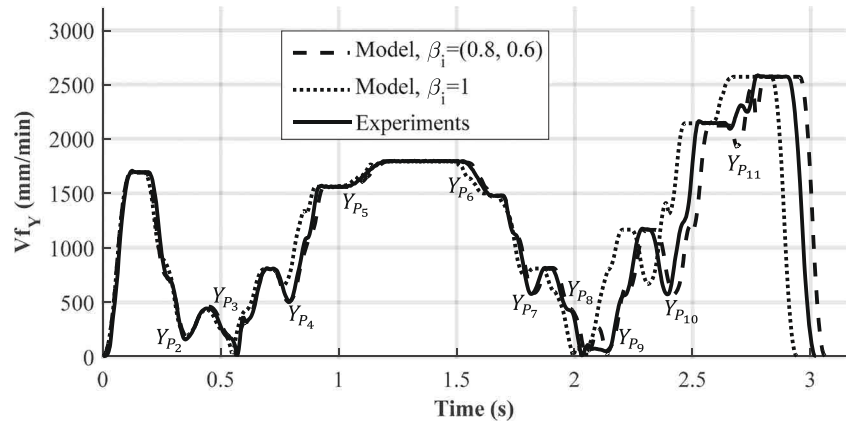
Table 2 Drive parameters

Symbol	Parameter	Value
$V_{M_x}, V_{M_y}$	Maximum axis speed (X and Y)	20m/min
$A_{M_x}, A_{M_y}$	Maximum axis acceleration (X and Y)	1 m/s <sup>2</sup>
$J_{M_x}, J_{M_y}$	Maximum axis jerk (X and Y) in linear interpolation mode	15 m/s <sup>3</sup>
$Jtb_{M_x}, Jtb_{M_y}$	Maximum axis jerk (X and Y) specific to the transition block	10 m/s <sup>3</sup>
$R_A$	The rate of acceleration allowed in radial acceleration	0.2
$R_J$	The rate of jerk allowed in radial jerk	0.2
$t_{cyl}$	Cycle time	8 ms
$f_s$	Sampling factor	0.125
	Anticipative mode	Speed
$N_{Antc}$	Anticipative capacity	8 blocks

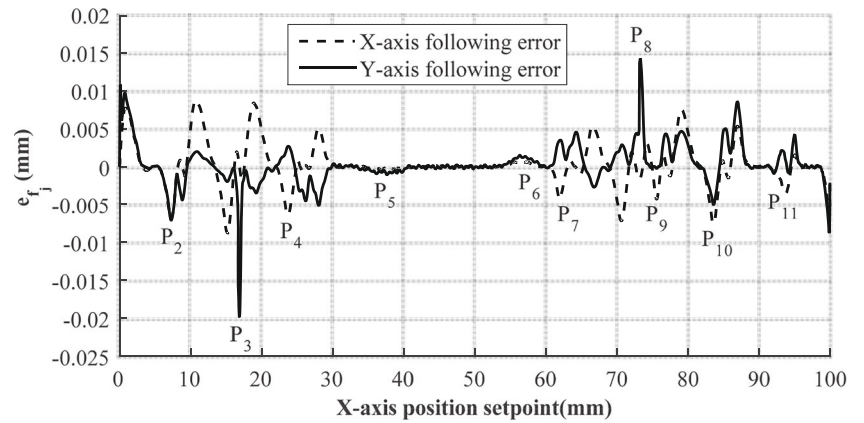
**Fig. 9** Theoretical and experimental variation of X-axis feed rate



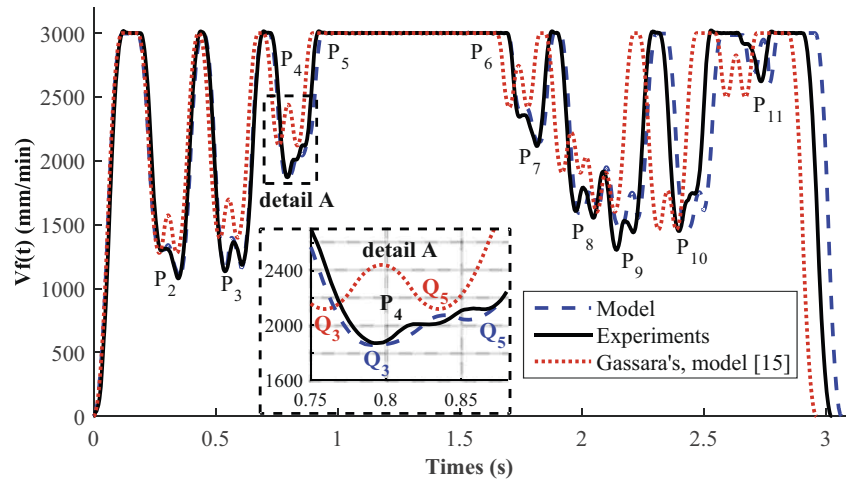
**Fig. 10** Theoretical and experimental variation of Y-axis feed rate



**Fig. 11** Axes following error along the tool path



**Fig. 12** Feed rate variation, model, experiments, and Gassara's model



Sinumerik CNC. Secondly, the feed rate limits are defined according to the kinematic axis capacity, as defined in Sinumerik [1]. Thirdly, the new analytical approach is based on the parametric synchronization between the axis kinematic parameters (axis position, feed rate, acceleration, and jerk).

By using the model developed in this paper, it has been demonstrated that taking into account the effect of the interpolator on the programmed tolerance, the parametric synchronization mode along the smoothing block, and the kinematic constraints defined by Sinumerik [1] ensure a relatively accurate estimate of the feed rate and the machining time.

## 5 Conclusion

In high-speed milling of free-form surface, the modeling of the tool path and the axis kinematics is necessary to optimize the manufacturing process parameters, the right choice of the smoothing mode according to the desired surface quality, and the accurate estimation of the machining time. The present study aims to model the axis kinematic along a locally smoothed trajectory with programmable contour tolerance.

The smoothing geometric hypotheses have led to identify the polynomial type used by Sinumerik CNC in local smoothing with programmed contour tolerance. The smoothing element can be assimilated to a third-degree Hermit polynomial. However, experimental tests showed that the effect of the axis motion inertia on the following error is mitigated by the reduction of the programmed tolerance: the interpolator reduces the programmed tolerance with a 0.6 coefficient if one of the axes changes direction and 0.8 otherwise.

The new kinematic model presented in this paper, specific to the transition phases, meets the axis kinematic parameters defined in Sinumerik: along the smoothing block, the axis synchronization mode is of parametric type and each axis is characterized by its radial acceleration and jerk.

The accuracy of the two analytical models reported in this article (geometrical and kinematic model) has given numerical results of the axis feed rate faithful to that of the experimental results and an estimate of the total machining time with a precision of 98%.

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