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# Dynamic simulation and optimal operation of district cooling networks via 2D orthogonal collocation

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37 size of real-world applications [4]. In general, mathematical optimisation of these systems is very  
38 much skewed in favour of district heating systems, as stated by Werner [5]. However, despite the lack  
39 of analysis of district cooling systems, most of the methodologies applied to the analysis of district  
40 heating systems (DHS) can also be used to study district cooling systems. It is important to point out  
41 that each kind of system presents its own issues, related not only to the kind of utility produced (hot or  
42 cold) but also to the way it improves system efficiency, as will be detailed in the objectives of this  
43 work. Hence the importance of studies focused on district cooling networks.

44

45 Before introducing the applications for optimising district energy systems, it is important to present  
46 a general classification of the type of problems we find in mathematical optimisation. An optimisation  
47 problem consists of one (or sometimes more) objective function that has to be minimised (e.g.  
48 Operational cost, CO<sub>2</sub> emissions) or maximised (efficiency, production), subject to the fulfilment of  
49 the physical or operational constraints of the system, which are represented as equality or inequality  
50 constraints, by manipulating a set of decision variables. Depending on the nature of the decision  
51 variables (continuous or discrete), a general categorisation of optimisation problems can be  
52 established, which is independent of the methods implemented to solve the problem as stated by  
53 Biegler and Grossmann [6]. If the problem is described using only continuous variables when  
54 considering the nature of the constraints and the objective function that describes the system (linear or  
55 non-linear), we have linear programming (LP) and non-linear programming (NLP) problems. When  
56 discrete variables are involved, they are classified as mixed-integer linear programming (MILP) and  
57 mixed-integer non-linear programming (MINLP) problems. Finally, when dealing with dynamic  
58 models, two approaches are possible. Either we represent the dynamic problem as a succession of  
59 steady-state problems, known as multi-period optimisation, or we deal with the dynamics of the  
60 system. In the latter case, we can use either Pontryagin's principle (optimal control) or discretisation,  
61 formulating the dynamic problem as an algebraic problem (NLP, MILP or MINLP), known as  
62 dynamic optimisation.

63

64 According to the classification described above, we can organise studies on the optimisation of  
65 district heating and cooling as presented in Table 1. This detail selected contributions in terms of the  
66 kind of problem that is solved and their main applications (due to the complexity of the models used to  
67 describe these systems, there are few instances of LP applied to district heating and cooling, which is  
68 why no LP problem is reported). More extensive reviews on the applications of optimisation in  
69 District energy systems are presented by Talebi *et al.* [7], Sameti *et al.* [4], Gang *et al.* [8]. Evely and  
70 Ayoub [9] also present optimisation applications specifically for DCS. They highlight the fact that most  
71 studies have focused on optimising distribution network infrastructure (the selection of technologies,  
72 the number and kinds of users connected to the network, the existence of network elements such as

73 pumps, chillers, storage tanks, pipes), considering steady-state models and leading to a MILP or  
 74 MINLP formulation. On the other hand, one of the most unexplored subjects of study in the field of  
 75 energy systems is the dynamic optimisation and control of these systems, as stated by Gang *et al.* [8]  
 76 and Allegrini *et al.* [10].

77  
 78

**Table 1. Classification of studies on DHC Optimisation**

<b>Continuous Variables</b>	<b>Integer variables</b>	
<b>NLP</b>	<b>MILP</b>	<b>MINLP</b>
	<b>Data based</b> Chow <i>et al.</i> [11] - Diversity factor	<b>Data based</b> Deng <i>et al.</i> [12] - Scheduling
	<b>Steady-State</b> Söderman [13] - Topology - Operation (flow rates)	<b>Steady-State</b> Mertz <i>et al.</i> [14] Marty <i>et al.</i> [15] - Topology - Sizing - Operation
	<b>Multiperiod</b> Khir <i>et al.</i> [16] - Sizing - Topology - Operation	
<b>Dynamic</b> Schweiger <i>et al.</i> [17] - Operation (Modelica) This contribution - Operation (2D-OCFE)		<b>Data based MIQCP</b> Schweiger <i>et al.</i> [17] - Scheduling

79

80 Among the applications used in district cooling, some studies are based on data. Chow *et al.* [11]  
 81 presented a MILP formulation that optimised the diversity factor, which is the proportion of diverse  
 82 types of building (office, residential, shops, hotels and mass transit railway stations), resulting in a  
 83 uniform cooling load to ensure a high stability in the cold production system to be installed. They first  
 84 calculated typical 24-hour demand profiles for five types of building for 36 typical days (three typical  
 85 days per month), using a freeware building energy analysis programme that can predict energy use and  
 86 cost for all types of building. With these data, they then implemented a genetic algorithm to solve a  
 87 MILP problem that aims to minimize a fluctuation index with respect to the maximum cooling  
 88 demand. Here, the optimisation variables are the number of buildings for each of the five categories. In  
 89 order to avoid local optimum, they used a genetic algorithm. However, their study does not consider  
 90 the topology of the network, which could have a considerable impact on the delay and the formulation  
 91 of the fluctuation index. Moving to MINLP applications, Deng *et al.* [12] presented an approach for  
 92 the optimal scheduling of an actual DHC system that minimised its daily operation cost. The system  
 93 was composed of an electric chiller system, a ground source heat pump, a thermal storage system and

94 a combined cooling, heating and power system. The nonlinearities of the continuous variables  
95 corresponded to the operational conditions of each of the components of the system, while the discrete  
96 variables corresponded to the use (on/off) of the chillers in different periods of energy demand. This  
97 contribution optimised the energy mixing of the system, aiming to cover a given total demand, but  
98 assumed that the equipment always operated at nominal levels. However, they did not consider the  
99 interactions of the clients with the distribution system nor their location with respect to the production  
100 site.

101

102 Continuing with steady-state studies, Söderman [13] presented an optimisation of the structure and  
103 operation of an existing cooling network, based on a steady-state model of the users' maximum  
104 cooling demand. He also presented a project in which the capacity of the network would be increased  
105 to serve almost twice as many customers. To expand the network, he computed the location of new  
106 energy storage and production sites, as well as the pipe connections of the new interconnected system.  
107 This work included the linearisation of the mathematical model of the network. The problems were  
108 solved using the MILP CPLEX solver. Although this contribution, contrary to the previous one,  
109 presented a detailed analysis of the network, some parameters (pipe diameters) were not reported.  
110 Also, the steady-state assumption could prevent the use of renewables to expand the network. Finally,  
111 the assumption of constant cooling demands could result in an overestimation of the production of  
112 cold. MINLP steady-state applications in heating networks are found in the works of Mertz *et al.* [14]  
113 and Marty *et al.* [15]. The former performed a combinatorial non-linear optimisation to find the  
114 topology and substation exchanger size that minimised the global cost of a district heating network.  
115 The resulting MINLP problem was solved using DICOPT within GAMS®. Marty *et al.* [15]  
116 implemented a strategy to simultaneously optimise the district heating network topology, the Organic  
117 Rankine Cycle (ORC) sizing of a geothermal plant, and the distribution of the geothermal fluid  
118 between the ORC and the DHN. To solve the proposed MINLP problem, they used the MINLP  
119 DICOPT solver in the GAMS® environment. Since the main critical point in solving an MINLP  
120 problem is its initialisation, Mertz *et al.* [14] and Marty *et al.* [15] also presented their strategies to  
121 overcome this point. However, all of these studies were performed for steady-state conditions,  
122 although the variable customer demand, the thermal storage or, sometimes, the use of intermittent  
123 renewable energy result in the district heating and district cooling networks becoming dynamic  
124 systems.

125

126 The multiperiod application presented by Khir and Haouari [16] developed an approximation for  
127 the optimal design of a DCS whose results comprised the chiller plant size, storage tank size, layout of  
128 the network and the quantities of energy produced and stored during each period. They used the ILOG  
129 CPLEX software package, with the aim of minimising the amount of investment and the operational

130 cost of the system. District planning included studies on the influence of the number and kind of  
131 buildings served by the cooling network. Their model considers the demand of the user at each period  
132 but does not consider the dynamics of temperature in the system pipes.

133

134 As already stated, one of the least explored subjects of study in the field of energy systems is the  
135 dynamic optimisation and control of these systems.

136

137 Recent advances in this field include the study by Schweiger *et al.* [17] dealing with optimal  
138 production planning in district heating systems. They presented a framework to represent on-grid  
139 energy systems and performed a dynamic thermo-hydraulic simulation of energy systems. The  
140 framework was based on the Modelica® modelling language, performing the continuous optimisation  
141 tasks with the OPTIMICA compiler toolkit, and the discrete optimisation in the Python open-source  
142 environment using the Pyomo module. They decomposed the resulting mixed-integer-optimal control  
143 problem into a Mixed Integer Quadratic Constrained Programming (MIQCP) problem (a particular  
144 form of MINLP problem) and a continuous problem. The results of the former provided the status and  
145 heat production of each unit. The discrete variables representing the status of each unit were thus fixed  
146 by this solution from the MIQCP, although the real heat production was calculated in the continuous  
147 problem which was transformed into a Nonlinear Programming (NLP) problem using a direct  
148 collocation method, then solved using the interior point algorithm IPOPT. The objective function  
149 proposed in this work (and which had to be minimised) was the supply temperature of the producer for  
150 the duration of the considered time span. Although this implementation is based on physical models, it  
151 is fully tool-oriented to Modelica users, offering few details on the mathematical modelling and  
152 treatment of the dynamic optimisation problem. This fact makes it difficult to replicate their  
153 methodology on other available modelling and optimisation tools. In the field of the dynamic  
154 optimisation of energy systems, we can also mention the dynamic optimisation of a hybrid Solar  
155 thermal and fossil Fuel system [18].

156 To our knowledge, studies on the dynamic optimisation of district energy systems are limited to the  
157 aforementioned works. We hope to contribute to this field and propose a dynamic optimisation of the  
158 return temperature of a district cooling network. The choice of this objective function will be  
159 discussed later; we will first introduce the dynamic problems and the way they can be solved.

160

161 Dynamic optimisation has been used for off-line tasks, including studies on operation in response  
162 to disturbances. As proposed by Schweiger *et al.* [17] a general form for optimisation problems of this  
163 kind can be represented as:

164

$$\min_{z,y,u,par} J = \phi(t_f, z(t_f)) + \int_{t_0}^{t_f} \mathcal{L}(t, z, y, u) dt \quad (1)$$

$$s. t. \quad \frac{dz(t)}{dt} = f(t, z(t), y(t), u(t), par), \quad z(t_0) = z_0, \quad (2)$$

$$g(z(t), y(t), u(t), par) = 0, \quad (3)$$

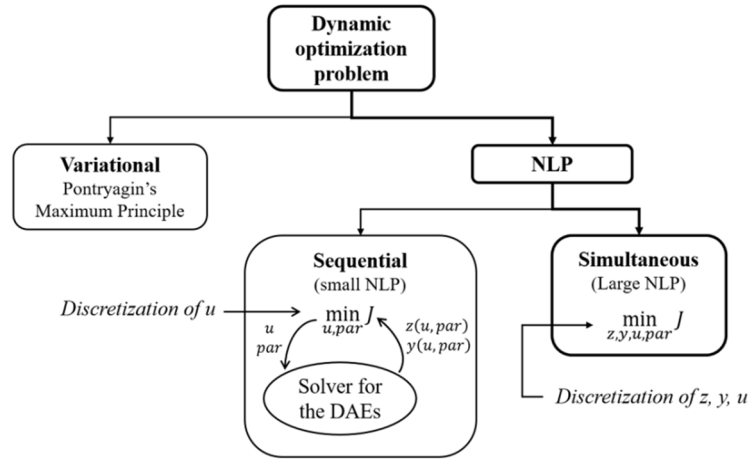
$$g_f(z(t_f)) = 0, \quad (4)$$

$$u_L \leq u(t) \leq u_U, \quad y_L \leq y(t) \leq y_U, \quad z_L \leq z(t) \leq z_U \quad (5)$$

165 where  $z(t)$  are the differential state variables,  $y(t)$  the algebraic state variables,  $u(t)$  the control  
 166 variables, all of which are functions of time  $t \in [t_0, t_f]$ , and  $par$  represents the time-independent  
 167 parameters. The constraints of this optimisation problem are the Differential and Algebraic Equations  
 168 (DAE) (2)-(4). This formulation is known as the problem of Bolza [19], where  $J$  is a scalar to be  
 169 minimised. The first term corresponds to the Mayer term and the integral term corresponds to the  
 170 Lagrange term. Thus, depending on the application, in dynamic optimisation, it is possible to  
 171 formulate objective functions of the form of Bolza, Mayer or Lagrange.

172

173 Biegler [20-21] reported different ways to solve the aforementioned problem. As shown in Figure  
 174 1, we can use the variational approach, based on the Pontryagin's Maximum Principle. However, this  
 175 approach could not handle properly with inequality constraints (in our case, we deal with such  
 176 constraints since the velocities are bounded). Other strategies applying an NLP solver can be used.  
 177 This involves replacing the time-dependent variables by discretised ones, such as coefficients of an  
 178 interpolation polynomial, for example, so that an NLP problem can be formulated and solved with  
 179 respect to these new discretised variables. The first strategy is the sequential approach: in this case,  
 180 only the control variables are discretised. For a set of control variables, a DAE solver in a loop solves  
 181 the state variables of the DAE system and returns the state and algebraic variables to the NLP  
 182 optimisation level. The control variables (in fact the discretised variables that represent them) are  
 183 updated by the NLP solver. This strategy can be time-consuming. In the second strategy, the  
 184 simultaneous approach, both state and control variables are discretised in time. Hence, the DAE  
 185 system is solved only once, at the optimal point, and therefore this can avoid computational effort to  
 186 obtain intermediate solutions for the DAE system.



187  
188  
189  
**Figure 1. Solution strategies for dynamic optimisation**

190 Many authors have suggested using Orthogonal Collocation on Finite Elements (OCFE) to  
191 discretise the state and/or control variables when DAEs are constraints of the dynamic optimisation  
192 problem [20-22] (or [23-25]).

193 With this in mind, in the present study, we use a simultaneous approach using 2D-OCFE. The  
194 equation that describes the transient temperature profiles in the pipes is a partial differential equation.  
195 Variables are then discretised in time and space.

196  
197 In order to contribute to the field of dynamic optimisation of district cooling system (DCS)  
198 operation, we propose a methodology that enables a simultaneous (oriented-equation) solution of this  
199 dynamic optimisation problem using 2D Orthogonal Collocation on Finite Elements (2D-OCFE). We  
200 apply this methodology to a medium-sized cooling system serving 20 consumers of different  
201 categories with fluctuating cooling demands, subject to variable external conditions. The dynamic  
202 simulation and optimisation were performed using insulated and non-insulated piping.

203  
204 First, the configuration of the studied cooling network is described, including the consumers'  
205 demand profile and the external conditions to which it is subject. Next, we present the resulting  
206 problem, then the proposed discretisation strategy, which consists in transforming the Partial  
207 Differential Algebraic Equation (PDAE) problem into a purely algebraic problem, by implementing  
208 2D Orthogonal Collocation on Finite Elements (2D-OCFE) for the dynamic simulation (DS) of the  
209 case study. Based on this formulation, we structure the operational objective function for the  
210 optimisation problem. Finally, we discuss the results of the simulation and optimisation problems.  
211 This work is the first stage of a project that aims to develop a methodology for the optimal  
212 management of a cooling network, considering the dynamics of the whole system, including  
213 conversion, storage, and energy distribution.

215 **2 Cooling system**

216 We develop a dynamic analysis of an academic case study, with conditions based on real data. The  
 217 system consists of 20 users distributed over an urban area in known locations. Based on this  
 218 distribution, we propose a set of nodes and pipes that connect the production site and the users. Next,  
 219 we build the cooling demand profile for each user based on typical performances for various kinds of  
 220 building, as reported by an industrial supplier of cooling services. Finally, we present the external  
 221 conditions to which the system will be subject.

222 **2.1 Configuration of the system**

223 The topology of the system is based on the illustrative example presented by Söderman [13]. From  
 224 the coordinates the author presented for the location of the users, it is possible to compute the lengths  
 225 of the pipes, as detailed in Table 2.

226

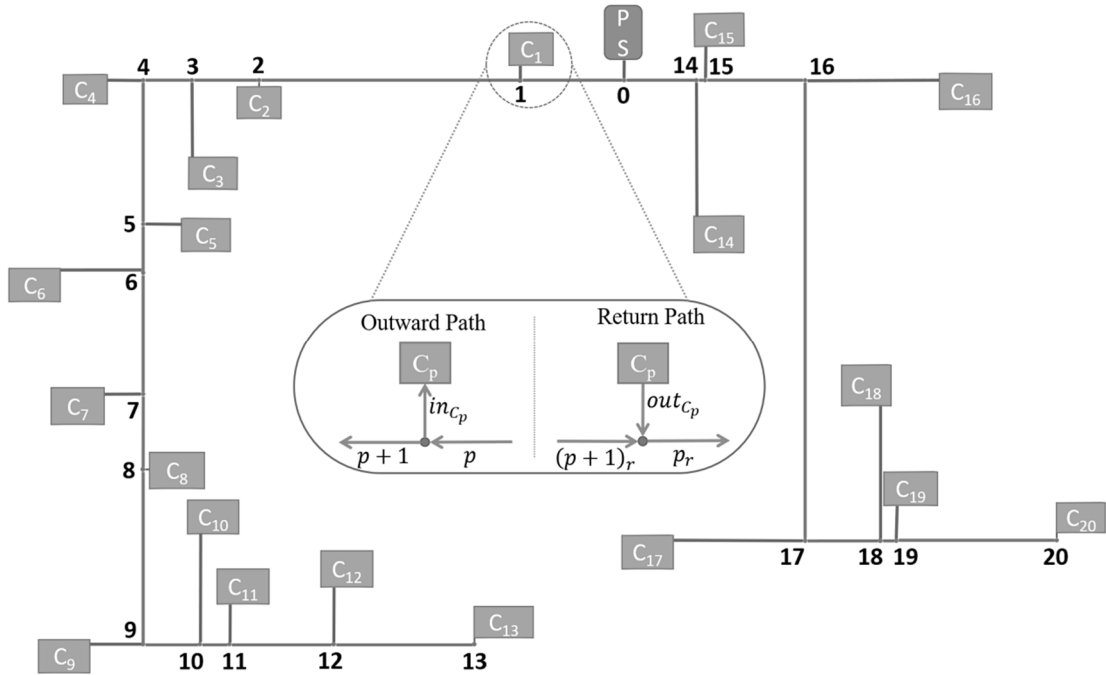
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**Table 2. Lengths of main and lateral pipes of the system**

Main pipes			Lateral pipes		
Pipe		Length (m)	Pipe		Length (m)
0	$0_r$	50	$in_{C_1}$	$out_{C_1}$	40
1	$1_r$	279.93	$in_{C_2}$	$out_{C_2}$	0.1
2	$2_r$	720.07	$in_{C_3}$	$out_{C_3}$	242.99
3	$3_r$	176.46	$in_{C_4}$	$out_{C_4}$	95.21
4	$4_r$	124.69	$in_{C_5}$	$out_{C_5}$	110.64
5	$5_r$	397.32	$in_{C_6}$	$out_{C_6}$	227.56
6	$6_r$	124.97	$in_{C_7}$	$out_{C_7}$	102.58
7	$7_r$	338.27	$in_{C_8}$	$out_{C_8}$	0.43
8	$8_r$	198.19	$in_{C_9}$	$out_{C_9}$	147.38
9	$9_r$	478.17	$in_{C_{10}}$	$out_{C_{10}}$	330.51
10	$10_r$	147.01	$in_{C_{11}}$	$out_{C_{11}}$	110.62
11	$11_r$	73.54	$in_{C_{12}}$	$out_{C_{12}}$	154.73
12	$12_r$	279.38	$in_{C_{13}}$	$out_{C_{13}}$	0.32
13	$13_r$	382.3	$in_{C_{14}}$	$out_{C_{14}}$	404.71
14	$14_r$	190.01	$in_{C_{15}}$	$out_{C_{15}}$	95.14
15	$15_r$	12.13	$in_{C_{16}}$	$out_{C_{16}}$	371.66
16	$16_r$	268.79	$in_{C_{17}}$	$out_{C_{17}}$	363.38
17	$17_r$	1280.6	$in_{C_{18}}$	$out_{C_{18}}$	371.94
18	$18_r$	198.28	$in_{C_{19}}$	$out_{C_{19}}$	91.03
19	$19_r$	32.89	$in_{C_{20}}$	$out_{C_{20}}$	0.1
20	$20_r$	438.04			

228

229 Using this information, Figure 2 presents a scheme of the cold network, main pipes (0-20), lateral  
 230 pipes ( $in_{C_1} - in_{C_{20}}$ ), the nodes and the users, in a simplified way that is useful for modelling  
 231 purposes. As shown in Table 2, the complete system also includes a return network, represented by the  
 232 return pipes ( $0_r - 20_r$  and  $out_{C_1} - out_{C_{20}}$ ) with the same lengths as the outward path pipes. Thus, the  
 233 whole network is an arrangement of 82 pipes with a total length of almost 19 km (18904.11 m).  
 234



235  
 236  
 237

**Figure 2. Representation of the cooling network**

238 To define the diameters of the pipes, we take into account the recommended flow velocities for  
 239 sizing cooling water pipes, as reported by Branan [26] and presented in Table 3. The system must  
 240 respect the velocity bounds during the simulation and optimisation analysis.

241  
 242

**Table 3. Maximum allowable speeds in pipes**

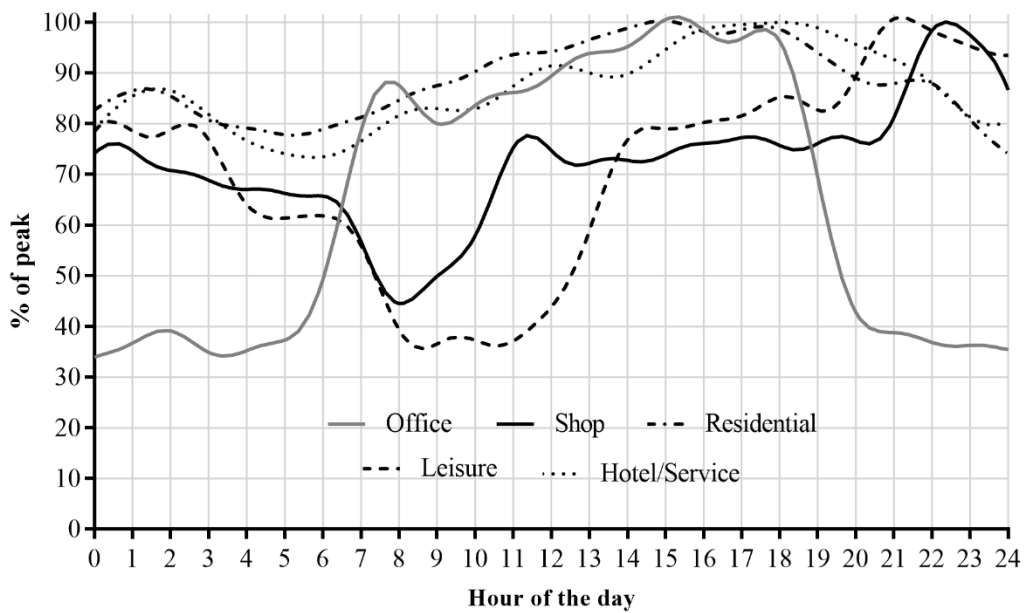
Pipe size (in)	Maximum velocity ( $ms^{-1}$ )	
	in mains pipes	in laterals pipes
2	--	1.31
3	0.94	1.32
4	1.08	1.54
6	1.29	1.69
8	1.27	1.76
10	1.37	1.86
12	1.56	2.08
14	1.56	2.19
16	1.80	2.41
18	1.90	2.53

Pipe size (in)	Maximum velocity (ms <sup>-1</sup> )	
	in mains pipes	in laterals pipes
20	2.03	--

243

## 244 2.2 Users' demand profiles

245 For this work, we built cooling demand profiles based on the daily cooling demand curves  
 246 presented by Olama [27] for different kinds of buildings including office, residential, hotel or service,  
 247 apartments, shopping and leisure. These profiles represent the variation in demand with respect to the  
 248 peak cooling load of the buildings, as seen in Figure 3.



249  
 250  
 251

Figure 3. Demand for different kinds of building

252 Using the peak cooling demands presented by Söderman for this system [13], which are based on  
 253 real data (Table 4), and the aforementioned profiles we can compute the demand profiles of the 20  
 254 consumers.

255

Table 4. Peak demands for users in the Network

Consumer	Type	Maximum load (kW)	Consumer	Type	Maximum load (kW)
C <sub>1</sub>	Shop	1640	C <sub>11</sub>	Residential	800
C <sub>2</sub>	Office	700	C <sub>12</sub>	Office	100
C <sub>3</sub>	Leisure	200	C <sub>13</sub>	Shop	180
C <sub>4</sub>	Office	780	C <sub>14</sub>	Office	1500
C <sub>5</sub>	Shop	100	C <sub>15</sub>	Hotel/Services	650
C <sub>6</sub>	Office	900	C <sub>16</sub>	Hotel/Services	380
C <sub>7</sub>	Office	100	C <sub>17</sub>	Leisure	455
C <sub>8</sub>	Residential	250	C <sub>18</sub>	Hotel/Services	900
C <sub>9</sub>	Hotel/Services	400	C <sub>19</sub>	Leisure	360

Consumer	Type	Maximum load (kW)	Consumer	Type	Maximum load (kW)
C <sub>10</sub>	Office	170	C <sub>20</sub>	Leisure	1220

256

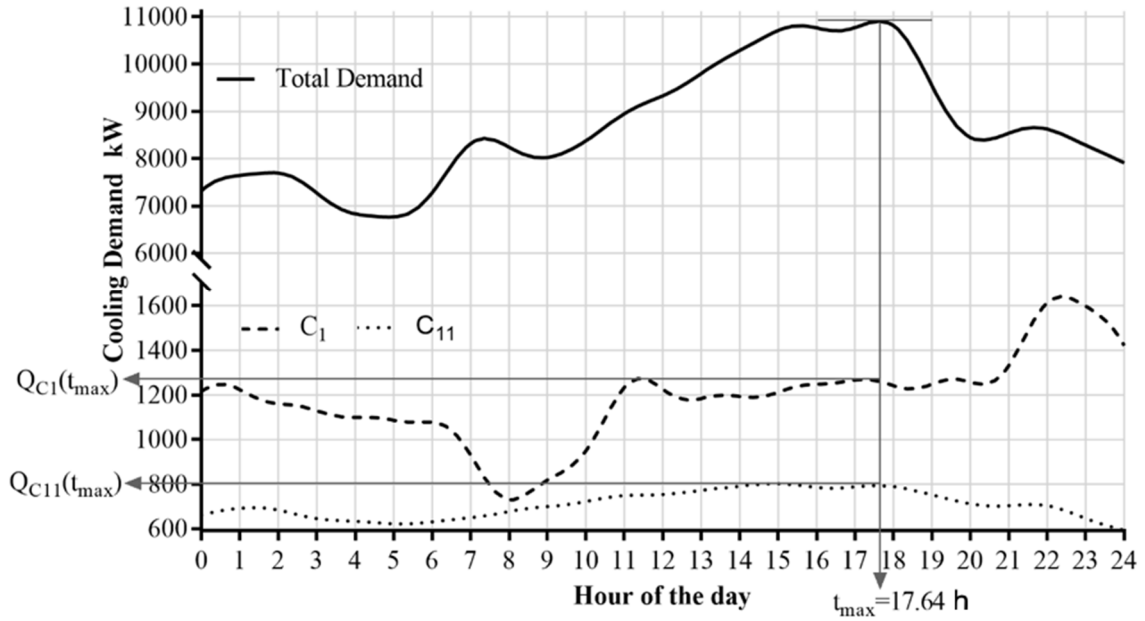
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258

259

260

Considering these data, Figure 4 presents the total cooling demand profile of the district cooling system and details the specific demand of two consumers (for the sake of clarity). The maximum demand of the network is 10911 kW and is reported at 17.64 h ( $t_{max}$ ). We will use the demand of each consumer reported at this time  $Q_{Cp}(t_{max})$  for dynamic simulation analysis.



261

262

263

Figure 4. Total and some specific cooling demands of the system

264

265

266

We will study the system under different external conditions, including various kinds and characteristics of soil as well as differentiated ambient temperatures corresponding to different climate zones.

### 267 2.3 Studied climate zones

268

269

270

The kind of soil and its moisture affect its thermal conductivity, as reported in the ASHRAE district cooling guide [28].

Table 5. Soil thermal conductivities

Soil Moisture (By mass)	Thermal conductivity ( $Wm^{-1}K^{-1}$ )		
	Sand	Silt	Clay
Low <4%	0.29	0.14	0.14
Medium 4%-20%	1.87	1.30	1
High >20%	2.16	2.16	2.16

271

272 In order to analyse the implementation of a district cooling network in different climate areas and  
273 its impact on the total thermal resistance and on the thermal distribution in the pipes, we chose three  
274 cases (cities) with different daily ambient temperature profiles and soil characteristics, as detailed  
275 below:

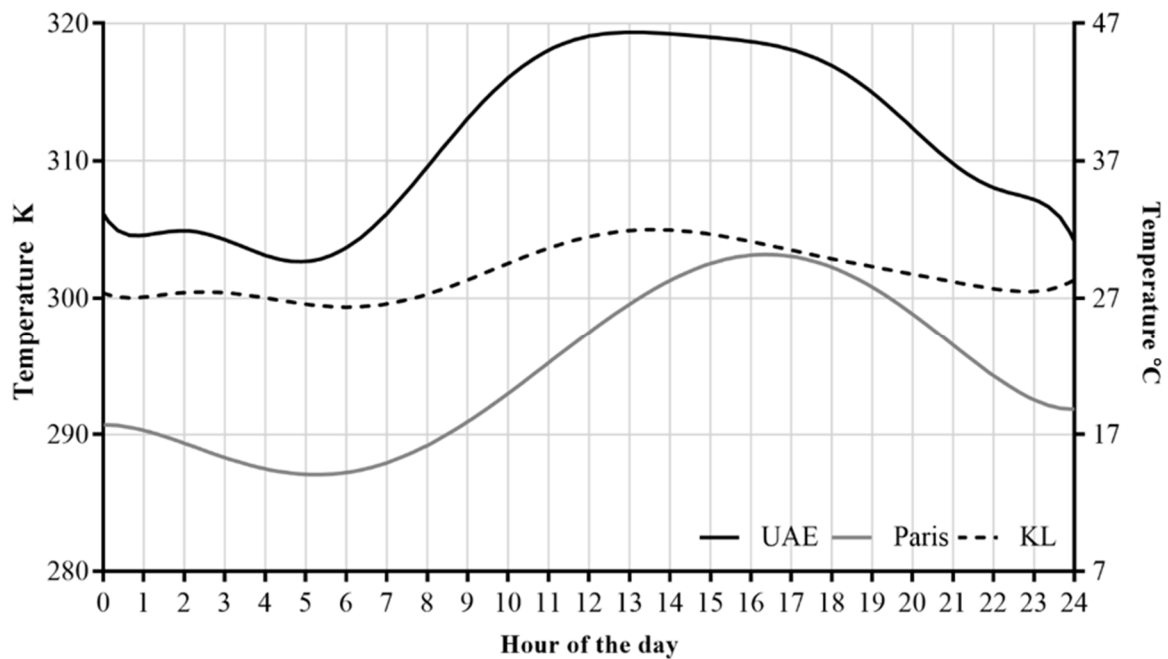
276

- 277 • Ras Al Khaimah (UAE): Low moisture; sandy soil.
- 278 • Paris: Medium moisture; clay soil
- 279 • Kuala Lumpur (KL): High moisture; clay soil

280

281 Figure 5 details the profile temperature on the hottest day in 2018 for each of the selected cities.  
282 These profiles were built using real data from the Weather Underground global community, which  
283 collects data from more than 250,000 weather stations around the world [29].

284



285

286

287 **Figure 5. Summer temperature profiles in the studied climate zones**

288

288 With the system configuration already defined, we can complete the mathematical model that  
289 describes the dynamic operation of the network and develop a proper strategy for its solution.

290

### 291 **3 Mathematical model**

292 The model of the DCS is constituted by the heat transfer equation in each pipe, together with mass  
293 and energy balances at each node of the system, under the following assumptions:

- 294 - The system uses water as cooling fluid ( $w$ ).

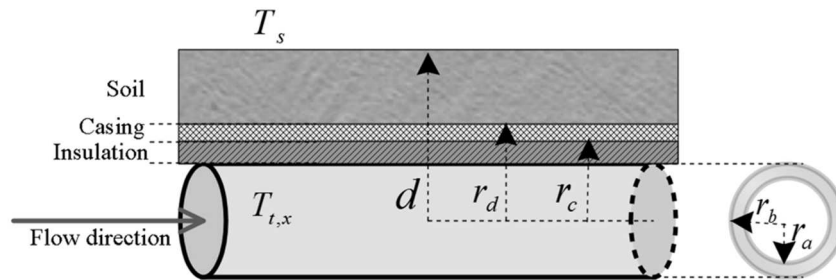
- 295 - The mass flow in each pipe is time-dependent and uniform.  
 296 - The physical properties of the fluid are constant.

297 **3.1 Heat balance in the pipes**

298 Most of the available literature on dynamic modelling and optimisation of district energy systems  
 299 focus their interest in the analysis of district heating systems. Nevertheless, these models can also be  
 300 applied to district cooling [30]. Studies on equation-based methods for the analysis of energy networks  
 301 [17,31,32] use a dynamic one-dimensional heat transfer equation to describe the temperature transients  
 302 in the pipes, defined as:

$$\rho_w \cdot C p_w \cdot A \cdot \frac{\partial T(t, x)}{\partial t} + \dot{m}(t) \cdot C p_w \cdot \frac{\partial T(t, x)}{\partial x} = \frac{T_s - T(t, x)}{R'} \quad (6)$$

303 where  $\rho_w$ ,  $C p_w$ ,  $A$ , and  $\dot{m}$  are the density, specific heat capacity, area (cross-section), and mass flow  
 304 rate of water in the pipe, respectively;  $R'$  is the total thermal resistance per unit length of pipe;  $T$  stands  
 305 for temperature in the pipe,  $T_s$  for the temperature of the soil surface, and  $t$  and  $x$  for time and  
 306 distance dependency.  
 307



308 **Figure 6. Representation of a buried pipe**

309 This heat equation is subject to the following assumptions:

- 310 ✓ Plug flow
- 311 ✓ Heat transfer is considered only in the radial direction
- 312 ✓ Conduction heat transfer is considered through the pipe, the insulation, the casing and the soil
- 313 ✓ Material properties are constant and independent of temperature.
- 314 ✓ It does not include thermal interactions between supply and return pipes
- 315 ✓ Thermal inertia of the pipes, the casing and the insulation is neglected
- 316 ✓ Conductive heat transfer in the fluid is neglected

317 The total thermal resistance per unit length of pipe,  $R'$ , is a function of the thermal conductivities  
 318 of the pipe,  $\lambda_{ab}$ , the insulation,  $\lambda_{bc}$ , the casing,  $\lambda_{cd}$ , and the soil,  $\lambda_s$ , as follows [31]:

321

$$R' = \frac{1}{2\pi r_a \bar{h}} + \frac{\ln\left(\frac{r_b}{r_a}\right)}{2\pi\lambda_{ab}} + \frac{\ln\left(\frac{r_c}{r_b}\right)}{2\pi\lambda_{bc}} + \frac{\ln\left(\frac{r_d}{r_c}\right)}{2\pi\lambda_{cd}} + \frac{1}{S\lambda_s} \quad (7)$$

322

323 Expressions to compute the conduction shape factor  $S$  are detailed in Chapter 4 of reference [33],

324 and the average convection heat transfer coefficient in the pipe  $\bar{h}$  can be computed as:

$$\bar{h} = \frac{\overline{Nu}\lambda_w}{2r_a} \quad (8)$$

325

326 where we can use the Dittus-Boelter equation [34] to compute the average Nusselt number  $\overline{Nu}$ .

327

328 The solution of equation (6) has been addressed using mainly discretisation (partial or total) or 1D  
 329 analytical solutions coupled with physical approximations. Discretisation strategies include the  
 330 implementation of finite volumes [35], finite elements [36], and finite differences [37]. On the other  
 331 hand, estimates are based on a succession of steady states, as proposed by Duquette *et al.* [31], or the  
 332 Lagrangian approach of Zhou *et al.* [38]. In this second group, we can also include the contributions of  
 333 Stevanovic *et al.* [39], van der Heijde *et al.* [32] and Schweiger *et al.* [17]. These last two  
 334 contributions use an implementation in Modelica®, where the fluid and temperature propagations are  
 335 calculated separately from the heat loss, combining a plug flow approach with an ideal mixed volume  
 336 model. These methods require a large number of grid points for discretisation, or a large storage  
 337 memory to compute the behaviour of the temperature in the pipes for the steady-state-based methods.  
 338 It could be pointed out here, that several of these studies ([31, 32, 38]) have validated the previous  
 339 model, based on reasonable assumptions, by comparison with experimental data.

340

341 However, for some dynamic modelling applications in chemical engineering [40,41], the  
 342 orthogonal collocation method has been used to handle Partial Differential Equation (PDE) problems.  
 343 If applied to the space domain, the orthogonal collocation method transforms a PDE system into an  
 344 Ordinary Differential Equation (ODE) system (where time is the only integration variable) which is  
 345 smaller in size than that obtained using a classical discretisation strategy (e.g. Finite differences). The  
 346 resulting ODE system can be solved using classical methods, like Runge-Kutta (RK) as carried out by  
 347 Ebrahimzadeh *et al.* [41], who reported computational times up to 90% lower compared to the Method  
 348 of Lines (i.e. finite differences in space and RK in time).

349 When the nature of the phenomenon demands more accurate measurements, the domain of  
 350 integration can be divided into subdomains or finite elements, where the orthogonal collocation  
 351 method is implemented, allowing the use of a large number of grid points. Biegler [42] reported better  
 352 convergence and lower computational requirements for the OCFE method compared to other  
 353 discretisation methods.

354

355 To describe the dynamics of the district system, we must couple the 82 Partial Differential  
356 Equations (PDE) (6) (one for each pipe) with the mass and energy balances in the connections  
357 between the network and the users, represented by linear and nonlinear algebraic equations presented  
358 in the next section. This leads to a Partial Differential Algebraic Problem (PDAE), which is solved in  
359 the present work using 2D-OCFE, transforming the PDAE system into a set of algebraic equations.  
360 Thus, we can solve the dynamic optimisation of the DCS using a simultaneous strategy.

361

362

### 363 3.2 Heat balances in the nodes

#### 364 Initial condition

365 For each pipe  $k$ , the initial spatial temperature profile is known from a steady-state simulation:

$$T_k(0, x) = \theta_k(x) \quad (9)$$

366 Here,  $\theta_k(x)$  represents the spatial distribution of temperature along pipe  $k$  at  $t = 0$ . The way to  
367 achieve this steady-state simulation will be discussed later.

368

#### 369 Boundary conditions

370 We assume that chilled water is produced at a constant temperature (277 K). For the pipe leaving  
371 the production site ( $p = 0$ ), we then have:

$$T_0(t, 0) = 277 \text{ K} \quad (10)$$

372 The nodes in the outward path are splitters, where the outlet temperature of the pipe entering the  
373 node is equivalent to the inlet temperature of the pipe leaving it:

374

$$\begin{aligned} T_{p-1}(t, L_{p-1}) &= T_p(t, 0) \quad , p = 1 \dots 13, 15, \dots 20 \\ T_0(t, L_0) &= T_{14}(t, 0) \\ T_p(t, L_p) &= T_{in_{C_p}}(t, 0) \quad , p = 1 \dots 20 \end{aligned} \quad (11)$$

375

376 where  $L_k$  is the length of pipe  $k$ .  $k$  may be  $p, p_r, in_{C_p}, out_{C_p}$ .

377 On the other hand, heat balances for the return path and the consumers ( $C_p = (C_1, C_2, \dots, C_{20})$ ) will  
378 define the boundary condition of the pipes leaving these elements of the system. As we consider  
379 constant properties, they will be:

380

$$\begin{aligned} \dot{m}_{p_r}(t) \cdot T_{p_r}(t, 0) &= \dot{m}_{(p+1)_r}(t) \cdot T_{(p+1)_r}(t, L_{(p+1)_r}) + \dot{m}_{out_{c_p}}(t) \cdot T_{out_{c_p}}(t, L_{out_{c_p}}), \\ & p = 1 \dots 12, 14, \dots 19 \\ \dot{m}_{0_r}(t) \cdot T_{0_r}(t, 0) &= \dot{m}_{1_r}(t) \cdot T_{1_r}(t, L_{1_r}) + \dot{m}_{14_r}(t) \cdot T_{14_r}(t, L_{14_r}) \\ \dot{m}_{p_r}(t) \cdot T_{p_r}(t, 0) &= \dot{m}_{out_{c_p}}(t) \cdot T_{out_{c_p}}(t, L_{out_{c_p}}), \quad p = 13, 20 \end{aligned} \quad (12)$$

381

$$Q_{c_p}(t) = \dot{m}_{in_{c_p}}(t) \cdot C_{p_w} \cdot \left( T_{out_{c_p}}(t, 0) - T_{in_{c_p}}(t, L_{in_{c_p}}) \right) \quad (13)$$

382

383 where  $T_{in_{c_p}}(t, L_p)$  and  $T_{out_{c_p}}(t, 0)$  are the inlet and outlet temperatures of the exchanger.

### 384 3.3 Mass Balances in the nodes

385 For the nodes and consumers, the mass balances are given by:

386 Outward path

$$\begin{aligned} \dot{m}_p(t) &= \dot{m}_{p+1}(t) + \dot{m}_{in_{c_p}}(t) \quad p = 1 \dots 12, 14 \dots 19 \\ \dot{m}_p(t) &= \dot{m}_{in_{c_p}}(t) \quad p = 13, 20 \\ \dot{m}_0(t) &= \dot{m}_1(t) + \dot{m}_{14}(t) \end{aligned} \quad (14)$$

387

388 Return path

$$\begin{aligned} \dot{m}_{p_r}(t) &= \dot{m}_{(p+1)_r}(t) + \dot{m}_{out_{c_p}}(t) \quad p = 1 \dots 12, 14 \dots 19 \\ \dot{m}_{p_r}(t) &= \dot{m}_{out_{c_p}}(t) \quad p = 13, 20 \\ \dot{m}_{0_r}(t) &= \dot{m}_{1_r} + \dot{m}_{14_r} \end{aligned} \quad (15)$$

389

390 Consumers

$$\dot{m}_{in_{c_p}}(t) = \dot{m}_{out_{c_p}}(t) \quad p = 1 \dots 20 \quad (16)$$

391

392 With the heat and energy balances of the system already formulated, the next section presents the  
393 analysis of degrees of freedom of the system and the supplementary relationships that we included in  
394 order to have zero degrees of freedom for simulation purposes.

### 395 3.4 Degrees of freedom and flow policy

396 From the set of equations (6) and (9) to (16), the degrees of freedom of the system can be analysed  
397 for the 82 pipes at each instant  $t$ , as detailed in Table 6. The number of degrees of freedom of the

398 system is 20 at each instant  $t$ , representing the profiles to be given for dynamic simulation or computed  
 399 via dynamic optimisation.

400

401

**Table 6. Analysis of degrees of freedom of the system**

Variables		Equations	
Variable	# of variables	Equation	# of equations
$T_k(x, t)$	82	(6)	<b>82</b>
		1 <sup>st</sup> order in $t$ and $x$ PDE requires per pipe: - <b>1</b> Initial Condition (9) - <b>1</b> Boundary Condition (10) (11) (12) (13)	82 1 40 21 20 } 82
$\dot{m}_k(t)$	82	(14) (15) (16)	21 21 20 } <b>62</b>

402

403 Considering this, for dynamic simulation purposes we must complete the degrees of freedom of the  
 404 system. We can do this by defining the flow policy we will use to achieve the cooling demand for each  
 405 consumer. Some systems operate under constant production conditions, as shown in Figure 7, where  
 406 the production of cold ( $\dot{m}_0$ ) and the mass flow in the main network ( $\dot{m}_p$  and  $\dot{m}_{in_{C_p}}$ ) are constant at  
 407 the level necessary to cover the peak of the total demand of the system. In this way, the producer  
 408 guarantees enough cold in the system during the studied period, but this results in cost overruns for  
 409 production and pumping of the chilled water. At each consumer substation, there is a common  
 410 pipe  $Cm(t)$  connecting the main and return networks, to regulate the flow to the consumer ( $vin_{C_p}(t)$ )  
 411 over time, sending large quantities of cold water directly to the return network in periods of low  
 412 demand, and the total flow of  $\dot{m}_{in_{C_p}}$  to the consumers when their demand corresponds to the peak. The  
 413 literature reports this policy as constant-primary secondary-variable flow [27,28].

414

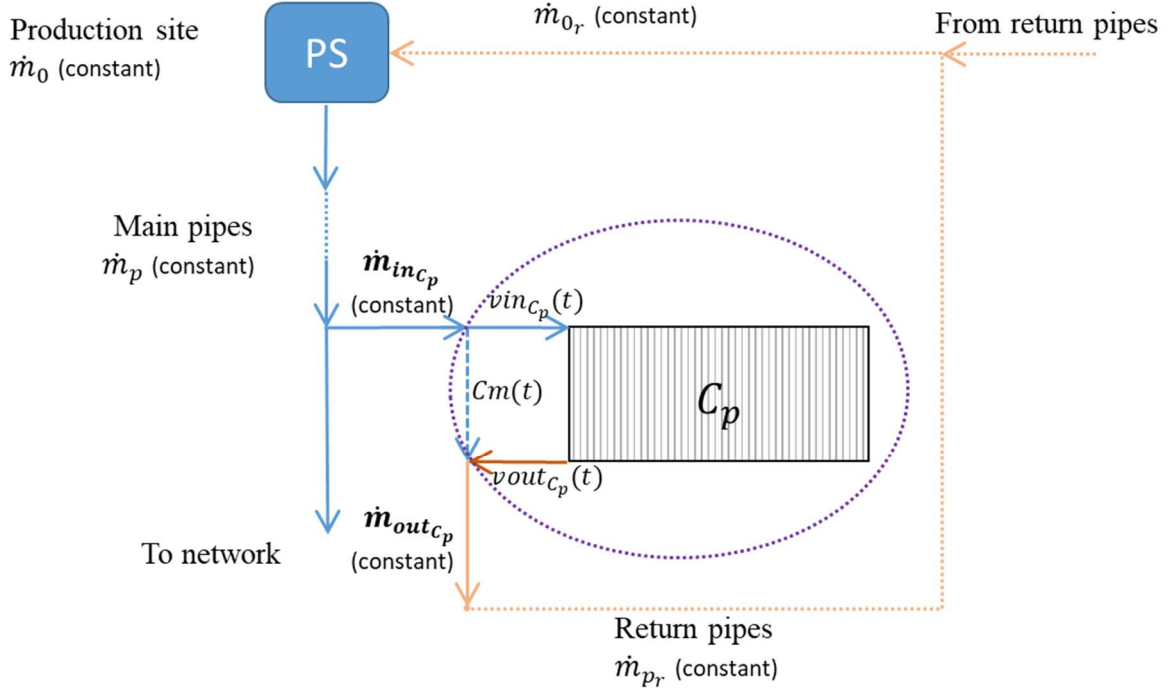


Figure 7. Constant flow policy diagram

415  
416  
417

418 To represent constant flow in the main network, we first define a constant mass flow leaving the  
419 production site by:

$$\dot{m}_0(t) = \dot{m}_{0Cte} \quad (17)$$

420

421 Furthermore, in the splitters, we assume that the mass flows entering the consumers at each time  
422 ( $\dot{m}_{in_{C_p}}(t)$ ) are proportional to their corresponding maximum peak demand ( $Peak(C_p)$  from Table 4).

423 We can do this by fixing the ratio between these variables for all the consumers over time as:

424

$$\frac{Peak(C_1)}{\dot{m}_{in_{C_1}}} = \frac{Peak(C_p)}{\dot{m}_{in_{C_p}}}, \quad \forall p \in \{2, \dots, 20\} \quad (18)$$

425

426 The mass and energy balances presented in section 3.3 are the balances for each consumer for the  
427 boundaries defined by the dotted borderline presented in Figure 7. Hence only the mass flows  $\dot{m}_{in_{C_p}}$   
428 are computed; the pipes  $C_m(t)$ ,  $v_{in_{C_p}}(t)$  and  $v_{out_{C_p}}(t)$  belong to the user's substation and their  
429 flows are not considered in this analysis.

430

431 The dynamic response of the system will initially be analysed under constant production mass flow  
432  $\dot{m}_{0Cte}$  as expressed in (17). This value corresponds to the value of the producer mass flow in steady-  
433 state, which is computed using relations (18) and imposing a return temperature (19) to complete the  
434 20 degrees of freedom.

$$T_{0_r}(L_{0_r}) = 287 \text{ K} \quad (19)$$

435

436 Then, to simulate the constant flow policy in the dynamic simulation, the 20 degrees of freedom at  
 437 each time  $t$  are completed using (17) and (18).

438

439 The optimisation analyses include the study of the constant flow policy and optimisation of the  
 440 system operation using a dynamic flow policy. In the former case, (17) and (18) will be constraints  
 441 and the value  $m_{0Cte}$  will be the only optimisation variable. In the latter case, these constraints are not  
 442 considered, giving the profiles  $\dot{m}_0(t)$  and  $\dot{m}_{inC_p}(t), p = 1 \dots 20$  as optimisation variables.

443

444 Finally, as mentioned in section 2.1, the flow velocity  $v_{k,e,i}$  inside a pipe  $k$ , cannot exceed its  
 445 maximum allowed velocity  $v_{max,k}$  reported in Table 3.

$$v_{k,e,i} \leq v_{max,k} \quad (20)$$

446 With this inequality, we complete the mathematical model that we will use to describe the  
 447 dynamics of the proposed district cooling distribution system.

448

#### 449 **4 Formulation of the optimisation problem**

450 With the mathematical model already defined, this section describes the operational optimisation  
 451 applications of the DCS. We present the operational objective function, which will be analysed for  
 452 different degrees of freedom. Lastly, we detail the methodology used to obtain the initialisation and  
 453 the solution of the resulting NLP problem.

454

455 For the present application, we will use a Lagrange problem type formulation, which will measure  
 456 the influence of the variations in cold demands  $Q_{C_p}(t)$  and of the soil surface temperature  $T_s(t)$  on the  
 457 systems, with the aim is to achieve a given operational condition over the studied time horizon.

458

459 In our case, the control variables are the mass flow in each pipe  $\dot{m}_k(t)$ . They are called control  
 460 variables because they are the variables which will have to be manipulated to manage the system  
 461 online. It should be noted, however, that the dynamic optimisation we do here is not an online control:  
 462 it consists in the offline calculation of the temporal profiles of these control variables. The known soil  
 463 temperature  $T_s(t)$  as well as the demand of each consumer  $Q_{C_p}(t)$  are treated as algebraic variables,  
 464 which are not optimisation variables. The velocities  $v_k(t)$  are optimised algebraic variables. The  
 465 lengths of the pipes  $L_k$  are time-independent parameters. In our case, the differential state variables are  
 466 the temperature in each pipe,  $T_k$ , which depend not only on  $t$  but also on  $x$ . Instead of dealing with

467 DAE constraints, we then deal with PDAE constraints. It can be noted that applying a space  
468 discretisation method to the temperatures leads to DAE constraints, while increasing the number of  
469 “state variables” dependent only on  $t$  (variable  $T_k(t, x)$  is replaced by  $nl$  variables  $T_{kl}(t)$ ).  
470

#### 471 **4.1 Objective function**

472 The efficiency of a DCS is measured in terms of the difference between the temperature of the fluid  
473 leaving the production site and the temperature of the fluid that returns to it ( $\Delta T$ ). Generally,  
474 maintaining a high  $\Delta T$  reduces the flow rates of the chilled water system and the costs of the  
475 distribution system due to the use of smaller pipe diameters. This results in savings in pumping energy  
476 costs and improves operating costs [27]. Typically,  $\Delta T$  in the DCS production site is maintained at  
477 around 8-12°C [28,43].  
478

479 When the  $\Delta T$  is not properly controlled, the DCS could present an important issue known as “*low*  
480  *$\Delta T$  syndrome*” [27]. Indeed, this low  $\Delta T$  at the production site is a consequence of a low  $\Delta T$  at each  
481 consumer, which is a symptom of the low efficiency at the consumer’s substation. Then, in order to  
482 satisfy the consumers’ demands, the system has to pump excess rates of chilled water although the  
483 plant is not designed to operate at this level. A high  $\Delta T$  design is generally economical to the operation  
484 of a district cooling station, the chilled water distribution network, and individual buildings’ heating,  
485 ventilating and air conditioning (HVAC) systems. This is because of savings in the size of piping and  
486 accessories in the plant and larger savings in piping, pre-insulation, and accessories in the chilled  
487 water distribution network.

488 To optimize  $\Delta T$  and meet customer demand, both the flow from the central plant and flow on the  
489 customer’s side must be varied [28]. These variations also represent savings in pumping energy. The  
490 complete dynamic flow policy of the system is represented in Figure 8. This operating policy  
491 eliminates the use of the common pipe shown in the case of the constant flow policy (Figure 7) and the  
492 resulting mixing and possible reduction in the temperature in the return network.  
493

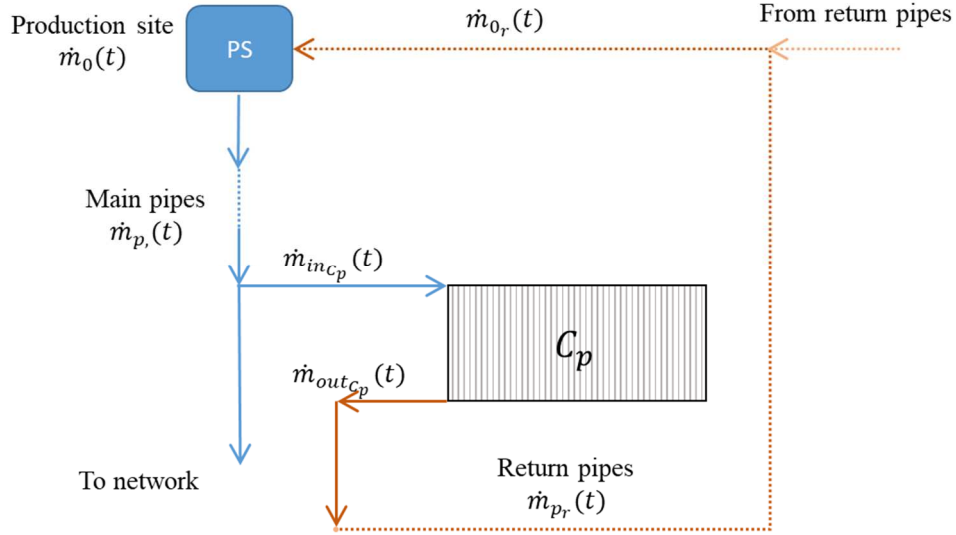


Figure 8. Dynamic flow policy diagram

494  
495  
496

497 The variations in temperature mentioned above represent not only a technical issue but also an  
498 economic impact on the customer. The consumer will be charged 3% for each degree Celsius of the  
499 monthly average return temperature below the system design return temperature [27]. On the other  
500 hand, it is important to avoid high temperatures that might compromise the proper operation of the  
501 production site technology.

502  
503  
504  
505  
506

Bearing this in mind, we define  $J$  as the quadratic error between the outlet temperature of the users  
and a set point. First, we analyse the system under the constant flow policy (21), and then we perform  
the optimisation for a dynamic operation (22), for a design outlet temperature of 287K.

$$\min_{\substack{\dot{m}_{0Cte} \\ Var}} \sum_p \left( \int_0^{24} (T_{outC_p}(t) - 287)^2 dt \right) \quad (21)$$

S. t. (6), (9) to (16), (17), (18), (20)

507 Since a constant flow policy is applied, equation (17) is embedded in the set of model equations.  
508 We also apply the flow policy equation (18), which requires that the ratios of the splitters are  
509 constant. In this optimisation, therefore, the only control variable is the constant flow leaving the  
510 producer  $\dot{m}_{0Cte}$ .  $Var$  represents all the other variables of the problem  $(T(t, x), \dot{m}_k(t), \dots)$  whose  
511 optimal paths minimise the difference between the consumers' outlet temperature and the proposed  
512 design temperature over time. The flow variables  $\dot{m}_k(t)$  are treated as algebraic variables.

513

$$\min_{\substack{\dot{m}_{in_c_p}(t) \\ var}} \sum_p \left( \int_0^{24} (T_{outC_p}(t) - 287)^2 dt \right) \quad (22)$$

S. t. (6), (9) to (16), (20)

514 In this case, none of the constant flow policy equations (17) and (18) are included in the model, so  
515 that the mass flow of the producer and the ratios of the splitters are time-dependent and will be the  
516 main control variables.

## 517 **4.2 Discretisation strategy**

518 As stated in section 1 in the present work, we use 2D-OCFE to transform equation (6) into a set of  
519 algebraic equations, which is highly advantageous and efficient for simultaneous optimisation  
520 applications[21]. To our knowledge, there are no studies related to the dynamic operation of DCSs that  
521 use OCFE to handle the resulting PDAE problem.

522  
523 The generalities of the numerical method and its implementation in discretising equation (6) are  
524 presented in Appendix A. Based on this implementation, we present the discretised mathematical  
525 model of the system in Appendix B.  $nf$  elements containing  $n_j + 1$  collocation points are used to  
526 discretise the space domain and  $ne$  elements containing  $n_i + 1$  collocation points are used to discretise  
527 the time domain. The choice of the value of these parameters is discussed in section 5. Finally, in  
528 order to verify the accuracy and the validity of the results obtained with this kind of discretisation,  
529 comparisons with other classical methods are proposed in Appendix C.

## 530 **4.3 Methodology for the solution of the dynamic simulation and optimisation problems**

531 A good initial guess for the optimisation of dynamic systems is crucial for a fast and reliable  
532 solution to a dynamic optimization problem, as stated by Safdarnejad *et al.* [44]. Figure 9 describes the  
533 procedure we implemented to obtain the initial guess and solve the PDAE problem.

534  
535 Due to the lack of piping data for the selected system [13], we define the diameters using an  
536 iterative procedure, which uses the maximum allowable speed flows as decision criteria. For this  
537 purpose, we first compute a theoretical maximum mass flow in the main pipe using a model without  
538 heat losses that includes only the heat and mass balances in the nodes and consumers, for the  
539 maximum demand (peak demand) of each consumer and a return temperature of 287 K (which is also  
540 the outlet temperature of each consumer exchanger since heat losses are neglected). We then evaluate  
541 the maximum speed constraints consecutively, looking for the smallest diameter of each pipe that  
542 could transport the computed flow without exceeding the allowable speeds. It is important to mention  
543 that the diameters chosen for this iterative step will lead us to a physically achievable operation, but  
544 they are not optimal regarding any economic or operational criteria. The computation of the optimal  
545 diameter of the pipes is beyond the scope of the present work.

546

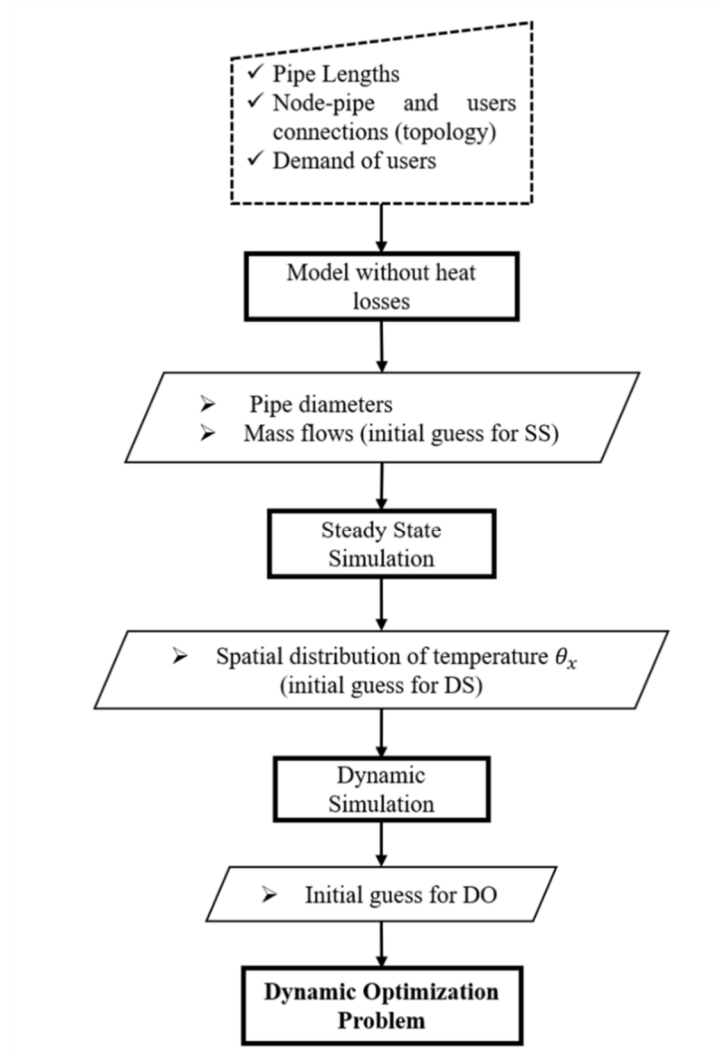


Figure 9. Schematic representation of the solution strategy

547  
548  
549

550 With the diameters already defined, we run the model without heat losses for the maximum  
551 demand of the system (demand in  $t = 17.64 \text{ h} = t_{max}$ , presented in Figure 4), and the result  
552 represents the initialisation to solve the steady-state (SS) problem. This problem is defined by the  
553 equation system (6), (9) to (18) and (20), with  $\frac{\partial T}{\partial t} = 0$ , external conditions in  $t = 0$  and user demand  
554 fixed to its value at  $t = t_{max}$ . The main result of this problem is the spatial distribution of  
555 temperature  $\theta_k(x)$  for each pipe  $k$  for the constant flow policy, which represents the initial condition  
556 for the dynamic problem and the initial guess for its solution. This distribution is given by the  
557 temperature values at the spatial collocation points  $\theta_{k,f,j}$ . With this, it is possible to solve the fully  
558 dynamic simulation (DS) problem (Equations (6), (9) to (18) and (20)), that describes the behaviour of  
559 the complete system, subject to the environmental and operational perturbations over the selected time  
560 horizon. Finally, we use the solution of the DS as the initial guess for the dynamic optimization (DO)  
561 problem. We implement all the described stages in the GAMS modelling environment and solve the

562 different problems using the CONOPT feasible path solver, on a 2.7 GHz quad-core CPU with 8 Gb of  
 563 RAM.

564

## 565 **5 Results and discussion**

566 We present the results according to the flowchart presented in Figure 9. First, we analyse the  
 567 computed pipe diameters of the cooling network pipes using the model without heat losses. These  
 568 diameters become inlet data (parameters) for the forthcoming problems. The second result is the  
 569 distribution of temperatures computed by the steady-state simulation for the system under the external  
 570 conditions described in section 2.3. Lastly, we present dynamic analyses (DS and DO) of the case with  
 571 major variations in the steady-state simulations.

### 572 **5.1 Pipe diameters**

573 Using the abovementioned iterative procedure with the model without heat losses, we define the  
 574 distribution of pipe diameters detailed in Table 7.

575

576

**Table 7. Pipe diameters for the DCS**

Pipe size (in)	Pipes	# of pipes
20	0, 0 <sub>r</sub>	2
16	1, 1 <sub>r</sub>	2
14	2, 2 <sub>r</sub> , 14, 14 <sub>r</sub>	2
12	3, 3 <sub>r</sub> , 4, 4 <sub>r</sub> , 5, 5 <sub>r</sub> , 6, 6 <sub>r</sub> , 15, 15 <sub>r</sub> , 16, 16 <sub>r</sub> , 17, 17 <sub>r</sub>	16
10	7, 7 <sub>r</sub> , 8, 8 <sub>r</sub> , 18, 18 <sub>r</sub>	6
8	9, 9 <sub>r</sub> , 10, 10 <sub>r</sub> , 11, 11 <sub>r</sub> , 19, 19 <sub>r</sub> , 20, 20 <sub>r</sub> , in/outC <sub>1</sub> , in/outC <sub>14</sub>	14
6	in/outC <sub>2</sub> , in/outC <sub>4</sub> , in/outC <sub>6</sub> , in/outC <sub>11</sub> , in/outC <sub>15</sub> , in/outC <sub>18</sub> , in/outC <sub>20</sub>	14
4	12, 12 <sub>r</sub> , in/outC <sub>9</sub> , in/outC <sub>16</sub> , in/outC <sub>17</sub> , in/outC <sub>19</sub>	10
3	13, 13 <sub>r</sub> , in/outC <sub>3</sub> , in/outC <sub>8</sub> , in/outC <sub>10</sub> , in/outC <sub>13</sub>	10
2	in/outC <sub>5</sub> , in/outC <sub>7</sub> , in/outC <sub>12</sub>	6
Total		82

577

578 These diameters ensure the operation of the system under the proposed demand profiles and they  
 579 will be fixed parameters for the subsequent problems.

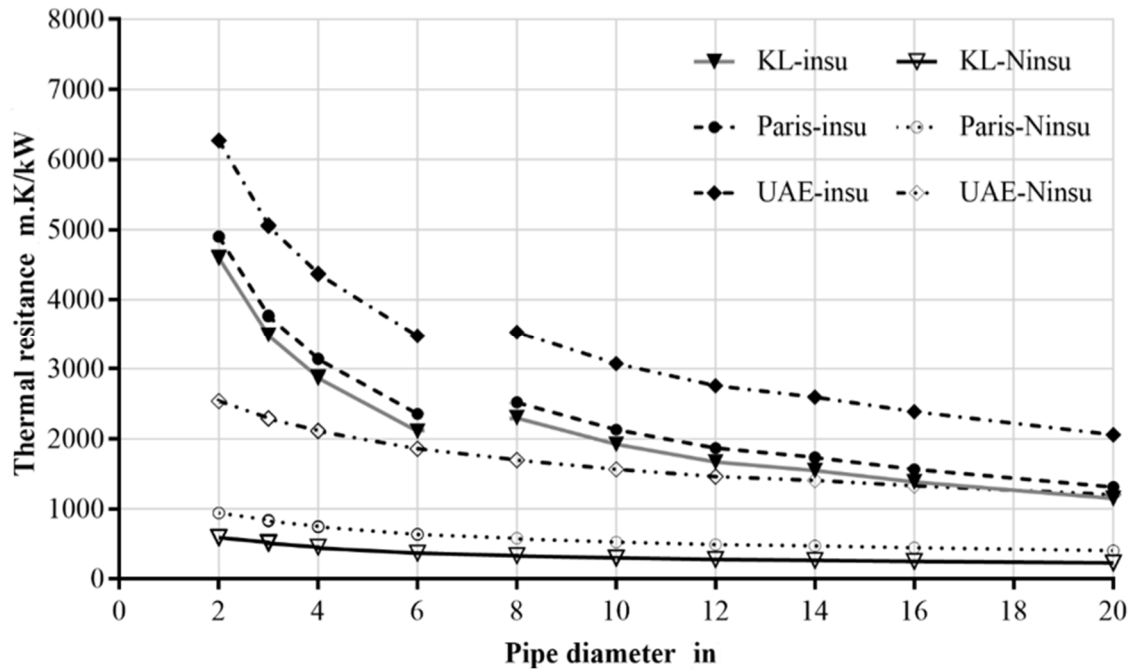
580

581 Under these conditions, the producer pumps a total flow of 259.66 kg s<sup>-1</sup> of chilled water, to supply  
 582 the cooling demand corresponding to the maximum demand of the network ( $t_{max}$ ). The distribution of  
 583 diameters presented here is consistent because the sizes of the pipes decrease the further away they are  
 584 from the production site. Furthermore, the smallest pipes feed those users with lower cooling  
 585 demands. With these data, we define the global thermal resistance per unit length  $R'$  of the pipes. To  
 586 compute the thermal resistances, we assume that all the pipes are buried at the same depth of one

587 meter ( $d$  in Figure 6). The thermal conductivities and thickness of the insulation correspond to the  
 588 values reported by the North American Insulation Manufacturers Association [45].

589  
 590 Figure 10 shows the variation in the global thermal resistance with respect to the pipe diameters  
 591 for insulated and non-insulated pipes, for the characteristic terrain and initial soil temperature for each  
 592 of the proposed climate zones detailed in section 2.3. These values will also be input parameters for  
 593 the forthcoming problems.

594



595  
 596 **Figure 10. Variation in total thermal resistance**  
 597

598 We can show that in all cases, as the pipe diameter increases, the value of the global thermal  
 599 resistance decreases, resulting in a major variation for insulated pipes. For both kinds of pipe,  
 600 installation under KL conditions has a lower thermal resistance. The discontinuity of  $R'$  for the  
 601 insulated pipes corresponds to a change in the insulation thickness, which is one inch for pipes with  
 602 diameters smaller than 8 inches and 1.5 inches for the others. Furthermore, for large insulated pipes  
 603 (16" and 20") the resistance in KL is equivalent to that computed for non-insulated pipes under UAE  
 604 conditions. These values will have a major influence on the spatial distributions of temperature, as we  
 605 will discuss in the next subsection.

606 **5.2 Steady-state simulations**

607 With all the elements of the distribution network already established, it is necessary to define the  
 608 number of elements ( $nf$ ) and points ( $nj + 1$ ) that will be used to solve the DS and DO problems. To  
 609 do this, we perform several simulations, changing the number of elements ( $nf = 1,2,3,5$ ) and points  
 610 inside each element ( $nj + 1 = 4,6,11$ ) (e.g. the degree of the Lagrange interpolation polynomial is =

611 3,5,10 ), and compare the results. We chose the combination of elements and points with the lowest  
 612 CPU time where the solution did not present significant variations compared with the solution  
 613 obtained with the maximum number of points. Using this procedure, we chose  $nf = 1$  and  $nj + 1 =$   
 614 11. Later, using the same procedure for dynamic simulations, we define  $ne = 24$  and  $ni + 1 = 6$ .

615

616 We completed the steady-state simulation for the climate (using  $T_s(t = 0)$  as external  
 617 temperature) and soil conditions presented in section 2.3 using insulated and non-insulated pipes. For  
 618 each climate zone, the simulation was first performed for a system with insulated pipes, fixing the  
 619 return temperature  $\theta_{0,r,nf,nj}$  to compute the producer mass flow  $\dot{m}_{0cte}$ . This latter is set as a parameter  
 620 for the system with non-insulated pipes, where we compute the return temperature in order to compare  
 621 the influence of the insulation under the same flow conditions. Table 8 details these results.

622

623

**Table 8. Results for steady-state simulations**

	Insulated pipes			Non-insulated pipes		
	KL	Paris	UAE	KL	Paris	UAE
$\dot{m}_{0cte} (kg s^{-1})$	263.6	261.4	262.9	263.6	261.4	262.9
$\theta_{0,r,nf,nj} (K)$	287	287	287	287.7	287.2	287.1

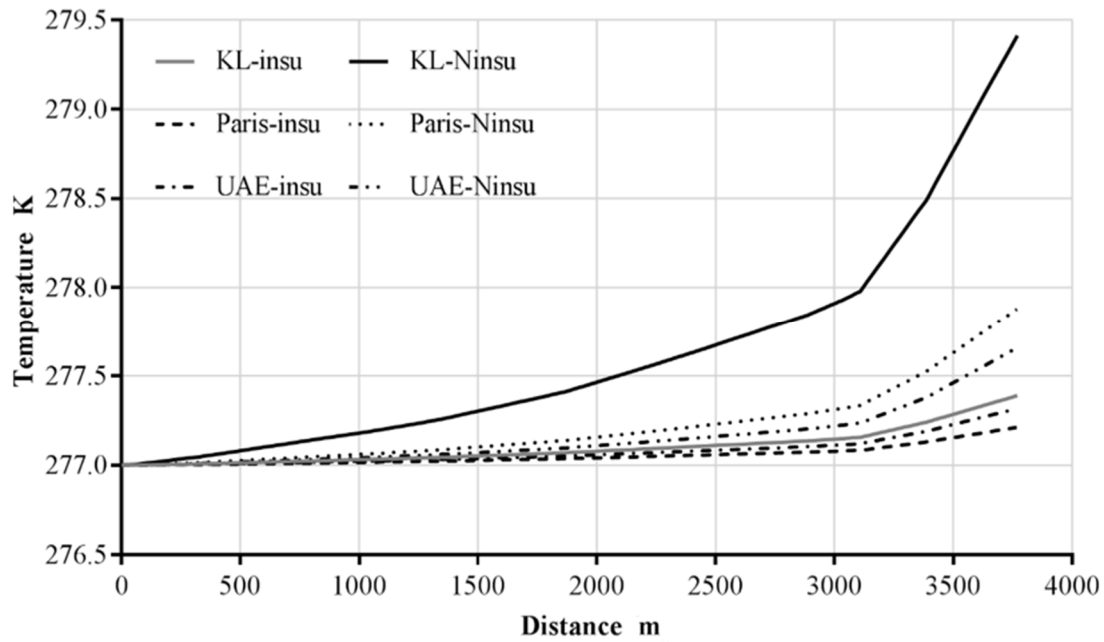
624

625 As expected, the system that reported the lowest thermal resistance (KL) demands more cold water  
 626 from the provider to achieve the proposed return temperature using insulated pipes. On the other hand,  
 627 although the UAE presents the biggest thermal resistance, this location also represents the hottest  
 628 external temperatures, resulting in a bigger mass flow compared with the network installed in Paris.  
 629 Moving to the systems with non-insulated pipes, the variation in the return temperature increases as  
 630 the thermal resistance decreases. Although the non-insulated system in UAE presents a slightly  
 631 smaller variation than that in Paris, the producer must pump more cold water, resulting ultimately in a  
 632 greater operational cost.

633

634 Figure 11 details the spatial distribution of temperature over the left outward branch of the network  
 635 ( $C_1$  to  $C_{13}$  in Figure 2; the production site is located at  $x = 0$ ). The solution of the steady-state model  
 636 for each climate condition takes less than 0.5 s.

637



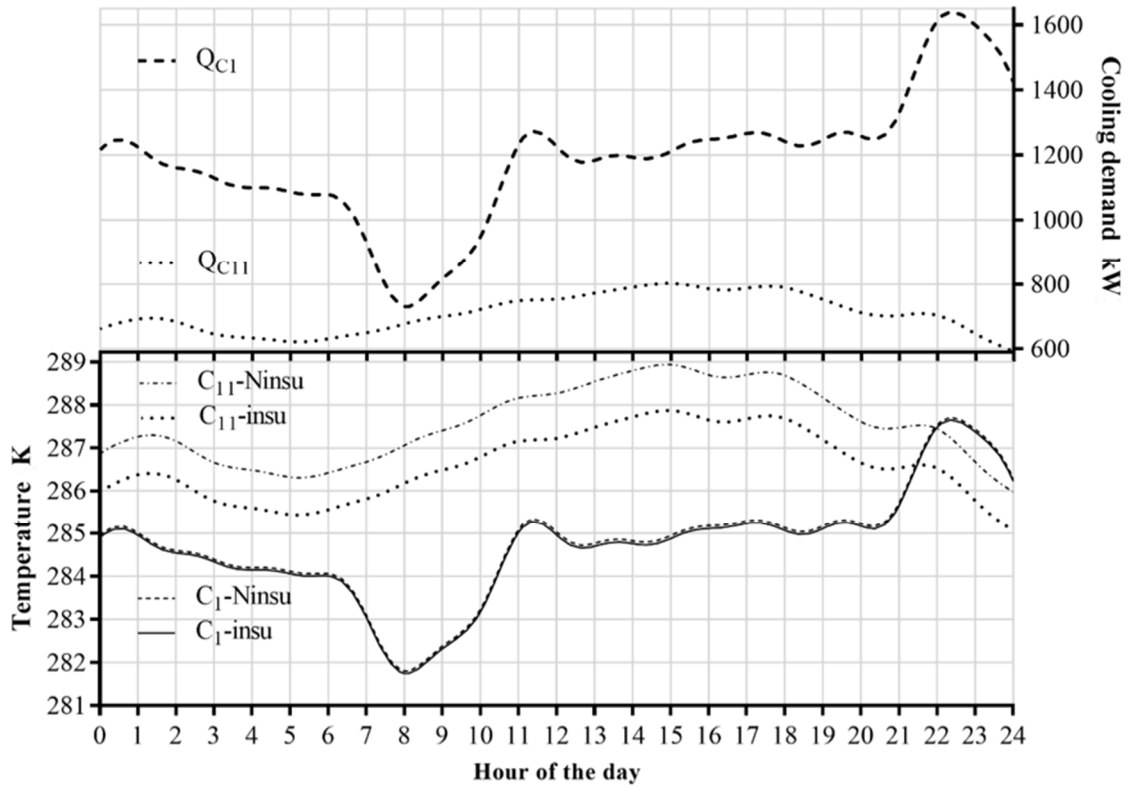
638 **Figure 11. Spatial distribution of temperature in the outward path for the left side of the network**  
 639  
 640

641 We observe in all cases that the water in the pipes increases in temperature as it moves further from  
 642 the production site. As expected, results under the humid conditions in Kuala Lumpur present the  
 643 biggest temperature increments. Although the variations in spatial temperature are of the order of 0.5  
 644 K or less, its computation will have a large impact on the dynamic study of the system under variable  
 645 flows. Furthermore, the computed mass flows aim to cover the maximum demand of the system.  
 646

647 These results agree with the total pipe resistances presented in Figure 10. The system with the  
 648 lowest thermal resistance values has the greatest spatial temperature variations. In the light of these  
 649 results, the dynamic analysis of the system will be performed for the external conditions in KL.

### 650 5.3 Dynamic simulation

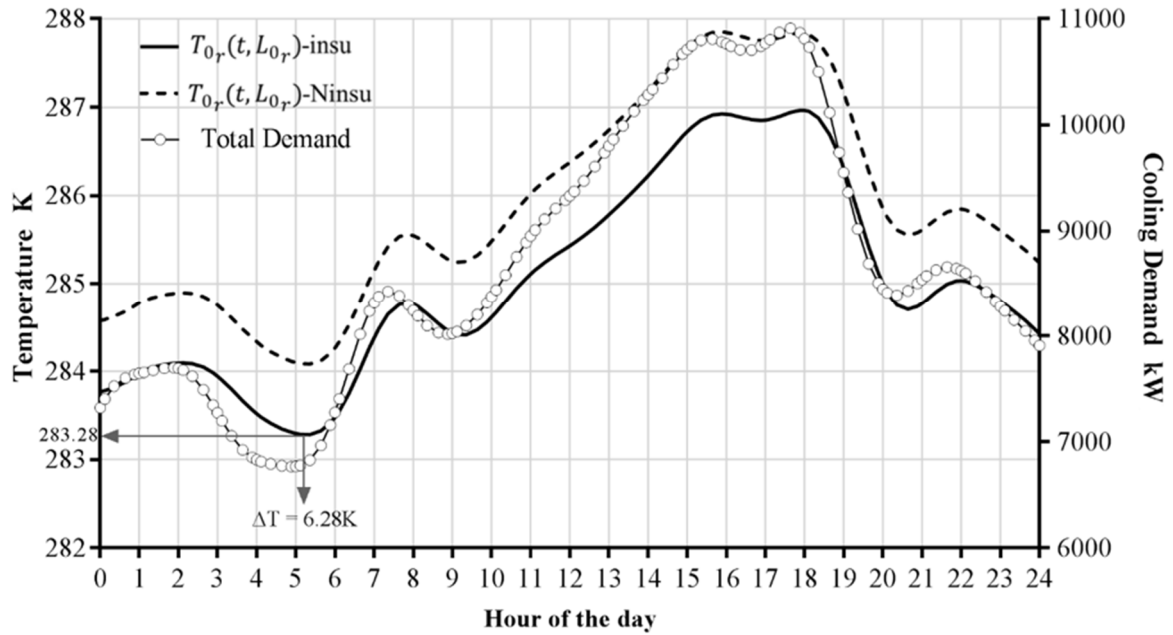
651 As stated in equation (17), the producer mass flow is fixed at its computed steady-state value  
 652 (Table 8) for the dynamic simulations. Figure 12 details the temperature profile at the outlet of  
 653 consumers 1 and 11, for both insulated and non-insulated pipes. We chose these consumers because of  
 654 their high demand and to show the impact of the distance from the producer site. The CPU times  
 655 reported for solving the dynamic simulation problem were 56.7 s for the system with insulated pipes  
 656 and 46.8 s for the network without insulation.



657  
658  
659

**Figure 12. Outlet temperature in consumers 1 and 11 for constant flow policy under KL conditions**

660 Using the constant flow policy, the consumers' outlet temperature will vary over time in line with  
 661 the corresponding demand. This results in a non-uniform return temperature to the production site, as  
 662 shown in Figure 13. For both insulated and non-insulated pipes, the return temperature ( $T_{0_r}(t, L_{0_r})$ ) is  
 663 influenced by the total demand profile. These profiles exhibit a lag time compared to the profile of the  
 664 total demand. This lag time represents the interval of time it takes for the fluid leaving each user to  
 665 arrive at the production site. These variations in return temperature represent a technical issue at the  
 666 production site due to the need for production at the cold utility to stay close to the design temperature  
 667 [46]. Furthermore, the operation under this flow policy drives the system to the undesired low  $\Delta T$   
 668 syndrome, with  $\Delta T$  values at the production site as low as 6.28K.



669  
670  
671

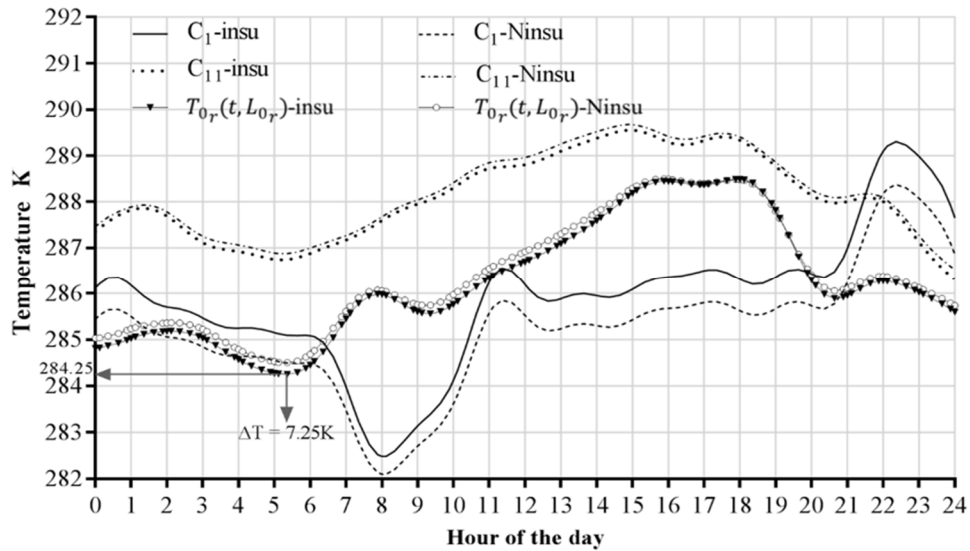
**Figure 13. Return temperature under constant flow policy for KL conditions**

672 As stated in section 4.3, the solution described for the dynamic simulation problem will be used as  
673 the initial guess for the optimisation problems, whose results are detailed in the next section.

#### 674 **5.4 Dynamic optimisation**

675 The DO problem stated in the formulation (21) aims to evaluate the potential of the constant flow  
676 policy to maintain the system under the desired conditions of operation. For this problem, the only  
677 control variable is the constant flow at the production site. Figure 14 presents the temperature profiles  
678 of the return and outlet pipes of the chosen consumers for both insulated and non-insulated pipes. In  
679 terms of CPU time, the solution of problem (21) required 70 s and 58 s for insulated and non-insulated  
680 pipes respectively.

681



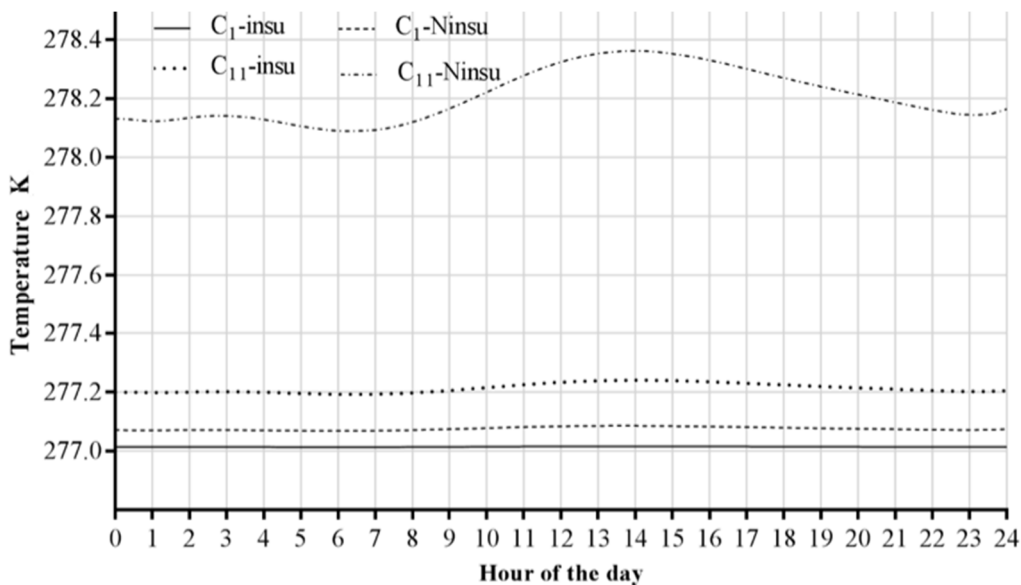
682  
683  
684 **Figure 14. Optimisation of the constant flow operation**

685 For insulated pipes, the optimal producer mass flow is  $228.36 \text{ kg s}^{-1}$ , while for the non-insulated  
686 system it is  $248.40 \text{ kg s}^{-1}$ , which represents an increment of 8.7% in the quantity of chilled water  
687 produced for the non-insulated system. Although the system presents a higher  $\Delta T$  (7.25 K) compared  
688 with the simulation results, the return temperature still presents variations, which would compromise  
689 the operation of the central cooling plant [46].

690

691 As expected, for consumer  $C_{11}$  the outlet temperature is higher when using non-insulated pipes. On  
692 the other hand, under the same conditions, consumer  $C_1$  presents a lower outlet temperature. This is  
693 due to the influence of distance on the inlet temperature of each consumer, and the larger mass flows  
694 for the non-insulated systems. Figure 15 details the inlet temperature profiles for consumers  $C_1$  and  
695  $C_{11}$ , located at 370 m and 3219 m from the producer, respectively.

696



697  
698 **Figure 15. Inlet Temperature profile for selected consumers under constant flow**

699

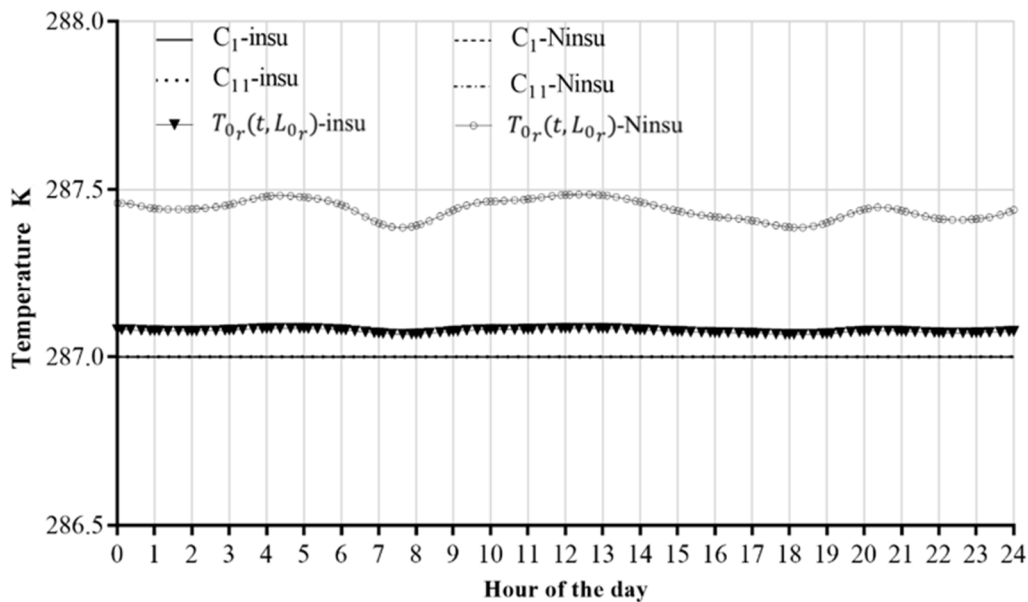
700 We can show the influence of distance from the source when we use non-insulated pipes. The  
701 difference in the values of the outlet temperature for  $C_1$  when the kind of pipe is changed ranges from  
702 0.05 K to 0.06 K, while for  $C_{11}$  this difference ranges from 0.89 K to 1.2 K.  $C_1$  is, therefore, operating  
703 at almost the same temperature in both cases, but due to the increment of the inlet mass flow to the  
704 consumer ( $31.77 \text{ kg s}^{-1}$  to  $34.57 \text{ kg s}^{-1}$ ), the outlet temperature is lower when using non-insulated  
705 pipes. This phenomenon was also observed in consumers  $C_2, C_3, C_4$  (1446 m from production site),  
706  $C_{14}, C_{15}$  and  $C_{16}$  (892 m from production site).

707

708 The previous results show that when working with the constant flow policy, it is not possible to  
709 operate the system under the desired parameters. Although the return temperature is warmer, its  
710 variation due to the demand profiles of the consumers prevents proper operation of the cooling  
711 network.

712

713 By implementing a dynamic flow policy, on the other hand, as stated in the optimisation problem in  
714 (22), the system will operate with more uniform return temperatures, using insulated and non-insulated  
715 pipes, as detailed in Figure 16.



716  
717  
718

Figure 16. Temperature profiles for dynamic flow policy

719 Solving this problem took 517 s and 334 s for insulated and non-insulated pipes respectively. It is  
720 important to note that this problem has a total of 2520 degrees of freedom, which corresponds to the  
721 value of the 20 inlet mass flows and production in the 120 collocation points in time.

722

723 As expected, the return pipe in the insulated network presents a lower temperature than the non-  
724 insulated one. Nevertheless, the only way to achieve the desired outlet temperature in the non-

725 insulated pipe for all consumers involved exceeding the maximum allowed speed in the pipes, as  
 726 shown in Table 9. Although the velocity violations are small (the largest is  $0.24 \text{ ms}^{-1}$  for pipes 13 and  
 727 13<sub>r</sub>) they should involve changing the corresponding pipes to implement a system without insulation.  
 728 However, for the sake of simplicity and to avoid having to recalculate previous simulations, we did not  
 729 change the diameters, but we increased the maximum velocity slightly. Contrary to Branam [26], the  
 730 American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) [47,48] does  
 731 not report a maximum allowable velocity depending on pipe size and indicates that in any case  
 732 (irrespective of application, size or material) the velocity in the pipes cannot exceed  $4.6 \text{ ms}^{-1}$ . As  
 733 detailed in Table 9, none of the pipes reports values even close to 2m/s. Although in these cases the  
 734 velocity bounds have been violated, the computed solution leads to reliable system operation. This  
 735 shows the lack of precision when using an approximation without heat losses to define the diameters  
 736 of the pipes, and the advantage of including the pipe diameter as an optimisation variable in future  
 737 studies.

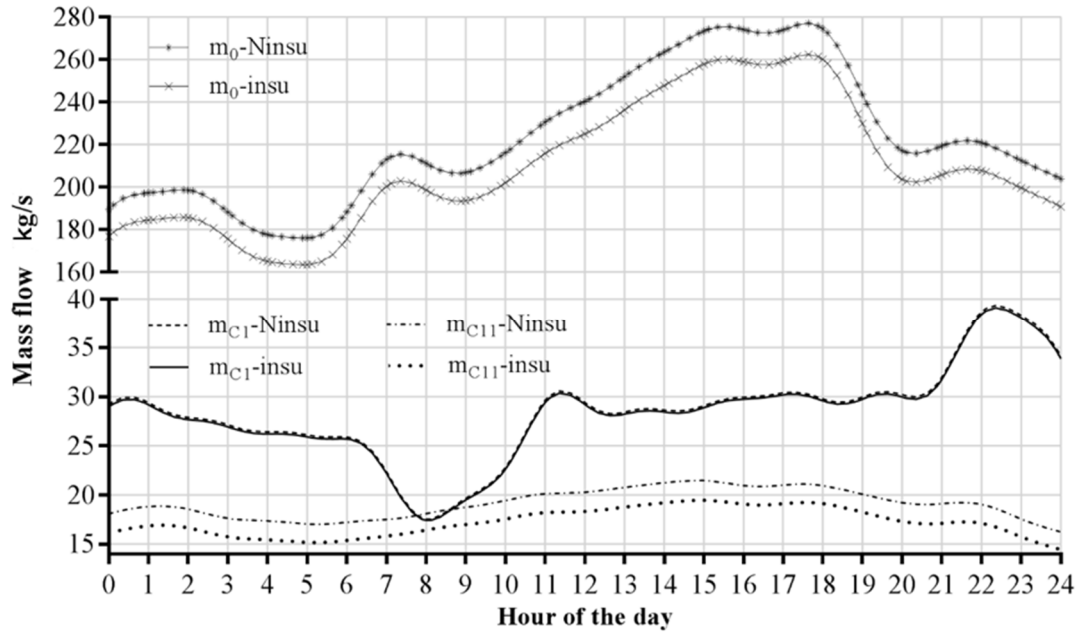
738  
 739 **Table 9. Pipes exceeding allowed velocities in non-insulated network**

<b>pipes</b>	<b>v (<math>\text{ms}^{-1}</math>)</b>	<b>maximum velocity (<math>\text{ms}^{-1}</math>)</b>	<b>pipes</b>	<b>v (<math>\text{ms}^{-1}</math>)</b>	<b>maximum velocity (<math>\text{ms}^{-1}</math>)</b>
9/9 <sub>r</sub>	1.37	1.27	in/out8	1.42	1.32
13/13 <sub>r</sub>	1.18	0.94	in/out12	1.36	1.31
19/19 <sub>r</sub>	1.28	1.27	in/out20	1.72	1.69

740

741 Figure 17 presents the optimal inlet mass flows for the selected consumers and for the production  
 742 site. The computed mass flows consider the heat gains required along the pipes to meet the fluctuating  
 743 consumer demand, reducing the variation in the outlet temperature for each consumer and hence, the  
 744 uncontrolled deviation in the return temperature.

745



**Figure 17. Optimal production and inlet mass flow for chosen consumers**

As expected, the non-insulated system requires more cold water. However, due to the heat gains and the kind of demand, this difference is not proportional among all the users, as can be seen when comparing the profiles of  $C_{11}$  (3219 m from production site) and  $C_1$  (370 m from production site).

Finally, it is possible to compute the total production required per day using the pumping methodologies presented here for the two kinds of piping as  $\int_0^{24} \dot{m}_0(t) dt$ . We present these results in Table 10, where DO.1 and DO.2 refer to the solution of problems (21) and (22) respectively.

**Table 10. Total chilled water production for the studied pumping methods**

Total production Ton				
DS	DO.1-insu	DO.1-Ninsu	DO.2-insu	DO.2-Ninsu
22775.13	19730.68	21462.09	18141.78	19329.01

The optimal dynamic policy (DO.2) represents a reduction of 8.06% (insulated) and 9.94% (non-insulated) compared with the computed productions using a constant flow (DO.1). Compared with the production using the constant flow policy in dynamic simulation, the reductions are 20.34% and 15.42% respectively. The system operates at the desired levels of temperature only if a complete dynamic policy is used.

## 6 Conclusions

In this contribution, we presented an innovative solution methodology, based on 2D-OCFE, for the dynamic simulation and dynamic optimisation with a non-restrictive computational time of district cooling systems. The chosen model was based on reasonable assumptions and was already validated with experimental data in other studies [31, 32, 38]. Using this method, it was possible to fully

768 discretise the initial differential problem, transforming the differential operators into algebraic  
769 combinations of the state variable at some collocation points in time and space. This results in a  
770 simultaneous solution of the optimisation problem, which involved solving steady-state and dynamic  
771 simulations to properly initialise it. Using this methodology, we analyse a medium-sized cooling  
772 network, including environmental and demand variations over a time horizon of 24 h.

773

774 The proposed strategy allowed solving the discretised model (that accounted with more than  
775 360000 variables) with CPU times around 50 s for dynamic simulation and less than 600 s for  
776 dynamic optimization. Via dynamic simulation, we computed the operational response of the system  
777 when a constant flow policy is implemented, resulting in undesirable levels of temperature of the  
778 water returning to the production site leading to a low  $\Delta T$  in the production site (as low as 6.28 K).  
779 Considering this, we proposed an objective function to control the temperature of the water leaving the  
780 clients implementing a dynamic flow policy to avoid the reported low  $\Delta T$  syndrome.

781 We computed the optimal paths for mass flow at the production site and the consumer substations  
782 to optimise the operation of the network using insulated and non-insulated pipes. Optimisation allowed  
783 to synchronize the energy production with the total demand of the system resulting in a lower  
784 production of chilled water to supply the consumer's demand for the two kinds of piping (20.34% and  
785 15.42% respectively) compared with the production computed via dynamic simulation.

786

787 The results of the optimisation presented here show the capability of the proposed methodology to  
788 improve the operating conditions of DCSs under varying cooling demands and external conditions.  
789 It might be interesting to make a comparison with experimental data in order to consolidate our  
790 methodology or to validate some numerical values used in the model (i.e. thermal conductivity of the  
791 soil, convective heat transfer coefficient...). Given the reported CPU times, this method represents a  
792 suitable starting point for a more complex analysis of DCSs, including optimal simultaneous operation  
793 and design.

794

795 We are currently working on the techno-economic analysis of the system, aiming to include the  
796 diameter of the pipes as optimisation variables and to compute properly the electrical requirement to  
797 pump the chilled water to the consumers. In future work, we intend to study the network when chilled  
798 water storage technology is included and to develop the MIDO model in order to choose the most  
799 appropriate technology for producing chilled water.

800

801

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803

804

## 805 **Appendix A. Generalities and implementation of OCFE**

806

807 A collocation method approximates the unknown solution (the state variable) of an ordinary  
808 differential equation as a finite sum of known trial functions and enforces the ordinary differential  
809 equation to be satisfied at some collocation points. If the trial functions are Lagrange basis  
810 polynomials, if the integration variable is normalised, and the collocation points are chosen as roots of  
811 orthogonal polynomials, then the method is called orthogonal collocation. When using the OCFE  
812 method, the state variable is approximated by a different interpolating polynomial on each finite  
813 element, and the state variable continuity must be ensured at the boundaries.

814

### 815 **A.1. One-dimensional case:**

816

817 Let us consider an example where the spatial variable is the only integration variable (1D case), it  
818 can be the model of our DCS in steady-state conditions. If the whole domain is divided into  $nf$   
819 elements ( $x_{f-1}$  and  $x_f$  being the boundaries of the  $f^{\text{th}}$  element and  $L_f$  its length) and if the state  
820 variable is approximated by a polynomial of order  $nj$  (e.g.  $nj + 1$  collocation points are used on each  
821 element), then the state variable (e.g. temperature) can be expressed on the  $f^{\text{th}}$  element as the following  
822 Lagrange interpolation polynomial:

823

$$T(\xi) = \sum_{j=0}^{nj} T_{f,j} \ell_j(\xi) \quad (\text{A-1})$$

824

825

826 with:

$$\xi = \frac{x - x_{f-1}}{x_f - x_{f-1}} = \frac{x - x_{f-1}}{L_f}, \xi \in [0,1] \quad (\text{A-2})$$

827 the normalised spatial variable, and

$$\ell_j(\xi) = \prod_{\substack{k=0 \\ k \neq j}}^{nj} \frac{\xi - \xi_k}{\xi_j - \xi_k} \quad (\text{A-3})$$

828 the  $j^{\text{th}}$  Lagrange basis polynomial of degree  $nj$ .

829

830 With such trial functions (e.g. Lagrange basis polynomial), the coefficient of the  $j^{\text{th}}$  trial function on  
831 the  $f^{\text{th}}$  element represents the variable at the collocation point  $\xi_j$  (i.e.  $T(\xi_j) = T_{f,j}$ ).

832

833 Then the implementation of the collocation method consists in writing that the differential equation  
834 is satisfied on each collocation point  $\xi_j$ . In each element  $f$ : the state variable is replaced by  $T_{f,j}$ , the  
835 derivative can be calculated by deriving the Lagrange interpolation polynomial, leading to a linear  
836 combination of the  $n_j + 1$  coefficients  $T_{f,k}$  ( $k = 0, \dots, n_j$ ). Then the differential equation satisfied at  
837 this collocation point  $\xi_j$  finally leads to an algebraic equation involving the  $n_j + 1$  coefficients  
838  $T_{f,k}$  ( $k = 0, \dots, n_j$ ). Finally, by solving the system of all these  $nf \times (n_j + 1)$  algebraic equations we  
839 obtain the  $nf \times (n_j + 1)$  values  $T_{f,j}$  of the temperature at these collocation points.

840 In order to estimate the derivative terms, Hedengren *et al.* [22] have proposed a convenient method,  
841 which includes the boundary condition. We have adapted it to collocation on finite elements and  
842 describe it below. It begins with a classical formulation of the Lagrange interpolation polynomial on  
843 the  $f^{\text{th}}$  element (equation (A-4)):

$$T(\xi) = \alpha_0 + \alpha_1\xi + \alpha_2\xi^2 + \alpha_3\xi^3 + \dots + \alpha_{n_j}\xi^{n_j} \quad (\text{A-4})$$

844 The first step is then to find a matrix  $M_x$  that relates the values of the derivative at the collocation  
845 points and the coefficients as follows:

$$\begin{bmatrix} \frac{dT}{d\xi}(\xi_1) \\ \frac{dT}{d\xi}(\xi_2) \\ \vdots \\ \frac{dT}{d\xi}(\xi_{n_j}) \end{bmatrix} = M_x \left( \begin{bmatrix} T_{f,1} \\ T_{f,2} \\ \vdots \\ T_{f,n_j} \end{bmatrix} - \begin{bmatrix} T_{f,0} \\ T_{f,0} \\ \vdots \\ T_{f,0} \end{bmatrix} \right) \quad (\text{A-5})$$

846 The coefficient  $\alpha_0$  corresponds to the boundary condition of the element  $f$ ,  $T_{f,0}$ , when the initial  
847 position is defined as zero. For the first element, this value ( $T_{1,0}$ ) must be given, whereas for the  
848 following elements it corresponds to the final value of the previous element. Then, substituting the  
849 approximation of the state variable and its derivative in the previous expression  $T_{f,0}$  will be cancelled  
850 and it follows:

$$\begin{bmatrix} 1 & 2\xi_1 & 3\xi_1^2 & \dots & n_j\xi_1^{n_j-1} \\ 1 & 2\xi_2 & 3\xi_2^2 & \dots & n_j\xi_2^{n_j-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2\xi_{n_j} & 3\xi_{n_j}^2 & \dots & n_j\xi_{n_j}^{n_j-1} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{n_j} \end{bmatrix} = M_x \begin{bmatrix} \xi_1 & \xi_1^2 & \dots & \xi_1^{n_j} \\ \xi_2 & \xi_2^2 & \dots & \xi_2^{n_j} \\ \vdots & \vdots & \ddots & \vdots \\ \xi_{n_j} & \xi_{n_j}^2 & \dots & \xi_{n_j}^{n_j} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{n_j} \end{bmatrix} \quad (\text{A-6})$$

851

852 Finally, considering the change of variable, this leads to:

$$\begin{bmatrix} \left(\frac{dT}{dx}\right)_{f,1} \\ \left(\frac{dT}{dx}\right)_{f,2} \\ \vdots \\ \left(\frac{dT}{dx}\right)_{f,n_j} \end{bmatrix} = \frac{1}{L_f} \cdot M_x \left( \begin{bmatrix} T_{f,1} \\ T_{f,2} \\ \vdots \\ T_{f,n_j} \end{bmatrix} - \begin{bmatrix} T_{f,0} \\ T_{f,0} \\ \vdots \\ T_{f,0} \end{bmatrix} \right) \quad (\text{A-7})$$

$$= \frac{1}{L_f} \cdot \begin{bmatrix} 1 & 2\xi_1 & 3\xi_1^2 & \dots & n_j \xi_1^{n_j-1} \\ 1 & 2\xi_2 & 3\xi_2^2 & \dots & n_j \xi_2^{n_j-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2\xi_{n_j} & 3\xi_{n_j}^2 & \dots & n_j \xi_{n_j}^{n_j-1} \end{bmatrix} \begin{bmatrix} \xi_1 & \xi_1^2 & \dots & \xi_1^{n_j} \\ \xi_2 & \xi_2^2 & \dots & \xi_2^{n_j} \\ \vdots & \vdots & \ddots & \vdots \\ \xi_{n_j} & \xi_{n_j}^2 & \dots & \xi_{n_j}^{n_j} \end{bmatrix}^{-1} \left( \begin{bmatrix} T_{f,1} \\ T_{f,2} \\ \vdots \\ T_{f,n_j} \end{bmatrix} - \begin{bmatrix} T_{f,0} \\ T_{f,0} \\ \vdots \\ T_{f,0} \end{bmatrix} \right)$$

853 Once the collocation points are defined, we can calculate the collocation matrix  $M_x$  and discretise  
854 the derivative terms using equation (A-7) to obtain a set of algebraic equations whose resolution will  
855 give the solution to the differential problem. For the present application, we use the nodes of the  
856 shifted Legendre Gauss Lobatto quadrature as collocation points [49]. Thus, 0 and 1 are nodes and  
857 there are  $n_j - 1$  internal nodes. It follows that:

$$T_{f,0} = T_{f-1,n_j} \quad f = 2, \dots, n_f \quad (\text{A-8})$$

858 Then, on each element, the ODE has to be written only on  $n_j$  points ( $\xi_j, j = 1 \dots n_j$ ) since the  
859 boundary condition (e.g.  $T_{f,0} = T(L_f \xi_0 + x_{f-1}) = T(x_{f-1})$ ) of each element is known. When dealing  
860 with a steady-state simulation, the above methodology has been applied to our DCS and the  
861 temperature, which depends only on  $x$ , has been noted  $T(t \text{ fixed}, x) = \theta(x)$ .

862

863 In the following subsection, we present the implementation of the method for both time ( $t$ ) and  
864 space domains ( $x$ ).

865

## 866 A.2. Implementation of 2D-OCFE

867 The orthogonal collocation on finite elements method in 2 dimensions can be formulated as an  
868 extension of the 1D derivation, as stated by Surjanhata[50], Finlayson[40] and Esche *et al.*[23], using  
869 the corresponding variables for each direction as well as a different polynomial or number of roots.  
870 First, the time horizon is divided into  $ne$  intervals (finite elements), and inside each interval,  $ni$   
871 collocation points are chosen. Similarly, each pipe has  $nf$  segments with  $n_j$  collocation points.  
872 Therefore, we define sets  $e \in (1, 2, \dots, ne), i \in (0, 1, \dots, ni), f \in (1, 2, \dots, nf), j \in (0, 1, \dots, n_j)$ . We  
873 also introduce the normalised time for element  $e$  (similarly to the normalised spatial variable):

$$\tau = \frac{t - t_{e-1}}{t_e - t_{e-1}} = \frac{t - t_{e-1}}{\Delta t_e}, \tau \in [0,1] \quad (\text{A-9})$$

874 Let us then consider the temperature of the spatial-time domain corresponding to element  $e$  in time  
 875 and element  $f$  in space. This is now approximated by the following function of  $\xi$  and  $\tau$  (degree  $ni$  in  $\tau$   
 876 and  $nj$  in  $\xi$ ):

$$T(\tau, \xi) = \sum_{i=0}^{ni} \sum_{j=0}^{nj} T_{e,i,f,j} \ell_i(\tau) \ell_j(\xi) \quad (\text{A-10})$$

877 The properties of the Lagrange basis polynomials lead to  $T(\tau_i, \xi_j) = T_{e,i,f,j}$  and therefore:

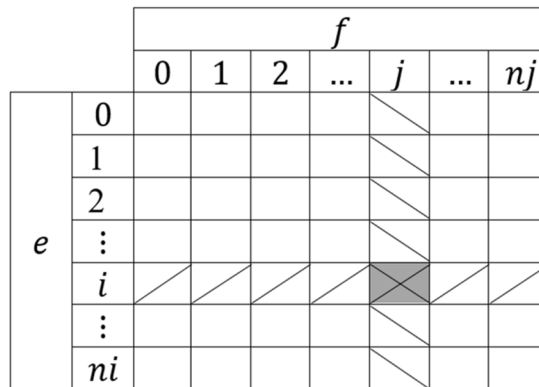
$$T(\tau, \xi_j) = \sum_{i=0}^{ni} T_{e,i,f,j} \ell_i(\tau) \quad (\text{A-11})$$

878 and:

$$T(\tau_i, \xi) = \sum_{j=0}^{nj} T_{e,i,f,j} \ell_j(\xi) \quad (\text{A-12})$$

879

880 To compute  $\frac{\partial T}{\partial \tau}(\tau_i, \xi_j)$  we will derive the Lagrange interpolation polynomial described by  
 881 equation (A-11). This represents the evolution in time of the temperature in the present element ( $e$ ) for  
 882 the given position ( $f, j$ ) and involves coefficients  $T_{e,l,f,j}$  with  $e, f, j$  fixed, and  $l$  varying from 0 to  $ni$   
 883 (and passing through the particular value  $i$ ), which are represented by  $\square$  in Figure A-1. The same  
 884 logic applies to compute  $\frac{\partial T}{\partial \xi}(\tau_i, \xi_j)$ : we will derive the Lagrange interpolation polynomial described  
 885 by equation (A-12) which represents the evolution in space of the temperature in the present element  
 886 ( $f$ ) for the given time ( $e, i$ ). It involves the coefficients  $T_{e,i,f,l}$  with  $e, i, f$  fixed, and  $l$  varying from 0  
 887 to  $nj$  (passing through the particular point  $j$ ), and represented by  $\square$  in Figure A-1.



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 889  
 890  
 891

**Figure A-1. Representation of the temperature discretisation (one element per domain)**

892 Finally, we add a subscript to take the issue of the pipe into account. Then the temperature of each  
 893 pipe  $k$  ( $k$  may be  $p, p_r, in_{C_p}, out_{C_p}$ )  $T_k(t, x)$  is expressed with the use of the  $ne \times (ni + 1) \times nf \times$   
 894  $(nj + 1)$  coefficients  $T_{k,e,i,f,j}$ .

896 Then, applying the same methodology as for 1D-OCFE, we can approximate  $\frac{\partial T_k}{\partial t}$  and  $\frac{\partial T_k}{\partial x}$  as:

$$\begin{bmatrix} \left(\frac{\partial T_k}{\partial t}\right)_{e,1,f,j} \\ \left(\frac{\partial T_k}{\partial t}\right)_{e,2,f,j} \\ \vdots \\ \left(\frac{\partial T_k}{\partial t}\right)_{e,ni,f,j} \end{bmatrix} = \frac{1}{\Delta t_e} \cdot M_t \left( \begin{bmatrix} T_{k,e,1,f,j} \\ T_{k,e,2,f,j} \\ \vdots \\ T_{k,e,ni,f,j} \end{bmatrix} - \begin{bmatrix} T_{k,e,0,f,j} \\ T_{k,e,0,f,j} \\ \vdots \\ T_{k,e,0,f,j} \end{bmatrix} \right) = \begin{bmatrix} D_{t_1}(k, e, f, j) \\ D_{t_2}(k, e, f, j) \\ \vdots \\ D_{t_{ni-1}}(k, e, f, j) \end{bmatrix} \quad (\text{A-13})$$

$$\begin{bmatrix} \left(\frac{\partial T_k}{\partial x}\right)_{e,i,f,1} \\ \left(\frac{\partial T_k}{\partial x}\right)_{e,i,f,2} \\ \vdots \\ \left(\frac{\partial T_k}{\partial x}\right)_{e,i,f,nj} \end{bmatrix} = \frac{1}{L_f} \cdot M_x \left( \begin{bmatrix} T_{k,e,i,f,1} \\ T_{k,e,i,f,2} \\ \vdots \\ T_{k,e,i,f,nj} \end{bmatrix} - \begin{bmatrix} T_{k,e,i,f,0} \\ T_{k,e,i,f,0} \\ \vdots \\ T_{k,e,i,f,0} \end{bmatrix} \right) = \begin{bmatrix} D_{x_1}(k, e, i, f) \\ D_{x_2}(k, e, i, f) \\ \vdots \\ D_{x_{nj-1}}(k, e, i, f) \end{bmatrix} \quad (\text{A-14})$$

897  
 898 In equations (A-13) and (A-14)  $M_t$  and  $M_x$  represent the collocation matrices for the normalised  
 899 domains with respective element sizes  $\Delta t_e$  and  $L_f$ . Finally,  $D_{t_i}(k, e, f, j)$  and  $D_{x_j}(k, e, i, f)$  denote the  
 900 algebraic combinations to describe the variational terms.

901  
 902 For the first element on each domain, the values of the boundary conditions,  $T_{k,1,0,f,j}$  and  $T_{k,e,i,1,0}$ ,  
 903 must be known. The way to achieve this will be explained in the next section. Then, for subsequent  
 904 elements, these values correspond to those of the final point of the prior element. For the present  
 905 application, we use the nodes of the shifted Legendre Gauss Lobatto quadrature as collocation points  
 906 so that 0 and 1 are nodes.

907  
 908 Then for the other spatial-time points (other than  $e, 0, f, j$  and  $e, i, f, 0$ ) (e.g.  $(\xi_j, \tau_i), j =$   
 909  $1 \dots nj, i = 1 \dots ni$ ) we assume that equation (6) is satisfied.  $T_k(\tau_i, \xi_j)$  is then replaced by  $T_{k,e,i,f,j}$   
 910 and the derivatives are expressed by equations (A-13) and (A-14).

911  
 912 In Appendix B, we describe the implementation of 2D-OCFE to transform the heat equation into a  
 913 set of algebraic equations, which, when coupled with the heat and mass balances, represent the  
 914 dynamics of the DCS.

915

916 **Appendix B. Discretised model**

917 This appendix details the discretised model based on the equations presented in section 3.

918

919 **B.1. Heat balances**

920 As described in the previous section, 2D-OCFE will be applied to the PDE (6). To do this, the mass  
921 flow in each pipe, which is only time-dependent, also needs to be discretised as follows:

922

$$\dot{m}_k(\tau) = \sum_{i=0}^{ni} \dot{m}_{k,e,i} \ell_i(\tau) \quad (\text{B-1})$$

923 The profile of the soil temperature  $T_s(t)$  is known and corresponds to one of the profiles in section  
924 2.3. When the PDE is applied in an element  $e$  in time at a collocation point  $\tau_i$  we use the  
925 corresponding soil temperature which has been noted  $T_{s,e,i}$ .

$$T_{s,e,i} = T_s(\tau_i \Delta t_e + t_{e-1}) \quad (\text{B-2})$$

926 In the same way, the temporal profile of the demand of each customer is known and we note:

$$Q_{C_p,e,i} = Q_{C_p}(\tau_i \Delta t_e + t_{e-1}) \quad (\text{B-3})$$

927 As explained in the previous section, before applying orthogonal collocation, boundary conditions  
928 must be known, which will be explained later. Once these boundary conditions are known  
929 ( $T_{k,1,0,f,j}$  ( $j = 1 \dots nj$ ) and  $T_{k,e,i,1,0}$  ( $i = 1, \dots ni$ )), using equations (A-13) and (A-14), we can  
930 represent equation (6) for each pipe at each spatial-time point other than the boundary points,  
931  $(\tau_i, \xi_j), i = 1 \dots ni, j = 1 \dots nj$ , by:

$$\rho_w \cdot C_{p_w} \cdot A_k \cdot D_{t_i}(k, e, f, j) + \dot{m}_{k,e,i} \cdot C_{p_w} \cdot D_{x_j}(k, e, i, f) = \frac{T_{s,e,i} - T_{k,e,i,f,j}}{R'} \quad (\text{B-4})$$

932 By implementing this, we represent the PDE (6) as a set of  $(ne \times ni \times nf \times nj)$  equations per  
933 pipe. The boundary conditions  $T_{k,e,0,f,j}$  and  $T_{k,e,i,f,0}$  are obtained from the configuration of the  
934 problem and from the heat and mass balances as follows:

- 935 ○ For elements  $e > 1$  and  $f > 1$ , for each pipe, the initial conditions correspond to the final  
936 point of the previous one, as already stated.

$$T_{k,e,0,f,j} = T_{k,e-1,ni,f,j}, \quad e = 2 \dots ne, \quad f = 1 \dots nf, \quad j = 1 \dots nj \quad (\text{B-5})$$

$$T_{k,e,i,f,0} = T_{k,e,i,f-1,nj}, \quad f = 2 \dots nf, \quad e = 1 \dots ne, \quad i = 1, \dots ni \quad (\text{B-6})$$

937 and

$$T_{k,e,0,f,0} = T_{k,e-1,ni,f-1,nj}, \quad e = 2 \dots ne, \quad f = 2 \dots nf \quad (\text{B-7})$$

- 938     ○ The element  $e = 1$  equation (9) leads to the following  $nf \times (nj + 1)$  equations in terms of  
939     discretised variables:

$$T_{k,1,0,f,j} = \theta_k(L_f \xi_j + x_{f-1}) = \theta_{k,f,j} \quad (\text{B-8})$$

- 940     ○ For element  $f = 1$ , this condition depends on the pipe.  
941         ▪ Equations (10) to (13) lead to the following:  $ne \times (ni + 1) - 1$  equations in terms of  
942     discretised variables:

$$T_{0,e,i,1,0} = 277 \text{ K} \quad (\text{B-9})$$

943

$$T_{p,e,i,nf,nj} = T_{p+1,e,i,1,0} = T_{in_{C_p},e,i,1,0}, \quad p = 1 \dots 12, 14, \dots 19$$

$$T_{0,e,i,nf,nj} = T_{1,e,i,1,0} = T_{14,e,i,1,0} \quad (\text{B-10})$$

$$T_{p,e,i,nf,nj} = T_{in_{C_p},e,i,1,0}, \quad p = 13, 20$$

944

$$\dot{m}_{p_r,e,i} \cdot T_{p_r,e,i,1,0} = \dot{m}_{p_r+1,e,i} \cdot T_{(p+1)_r,e,i,nf,nj} + \dot{m}_{out_{C_p},e,i} \cdot T_{out_{C_p},e,i,nf,nj},$$

$$p = 1 \dots 12, 14, \dots 19$$

(B-11)

$$\dot{m}_{(0)_r,e,i} \cdot T_{(0)_r,e,i,1,0} = \dot{m}_{(1)_r,e,i} \cdot T_{(1)_r,e,i,nf,nj} + \dot{m}_{(14)_r,e,i} \cdot T_{(14)_r,e,i,nf,nj}$$

$$T_{(p)_r,e,i,1,0} = T_{out_{C_p},e,i,nf,nj}, \quad p = 13, 20$$

$$Q_{C_p,e,i} = \dot{m}_{in_{C_p},e,i} \cdot C_p w \cdot (T_{out_{C_p},e,i,1,0} - T_{in_{C_p},e,i,nf,nj}) \quad (\text{B-12})$$

945     Finally, Figure B-1 summarises variables  $T_{k,e,i,f,j}$  and their associated equations.

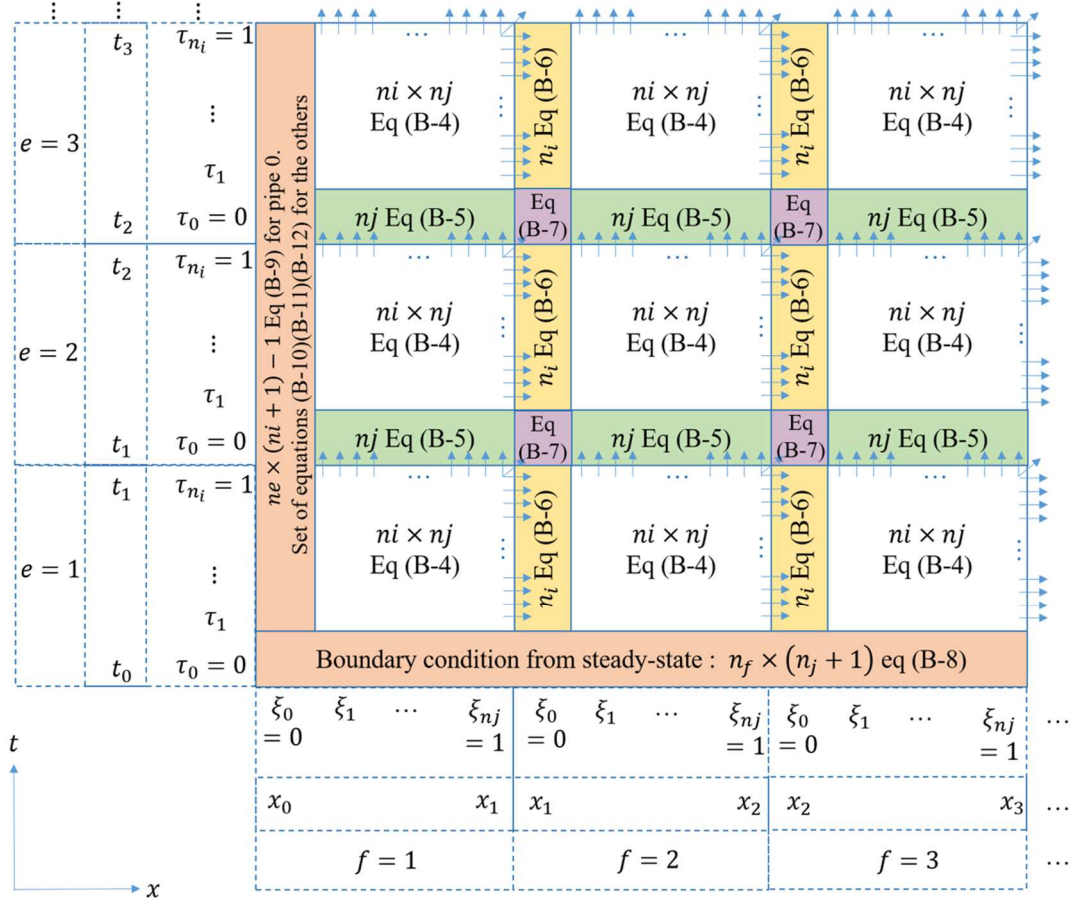


Figure B-1. Associated equations for the solution of the PDE on a pipe

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## B.2. Mass balances

951 In this section, the mass balance equations and the constraints concerning the velocity and the flow  
952 policy involve only time-dependent variables. Therefore, each of these equations from the original  
953 PDAE problem leads to a corresponding set of  $ne \times (ni + 1)$  equations, replacing  $\dot{m}_k(t)$  and  $v_k(t)$   
954 by  $\dot{m}_{k,e,i}$  and  $v_{k,e,i}$ . In this section, these equations are written in terms of discretised variables.

955

956 Mass balance equations (14), (15) and (16) lead to:

$$\begin{aligned} \dot{m}_{p,e,i} &= \dot{m}_{p+1,e,i} + \dot{m}_{in_{C_p},e,i} \quad p = 1 \dots 12, 14 \dots 19 \\ \dot{m}_{p,e,i} &= \dot{m}_{in_{C_p},e,i} \quad p = 13, 20 \\ \dot{m}_{0,e,i} &= \dot{m}_{1,e,i} + \dot{m}_{14,e,i} \end{aligned} \quad (B-13)$$

957

$$\begin{aligned} \dot{m}_{p,r,e,i} &= \dot{m}_{(p+1),r,e,i} + \dot{m}_{out_{C_p},e,i} \quad p = 1 \dots 12, 14 \dots 19 \\ \dot{m}_{p,r,e,i} &= \dot{m}_{out_{C_p},e,i} \quad p = 13, 20 \\ \dot{m}_{0,r,e,i} &= \dot{m}_{1,r,e,i} + \dot{m}_{14,r,e,i} \end{aligned} \quad (B-14)$$

958

$$\dot{m}_{in_{C_p,e,i}} = \dot{m}_{out_{C_p,e,i}} \quad p = 1 \dots 20 \quad (\text{B-15})$$

959 Flow policy equations (17) and (18) lead to:

$$\dot{m}_{0,e,i} = \dot{m}_{0Cte} \quad (\text{B-16})$$

960

$$\frac{Peak(1)}{\dot{m}_{in_{C_1,e,i}}} = \frac{Peak(p)}{\dot{m}_{in_{C_p,e,i}}}, \quad \forall p \in \{2, \dots, 20\} \quad (\text{B-17})$$

961

962 and the flow velocity equation (20) leads to:

$$v_{k,e,i} \leq v_{max,k} \quad (\text{B-18})$$

963

964 Finally, the  $n_{pipes} \times n_e \times (n_i + 1)$  variables  $\dot{m}_{k,e,i}$ , with  $n_{pipes} = 82$ , are associated with:

965  $21 \times n_e \times (n_i + 1)$  equations (B-13).

966  $21 \times n_e \times (n_i + 1)$  equations (B-14).

967  $20 \times n_e \times (n_i + 1)$  equations (B-15).

968  $1 \times n_e \times (n_i + 1)$  equations (B-16).

969  $19 \times n_e \times (n_i + 1)$  equations (B-17).

970

971 Together with the variables  $T_{k,e,i,f,j}$  associated with the equations described in the previous section,  
972 they constitute the algebraic system that must be solved.

973

974 The set of equations (B-4) to (B-18) results from the implementation of 2D-OCFE in the original  
975 PDAE problem. This formulation will allow us to perform the dynamic simulation analysis of the case  
976 study, whose main output will be the temperature profiles of the pipes that make up the system.

### 977 **Appendix C. Validation of the discretization method**

978

979 To validate the proposed 2D-OCFE method for solving the dynamic one-dimensional heat transfer  
980 equation in pipes, we analyse a simple case study. The problem is solved using a well- known  
981 discretisation strategy (second-order centred finite differences for time and space) as a reference  
982 scenario, which will be compared to the solution obtained via 2D-OCFE.

983 We propose to study the propagation of a temperature wave in the inlet of a 12-inch non-insulated  
984 pipe with a length of 1km, a constant flow velocity of 0.97m/s, and external temperature ( $T_s$ ) varying  
985 linearly from 25°C to 27°C, for 1 hour.

986

987 First, we defined the number of discretisation points for the reference scenario. For this, we made  
988 several runs by incrementing the number of points on each domain ( $x, t$ ) and comparing the results  
989 from each run until the change in the computed values was negligible. Next, we chose a reference  
990 scenario computed with 1200 discrete points in time and 200 in distance using the method of second-  
991 order centred finite differences. The computational time for this scenario was 24.84 s.

992

993 Then we implemented the proposed 2D-OCFE method to solve this problem using different  
994 combinations of the number of elements ( $ne$  and  $nf$ ) and collocation points ( $(ni + 1)$  and  $(nj + 1)$ )  
995 on each domain, and comparing each result with the reference scenario. Table C-1 details some of the  
996 implemented combinations, their corresponding % of error with respect to the reference scenario and  
997 the computational time.

998

999

**Table C-1. 2D-OCFE combinations results**

$ne$	$ni+1$	$nf$	$nj + 1$	Error [%]	CPU time (s)
30	6	2	6	[-0.0056, 0.0044]	0.047
30	6	3	6	[-0.0027,0.0054]	0.047
30	6	4	6	[-0.0049, 0.0056]	0.062
30	6	5	6	[-0.0028, 0.0052]	0.094

1000

1001 We can see that via 2D-OCFE the number of points required is considerably reduced, from 1200 to  
1002 180 in time and from 200 to 30 (at most) in distance, achieving accurate results with respect to the  
1003 reference scenario, given the reported errors, with computational times more than 250 times smaller  
1004 than implementing a conventional discretisation strategy.

1005

1006 Finally, Figure C-1 shows the inlet temperature wave and the variation in temperature at the pipe  
1007 outlet for some of the runs made to define the reference scenario (1200 t) using 200 discretisation  
1008 points in distance. These are compared to the response using 2D-OCFE (last case in Figure C-1)Here,  
1009 we show that by using 2D-OCFE it is possible to compute more accurate results than with some  
1010 conventional discretisation using 150 and 500 points in time.

1011

1012 These results validate the accuracy of the proposed 2D-OCFE, and thus we can replicate it for the  
1013 analysis of district cooling systems.

1014

1015

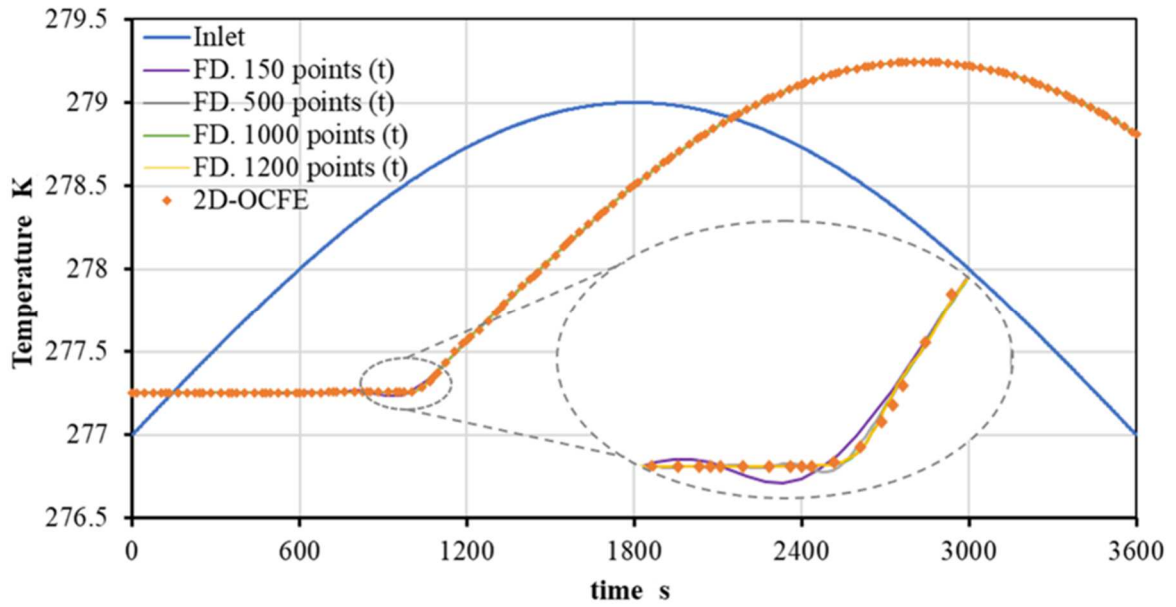


Figure C-1. Pipe inlet temperature variation and comparison of outlet responses

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## 1020 Nomenclature

### 1021 Symbols

1022  $\bar{h}$  Convective heat transfer coefficient,  $\text{W m}^{-2}\text{K}^{-1}$

1023  $C_p$  Specific Heat Capacity,  $\text{J kg}^{-1}\text{K}^{-1}$

1024  $L$  Pipe length, m

1025  $L_f$  Length of spatial finite element

1026  $\dot{m}$  Mass flow rate,  $\text{kg s}^{-1}$

1027  $N_x$  Spatial collocation matrix

1028  $N_x$  Temporal collocation matrix

1029  $r$  radius, m

1030  $R'$  Total thermal resistance,  $\text{m K W}^{-1}$

1031  $t$  Time, s

1032  $T$  Temperature, K

1033  $v$  Velocity,  $\text{m s}^{-1}$

1034  $\Delta T$  Temperature difference, K

1035  $\Delta t_e$  Size of temporal finite element

### 1036 Greek symbols

1037  $\lambda$  Conductivity,  $\text{W m}^{-1}\text{K}^{-1}$

1038  $\rho$  Density,  $\text{kg m}^{-3}$

1039  $\xi$  Normalised spatial variable

1040  $\tau$  Normalised time variable

1041  $\theta$  Steady-state temperature

1042 **Sets and indices**

1043  $C_p$  Set of consumers

1044  $D_{t_i}$  Linear transformation for time derivative

1045  $D_{x_j}$  Linear transformation for space derivative

1046  $e$  Index for time elements

1047  $f$  Index for distance elements

1048  $i$  Index for time collocation points in time

1049  $j$  Index for distance collocation points

1050  $k$  Index of pipes

1051  $p$  Sub-index for main pipes

1052  $in_{cp}$  Sub-index for pipes entering clients

1053  $out_{cp}$  Sub-index for pipes leaving clients

1054 **Subscripts**

1055  $r$  Return pipe

1056  $s$  Soil

1057  $w$  Water

1058

1059

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