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## BRIEF COMMUNICATIONS

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### Experiments on the motion of a solid sphere toward a wall: From viscous dissipation to elasto-hydrodynamic bouncing

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In the present study, we investigate experimentally the motion of a macroscopic (non-Brownian and noncolloidal) solid sphere falling under gravity into a viscous liquid toward a solid wall. We observe the transition from a nonbouncing to a bouncing regime when increasing the Stokes number  $St$  which characterizes the particle inertia. In the bouncing regime, the recording of the particle trajectories allows us to determine the coefficient of 'elastic' restitution  $e$  above the transition. We observe that  $e$  first increases with  $St$  as predicted by Davis, Serayssol, and Hinch (1986) and seems to reach a plateau at high  $St$ . © 1999 American Institute of Physics. [S1070-6631(99)01509-3]

In many multiphase flow situations (e.g., sedimentation, fluidization, filtration), a key role is played by collisions between a particle and a solid surface such as the wall of the container or the surface of a filter. Such collisions imply both solid mechanics and fluid dynamics.

When an elastic particle collides a solid wall in a vacuum or under conditions of negligible fluid resistance, its incoming kinetic energy is converted into elastic strain energy as it deforms in the vicinity of contact and is then restored as the particle rebounds. The theory describing the motion and deformation during such a collision is known as Hertz contact theory;<sup>1</sup> the force  $F$  acting during the contact is  $F = (4/3)R^{1/2}\delta^{3/2}/\theta$ , where  $R$  is the sphere radius,  $\delta$  the surface deformation at the axis of symmetry and  $\theta = (1 - \nu_s^2)/E_s + (1 - \nu_w^2)/E_w$  is a coefficient depending on the Poisson's ratio  $\nu_s$ ,  $\nu_w$  and on the Young's elastic moduli  $E_s$ ,  $E_w$  of the sphere and the wall, respectively. The maximum sphere deformation is given by  $\delta_{\max} = ((5/4)\theta\rho_s U^2)^{2/5}R$ , where  $U$  is the impact velocity. The elastic bouncing of a solid sphere on a wall can be characterized by the coefficient of elastic restitution  $e$  which is the ratio  $u_r/U$  of the velocities, respectively, just after and before the rebound. Under conditions of negligible fluid resistance, this coefficient can be determined easily by measuring the height of fall and the height of rebound. For an elastic bouncing on a thick plate,  $e$  is of the order of 0.99, only one percent of energy being absorbed by elastic waves in the

target.<sup>2</sup> If the thickness  $b$  of the plate is not large compared to  $R$ ,  $e$  decreases when  $R/b$  or  $U$  increases,<sup>3</sup> as some energy is dissipated in flexible modes of the target. For large  $U$ , plastic deformations occur and  $e$  decreases when  $U$  increases.<sup>4</sup>

At the opposite extreme, when a perfectly rigid sphere moves into a viscous fluid, its kinetic energy is dissipated by viscous forces as it approaches the wall. In the case of creeping motion (zero Reynolds number) of a rigid sphere toward a rigid plane wall, Brenner<sup>5</sup> has calculated the viscous force of resistance  $F$  acting on the sphere moving at the velocity  $U$  into a fluid of viscosity  $\mu$  at a distance  $h$  between its bottom apex and the wall,  $F = 6\pi\mu R U \lambda$ , where  $\lambda = \lambda(h/R)$  is a correction to Stokes' law given by an infinite series. Note that  $\lambda \rightarrow 1$  when  $h/R \rightarrow \infty$  and that  $\lambda \rightarrow R/h$  when  $h/R \rightarrow 0$ . The rate of close approach is asymptotically slow and the sphere does not rebound. Note that if the sphere is perfectly rigid, no rebound will occur even if small or arbitrary inertia is introduced, as the kinetic energy cannot be stored by elastic deformation.<sup>6</sup> The lubrication regime has been tested experimentally and the results are in good agreement with Brenner's theory.<sup>7</sup>

In between these two limiting cases, depending on the part of the particle kinetic energy that will be dissipated by fluid forces and internal solid friction, the elastic deformation may be significant and rebound may occur for an elastic sphere. The first analysis that attempts to address this diffi-

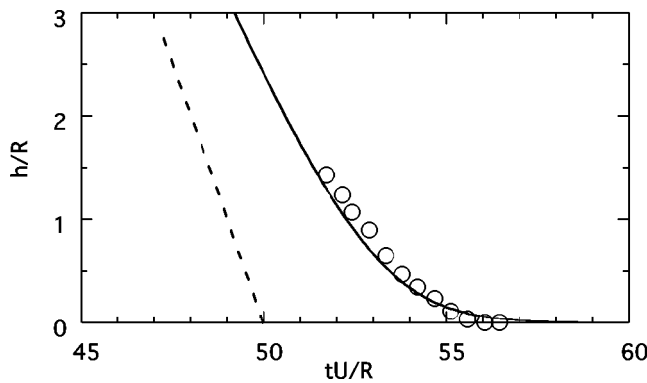


FIG. 1. Normalized distance sphere-wall  $h/R$  as a function of the normalized time  $tU/R$ . Experimental trajectories of glass bead falling in glycerol toward a glass wall for  $Re=5.10^{-4}$  ( $\circ$ ), line of slope  $-1$  (---), and lubrication theory (—) of Brenner (Ref. 5).

cult problem is the one of Davis, Serayssol, and Hinch.<sup>8</sup> In this analysis, a thin lubrication layer prevents the solid contact, but the elastic sphere deforms as a result of the high pressure in the lubrication layer, and may rebound (the fluid layer then reduces to a typical thickness of  $10^{-3}R$ ). The relevant parameter for the bouncing transition is the Stokes number  $St$ , which characterizes the particle inertia relative to viscous forces. The critical Stokes number  $St_c$  for the transition was found to be roughly between 1 and 10, depending on the elasticity parameters of the sphere and the wall ( $St_c$  decreases when the solids are less rigid). These authors have calculated also the ratio  $u_r/U$ , which can be thought of as a restitution coefficient  $e$ , but it measures here the energy dissipated in the solid and in the fluid. This ratio first increases with  $St$  above  $St_c$  and seems to saturate around the value 0.6 for higher  $St$ . To our knowledge, only one experimental study deals with the influence of the surrounding fluid on the bouncing transition of a macroscopic sphere on a wall.<sup>9</sup> In this study, solid spheres were dropped one at a time from varying heights onto a smooth or rough plate covered with a thin layer of a Newtonian oil, and the spheres are observed to rebound or not. This allowed the authors to check the previous theoretical predictions<sup>8</sup> for  $St_c$ , but they have not measured the coefficient of restitution  $e$ . Otherwise, numerous authors have studied the bouncing regime of small latex spheres in vacuum or in low density fluid like gas but only in

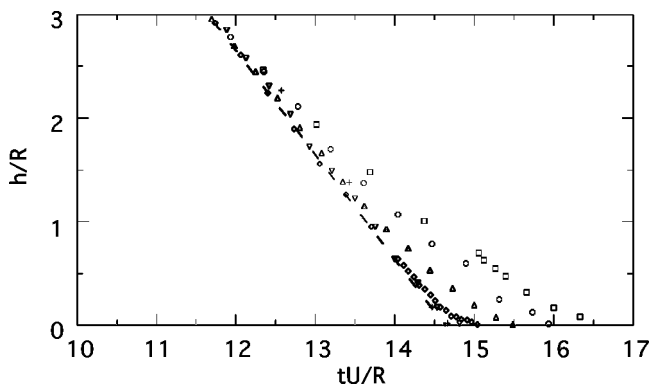


FIG. 2. Experimental trajectories of steel sphere falling in glycerol toward a glass wall for  $Re=0.2$  ( $\square$ ),  $0.33$  ( $\circ$ ),  $1.1$  ( $\triangle$ ), and  $2.5$  ( $\diamond$ ) or in silicone oil for  $Re=4$  and  $St=7$  ( $\nabla$ ), and for  $Re=7$  and  $St=12$  ( $+$ ), together with the line of slope  $-1$  (---).

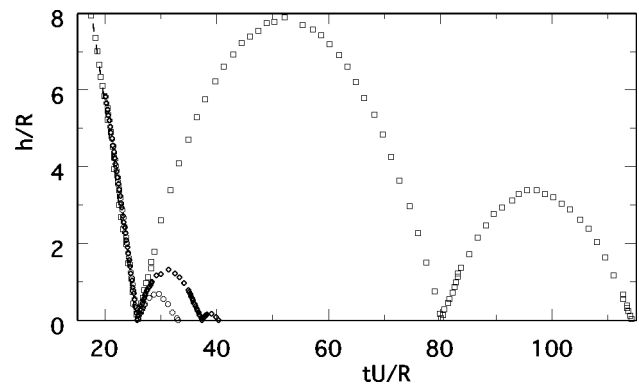


FIG. 3. Experimental trajectories of steel sphere falling in silicone oil toward a glass wall for  $Re=16$  and  $St=29$  ( $\circ$ ), and for  $Re=30$  and  $St=55$  ( $\diamond$ ) in water with  $Re=2080$  and  $St=3600$  ( $\square$ ), and line of slope  $-1$  before rebound (---).

term of sticking probabilities, as they are interested in the efficiency of aerosol filtration.<sup>10</sup>

In the present study, we investigate experimentally the motion of a macroscopic (non-Brownian and noncolloidal) solid sphere falling under gravity into a viscous liquid toward a solid wall, from viscous dissipation to elasto-hydrodynamic bouncing. In this last regime, we measure the coefficient of “restitution  $e$ ” as a function of  $St_c$ .

We used glass beads or steel spheres, of radius  $R$  ranging from 0.05 mm to 2.5 mm, in water, glycerol or silicone oil contained in a rectangular vessel ( $10\text{ cm} \times 10\text{ cm} \times 30\text{ cm}$ ). The mass density of the solid spheres  $\rho_s$  is  $2.5 \times 10^3$  and  $7.8 \times 10^3\text{ kg/m}^3$  for glass and steel, respectively, the Young’s modulus of elasticity  $E$  is  $60 \times 10^9$  Pa and  $214 \times 10^9$  Pa, respectively, and the Poisson’s ratio  $\nu$  is 0.24 and 0.30. The mass density of the fluids  $\rho_f$  is  $10^3$ ,  $1.25 \times 10^3$ , and  $0.97 \times 10^3\text{ kg/m}^3$  for water, glycerol, and silicone oil, respectively, whereas the dynamic viscosity  $\mu$  is 0.001, 0.63, and 0.1 Pa s (at  $T=20^\circ\text{C}$ ). The lateral walls of the vessel are glass plates allowing visualization. The bottom wall toward which the sphere moves is of glass too and of thickness  $b = 1\text{ cm}$ . The sphere trajectory is recorded by a high speed video camera (Kodak EKTAPRO with up to 6000 images/s) and analyzed by video means. The sphere position is known within an error of 4% relative to the sphere radius. With the

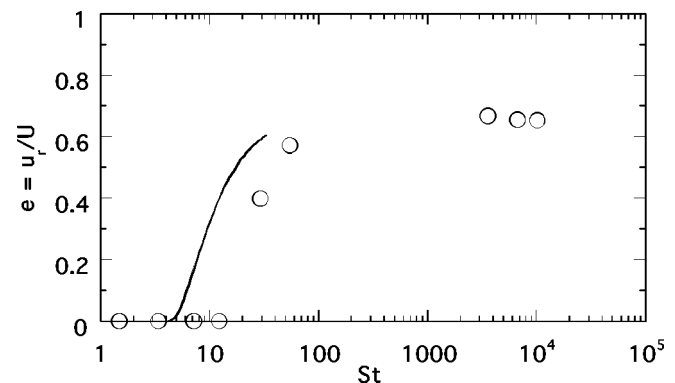


FIG. 4. Coefficient of restitution  $e$  (ratio of the maximum rebound velocity  $u_r$  to the incident velocity  $U$ ) as a function of the Stokes number  $St$ . Experimental measurements for the first rebound of steel sphere falling in liquid toward a glass wall ( $\circ$ ) and calculations of Davis *et al.* (Ref. 9) (—) for  $\epsilon=10^{-5}$  ( $\xi_v \approx 10^{-4}$ ).

different spheres and fluids, the obtained values of  $U$  are between 1 mm/s and 1 m/s. Due to the large size of the spheres ( $R > 100 \mu\text{m}$ ), surface forces can be neglected during the rebound.

A proper scaling of the equations governing the particle dynamics yields essentially four dimensionless parameters which provide measures of the relative magnitudes of the different physical effects far from the wall and during the impact. (i) The particle Reynolds number,  $\text{Re} = \rho_f R U / \mu$ , ratio of fluid inertia to viscous forces; (ii) the Stokes number,  $\text{St} = (2/9) \rho_s R U / \mu$ , ratio of *particle* inertia to viscous forces. (Note that the ratio  $\text{St}/\text{Re}$  corresponds to the ratio  $\rho_f / \rho_s$ ); (iii) the Froude number,  $\text{Fr} = \rho_s U^2 / (\rho_s - \rho_f) g R$ , ratio of the particle inertia to the apparent gravitational force; (iv) is the relative deformation,  $\xi$ , which provides a measure of the tendency of the sphere to deform during the impact. For a Hertz contact it writes<sup>1</sup>  $\xi_i = \delta_{\text{max}} / R = ((5/4) \theta \rho_s U^2)^{2/5}$ . When expressing the tendency of the sphere to deform due to viscous forces (by the same kind of parameter  $\xi_v$ ), the term  $\rho_s U^2$  has to be replaced by the term  $\mu U / R$  in  $\xi_i$ .<sup>8</sup>

Using different spheres and fluids,  $\text{Re}$  and  $\text{St}$  have been varied experimentally in a large range, roughly between  $10^{-4}$  and  $10^4$ , whereas  $\xi_i$  ( $\xi_i$  or  $\xi_v$ ) remains always smaller than  $10^{-3}$ .

The experimental sphere trajectories, i.e., the distance  $h$  between the bottom apex of the sphere and the wall as a function of time  $t$ , are presented in Figs. 1–3 in a normalized form;  $h$  is normalized by  $R$  and  $t$  by the ratio  $R/U$ , where  $U$  is the limiting fall particle velocity far from the bottom wall. In this normalized representation, the trajectory far from the wall thus corresponds to a line of slope  $-1$ .

In the case of very low  $\text{Re}$  ( $\text{Re} < 0.01$ ), we have checked that the experimental trajectory is well fitted by the lubrication theory (Fig. 1). The sphere feels the wall far from it and the trajectory deviates strongly from the one it would be without the bottom wall.

For larger  $\text{Re}$  numbers, fluid inertia needs to be considered and thus the balance between diffusion and convection. The time of diffusion for the momentum on the lengthscale  $h$  is  $t = h^2 \rho_f / \mu_f$  whereas the time for the particle advection on the same lengthscale is  $t = h/U$ . These two times are equal for a critical distance  $h_c$  that scales as  $h_c/R = 1/\text{Re}$ . Hence the distance from the wall at which the sphere feels the wall, and so where the normalized trajectory deviates from the line of slope  $-1$ , is smaller when the  $\text{Re}$  is larger. This is what we observe for Reynolds numbers roughly ranging from 0.1 to 5 (Fig. 2). For  $\text{Re} = 7$  ( $\text{St} = 12$ ), the sphere does not feel the wall until near physical contact, but no rebound is still observed after impact as  $\text{St}$  is still too low.

For even larger  $\text{Re}$  and  $\text{St}$  ( $\text{Re} > 10$  and  $\text{St} > 20$ ), there is one or more rebounds (Fig. 3). After the falling motion at constant velocity  $U$  (line of slope  $-1$ ), the sphere rebounds with a velocity  $u_r$  smaller than the impact velocity  $U$ . Note that the rebound trajectory is not parabolic as the force acting on the sphere is not only the constant gravitational force but also the velocity dependent drag force together with the effects of added mass and Basset history integral. The history Basset force is certainly important for this kind of reverse motion as the bouncing sphere travels toward its old wake.<sup>11</sup>

The coefficient of restitution  $e = u_r/U$  cannot be extracted here by measuring the height of rebound relative to the height of fall as there is a nonnegligible fluid dissipation; the height of rebound is independent of the height of fall as soon as the sphere has reached its limiting fall velocity before the rebound. Note that our measurement precision for the sphere position (4% relative to  $R$ ) does not allow us to discriminate between the occurrence of solid contact or the existence of a thin lubrication layer during the rebound.

We find that the critical Stokes number  $\text{St}_c$  for the bouncing transition is about 20. For  $\text{St} < \text{St}_c$ , no rebound occurs ( $e = 0$ ) whereas for  $\text{St} > \text{St}_c$ ,  $e = 0$  and there is one or more rebounds. We observe that  $e$  increases first with  $\text{St}$  for  $\text{St} > \text{St}_c$ , and then seems to saturate the value  $e \sim 0.65$  for large  $\text{St}$  (Fig. 4). Our value of  $\text{St}_c$  is a little above the range predicted by Davis *et al.*,<sup>8</sup> but the increase of  $e$  we observe just above the transition is however very similar to their calculations. Note that their calculations have been made for fixed values of their deformation parameter  $\epsilon = (4 \theta \mu U_0 R^{3/2}) / (\pi h_0^{5/2})$ , where  $U_0$  is the imposed velocity at the small distance  $h_0$  ( $h_0$  is typically  $10^{-2} R$ , and this consideration leads to the relation  $\xi_v = 10^{-2} (5 \pi / 16 \epsilon)^{2/5} \approx 10^{-2} \epsilon^{2/5}$ ).<sup>8</sup> In our experiments, the variation of  $\text{St}$  cannot be made at fixed  $\epsilon$ , but the experimental value of  $\xi_v$  does not vary much close to the transition and is  $\xi_v \approx 10^{-4}$ , which corresponds to  $\epsilon \approx 10^{-5}$ . The strong dependence of  $e$  upon  $\text{St}$  close to the transition clearly shows the importance of the viscous forces of the fluid medium. The saturation of  $e$  that we observe at high  $\text{St}$  would indicate that the viscous forces can be considered as negligible. But our saturation value of  $e$  at high  $\text{St}$  is far less than the value  $e \approx 0.9$  that is measured for an equivalent but dry sphere at the same impact velocity ( $U \approx 1$  m/s).<sup>2,3</sup> This means that there still would be an hydrodynamic effect, but in this case of inertial origin, which could be tested by measuring the high  $\text{St}$  value of  $e$  for different densities.

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