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Non-binary turbo codes: design, simplified decoding and comparison with SoA codes

Rami Klaimi, Charbel Abdel Nour, Catherine Douillard



The EPIC project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 760150.



OUTLINE

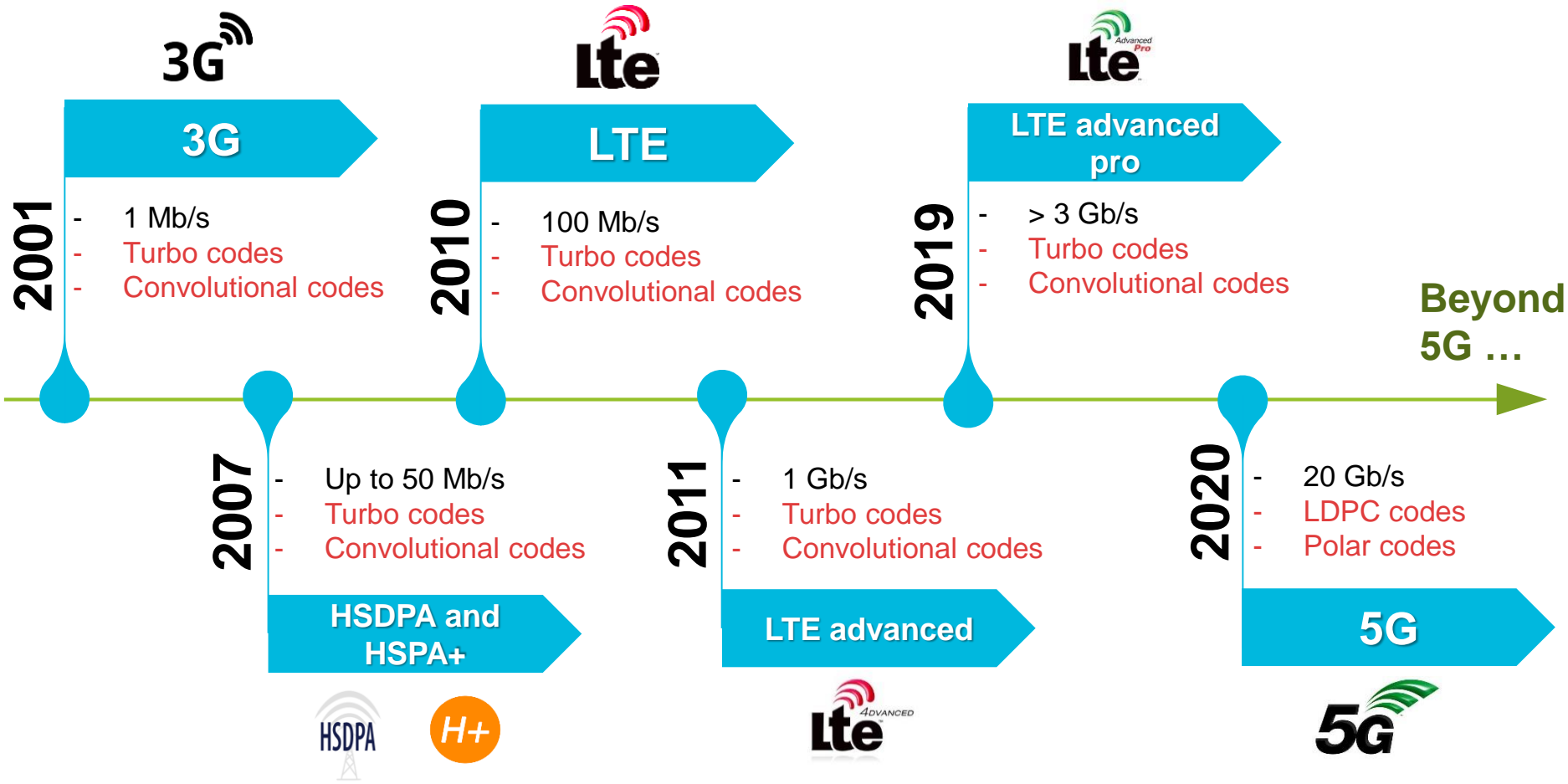
1. **Context and state of the art**
2. **Design of NB-TCs**
3. **Union bound evaluation for NB-TCs**
4. **Low-complexity decoding algorithm**
5. **Performance comparison**
6. **Conclusion and future work**



OUTLINE

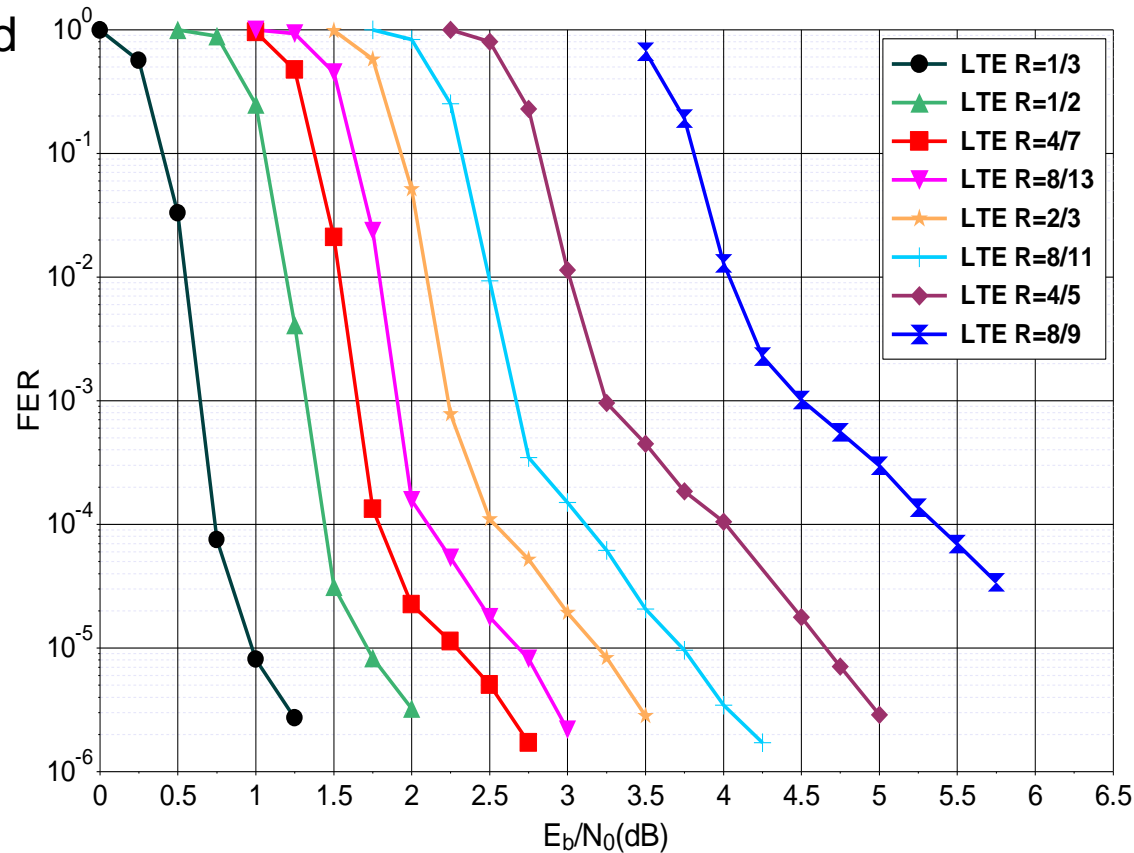
1. **Context and state of the art**
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Why new families of error correcting codes should be considered?

- ▶ The performance of standardized codes is to be improved:
 - at high SNRs for high coding rates,
 - for their association with high order modulations,
 - for short block sizes targeting low latency applications.

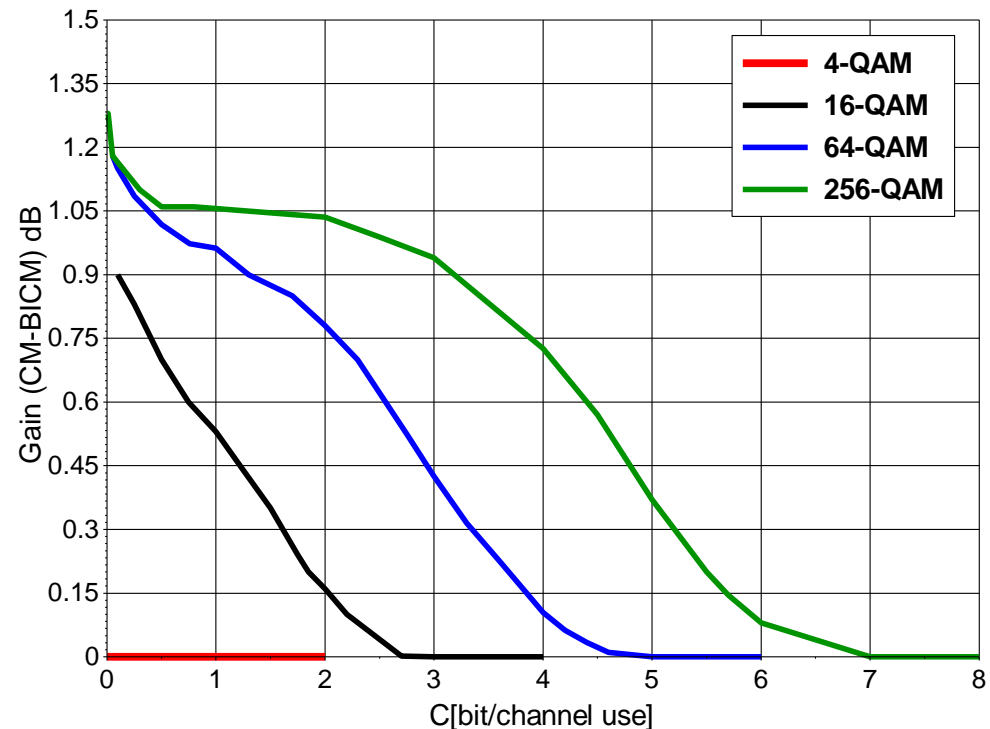


Advantages of NB codes

- ▶ Lower correlation in the decoding process [1]
- ▶ Theoretical performance gains for high modulation orders [2]
 - Gap closed by Probabilistic Shaping (PS) for optical channels [3]
 - PS + NB codes better than PS + binary codes [4]
- ▶ More freedom degrees can be investigated

Existing NB codes

- ▶ Reed-Solomon codes
- ▶ NB-LDPC codes
- ▶ Some studies targeted NB-CC and NB-TC



[1] C. Berrou, M. Jezequel, C. Douillard, and S. Kerouedan, "The advantages of non binary turbo codes," in IEEE Inform. Theory Workshop, Cairns, Queensland, Australia, 2001, pp. 61–63.

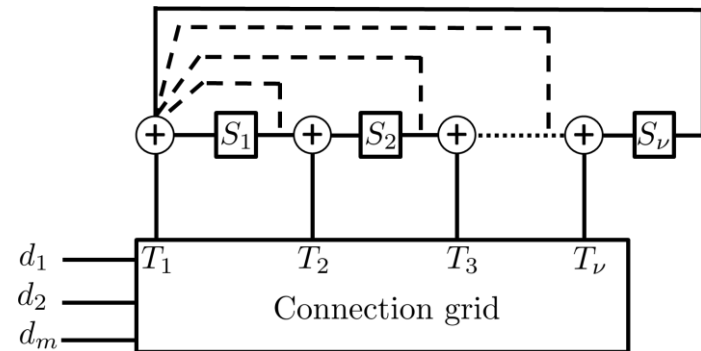
[2] G. Caire, G. Taricco, and E. Biglieri, "Bit-interleaved coded modulation," IEEE Trans. Inform. Theory, vol. 44, no. 3, pp. 927–946, 1998

[3] Böcherer, G., Steiner, F., & Schulte, P. (2015). Bandwidth efficient and rate-matched low-density parity-check coded modulation. *IEEE Transactions on Communications*, 63(12), 4651-4665.

[4] Steiner, F., Liva, G., & Böcherer, G. (2017, December). Ultra-sparse non-binary LDPC codes for probabilistic amplitude shaping. *IEEE Global Commun. Conf* (pp. 1-5).

m-binary codes [5]

- ▶ Better convergence.
- ▶ Larger minimum distance than binary codes.
- ▶ Less sensitivity to puncturing.
- ▶ Reduced latency.
- ▶ Robustness of the decoder.

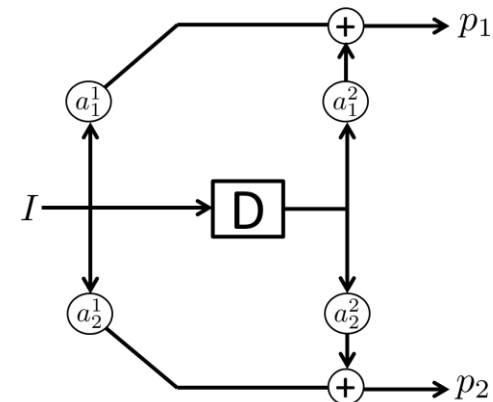


NB-CC over rings [6]

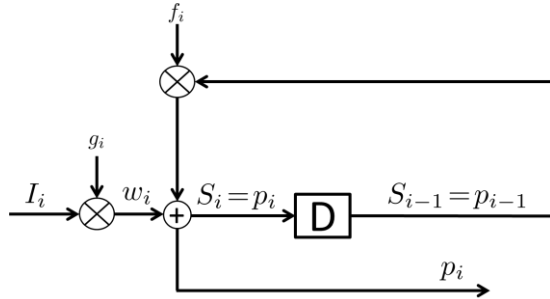
- ▶ Good performance with phase modulations.
- ▶ Adapted to perform with m-QAM constellations.
- ▶ Not suitable for NB-TCs: non-systematic and non-recursive.

[5] C. Douillard and C. Berrou, "Turbo codes with rate- $m/(m+1)$ constituent convolutional codes," IEEE Trans. Commun., vol. 53, no. 10, pp. 1630–1638, 2005.

[6] J. L. Massey and T. Mittelholzer, "Convolutional codes over rings," in 4th Joint Swedish Soviet Int. Workshop Inform. Theory, Gotland, Sweden, 1989, pp. 14–18.



NB-TC based on time-variant NB-CCs [7][8]



Parallel concatenation

- ▶ A protograph sub-ensemble of a regular NB-LDPC code is derived to define the NB-TCs.

$$H = \begin{bmatrix} \tilde{I} & \tilde{P}^{(1)} & 0 \\ \tilde{\Pi} & 0 & \tilde{P}^{(2)} \end{bmatrix}$$

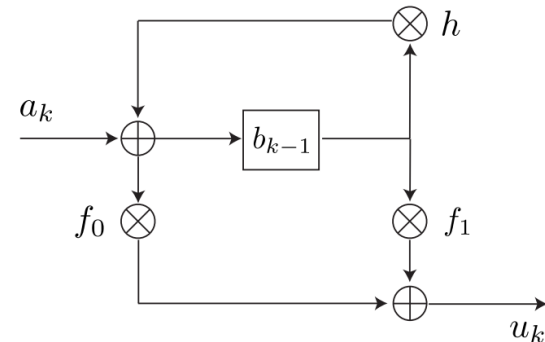
Serial concatenation

- ▶ The protograph is stretched to form a serial structure:

$$H = \left[\tilde{\Pi}^{-1} \quad \tilde{P}^{(2)} \mid \tilde{I}^{-1} \quad \tilde{P}^{(1)} \right]$$

NB-TC over GF(q), q > 2 [9]

- ▶ Based on NB-CCs over GF(q)



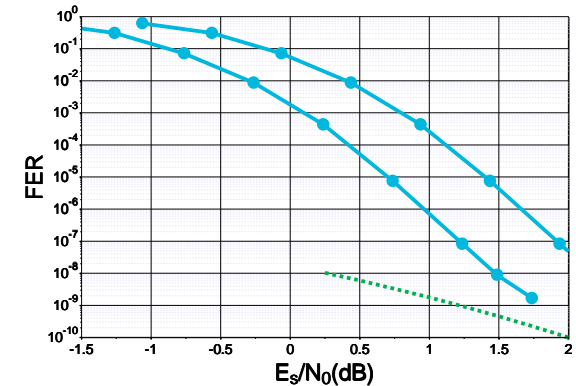
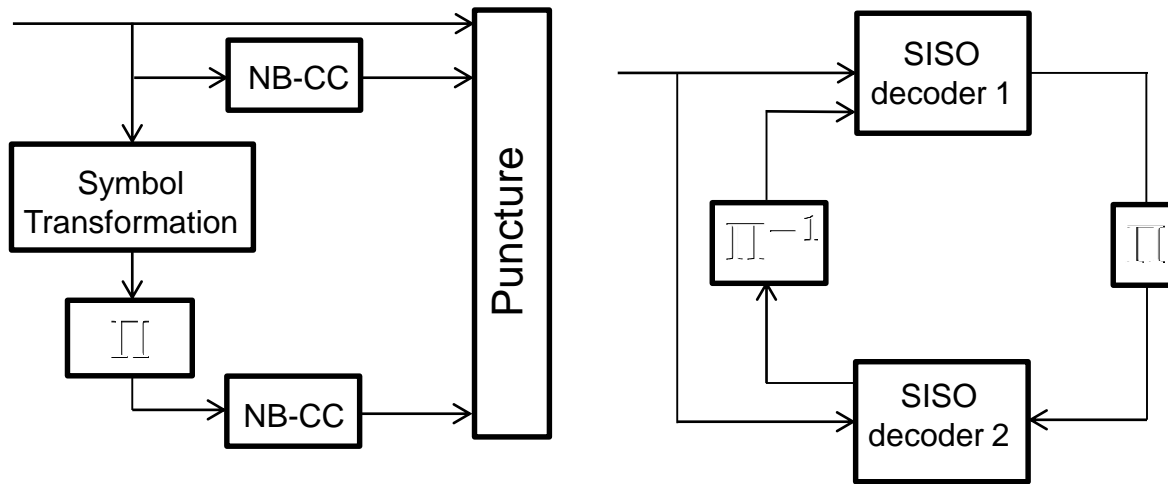
- ▶ Good NB-TCs using:

- union bounds,
- EXIT charts.

[7] G. Liva, S. Scalise, E. Paolini, and M. Chiani, "Turbo codes based on time-variant memory-1 convolutional codes over F_q ," in *IEEE Int. Conf. Commun.*, Kyoto, Japan, June 2011, pp. 1–6.

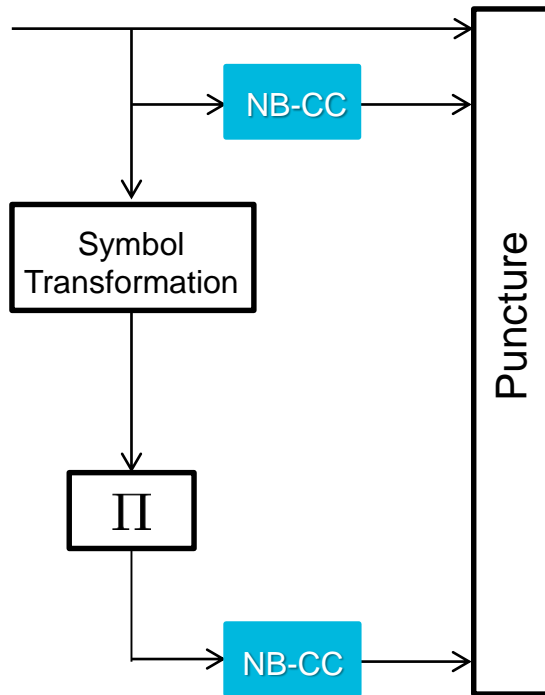
[8] G. Liva, E. Paolini, B. Matuz, S. Scalise, and M. Chiani, "Short turbo codes over high order fields," *IEEE Trans. Commun.*, vol. 61, no. 6, pp. 2201–2211, June 2013.

[9] Matsumine, T., & Ochiai, H. (2018). Capacity-Approaching Non-Binary Turbo Codes: A Hybrid Design Based on EXIT Charts and Union Bounds. *IEEE Access*, 6, 70952-70963.



Main contributions of the work:

- ❖ New structure of NB-CCs
- ❖ Interleaving constraints
- ❖ Two puncturing techniques
- ❖ Bijective symbol transformation block
- ❖ Estimation of the distance spectra and evaluation of union bounds
- ❖ A low-complexity decoder
- ❖ Performance comparison with existing codes



1. Context and state of the art

2. Design of NB-TCs:

- ❑ Proposed structure of NB-CCs
- ❑ Interleaver design for NB-TCs
- ❑ Proposed puncturing techniques
- ❑ Introduction of a symbol transformation block

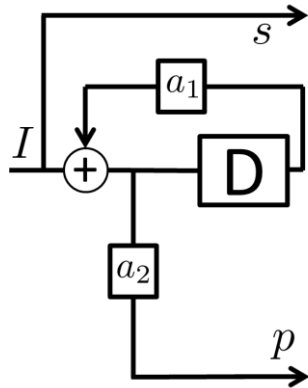
3. Union bound evaluation for NB-TCs

4. Low-complexity decoding algorithm

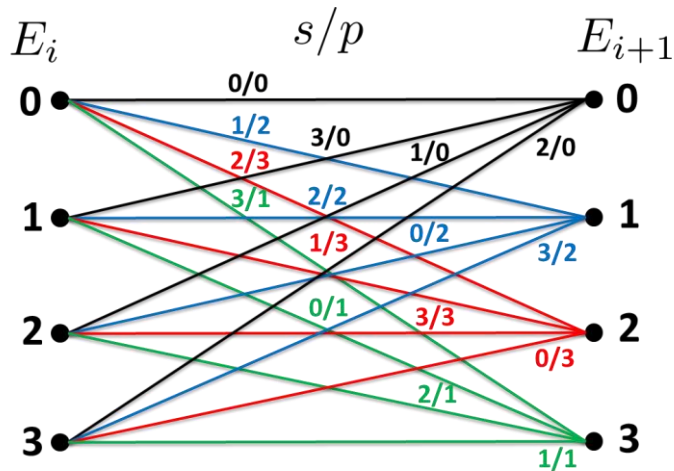
5. Performance comparison

6. Conclusion and future work

Accumulator encoder over $GF(q)$, $q > 2$



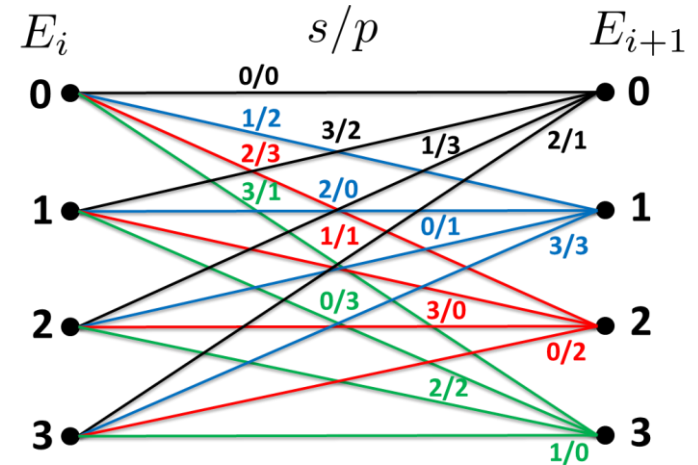
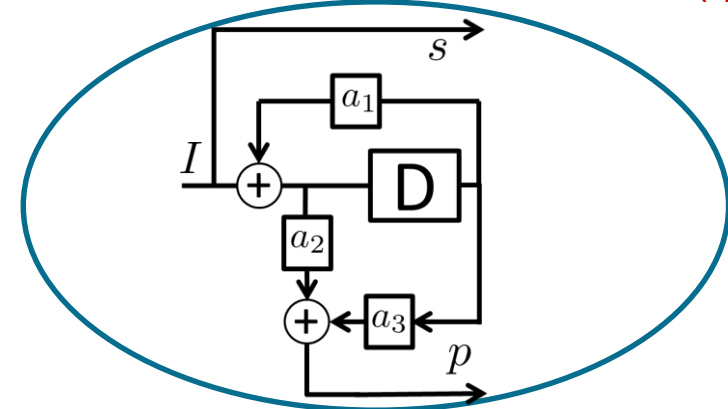
25% gain in the minimum distance



$q = 4$

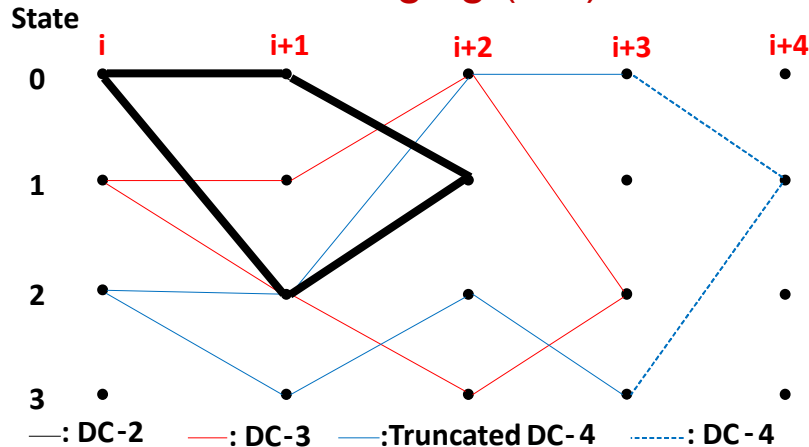
Same Parity when arriving to a state
 → systematic value responsible for the distance

Proposed constituent encoder over $GF(q)$

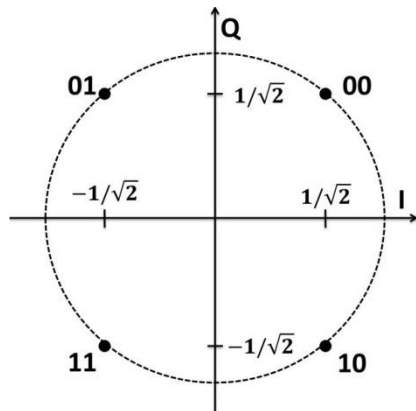


Distinct parity when arriving to a state
 → systematic and parity symbols contribute to the distance

Error-prone sequences: Diverging-Converging (DC)



4-QAM Constellation

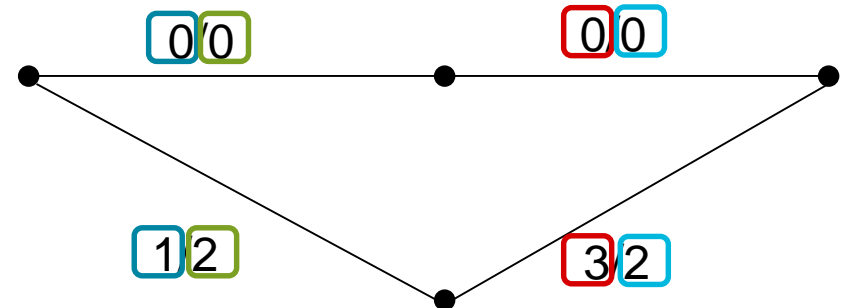


Non-linearity of the non-binary coded modulation

For high constellation orders:

- Non-binary coded modulations are non-linear in signal space
- Different protection levels for constellation symbols
- All-zero sequence cannot be used as reference

Euclidian Distance Calculation



$$d_{euc}^2 = d_{01}^2 + d_{02}^2 + d_{03}^2 + d_{02}^2$$

Error rates mainly depend on shortest sequences !

Code enumeration over GF(q) for the proposed structure

When the constellation mapping varies:

- $q! * (q - 1)^3$ possible codes to test

⇒ Prohibitive search complexity for high values of q .

⇒ Question: Do we really need to consider all these combinations ?

Answer: No, for the proposed structure.

When the constellation mapping is fixed:

- $(q - 1)^3$ possible codes to test

Proposal: Limitation of the enumerated DC sequences to the short ones

- Acceptable complexity
- Accurate enumeration of the two lowest distances of the code

Consider a q -QAM constellation mapping

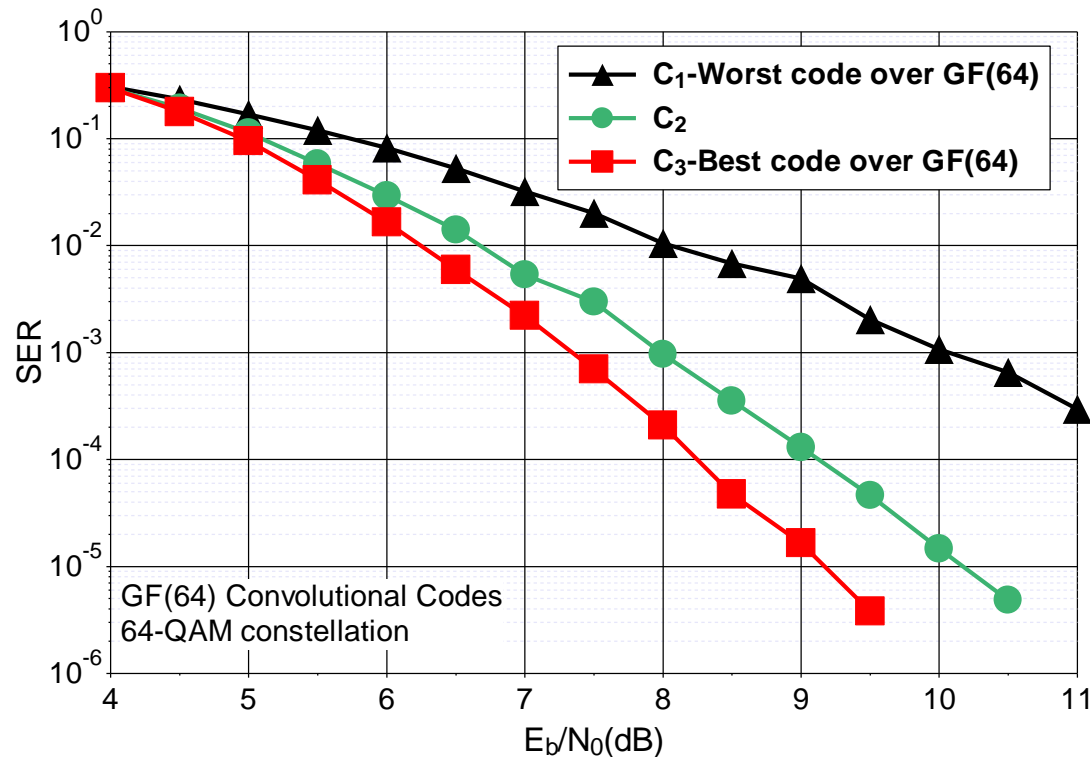
Define NB-CC by changing $a_i \in GF(q)$

Calculate the distance spectrum by enumerating all candidate DC sequences

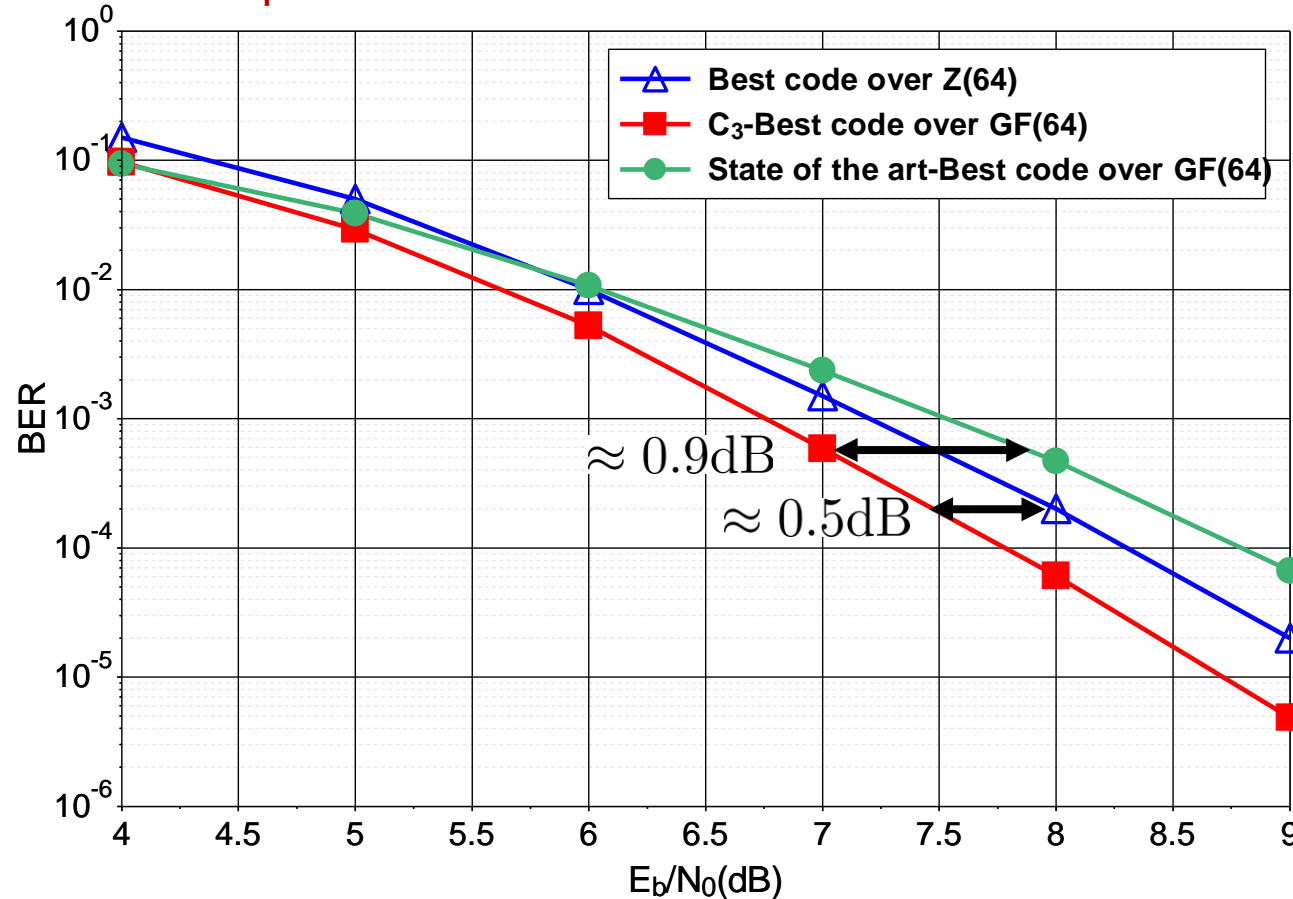
Choose the code with the best distance spectrum

Results over GF(64)

Code	C_1	C_2	C_3
(a_1, a_2, a_3)	(41, 2, 0)	(41, 1, 24)	(31, 5, 18)
$d_1^2 (= d_{min}^2)$	0.38	1.14	1.52
$n(d_1)$	238422	1542390	652698
d_2^2	0.57	1.23	1.61
$n(d_2)$	230886	4111444	1084014

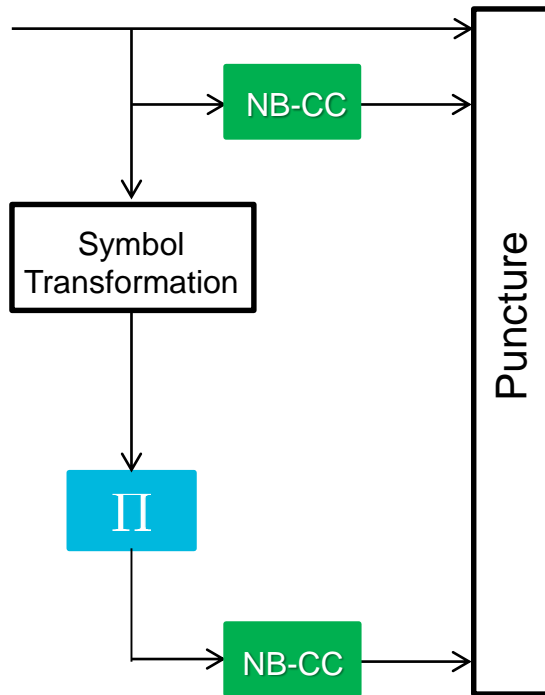


Bit error rate performance evaluation over a 64-QAM constellation



[10] T. Konishi, "A coded modulation scheme for 64-QAM with a matched mapping," in *IEEE Int. Symp. Inform. Theory and its Applications (ISITA)*, Oct. 2014, pp. 191–195.

[11] Matsumine, T., & Ochiai, H. (2018). Capacity-Approaching Non-Binary Turbo Codes: A Hybrid Design Based on EXIT Charts and Union Bounds. *IEEE Access*, 6, 70952-70963.



1. Context and state of the art

2. Design of NB-TCs:

- Proposed structure of NB-CCs
- Interleaver design for NB-TCs
- Proposed puncturing techniques
- Introduction of a symbol transformation block

3. Union bound evaluation for NB-TCs

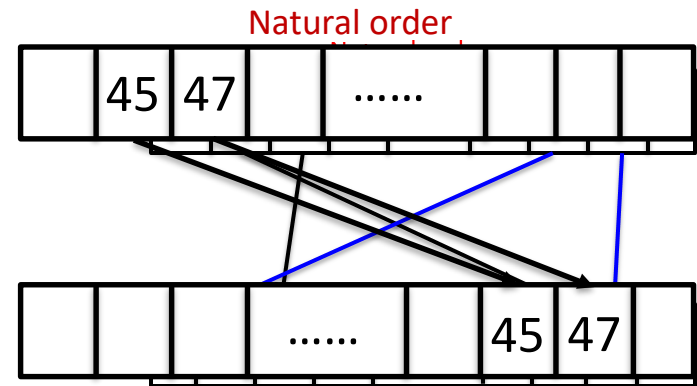
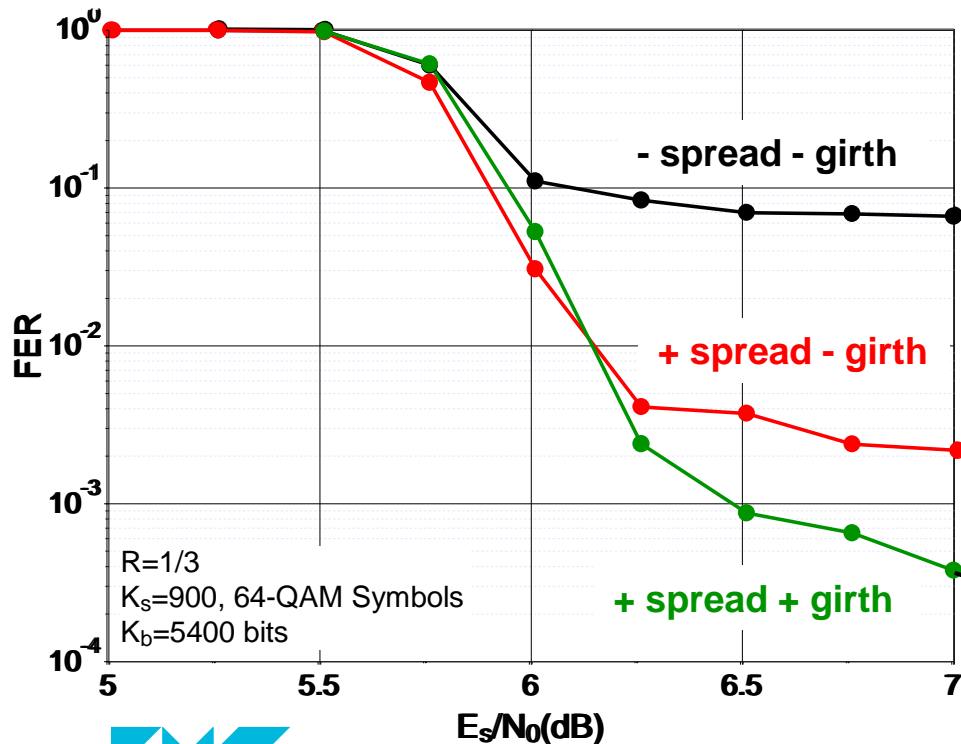
4. Low-complexity decoding algorithm

5. Performance comparison

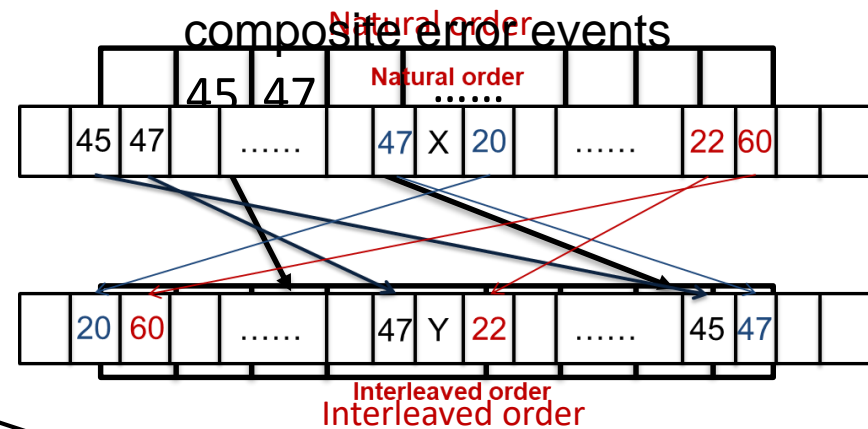
6. Conclusion and future work

Interleaving design: criteria to choose the best interleaver for NB-TCs

- ▶ **Spreading constraint:** avoid same short-DC sequence at both decoders.
- ▶ **Girth constraint:** short correlation cycles => propagated errors.



Problem to be solved:
composite error events



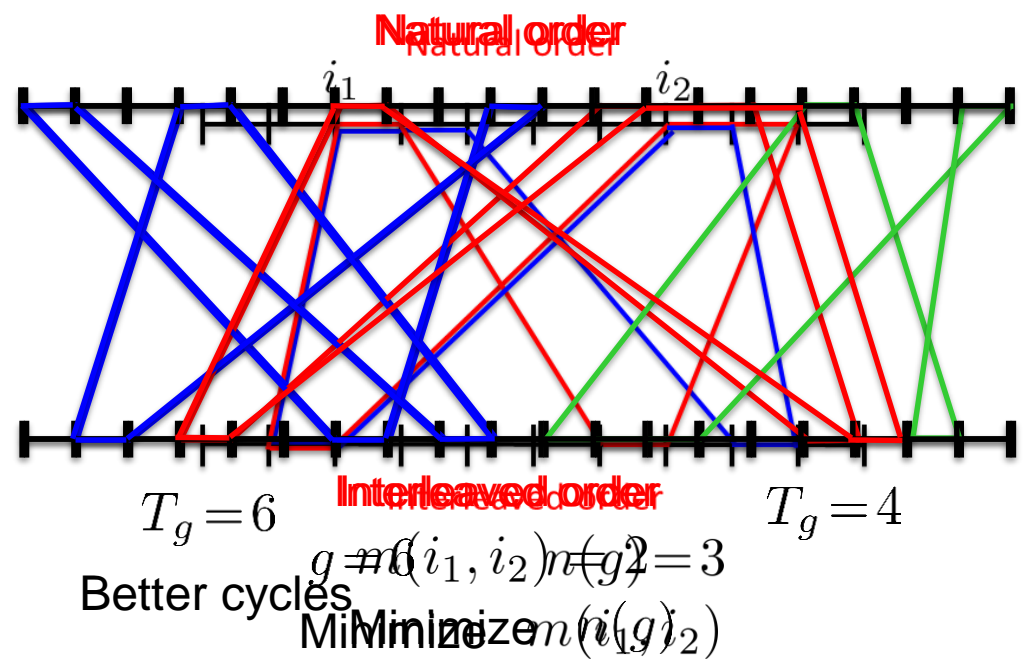
ARP interleaver :

$$\Pi(i) = (P \cdot i + S(i \bmod Q)) \bmod K$$

Interleaving design: reducing correlation effect for short frame sizes

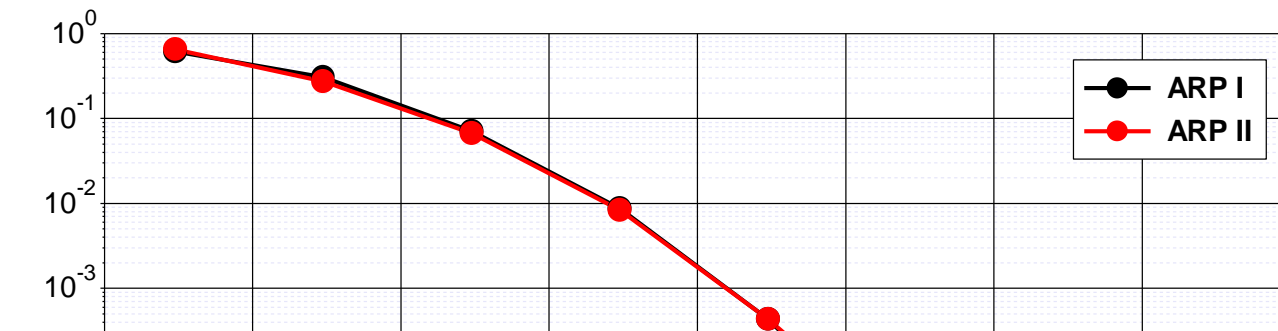
Correlation cycle possible problems:

- ▶ High minimum cycle occurrence $n(g)$
- ▶ Low information exchange between component decoders
- ▶ Couples of symbols occurrence in minimum cycles $m(i_1, i_2)$.



Proposed interleaver design for NB-TC over GF(64)

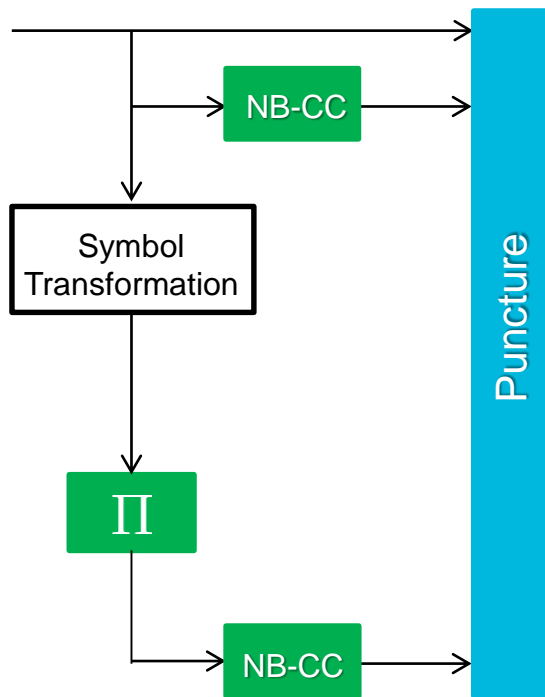
ARP	Q	P	$(S(0), \dots, S(Q-1))$	$n(g)$	$T_g(\mathbf{d}, \mathbf{d}')$	$m(i_1, i_2)$
I	3	19	(0,25,14)	2	4	1
II	5	7	(0,17,3,22,28)	4	4	2



Summary

- ▶ ARP interleavers are suited for NB-TCs.
- ▶ Short correlation cycles were optimized.
- ▶ Promising asymptotic performance.
- ▶ Publication in ISTC 2018:

R. Garzon Bohorquez, R. Klaimi, C. Abdel Nour and C. Douillard, "Mitigating Correlation Problems in Turbo Decoders", in 10th International Symposium on Turbo Codes and Iterative Information Processing (ISTC 2018), Hong Kong, Dec. 2018.



1. Context and state of the art

2. Design of NB-TCs:

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- Interleaver design for NB-TCs
- Proposed puncturing techniques**
- Introduction of a symbol transformation block

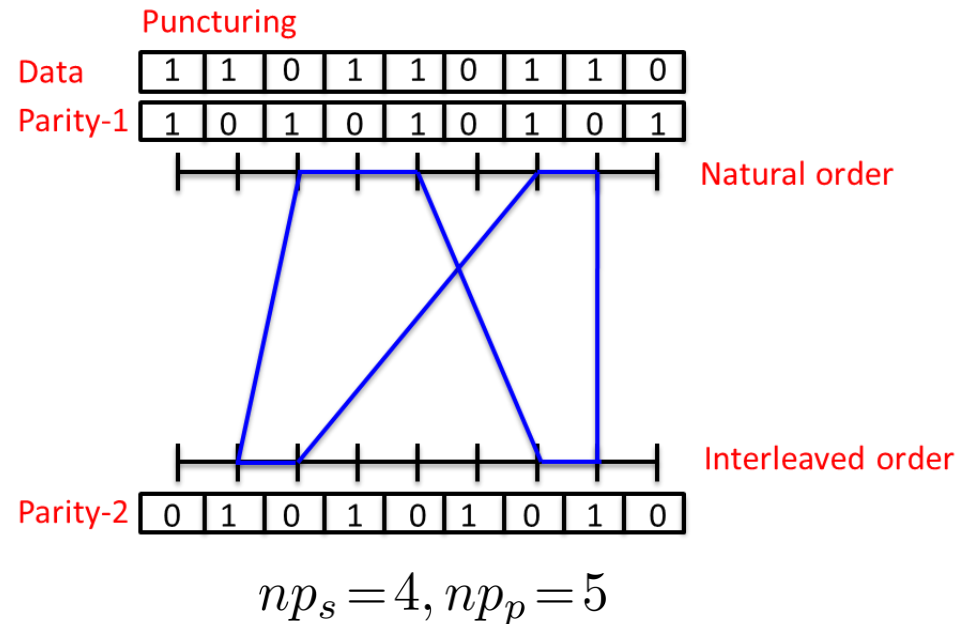
3. Union bound evaluation for NB-TCs

4. Low-complexity decoding algorithm

5. Performance comparison

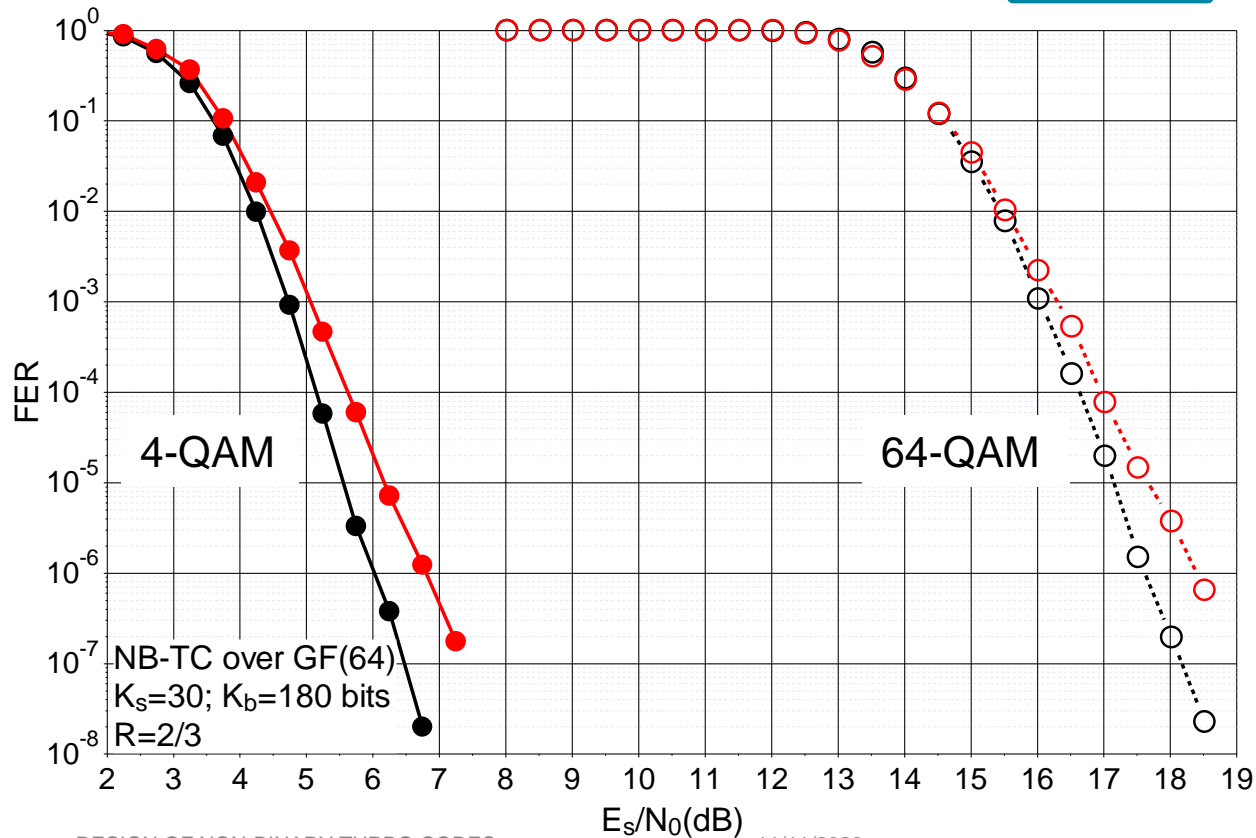
6. Conclusion and future work

- ▶ Low number of non-punctured symbols in correlation cycles => possible error event.
- ▶ Avoid to puncture symbols from short correlation cycles.
- ▶ Higher mutual information propagated in short correlation cycles.



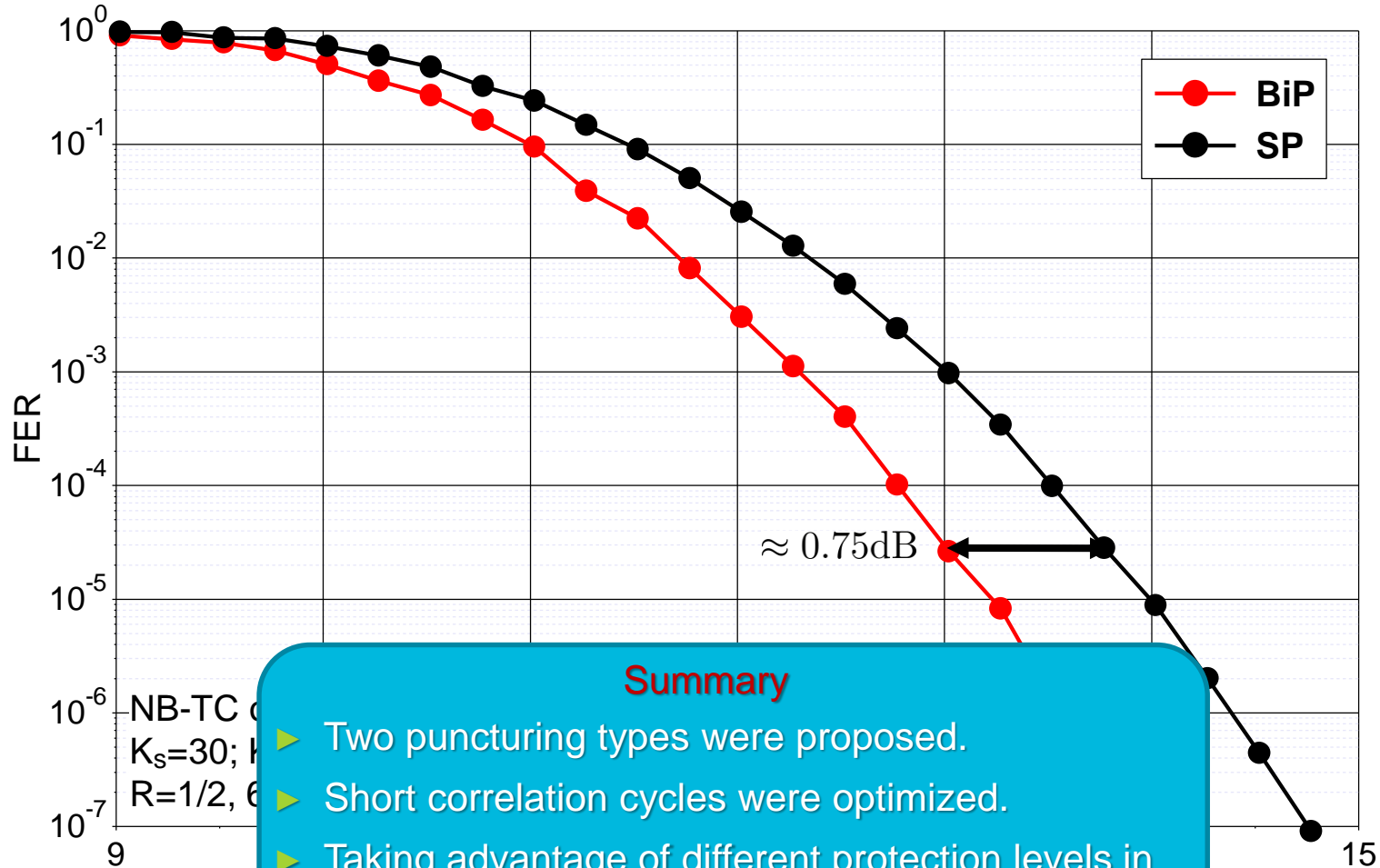
Proposed symbol puncturing design for NB-TC over GF(64)

Puncturing mask	Systematic d	Parity r_1	Parity r_2	np_s	np_p
I	11111 11111	00100 01000	01000 10100	4	2
II	11111 11111	01000 01100	00110 00000	4	0



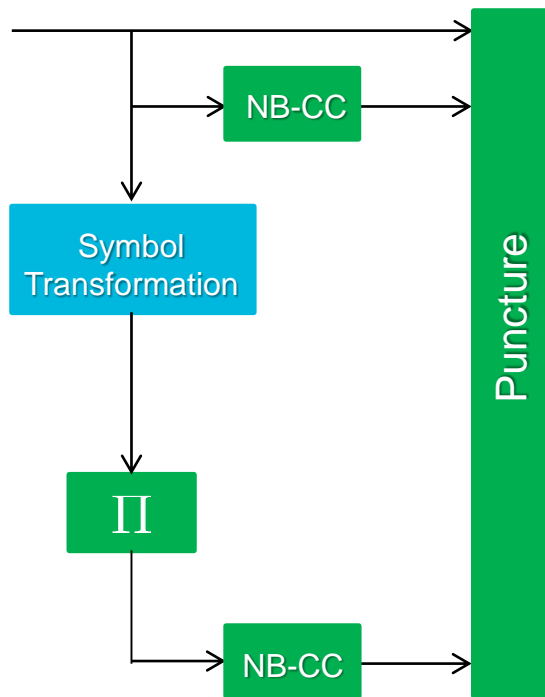
- ▶ Applied method:
 - Bits are protected differently in high order constellations.
 - Several symbol protection levels.
- ▶ Under consideration for possible patent filing.

Proposed bit puncturing design for NB-TC over GF(64), R=1/2



Summary

- ▶ Two puncturing types were proposed.
- ▶ Short correlation cycles were optimized.
- ▶ Taking advantage of different protection levels in high order constellation.



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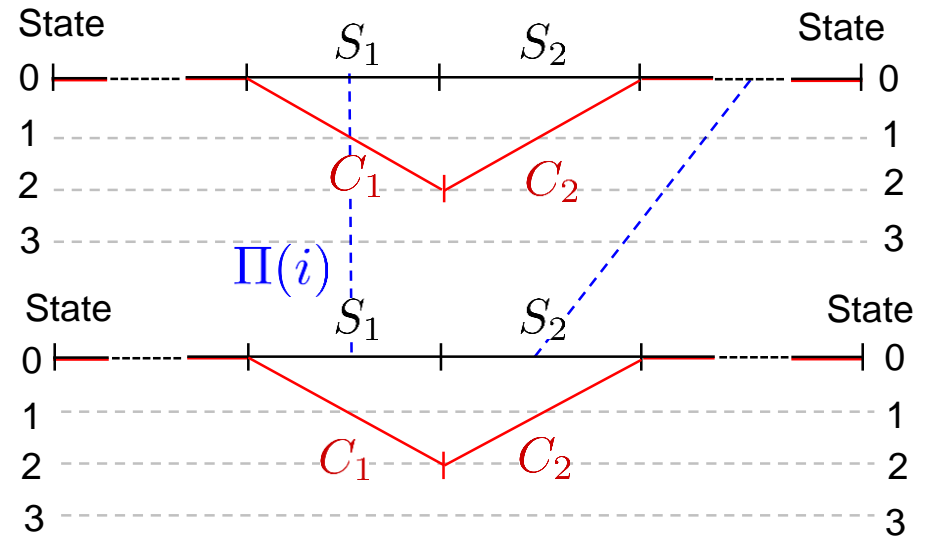
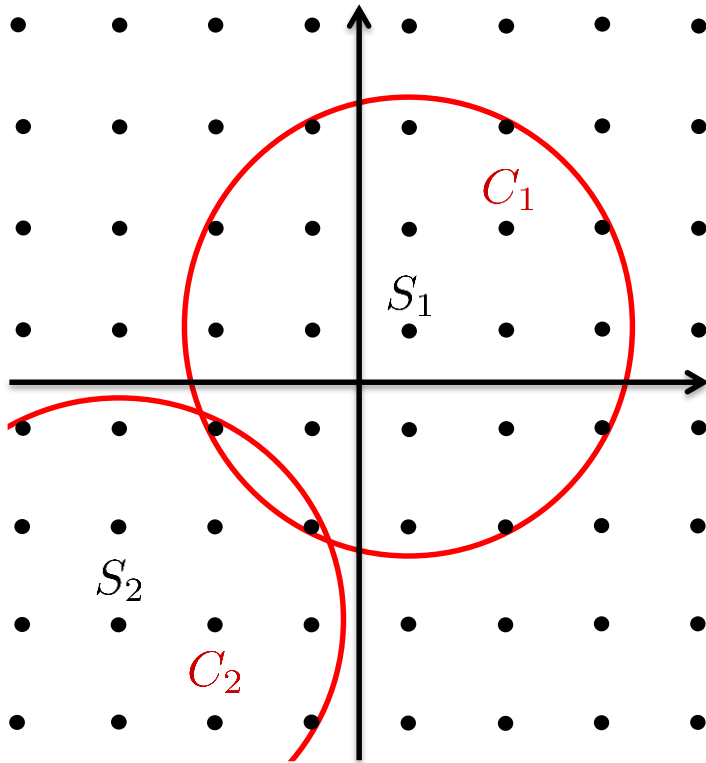
3. Union bound evaluation for NB-TCs

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Error prone sequences of NB-TCs: neighboring systematic symbols

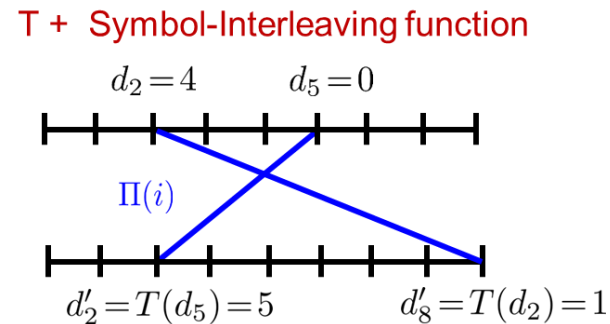
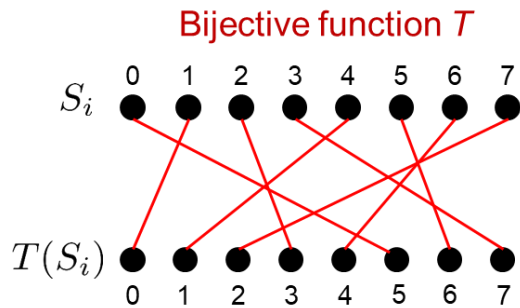
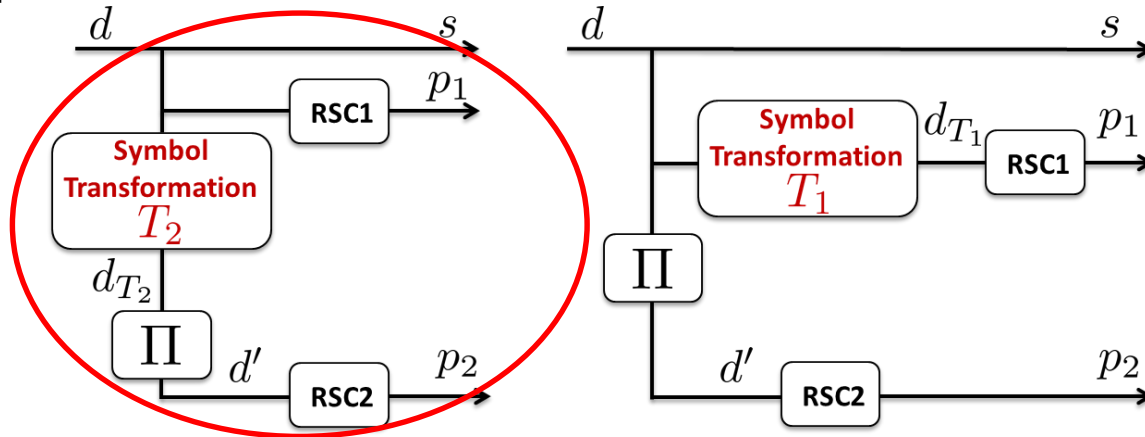


Repeated error pattern at both decoders => same false decoded sequence at both SISO decoders.

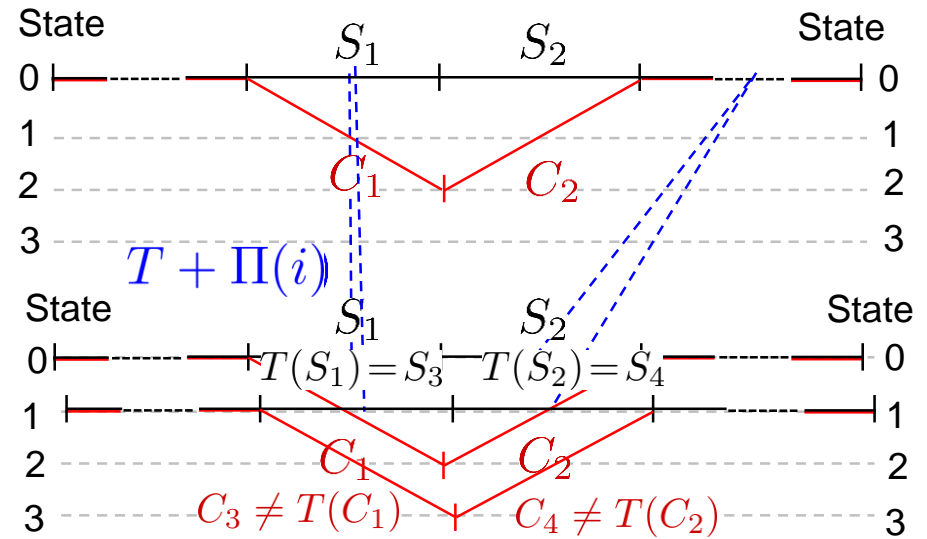
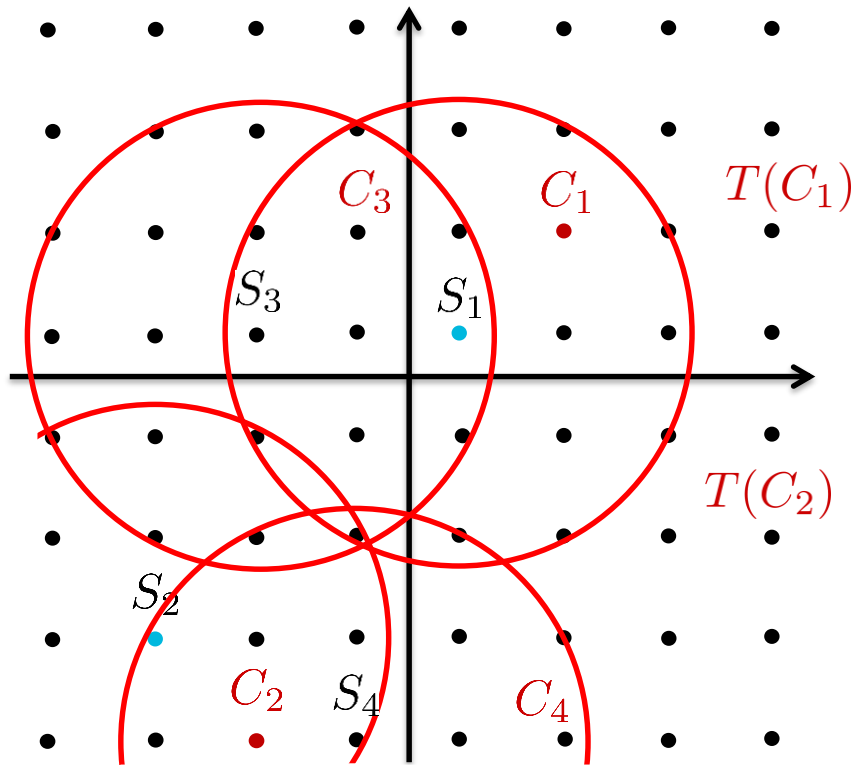
Neighboring constellation symbols
=> low CSED.
How to avoid them?

Symbol transformation: an additional freedom degree for NB-TCs

- ▶ New freedom degree can be investigated in the case of NB-TCs.
- ▶ A bijective symbol transformation can be added before one of the constituent encoders.
- ▶ The repetition structure is replaced by the transformation block to enhance the error correcting capabilities.



Why is it beneficial?



Bijective transformation results in different sequences at both decoders
 => different candidates at decoders
 => better information exchange.

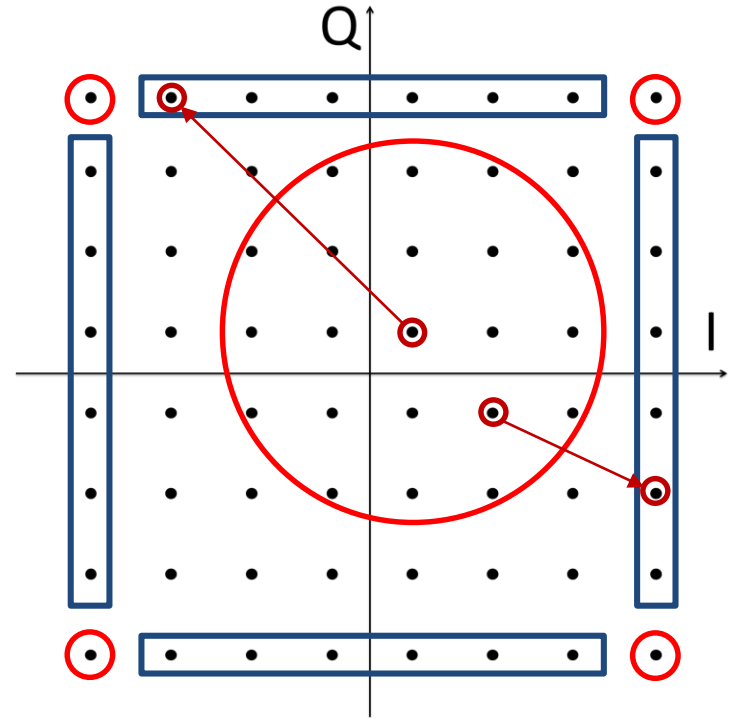
Neighboring constellation symbols
 Initial design constraint: maximize $\delta(S_i, S_j) = d^2(S_i, S_j) + d^2(T(S_i), T(S_j))$
 => low CSED
 How to avoid them?

GF(64) code mapped to a 64-QAM constellation

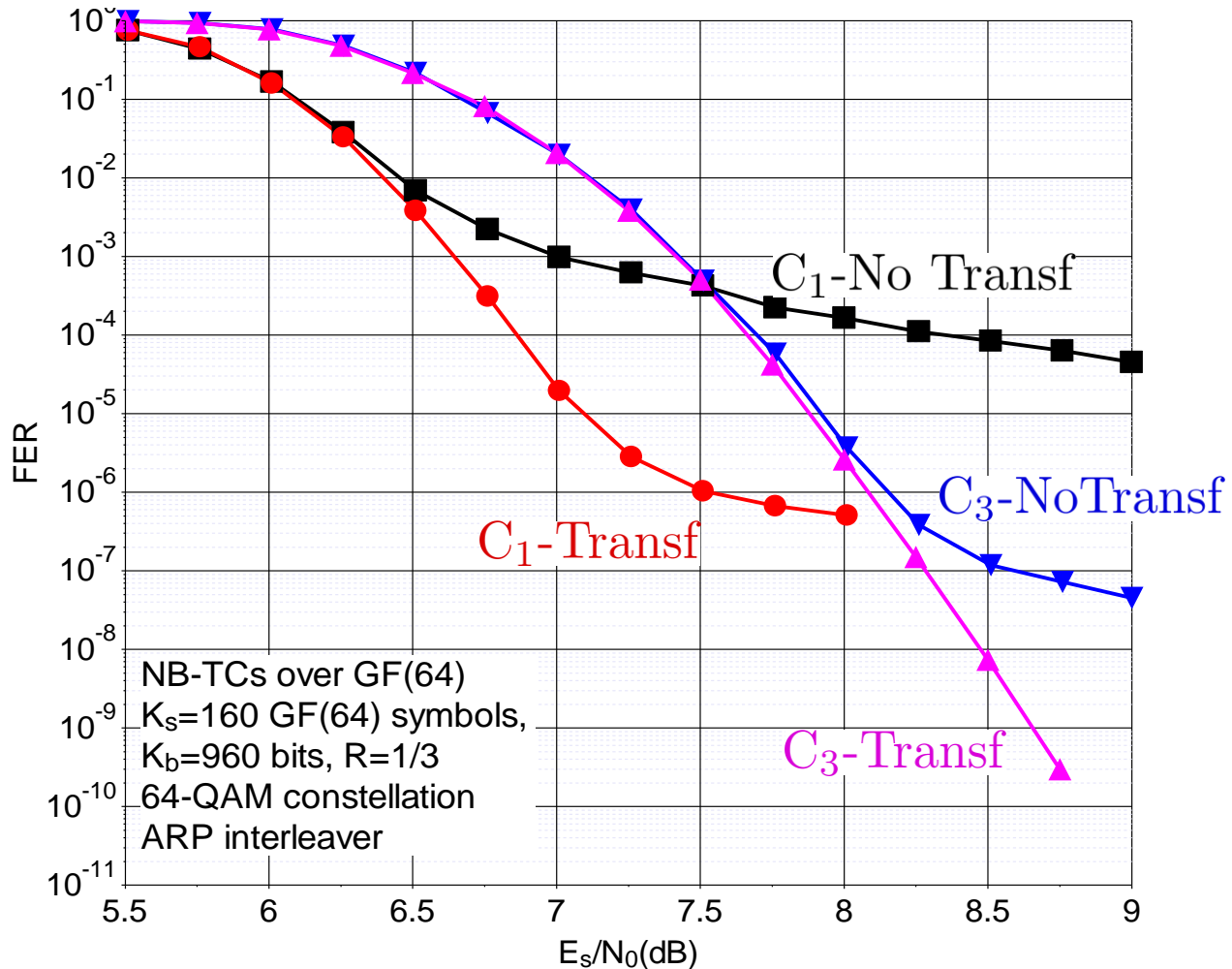
- ▶ Different protection levels for symbols in a 64-QAM constellation
- ▶ Neighboring symbols lead to short DC sequences.

Proposed constraints to design a good transformation function

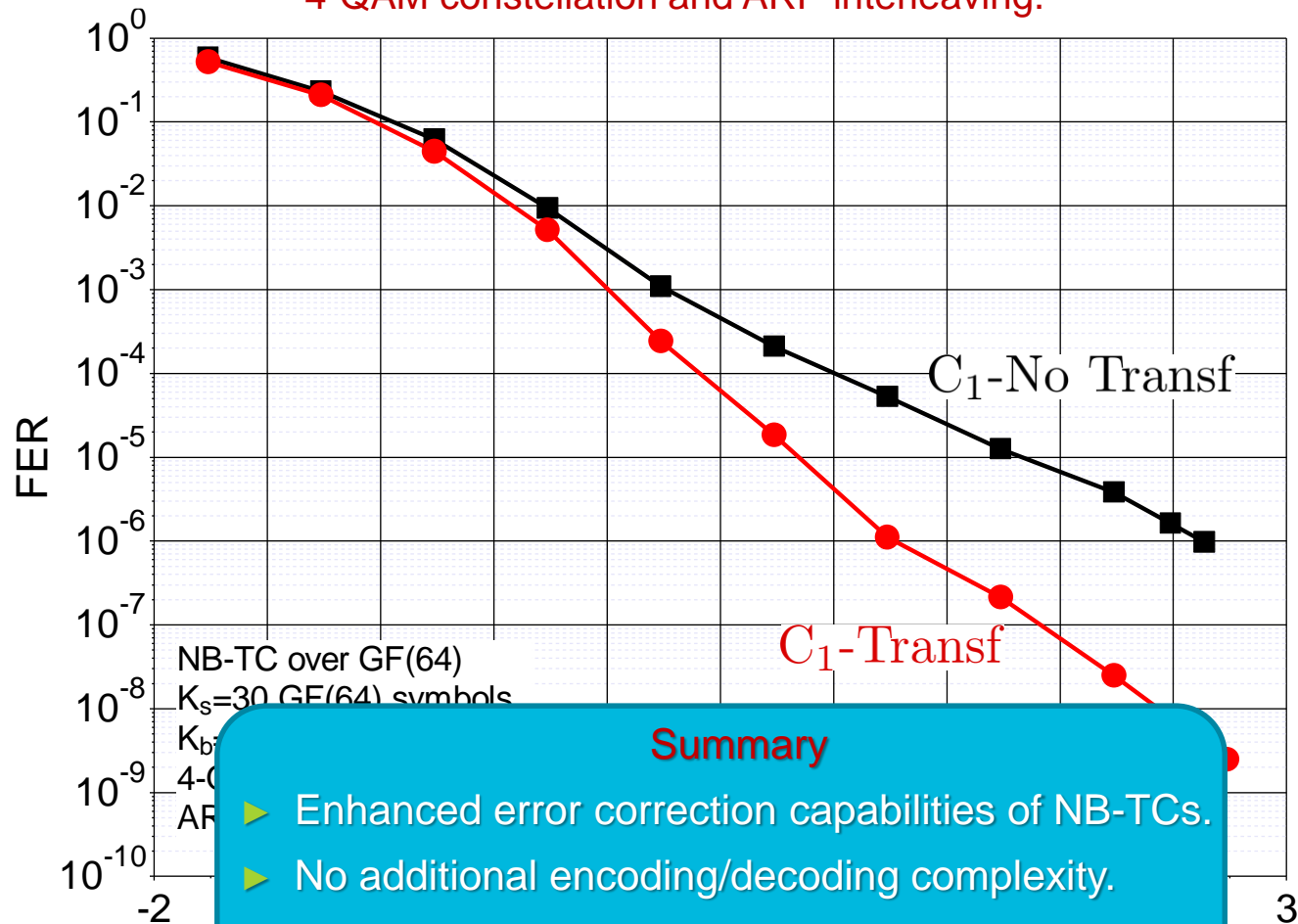
- ▶ Highly protected symbols => low protected symbols.
- ▶ Neighboring symbols => non-neighboring ones.

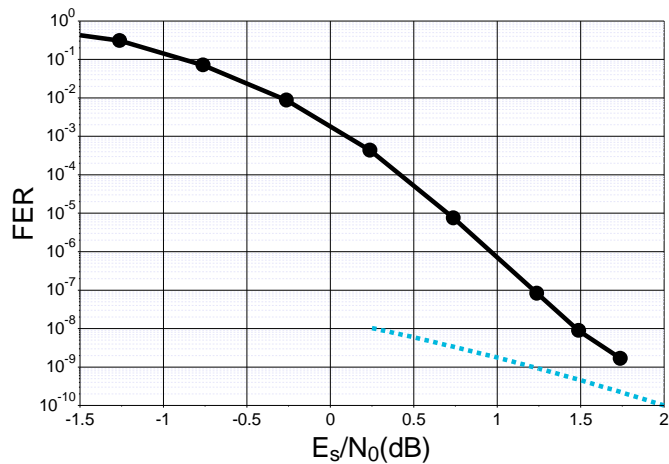


Performance evaluation of the transformation block for NB-TC over GF(64), 64-QAM constellation and ARP interleaving.



Performance evaluation of the transformation block for NB-TC over GF(64), 4-QAM constellation and ARP interleaving.





1. Context and state of the art
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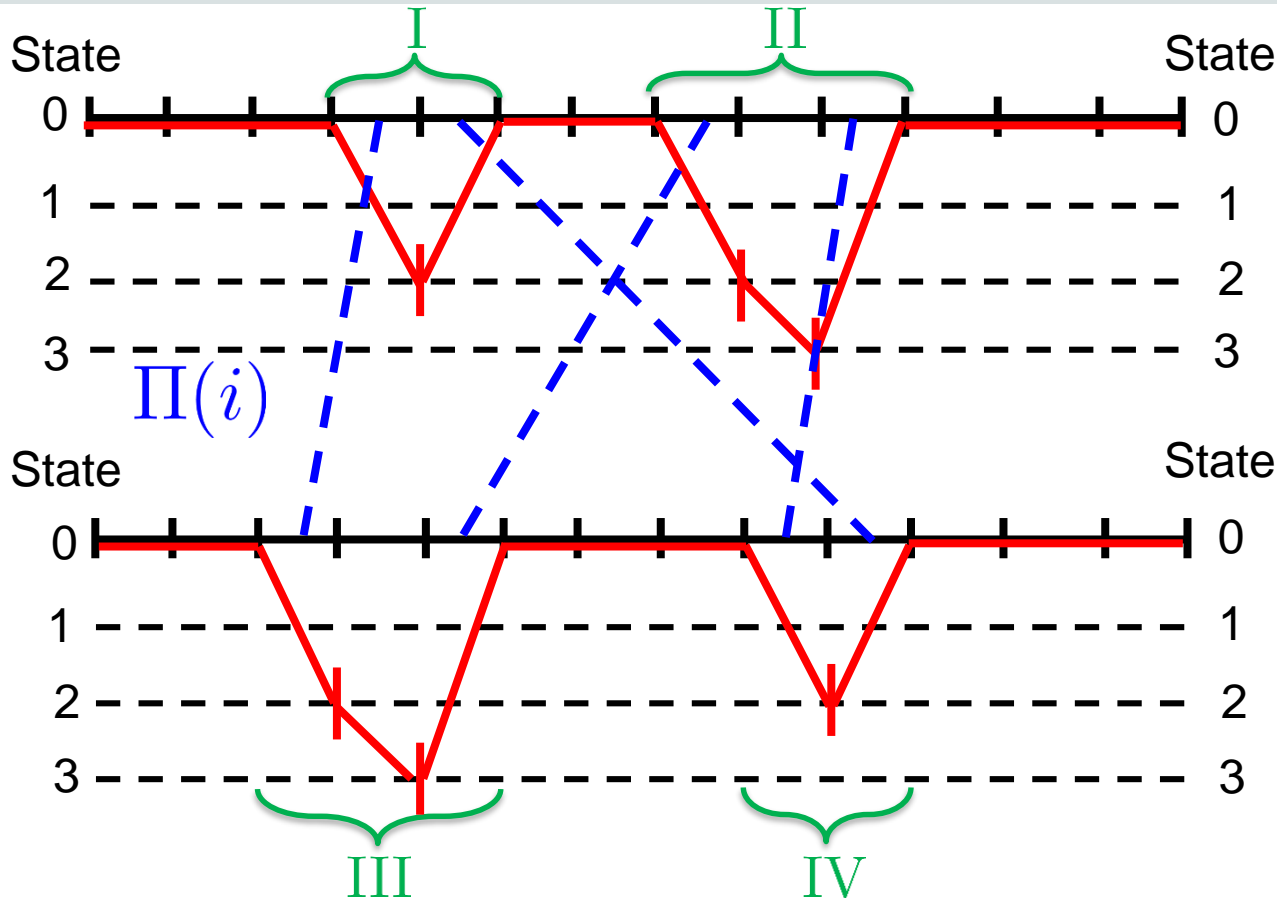
- ▶ Union bounds are used to predict the asymptotic behavior of a FEC.

	B-TC	NB-TC
B modulation	✓	✓
NB modulation	✓	×

- ▶ Union bound in the case of NB-TCs:

$$P_f \leq 2 \sum_{i=1}^{i_{max}} r(d_i) Q\left(\frac{d_i}{2\sigma}\right)$$

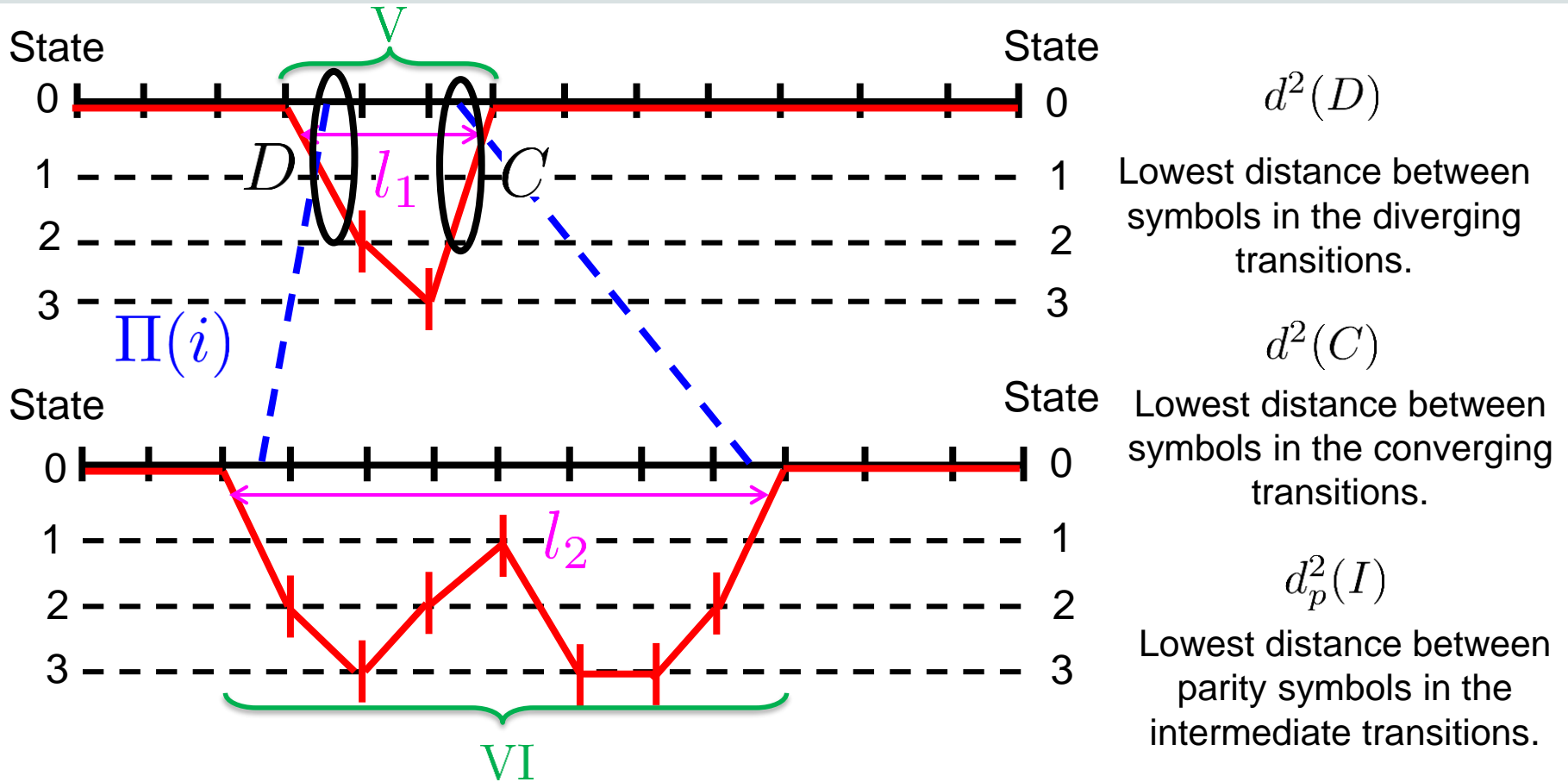
- ▶ **Problem:** identification of the cumulated Euclidean distance spectrum of NB-TCs.
- ▶ Two types of error-prone sequences of NB-TCs are observed:
 - error events caused by short correlation cycles,
 - error events caused by low interleaving spread.



$d^2(\text{DC} - 2)$
 Lowest distance for
 DC-2 sequences

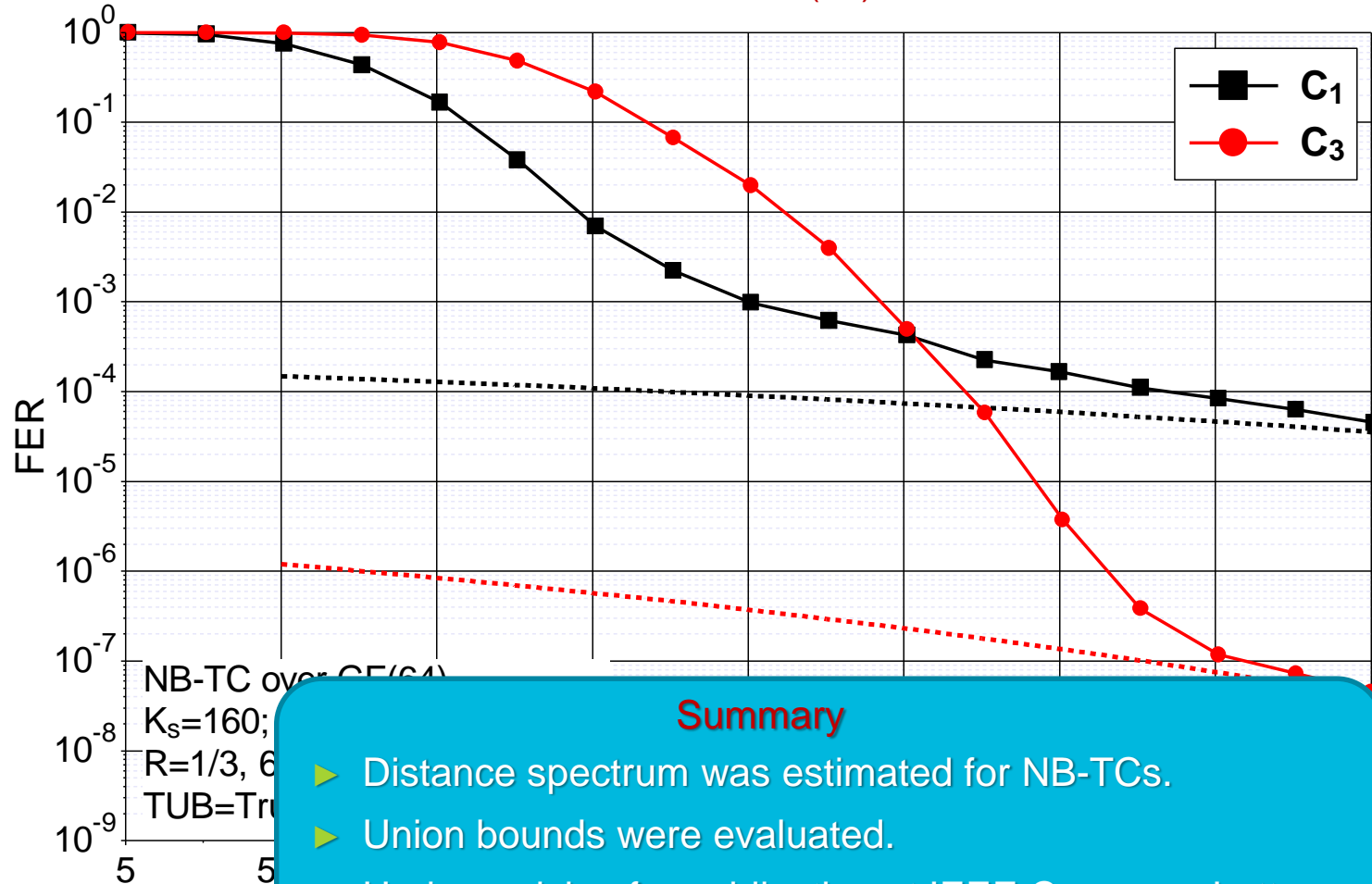
$d^2(\text{DC} - 3)$
 Lowest distance for
 DC-3 sequences

$$d_{g=6}^2 = \underbrace{d^2(\text{DC} - 2)}_I + \underbrace{d^2(\text{DC} - 3)}_{II} + \underbrace{d_p^2(\text{DC} - 3)}_{III} + \underbrace{d_p^2(\text{DC} - 2)}_{IV}$$



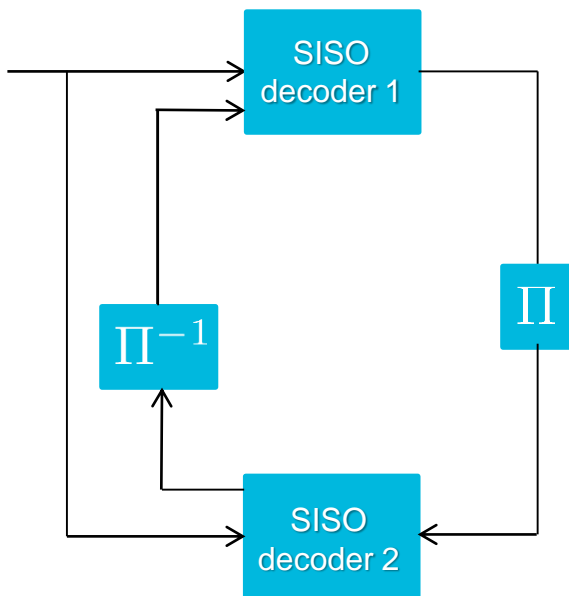
$$d_{l_1+l_2}^2 = \underbrace{d^2(D)}_V + \underbrace{d^2(C)}_V + \underbrace{(l_1 - 2)d_p^2(I)}_V + \underbrace{d_p^2(D) + (l_2 - 2)d_p^2(I) + d_p^2(C)}_{VI}$$

Union bound evaluation for NB-TC over GF(64), R=1/3, 64-QAM constellation



Summary

- ▶ Distance spectrum was estimated for NB-TCs.
- ▶ Union bounds were evaluated.
- ▶ Under revision for publication at IEEE Commun. Letters.



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4. **Low-complexity decoding algorithm**
5. Performance comparison
6. Conclusion and future work

Forward state metric

$$\alpha_i(j) = \min_{j' \in \{0 \dots q-1\}} (\alpha_{i-1}(j') + \gamma_{s,i-1}(j', j) + \gamma_{p,i-1}(j', j))$$

Complexity

q^2 ACS operations

Backward state metric

$$\beta_i(j) = \min_{j' \in \{0 \dots q-1\}} (\beta_{i+1}(j') + \gamma_{s,i}(j, j') + \gamma_{p,i}(j, j'))$$

q^2 ACS operations

Extrinsic LLR

$$L_i^e(a) = \min_{(j,j') \in \{0 \dots q-1\}^2 \mid s(j,j')=a} (\alpha_i(j) + \beta_{i+1}(j') + \gamma_{p,i}(j, j'))$$

q^2 ACS operations

$3q^2$ ACS operations
at each trellis stage

Complexity reduction for NB-LDPC decoders

- ▶ Extended Min-Sum (EMS) decoding algorithm
- ▶ Limiting the number of considered LLRs to the (n_m) most reliable ones [\[12\]](#)
- ▶ Limiting the number of Add-Compare-Select (ACS) operations through the use of *bubble check algorithm*. [\[13\]](#)

		U_1 U_2 U_3		
		0	7	15
V_1	0	0	7	15
V_2	6	6	13	21
V_3	13	13	20	28

$$E_1 = U_1 + V_1$$

		U_1 U_2 U_3		
		0	7	15
V_1	0	0	7	15
V_2	6	6	13	21
V_3	13	13	20	28

$$E_2 = \min(U_2 + V_1, U_1 + V_2)$$

[\[12\]](#) A. Voicila, D. Declercq, F. Verdier, M. Fossorier, and P. Urard, "Low-complexity, low-memory EMS algorithm for non-binary LDPC codes" in *IEEE International Conference on Communications. IEEE, 2007*, pp. 671–676.

[\[13\]](#) E. Boutillon and L. Conde-Canencia, "Bubble check: a simplified algorithm for elementary check node processing in extended min-sum non-binary LDPC decoders" *Electronics Letters*, vol. 46, no. 9, pp. 633–634, 2010.

Bubble check: not directly applicable for Min-Log-MAP

$$\alpha_i(j) = \min_{j' \in \{0 \dots q-1\}} (\alpha_{i-1}(j') + \gamma_{s,i-1}(j', j) + \gamma_{p,i-1}(j', j))$$

- ▶ Output sorting is not required.
- ▶ Three terms in each calculation => 3D bubble check.
- ▶ Not all transition/state metric combinations are possible: some cells in the table remain empty.

Proposal

- ▶ 2D-bubble proposed => compute forward, backward recursions and extrinsic info.
- ▶ n_m LLRs are used.
- ▶ No connections between highly reliable values => exit algorithm.
- ▶ Bubble identified => introduction of a virtual bubble => possible early stopping.

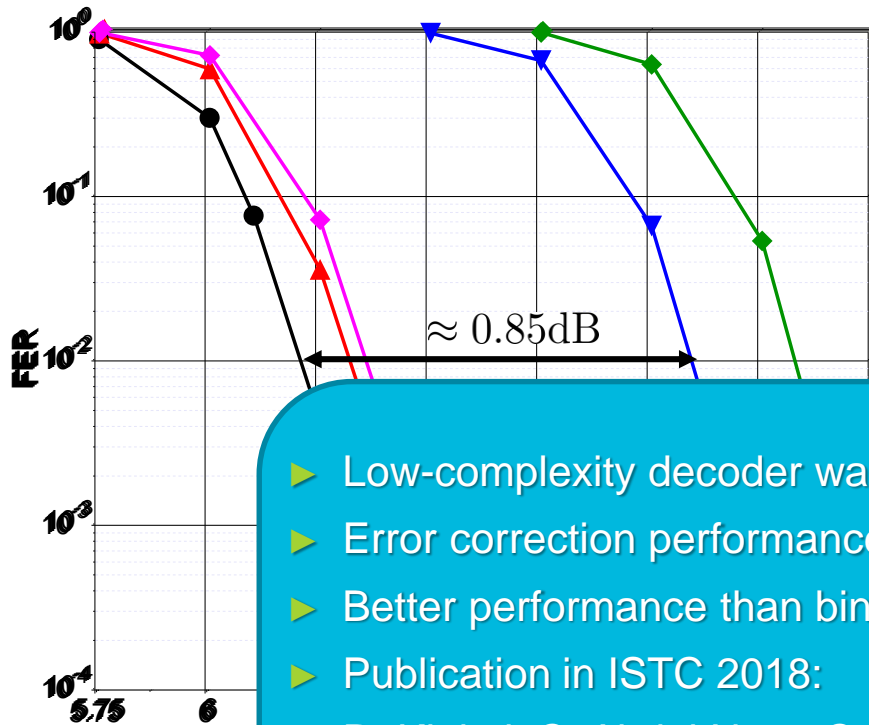
Decreasing reliabilities

	α_1	α_2	α_3	α_4	α_{n_m}
γ_s^1		B			
γ_s^2	B				
γ_s^3		B		B	
γ_s^4			B		
$\gamma_s^{n_m}$					B

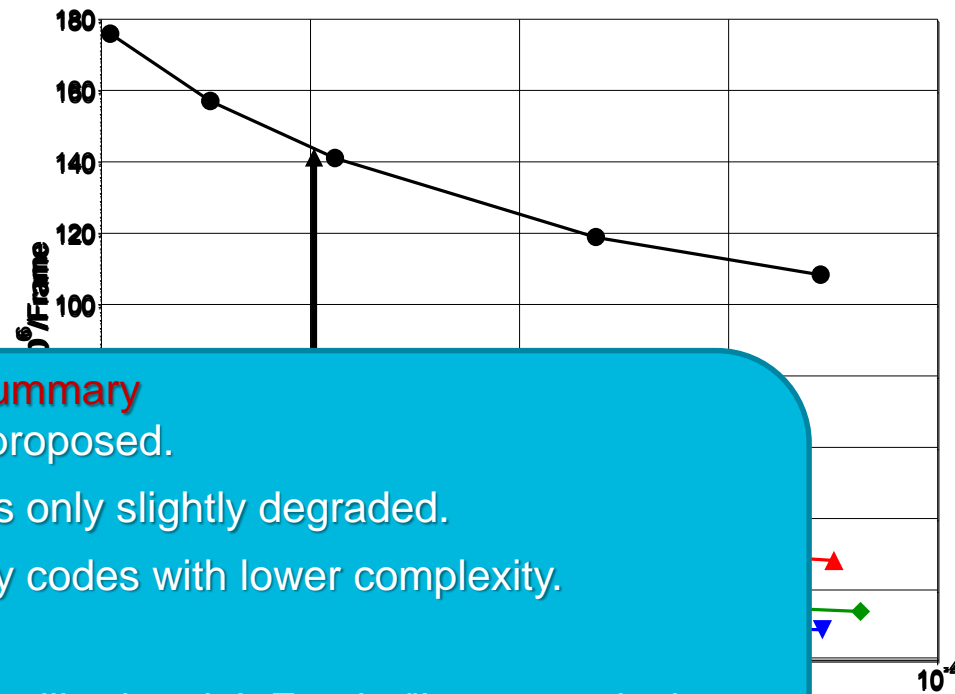
$$\alpha_i(j) = \min(\alpha_i(j), B)$$

Error correction performance and decoding complexity 42/54

Frame error rate performance for
 $K_s=900$, 64-QAM Symbols, rate=1/3,
8 decoding iterations, AWGN channel.



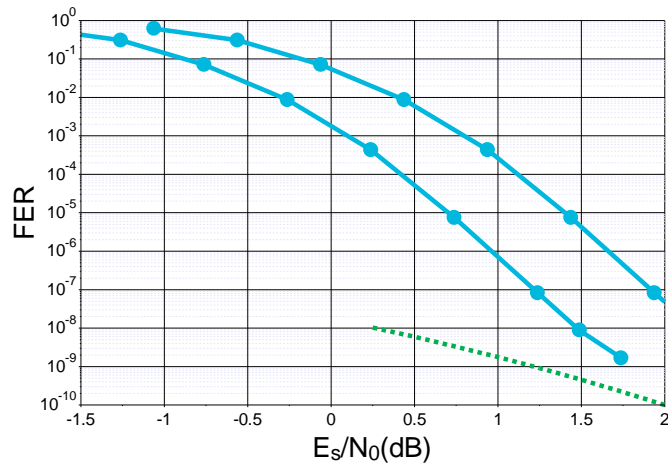
Performed ACS operations to achieve a particular FER for different algorithms



Summary

- ▶ Low-complexity decoder was proposed.
- ▶ Error correction performance is only slightly degraded.
- ▶ Better performance than binary codes with lower complexity.
- ▶ Publication in ISTC 2018:

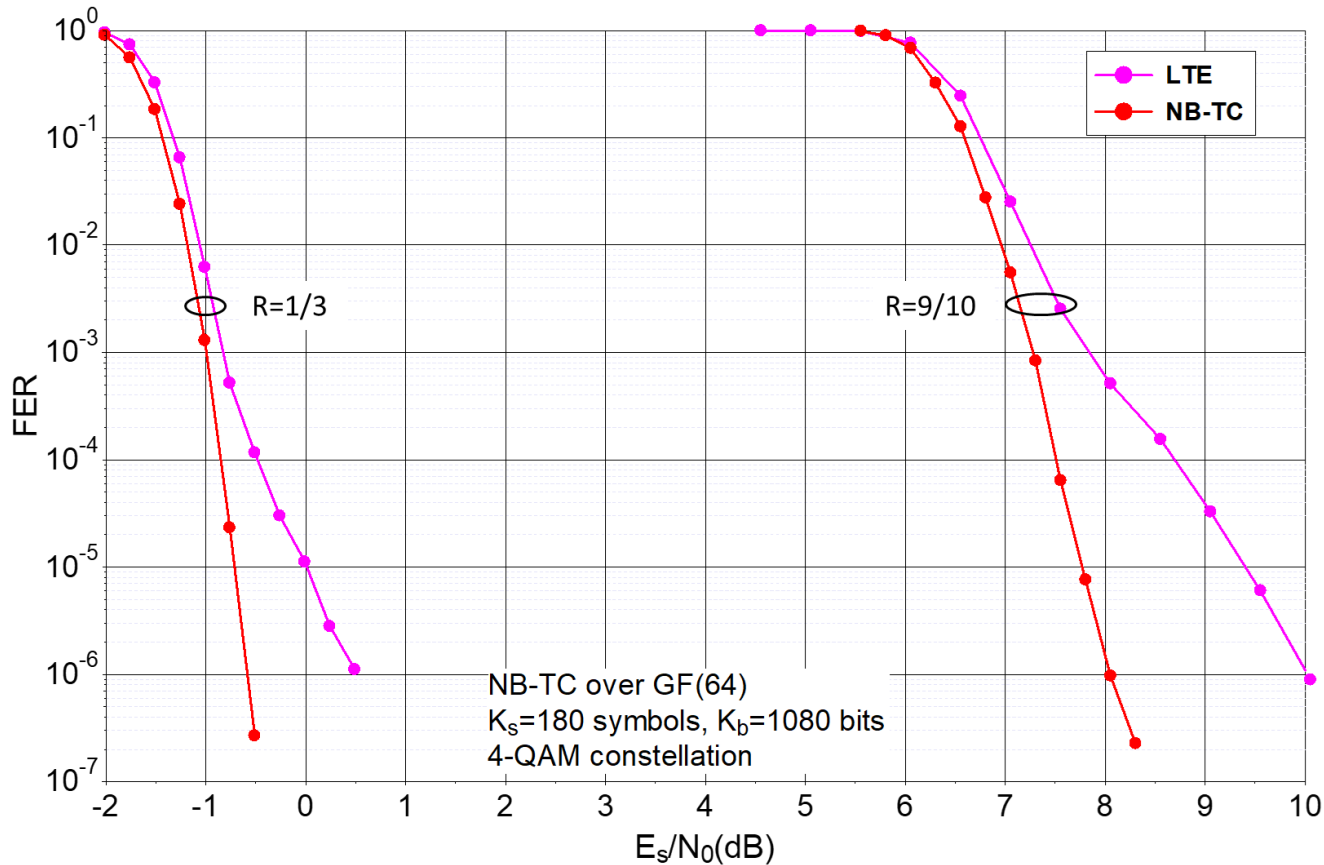
R. Klaimi, C. Abdel Nour, C. Douillard and J. Farah, "Low-complexity decoders for non-binary turbo codes", in 10th International Symposium on Turbo Codes and Iterative Information Processing (ISTC 2018), Hong Kong, Dec. 2018.



1. Context and state of the art
2. Design of NB-TCs
3. Union bound evaluation for NB-TCs
4. Low-complexity decoding algorithm
- 5. Performance comparison**
6. Conclusion and future work

Performance comparison with existing codes: comparison with LTE code

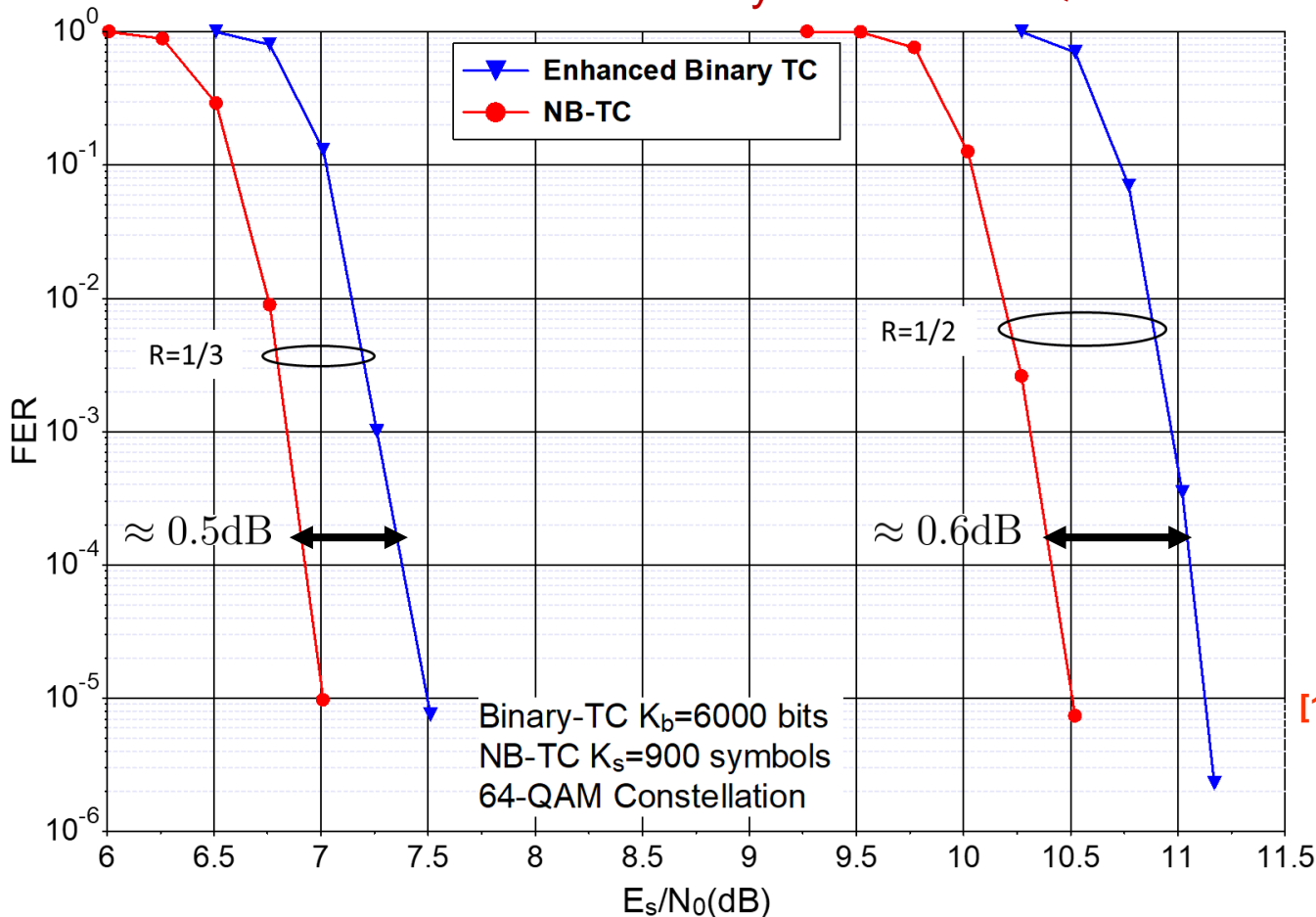
Frame error rate performance comparison of NB-TCs and LTE binary TC over 4-QAM constellation



Code	NB-TC	LTE
Channel	AWGN	
Decoder	MLM	MLM
Nb. It	8	8

Performance comparison with existing codes: comparison with enhanced binary turbo code

Frame error rate performance comparison of NB-TCs and enhanced binary TC over 64-QAM constellation [14]

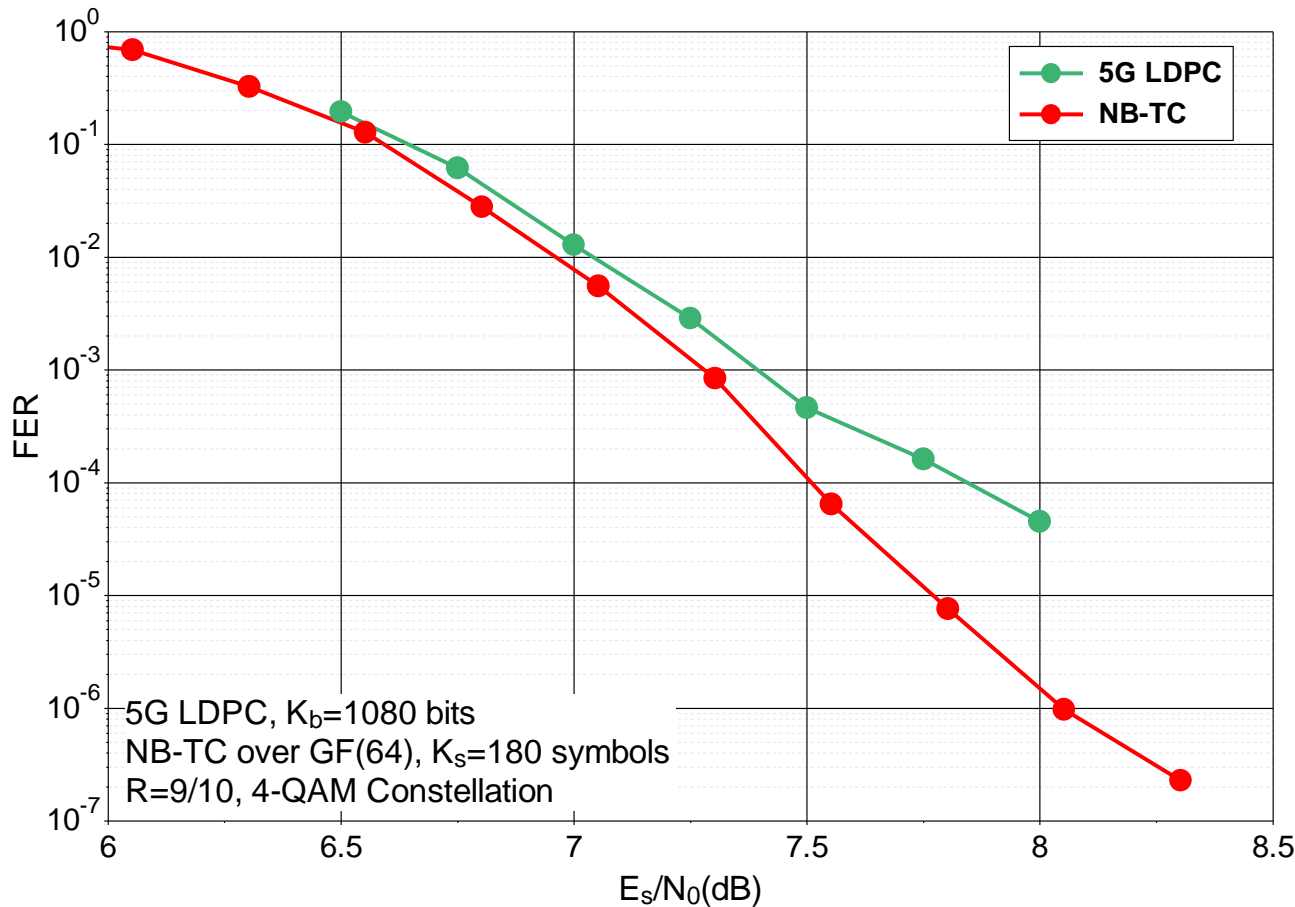


Code	NB-TC	Bin-TC
Channel	AWGN	
Decoder	MLM	MLM
Nb. It	8	8

[14] R. Garzón-Bohórquez, C. Abdel Nour, and C. Douillard, "Protograph-based interleavers for punctured turbo codes," IEEE Trans. Commun., vol. 66, no. 5, pp. 1833–1844, 2018.

Performance comparison with existing codes: comparison with 5G-LDPC code

Frame error rate performance comparison of NB-TCs and 5G-LDPC code over 4-QAM constellation [15]



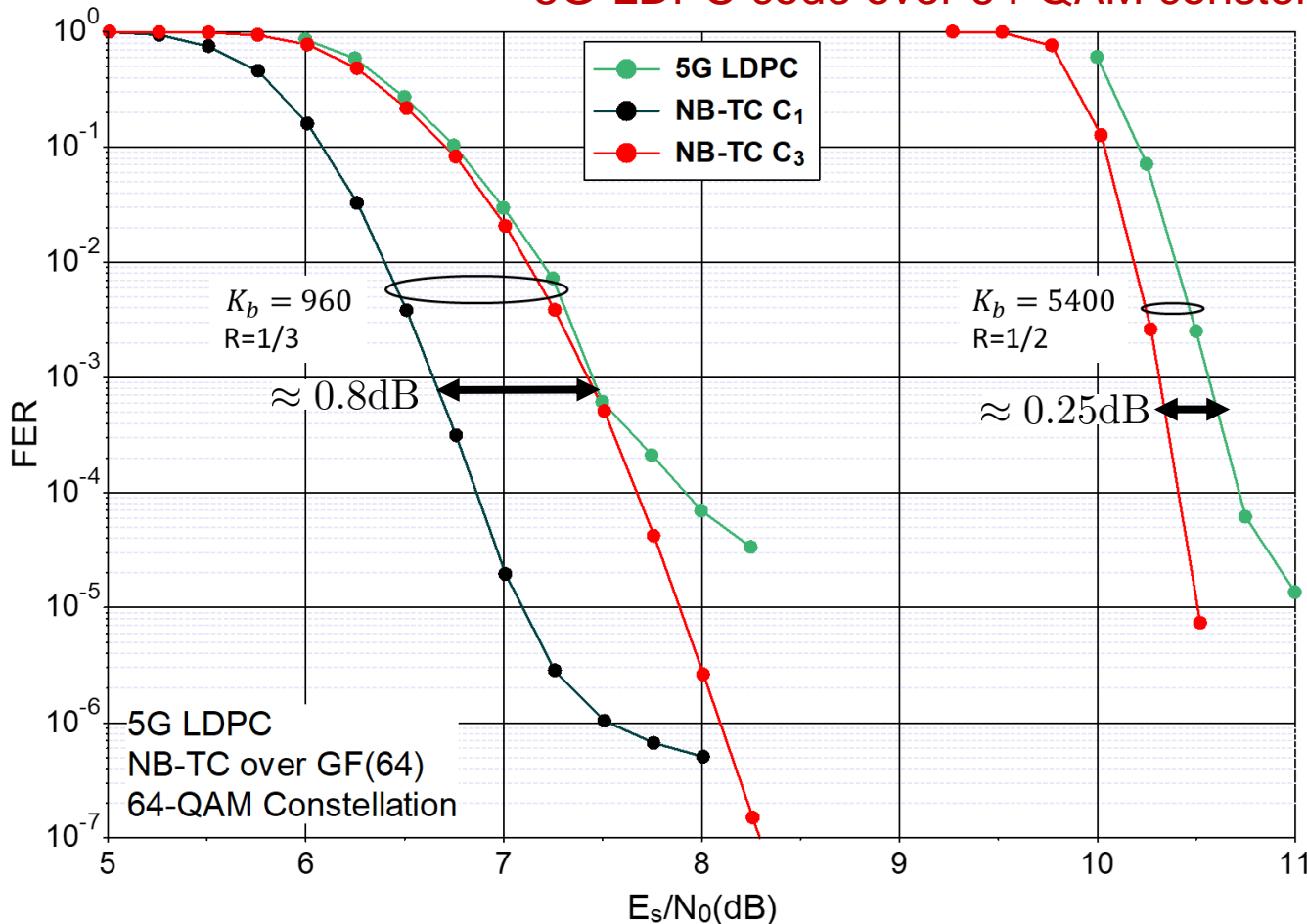
Code	NB-TC	LDPC
Channel	AWGN	
Decoder	MLM	BP
Nb. It	8	50

[15] R. Maunder, "Matlab simulations of the encoder and decoder for the New Radio LDPC code from 3GPP Release 15," Nov 2018. [Online]. Available: <https://github.com/robmaunder/ldpc-3gpp-matlab>

Performance comparison with existing codes: comparison with 5G-LDPC code

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Frame error rate performance comparison of NB-TCs and 5G-LDPC code over 64-QAM constellation [15]

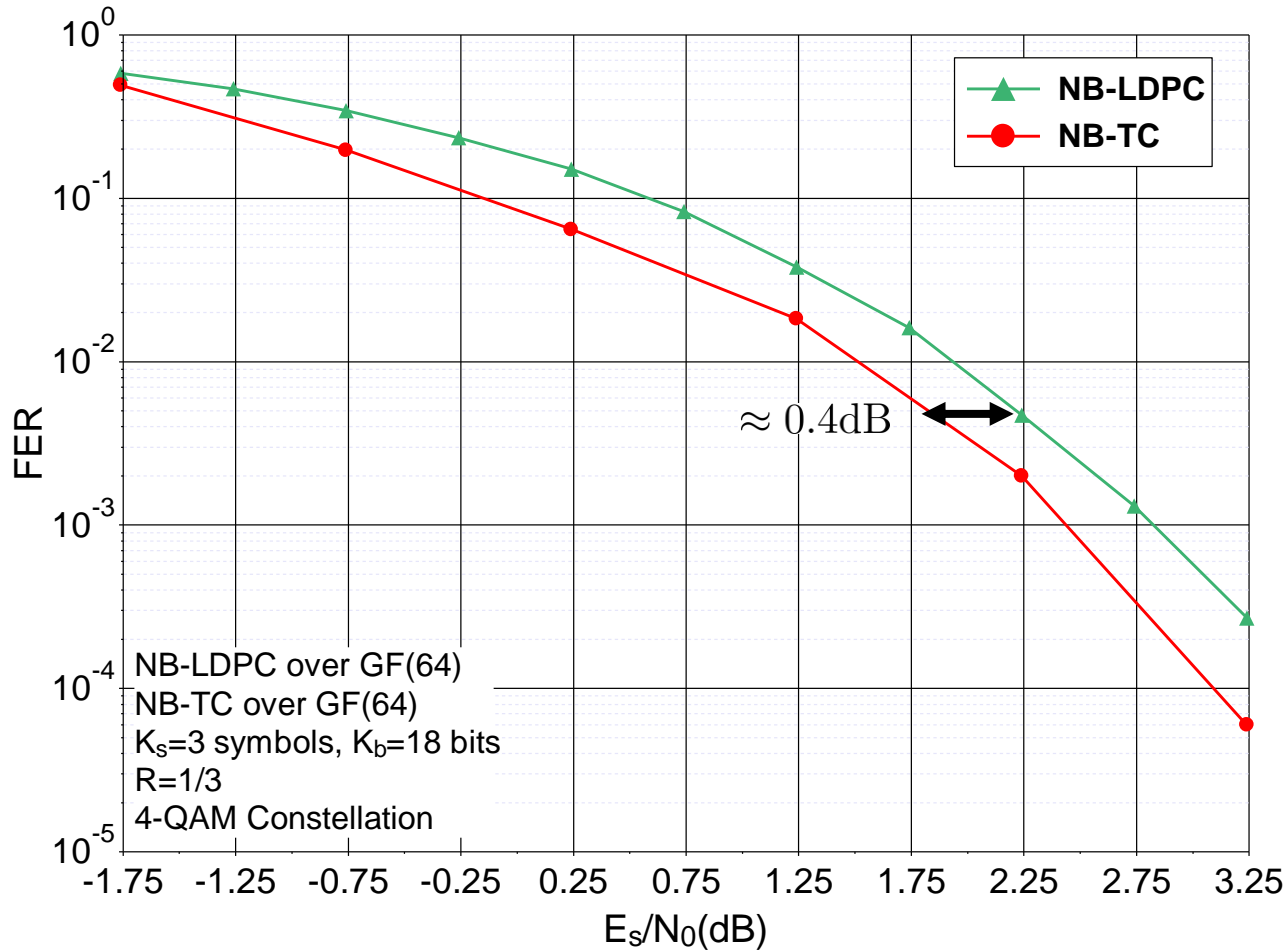


Code	NB-TC	LDPC
Channel	AWGN	
Decoder	MLM	BP
Nb. It	8	50

[15] R. Maunder, "Matlab simulations of the encoder and decoder for the New Radio LDPC code from 3GPP Release 15," Nov 2018. [Online]. Available: <https://github.com/robmaunder/ldpc-3gpp-matlab>

Performance comparison with existing codes: comparison with NB-LDPC code

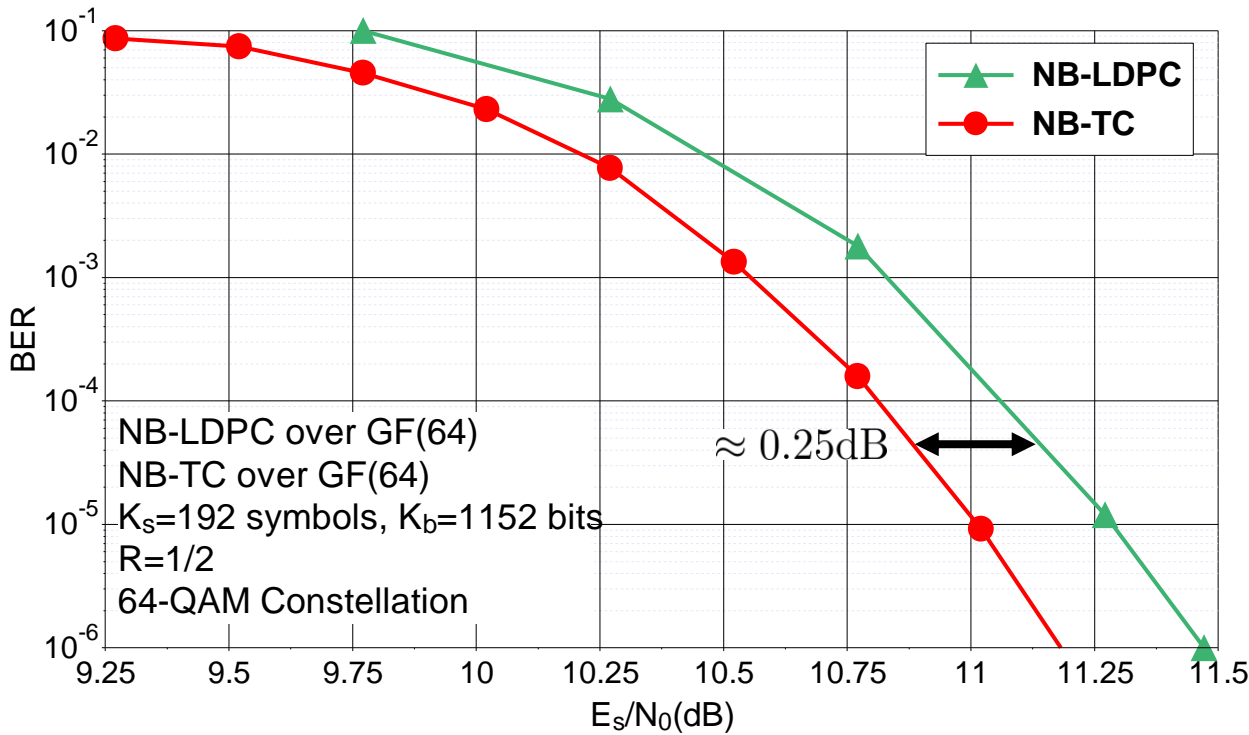
Frame error rate performance comparison of NB-TCs and NB-LDPC code over 4-QAM constellation [16]



[16] Lab-STICC, "NB-LDPC matrices and simulation results." [Online]. Available: <http://www-labsticc.univ-ubs.fr/nb-ldpc/>

Performance comparison with existing codes: comparison with NB-LDPC code

Bit error rate performance comparison of NB-TCs and NB-LDPC code over 64-QAM constellation [17]



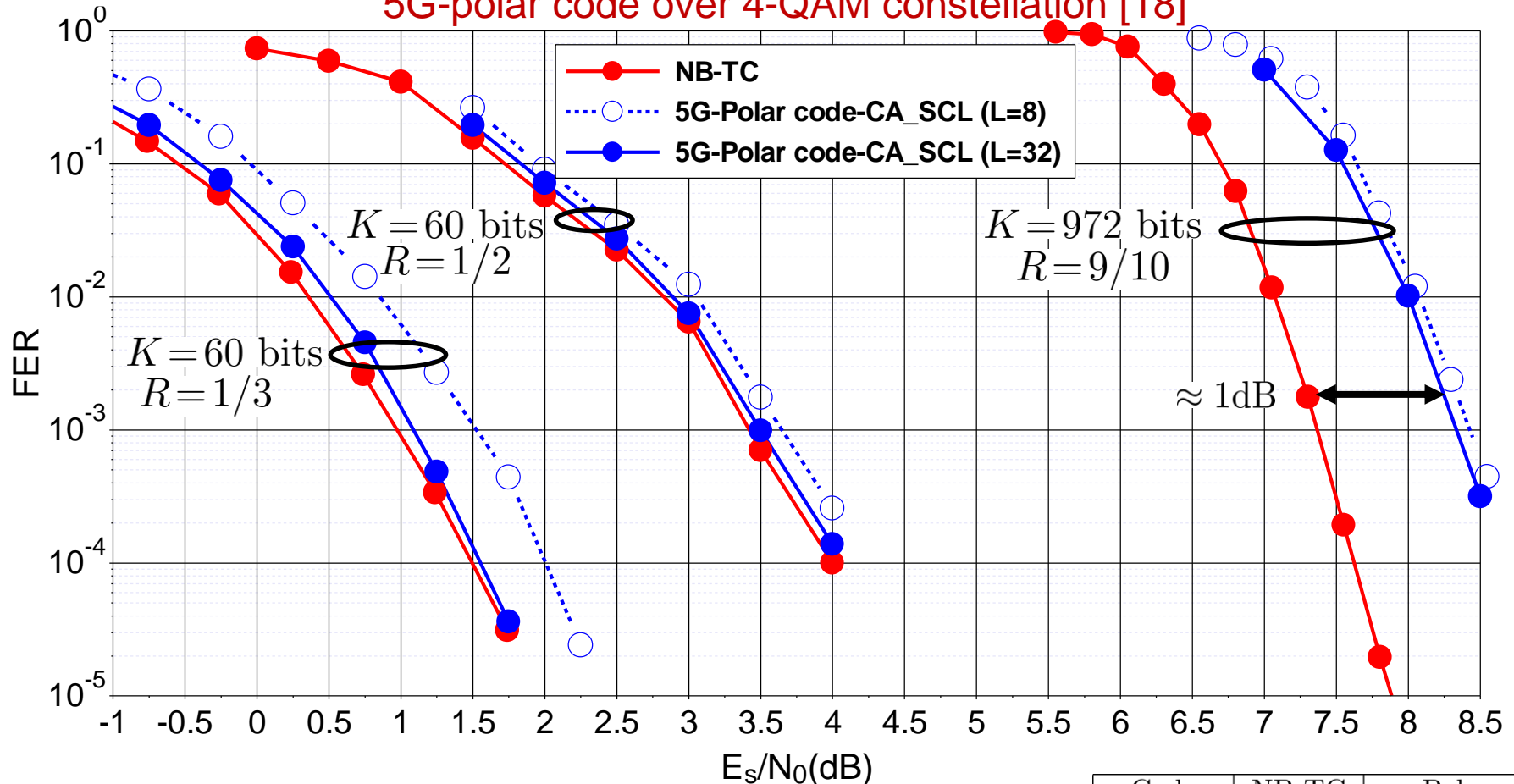
Code	NB-TC	NB-LDPC
Channel	AWGN	
Decoder	MLM	FFT-QSPA
Nb. It	8	50

[17] D. Feng, H. Xu, Q. Zhang, Q. Li, Y. Qu, and B. Bai, "Nonbinary LDPC-Coded Modulation System in High-Speed Mobile Communications," IEEE Access, vol. 6, pp. 50994–51001, 2018.

Performance comparison with existing codes: comparison with 5G-polar code

50/54

Frame error rate performance comparison of NB-TCs and 5G-polar code over 4-QAM constellation [18]



[18] R. Maunder, "Matlab simulations of the encoder and SCL decoder for the New Radio polar code from 3GPP Release 15," Aug 2018. [Online]. Available: <https://github.com/robmaunder/polar-3gpp-matlab>

Code	NB-TC	Polar
Channel	AWGN	
Decoder	MLM	SCL (8, 32)
Nb. It	8	

OUTLINE

1. Context and state of the art
2. Design of NB-TCs
3. Union bound evaluation for NB-TCs
4. Low-complexity decoding algorithm
5. Performance comparison
- 6. Conclusion and future work**



- ▶ Proposal of a new low-complexity structure of NB-CC with largely improved CSED ($> 25\%$).
- ▶ Error patterns analyzed \Rightarrow design of suitable ARP interleavers.
- ▶ Study of puncturing schemes \Rightarrow bit puncturing with largely improved performance (≈ 0.75 dB).
- ▶ Introduction of a transformation block \Rightarrow very low error floors ($< 10^{-10}$ of FER).
- ▶ Union bounds for NB-TC+high order modulations can now be evaluated.
- ▶ First low-complexity decoding algorithm for NB-TCs
 - E.g. 7x reduction in nb of ACS vs MLM for 0.15dB perf penalty.
 - Better perf than 64-state binary codes with lower complexity.
 - Several perf/complexity tradeoffs.
- ▶ Proposed NB-TCs are shown to outperform existing binary and non-binary FEC codes under different simulation conditions.

- ▶ Towards a complete FEC solution
 - Rate adaptability mechanism.
 - Optimization of the proposed blocks depending on the application.
- ▶ Further design exploration
 - Time variant NB-CCs => break short DC sequences.
 - Enhanced design of the transformation block.
 - NB-TCs with probabilistic shaping.
- ▶ Hardware implementation
 - Investigation of different perf/complexity tradeoffs.
 - Design space exploration of HW architectures.

► Journal papers:

- [1] R. Klaimi, C. Abdel Nour, C. Douillard and J. Farah "Union bound evaluation for Non-Binary Turbo Codes", submitted to IEEE Commun. Letters.
- [2] R. Klaimi, C. Abdel Nour, C. Douillard and J. Farah "Enhancing performance of non-binary turbo codes via symbol transformation", to be submitted to IEEE Trans. on Commun.

► Conference papers:

- [1] R. Klaimi, C. Abdel Nour, C. Douillard and J. Farah, "Design of Low-Complexity Convolutional Codes over $GF(q)$ ", IEEE Global Commun. Conf.: Communication Theory (Globecom2018 CT), Abu Dhabi, United Arab Emirates, Dec. 2018.
- [2] R. Garzon Bohorquez, R. Klaimi, C. Abdel Nour and C. Douillard, "Mitigating Correlation Problems in Turbo Decoders", in 10th International Symposium on Turbo Codes and Iterative Information Processing (ISTC 2018), Hong Kong, Dec. 2018.
- [3] R. Klaimi, C. Abdel Nour, C. Douillard and J. Farah, "Low-complexity decoders for non-binary turbo codes", in 10th International Symposium on Turbo Codes and Iterative Information Processing (ISTC 2018), Hong Kong, Dec. 2018.

► Patent filing:

- [1] R. Klaimi, C. Abdel Nour and C. Douillard "Procédé de génération d'un signal mettant en œuvre un turbo-codeur, dispositif et programme d'ordinateur correspondants (in French)", Patent application FR 1873615, Dec. 2018.
- [2] Patent filing under consideration for the bit puncturing technique.

- [1] C. Berrou, M. Jezequel, C. Douillard, and S. Kerouedan, “The advantages of non binary turbo codes,” in IEEE Inform. Theory Workshop, Cairns, Queensland, Australia, 2001, pp. 61–63.
- [2] G. Caire, G. Taricco, and E. Biglieri, “Bit-interleaved coded modulation,” IEEE Trans. Inform. Theory, vol. 44, no. 3, pp. 927–946, 1998
- [3] Böcherer, G., Steiner, F., & Schulte, P. (2015). Bandwidth efficient and rate-matched low-density parity-check coded modulation. *IEEE Transactions on Communications*, 63(12), 4651-4665.
- [4] Steiner, F., Liva, G., & Böcherer, G. (2017, December). Ultra-sparse non-binary LDPC codes for probabilistic amplitude shaping. *IEEE Global Commun. Conf* (pp. 1-5).
- [5] C. Douillard and C. Berrou, “Turbo codes with rate- $m/(m+1)$ constituent convolutional codes,” IEEE Trans. Commun., vol. 53, no. 10, pp. 1630–1638, 2005.
- [6] J. L. Massey and T. Mittelholzer, “Convolutional codes over rings,” in 4th Joint Swedish Soviet Int. Workshop Inform. Theory, Gotland, Sweden, 1989, pp. 14–18.
- [7] G. Liva, S. Scalise, E. Paolini, and M. Chiani, “Turbo codes based on time-variant memory-1 convolutional codes over F_q ,” in IEEE Int. Conf. Commun., Kyoto, Japan, June 2011, pp. 1–6.
- [8] G. Liva, E. Paolini, B. Matuz, S. Scalise, and M. Chiani, “Short turbo codes over high order fields,” IEEE Trans. Commun., vol. 61, no. 6, pp. 2201–2211, June 2013.
- [9] Matsumine, T., & Ochiai, H. (2018). Capacity-Approaching Non-Binary Turbo Codes: A Hybrid Design Based on EXIT Charts and Union Bounds. *IEEE Access*, 6, 70952-70963.
- [10] R. Garzon Bohorquez, C. Abdel Nour, and C. Douillard, “On the Equivalence of Interleavers for Turbo Codes”, IEEE Wireless Communications Letters, 4(1):58 – 61, February 2015.
- [11] R. Garzon Bohorquez, C. Abdel Nour, and C. Douillard, “Protograph-based interleavers for punctured turbo codes”, IEEE Transactions on Communications, 66(5):1833–1844, 5 2018.
- [12] R. Garzon Bohorquez, C. Abdel Nour, and C. Douillard, “Improving Turbo Codes for 5G with Parity Puncture-Constrained interleavers”, ISTC 2016 : 9th International Symposium on Turbo Codes & Iterative Information Processing, pages 151 – 155, Brest, France, September 2016.