

**IRSN**

INSTITUT  
DE RADIOPROTECTION  
ET DE SÛRETÉ NUCLÉAIRE

*Faire avancer la sûreté nucléaire*

# RESONANCE PARAMETERS AND COVARIANCE EVALUATIONS FOR THE GADOLINIUM ISOTOPES

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# OUTLINE

- ❑ Evaluation of Gadolinium isotopes:  $^{155}\text{Gd}$ ,  $^{156}\text{Gd}$ ,  $^{157}\text{Gd}$ ,  $^{158}\text{Gd}$ , and  $^{160}\text{Gd}$
- ❑ Extension of the Resolved Resonance Region
- ❑ New and Old Measurements: Transmission and Capture
- ❑ Tools used in the analysis: SAMMY, SAMDIST, and SAMINT
- ❑ Statistical Distribution: Porter-Thomas, Wigner,  $\Delta_3$ -statistics Test
- ❑ Benchmark Results and uncertainty propagation
- ❑ Concluding remarks

# Experimental Data Base

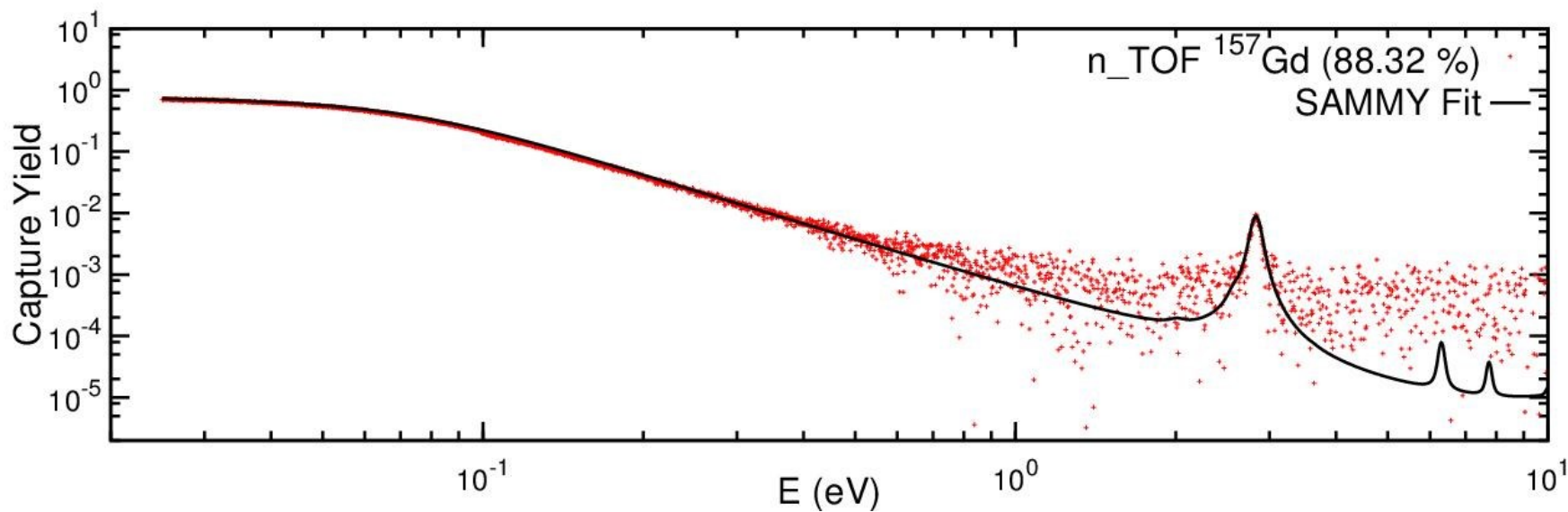
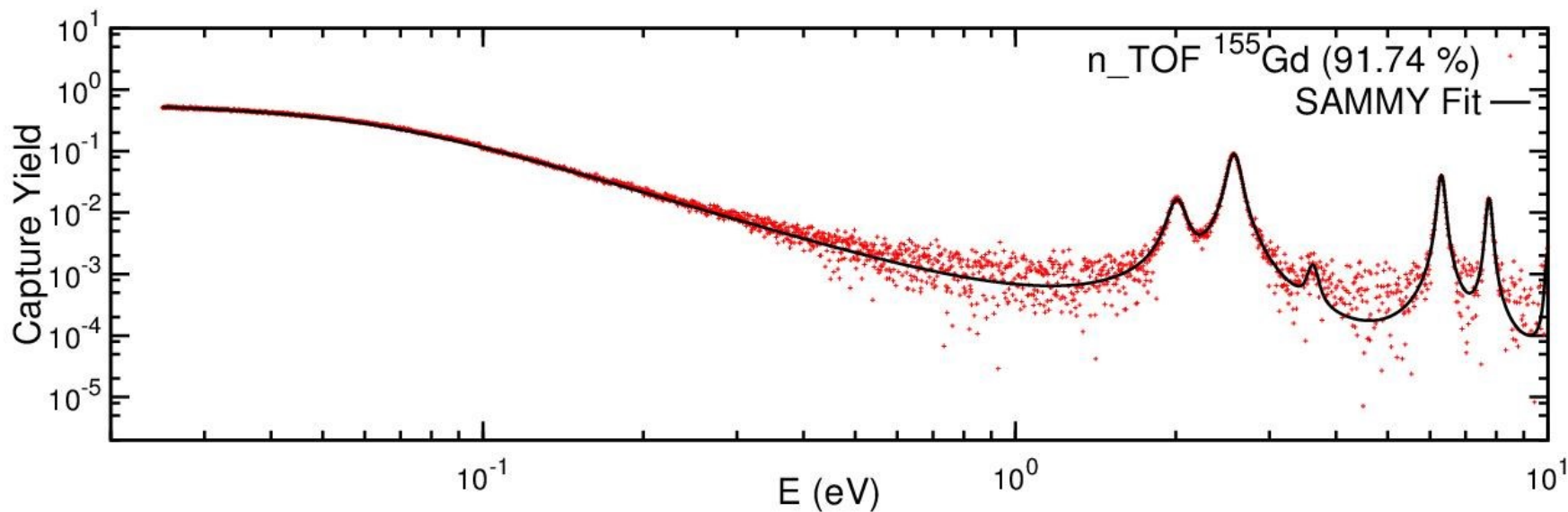
## Experimental transmission and capture data

Data Set	Enrichment (%)	Energy Range (eV)	Flight Path (m)	Density (at/b)
<b>Natural Gadolinium</b>				
Transmission (RPI)		0.2 - 300.0	25.585	$7.806 \times 10^{-4}$
Transmission (RPI)		0.3 - 500.0	25.597	$1.566 \times 10^{-3}$
Transmission (RPI)		0.3 - 1000.0	25.597	$1.566 \times 10^{-3}$
Capture (RPI)		0.2 - 1000.0	25.585	$7.806 \times 10^{-4}$
<b><sup>155</sup>Gd</b>				
Capture (RPI)	91.74	0.2 - 1000.0	25.567	$3.083 \times 10^{-4}$
Capture (n_TOF)	91.74	0.025 - 50.0	183.90	$1.236 \times 10^{-6}$
Capture (n_TOF)	91.74	1.0 - 1000.0	183.90	$1.244 \times 10^{-4}$
<b><sup>157</sup>Gd</b>				
Capture (RPI)	90.96	0.2 - 1000.0	25.569	$5.820 \times 10^{-4}$
Capture (n_TOF)	88.32	0.025 - 50.0	183.90	$5.753 \times 10^{-6}$
Capture (n_TOF)	88.32	1.0 - 1000.0	183.90	$2.340 \times 10^{-4}$

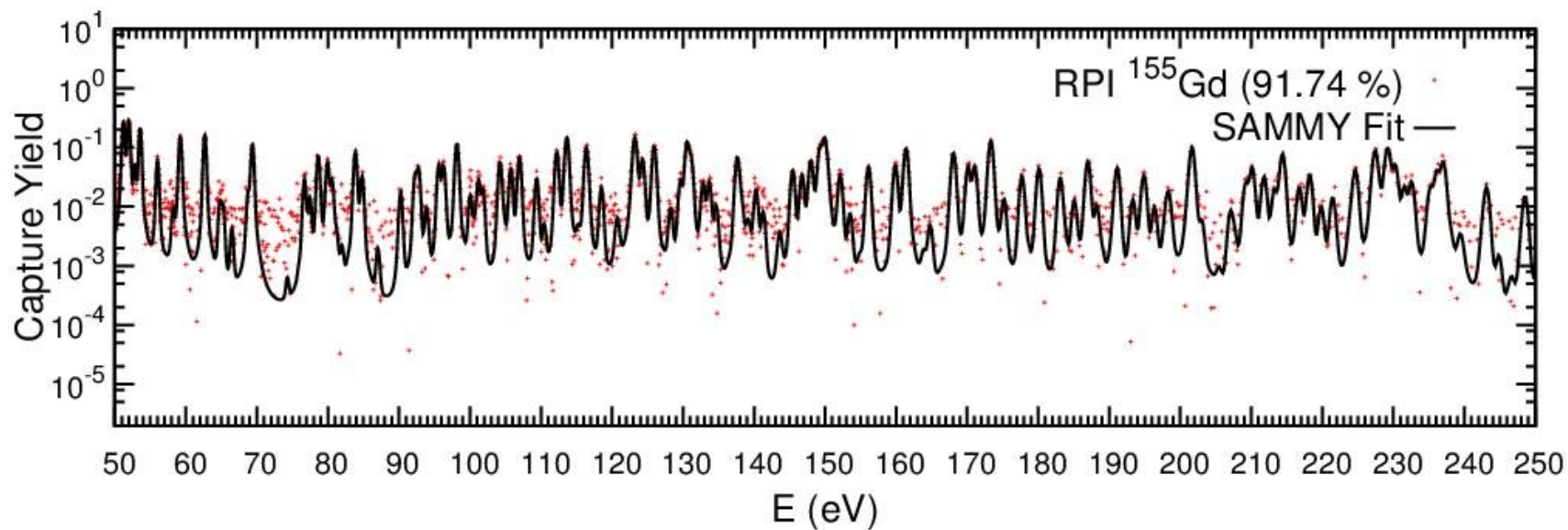
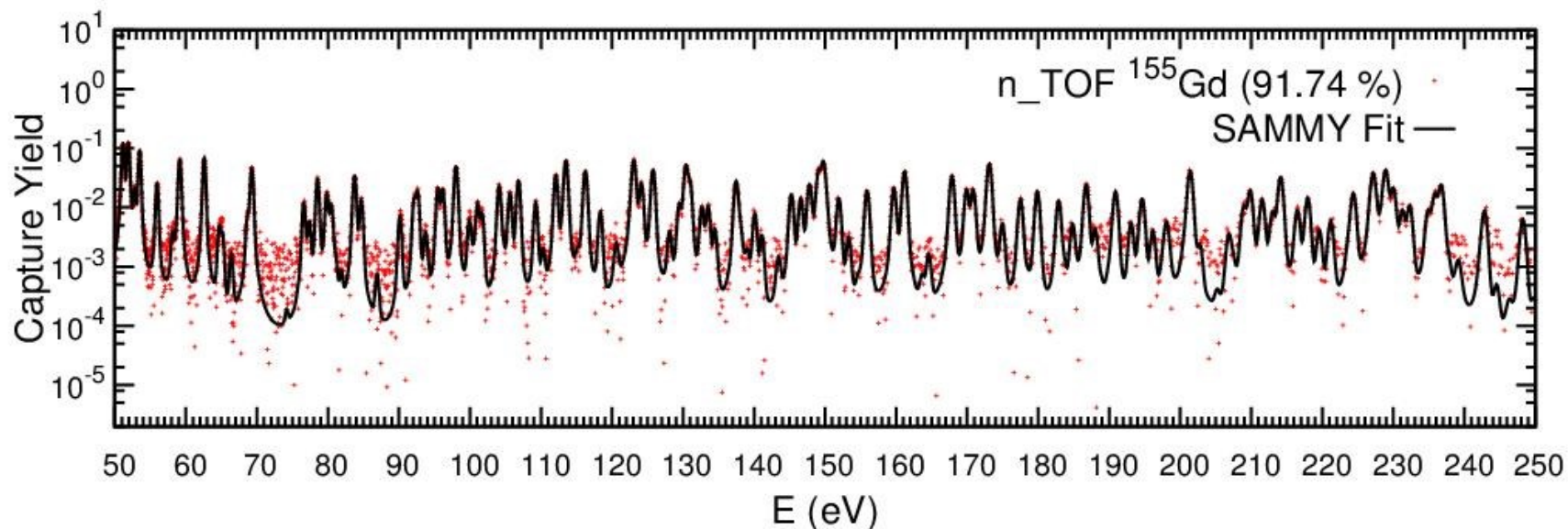
# New Evaluation Resonance Region

- **SAMMY**: Evaluation tool based on the R-matrix approach together with the generalized least square methodology (Bayes' Theorem)
- **SAMDIST**: Tool used for testing the statistic in connection to the R-matrix Parameters
- **SAMINT**: Tool that allows combining differential and integral data;

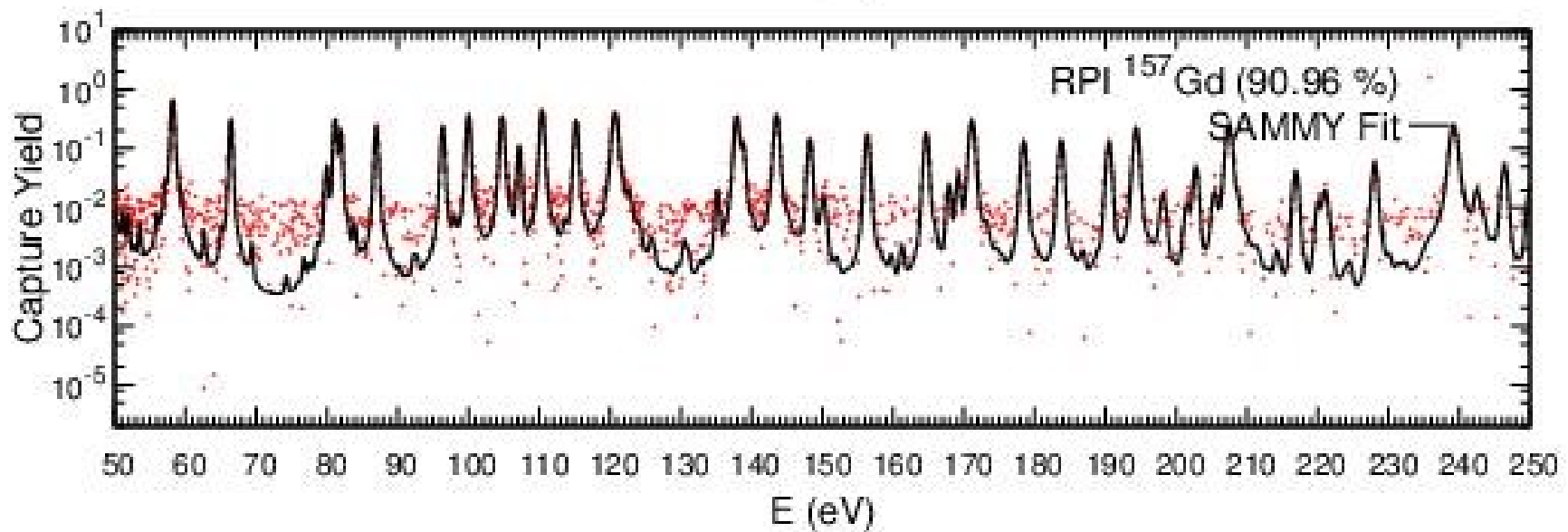
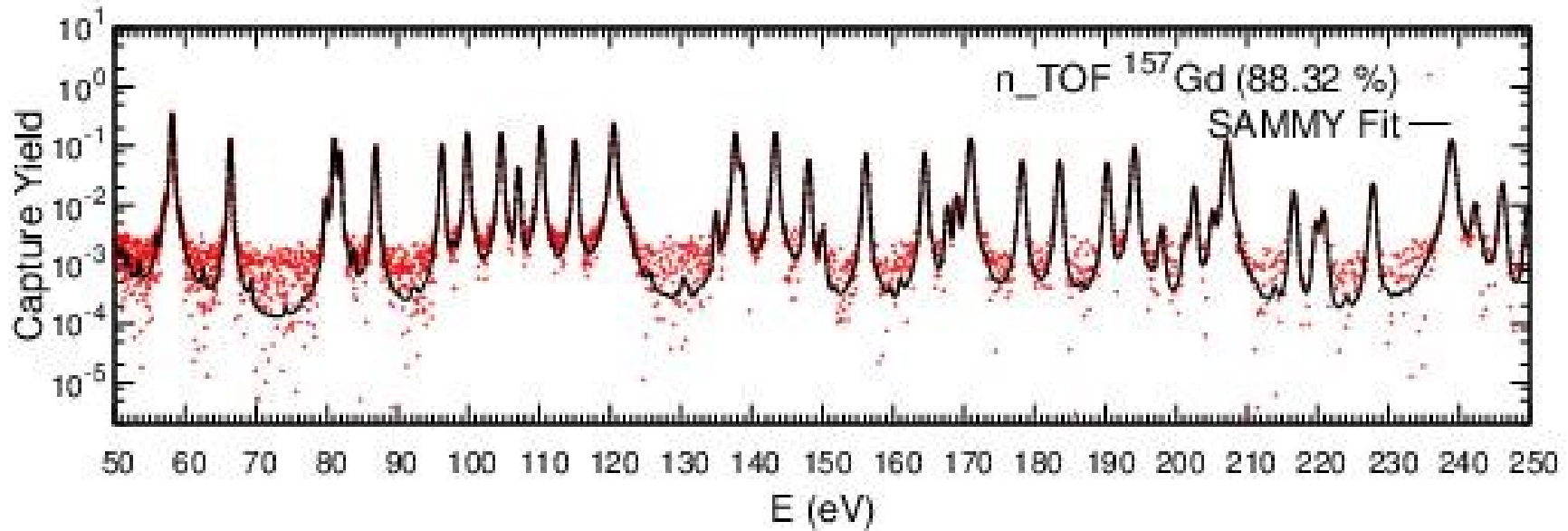
## n\_TOF data (SAMMY Fitting) for $^{155,157}\text{Gd}$



## N\_TOF and RPI data (SAMMY Fitting) for $^{155}\text{Gd}$ (capture)

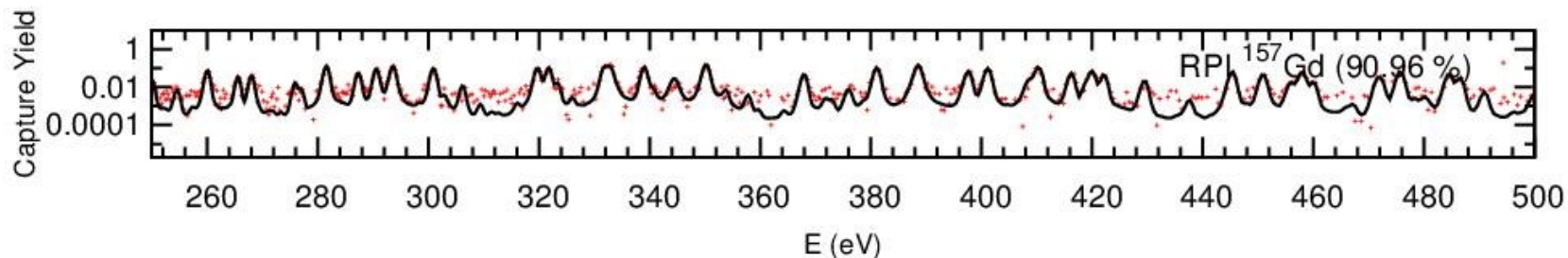
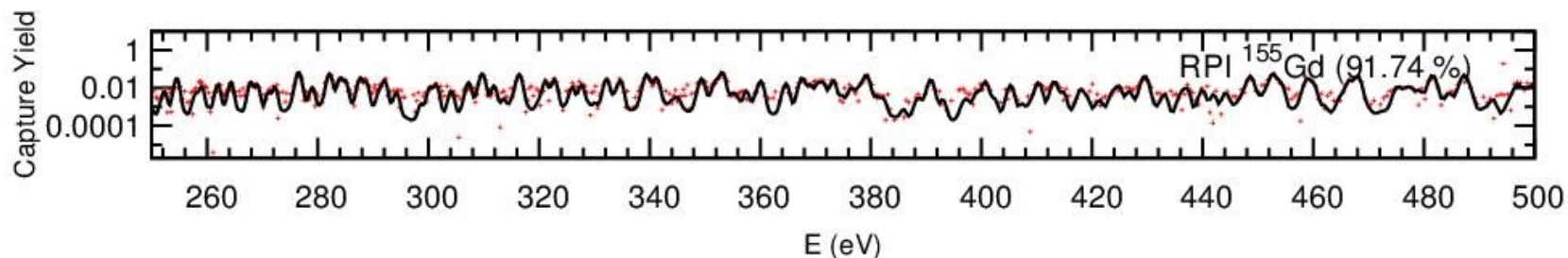
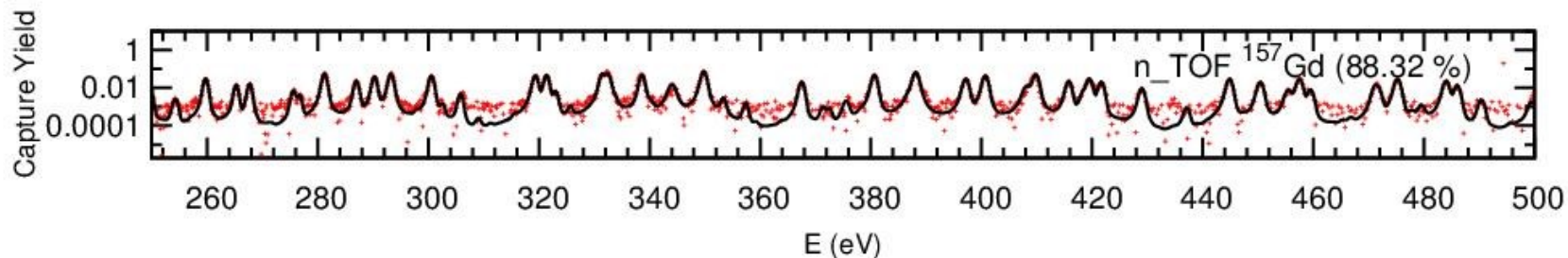
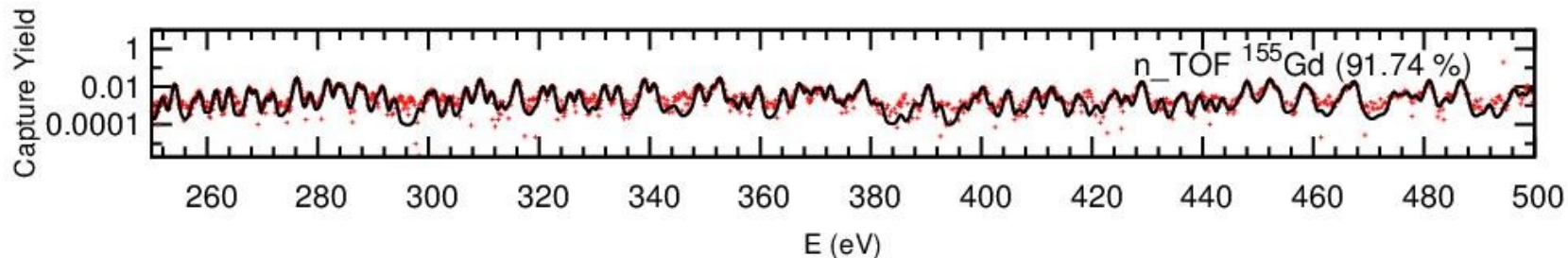


## n\_TOF and RPI data (SAMMY Fitting) for $^{157}\text{Gd}$ (capture)



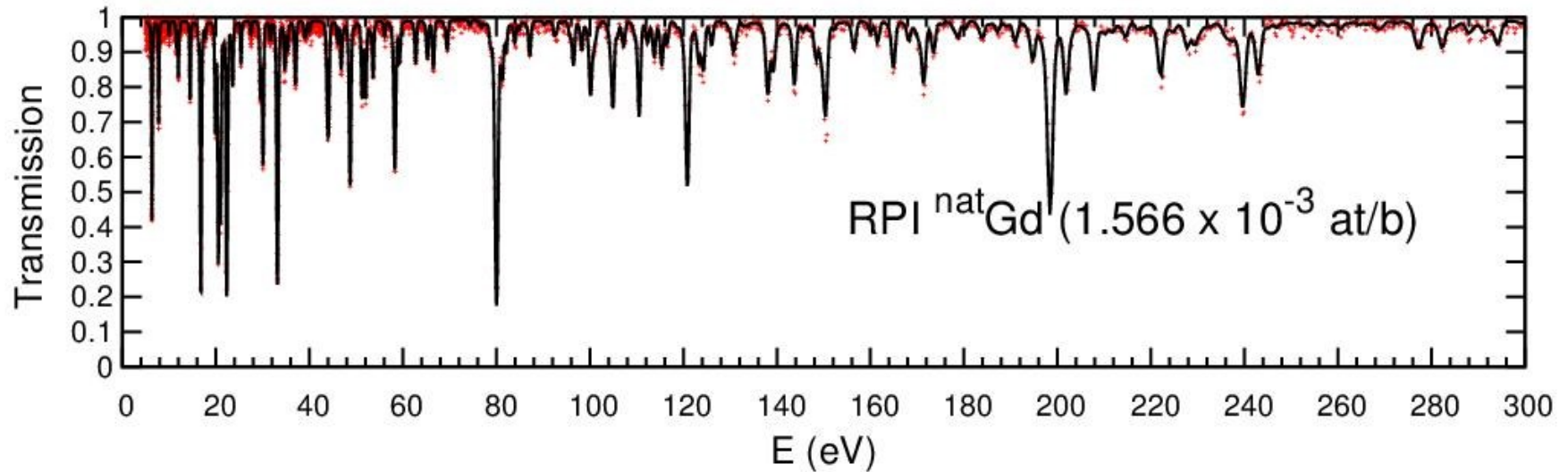
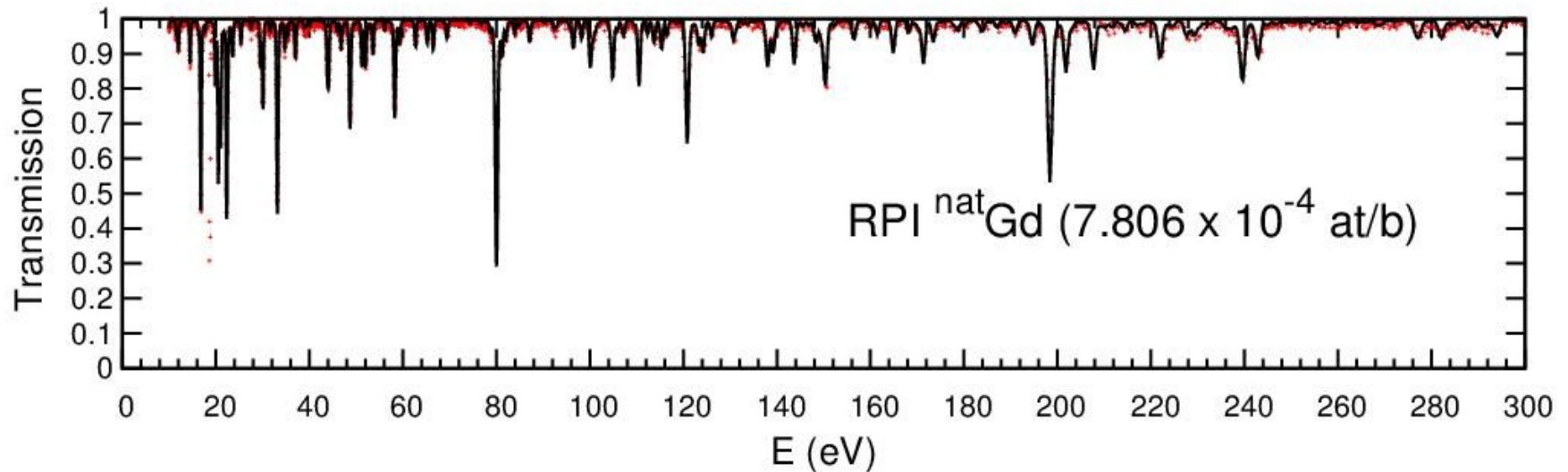


## n\_TOF and RPI data (SAMMY Fitting) for $^{155,157}\text{Gd}$ (capture)





# n\_TOF and RPI data (SAMMY Fitting) for natural Gadolinium (transmission)



# Resonance region upper energy

Isotope	JEFF3.3 (eV)	IRSN Evaluation (eV)
$^{152}\text{Gd}$	2658	2658
$^{154}\text{Gd}$	2760	2760
$^{155}\text{Gd}$	181.8	500
$^{156}\text{Gd}$	1580	2250
$^{157}\text{Gd}$	215	500
$^{158}\text{Gd}$	6037.6	10000
$^{160}\text{Gd}$	2883.7	10000

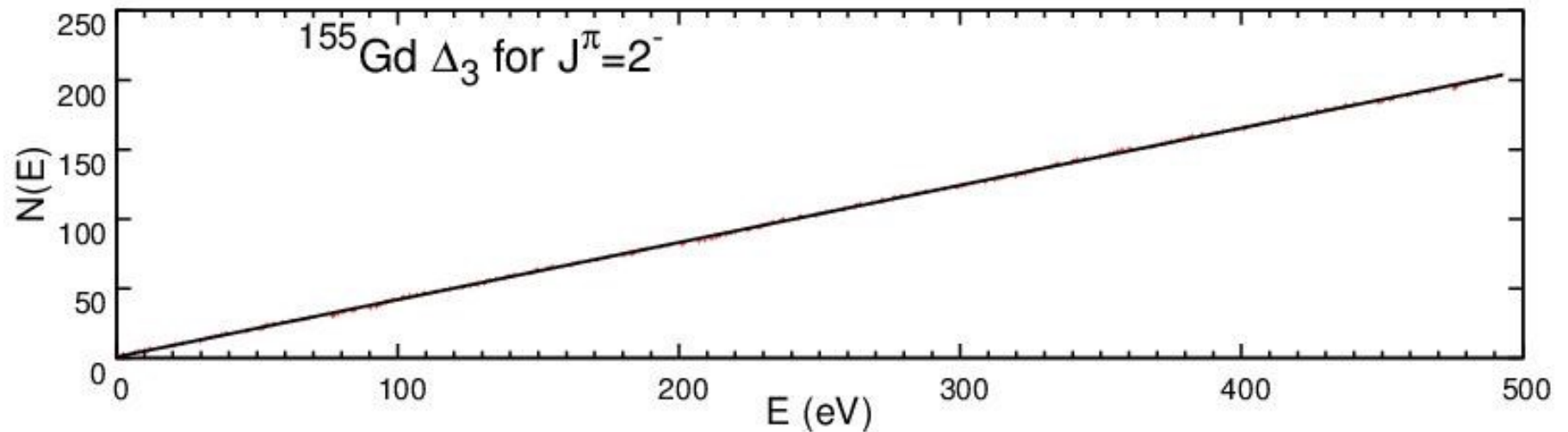
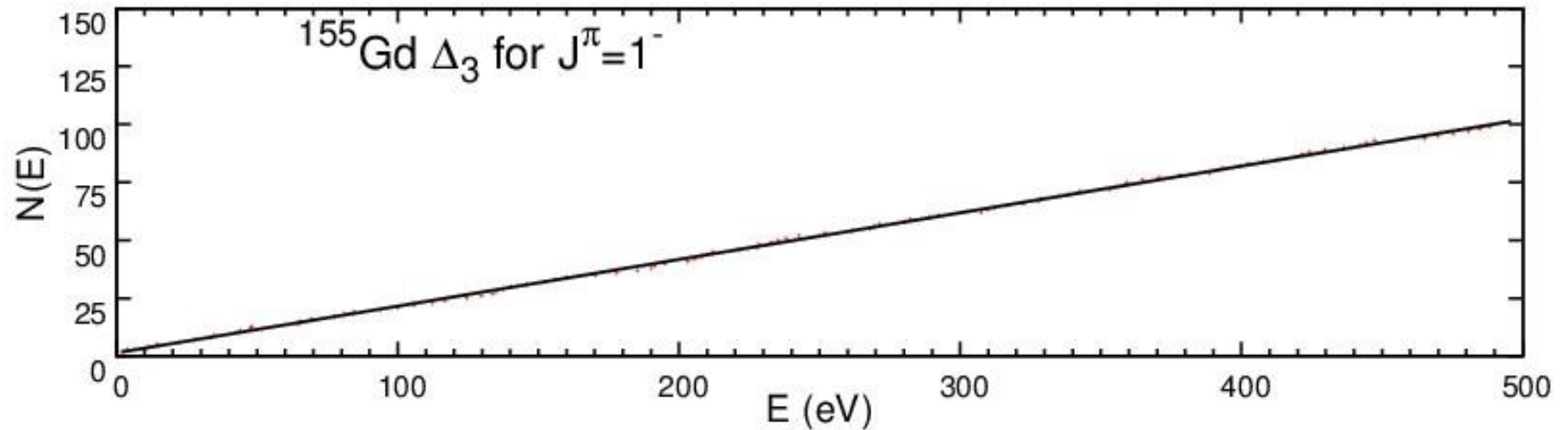
# Statistical Properties and Testing

Average level spacing:

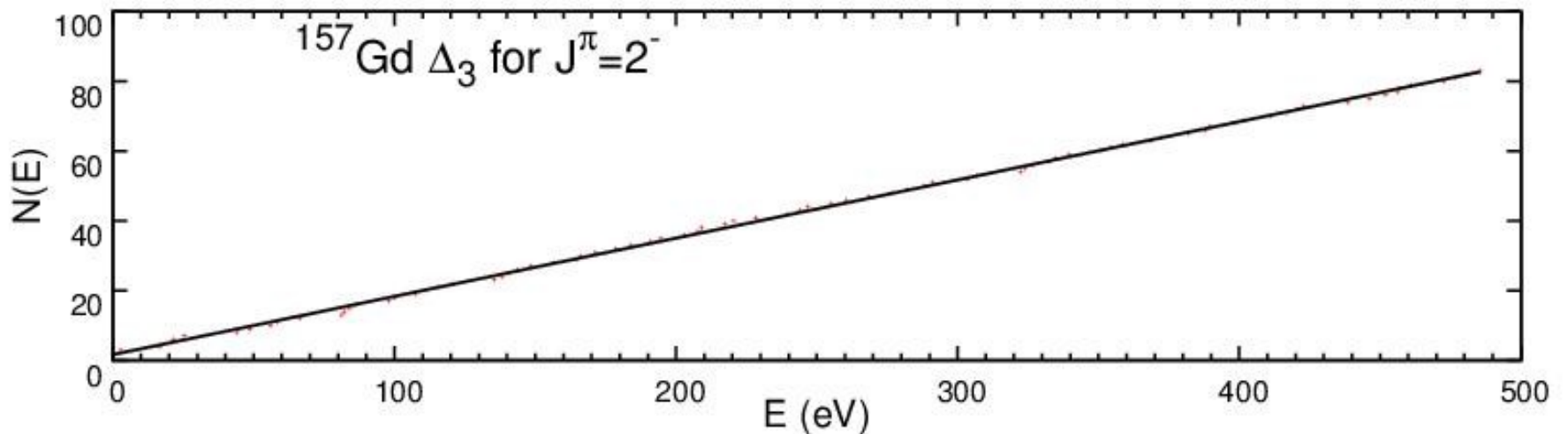
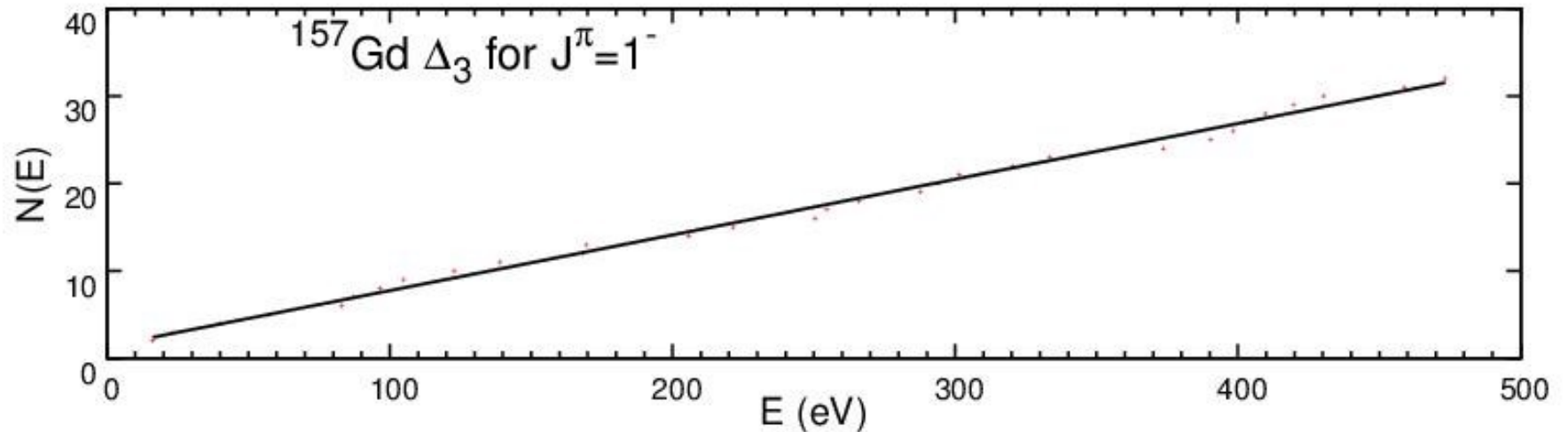
For a level spacing density proportional to  $2J+1$  leads to  $D_1^-/D_2^- \sim 5/3$

$D(J^\pi)$	$^{155}\text{Gd}$ (eV)	$^{157}\text{Gd}$ (eV)
$D(1^-)$	$5.00 \pm 0.32$	$15.20 \pm 1.75$
$D(2^-)$	$2.46 \pm 0.12$	$5.99 \pm 0.43$
$D$	$1.64 \pm 0.35$	$4.31 \pm 1.80$

# Cumulative number of energy levels in the energy 0 eV to 500 eV for $^{155}\text{Gd}$



# Cumulative number of energy levels in the energy 0 eV to 500 eV for $^{157}\text{Gd}$



# $\Delta_3$ -statistics Test

	$^{155}\text{Gd}$		$^{157}\text{Gd}$	
$J^\pi$	Theory	Experimental	Theory	Experimental
$1^-$	$0.459 \pm 0.109$	0.744	$0.344 \pm 0.109$	0.519
$2^-$	$0.531 \pm 0.109$	0.904	$0.441 \pm 0.109$	0.586



# $\Delta_3$ -statistics Test

## Observations:

- It can be noted from that in the energy range from 0 to 500 eV the  $\Delta_3$ -statistics results are in better shape for  $^{157}\text{Gd}$  as compared to the results for  $^{155}\text{Gd}$ .
- This observation does not come as a surprise since the resonance spacing for  $^{155}\text{Gd}$  is smaller than that for  $^{157}\text{Gd}$ , that is, more energy levels in  $^{155}\text{Gd}$  increases the chance of missing levels.

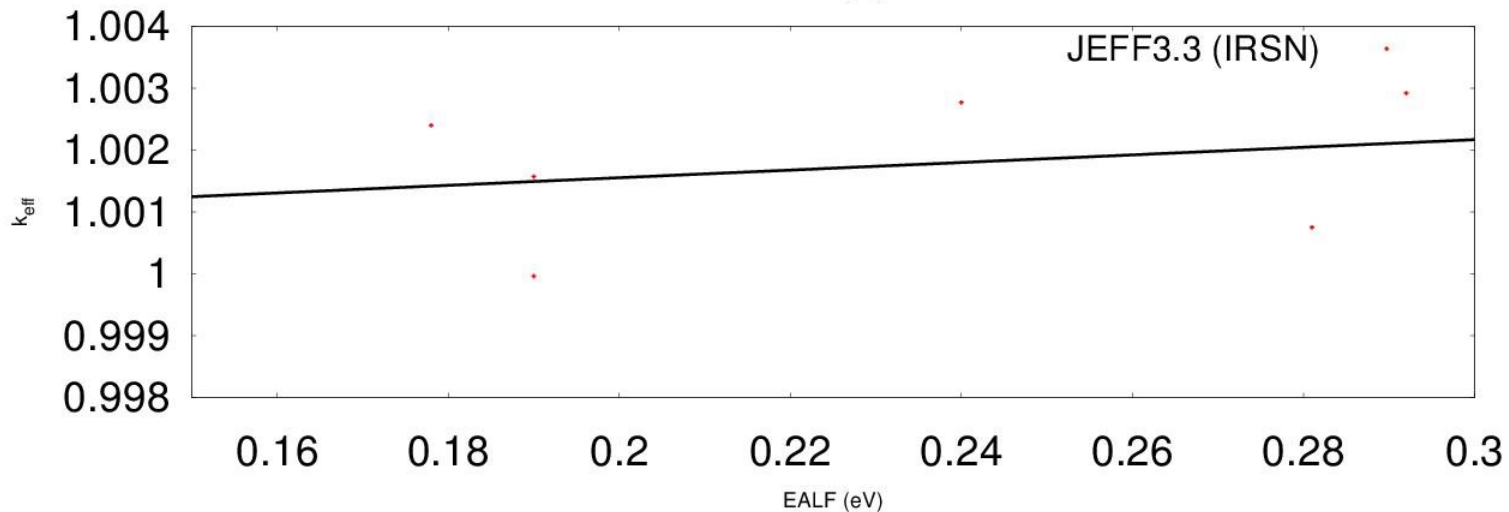
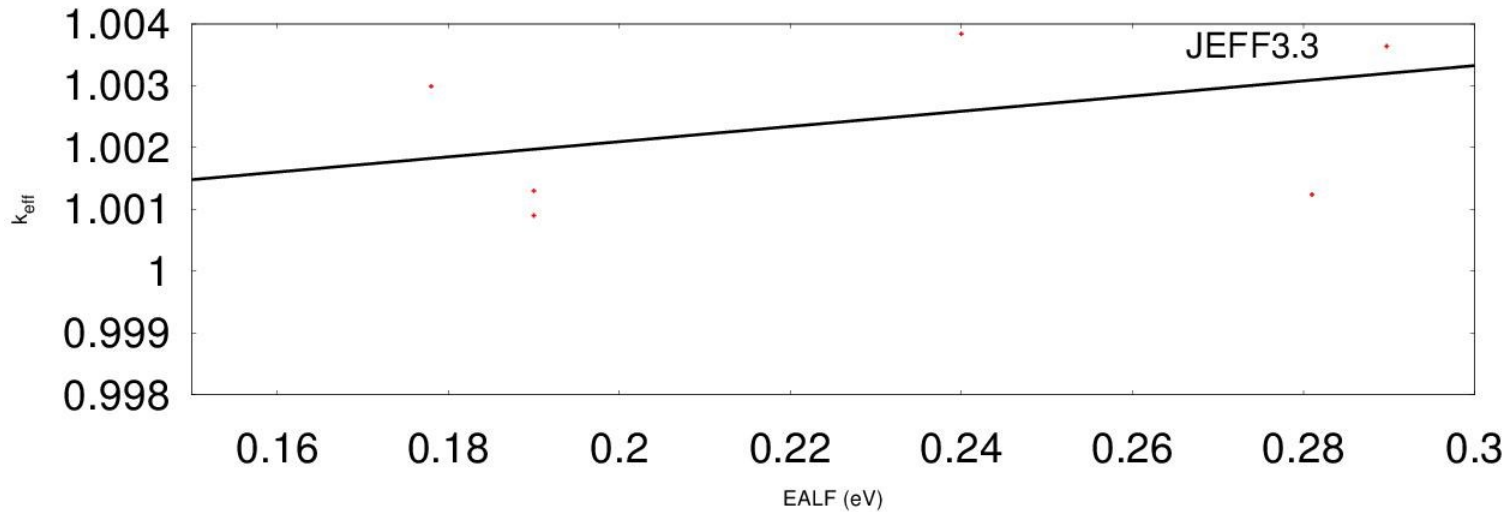
# Thermal values, resonance integral and Westcott factors

Quantities	Gadolinium Isotopes				
	<sup>155</sup> Gd	<sup>156</sup> Gd	<sup>157</sup> Gd	<sup>158</sup> Gd	<sup>160</sup> Gd
$\sigma_s$ (barns)	61.58	4.88	1018.58	5.28	10.31
$\sigma_\gamma$ (barns)	61857.42	1.93	254570.90	2.25	1.52
$I_\gamma$ (barns)	1567	99.40	814.98	78.27	10.32
$g_\gamma$	0.83326	1.00037	0.85204	1.00046	0.99994

# Benchmark Testing (ICSBEP)

Benchmark ID	EALF (eV)	Gd concentration (ppm)
LEU-COMP-THERM-042-C02	0.178	-
LEU-COMP-THERM-006-C04	0.190	68
MIX-SOL-THERM-006.C04	0.190	359
PU-SOL-THERM-034.C04	0.240	1131
LEU-COMP-THERM-005-C04	0.281	480
PU-SOL-THERM-034.C06	0.292	1896
PU-SOL-THERM-034.C09	1.792	3692

# Results: biases in $k_{eff}$ have been reduced with the new gadolinium evaluations



# Covariance Generation and Uncertainty Propagation

Result of the SAMMY fitting of the Experimental data lead to resonance parameter covariance

Experimental transmission and capture data				
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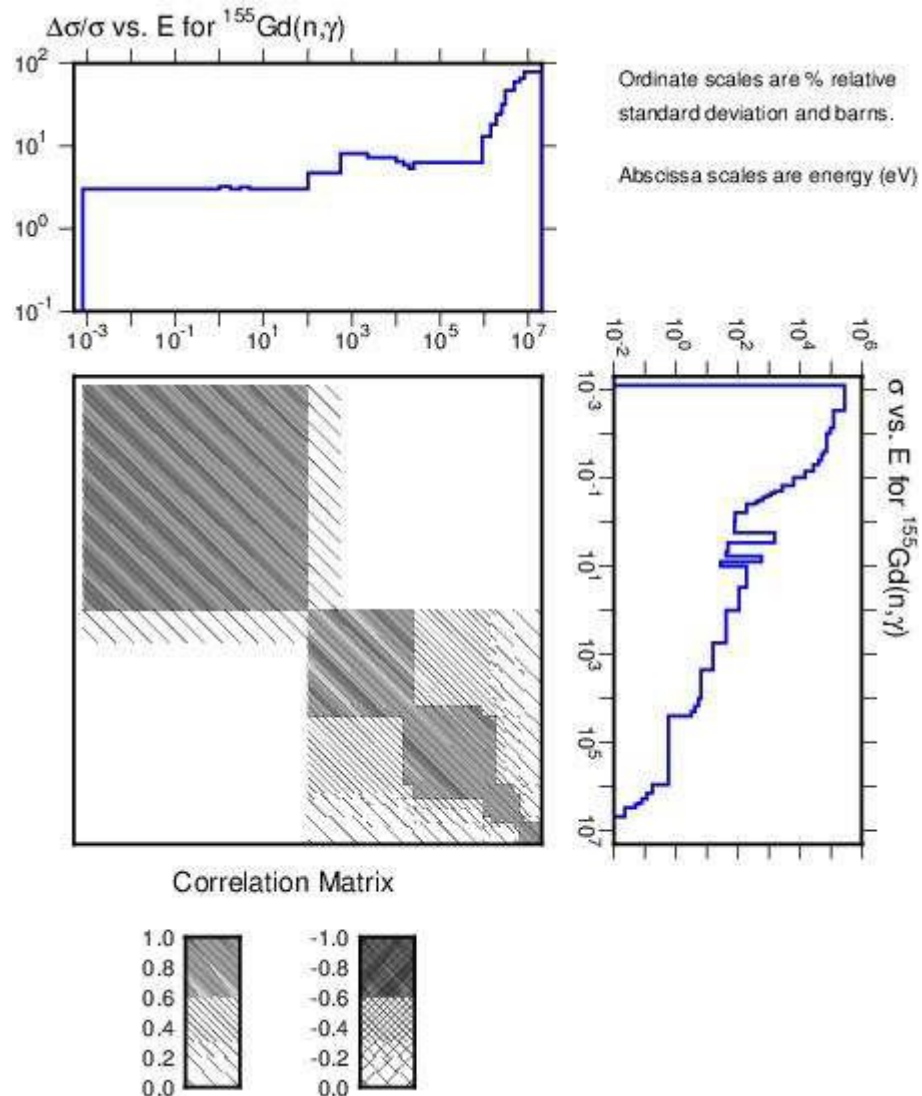
# Covariance Generation and Uncertainty Propagation

## Resonance Parameter Covariance for $^{155}\text{Gd}$ and $^{157}\text{Gd}$

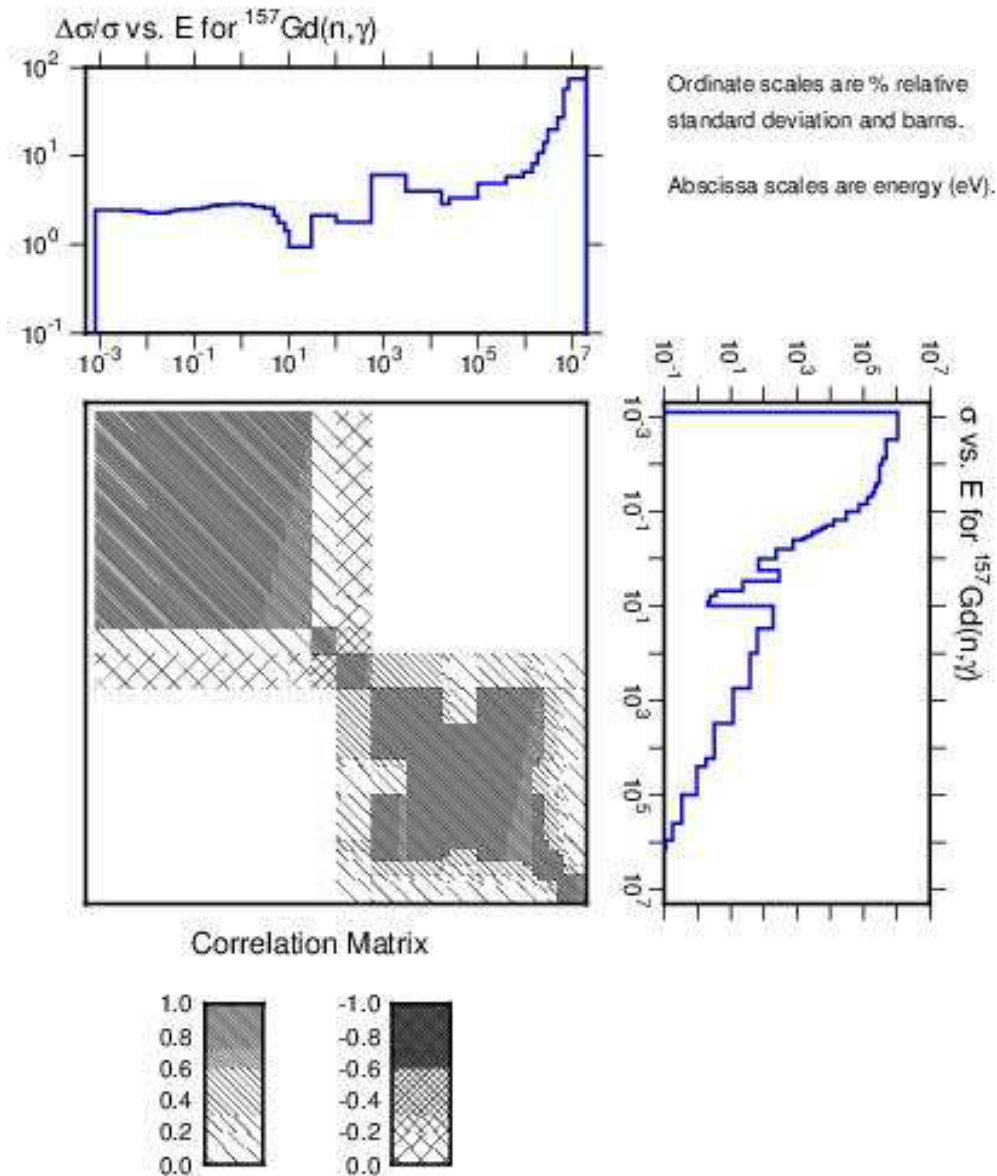
	Gadolinium Isotopes	
	$^{155}\text{Gd}$	$^{157}\text{Gd}$
$\sigma_s$ (barns)	$61.58 \pm 1.94$	$1018.58 \pm 20$
$\sigma_y$ (barns)	$61857.42 \pm 950.0$	$254570.90 \pm 5600.51$
$I_y$ (barns)	$1567.01 \pm 23$	$814.98 \pm 19.81$



# Capture cross section covariance for $^{155}\text{Gd}$



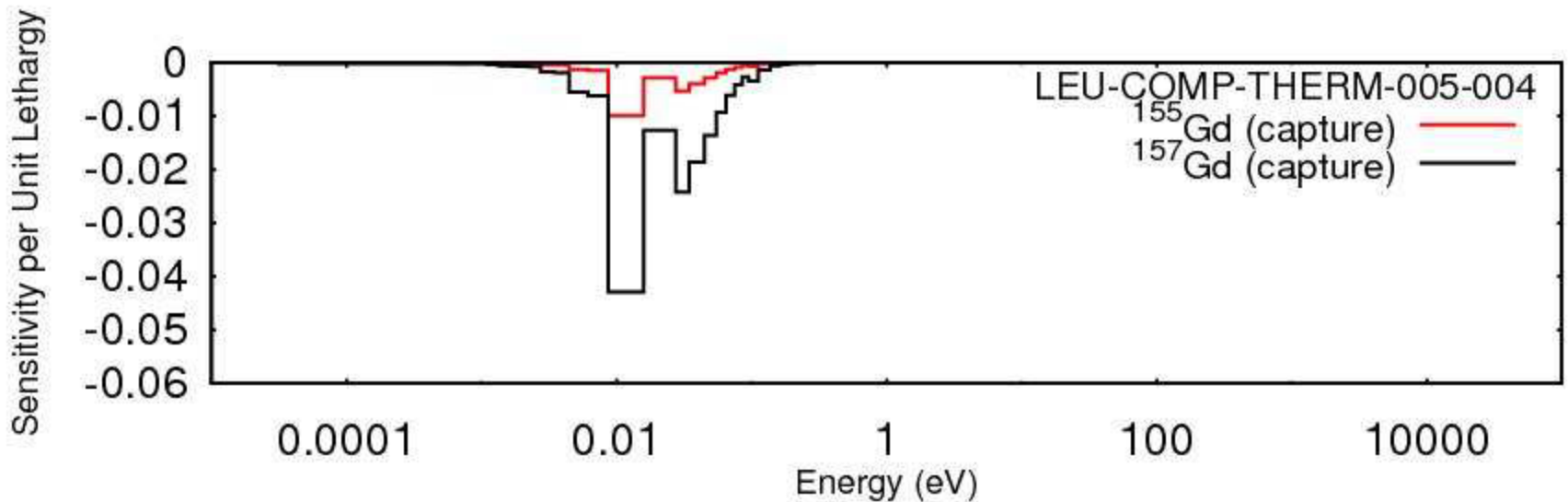
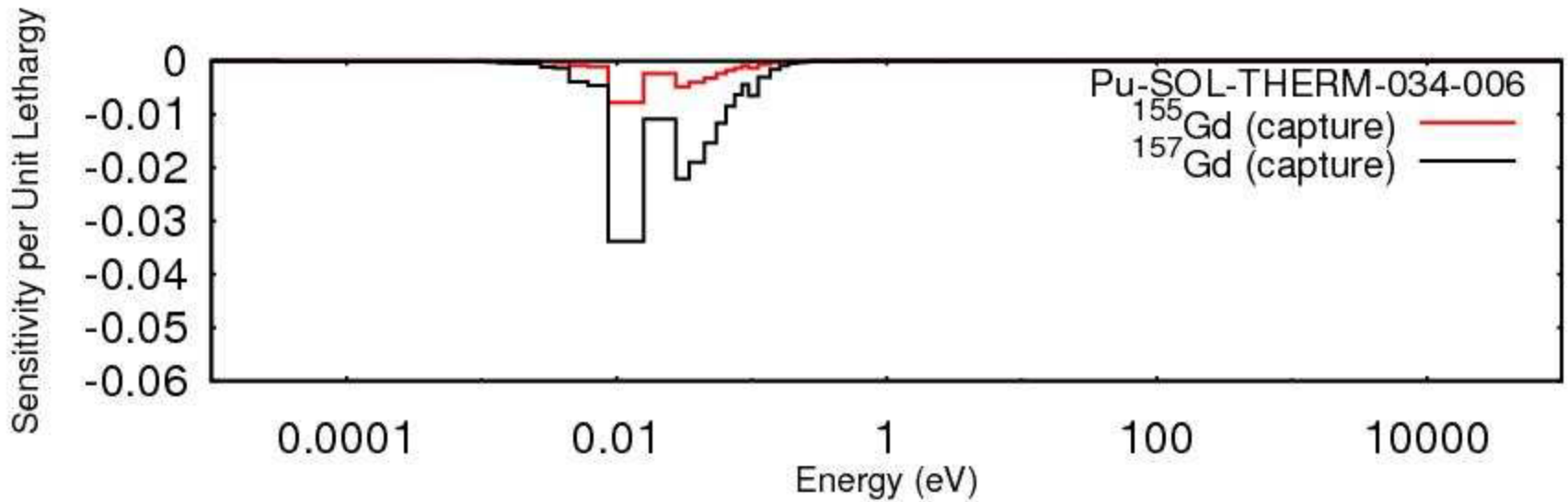
# Capture cross section covariance for $^{157}\text{Gd}$



# Benchmark Testing (ICSBEP)

Benchmark ID	EALF (eV)	Gd concentration (ppm)
LEU-COMP-THERM-042-C02	0.178	-
LEU-COMP-THERM-006-C04	0.190	68
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LEU-COMP-THERM-005-C04	0.281	480
PU-SOL-THERM-034.C06	0.292	1896
PU-SOL-THERM-034.C09	1.792	3692

# Capture cross section sensitivities for $^{155}\text{Gd}$ and $^{157}\text{Gd}$



## LEU-COMP-THERM-005-004

$k_{eff}$ (calculated)	$\% dk_{eff} / k_{eff}$	(n, $\gamma$ )
0.9956 $\pm$ 0.0002*	4.7021 $\times 10^{-1} \pm 3.4111 \times 10^{-5}$	5.5991 $\times 10^{-2} \pm 2.7469 \times 10^{-7}$ ( <sup>155</sup> Gd)
		3.0433 $\times 10^{-2} \pm 4.1673 \times 10^{-6}$ ( <sup>157</sup> Gd)

\*Calculated  $k_{eff}$  with a Monte Carlo statistical error of 0.0002

# Uncertainty propagation on $k_{eff}$ due to the Gd nuclear data uncertainty

## PU-SOL-THERM-03-004

$k_{eff}$ (calculated)	% $dk_{eff} / k_{eff}$	(n, $\gamma$ )
0.9986±0.0002	1.2324 x 10 <sup>-1</sup> ± 1.8551 x 10 <sup>-5</sup>	5.7008 x 10 <sup>-2</sup> ± 8.1583 x 10 <sup>-8</sup> ( <sup>155</sup> Gd)
		6.1649 x 10 <sup>-2</sup> ± 1.3802 x 10 <sup>-6</sup> ( <sup>157</sup> Gd)



# Concluding Remarks

- Resolved resonance evaluation of the gadolinium isotopes  $^{155}\text{Gd}$ ,  $^{156}\text{Gd}$ ,  $^{157}\text{Gd}$ ,  $^{158}\text{Gd}$ , and  $^{160}\text{Gd}$  have been completed;
- SAMMY, SAMDIST and SAMINT codes have been used in the evaluation process;
- Experimental data for enriched samples were used;
- New evaluation extends the resolved resonance energy region;
- Statistical tests of the resonances parameters for  $^{155}\text{Gd}$  and  $^{157}\text{Gd}$  were carried;
- The resulting resonance parameters fit well the capture cross section and the transmission data;
- Resonance parameter covariance generation in the SAMMY fitting process.