Parallelized approaches to solve the capacitated lot-sizing problem with lost sales and setup times

Mehdi Charles, Stéphane Dauzère-Pérès, Safia Kedad-Sidhoum, Issam Mazhoud

Parallelized approaches to solve the capacitated lot-sizing problem with lost sales and setup times

Charles Mehdi
Mines Saint-Etienne/DecisionBrain
mehdi.charles@decisionbrain.com

Dauzère-Pérès Stéphane
Mines Saint-Etienne
dauzere-peres@emse.fr

Kedad-Sidhoum Safia
Conservatoire National des Arts et Métiers
safia.kedad_sidhoum@cnam.fr

Mazhoud Issam
DecisionBrain
issam.mazhoud@decisionbrain.com

Abstract

Multi-item lot-sizing problems faced in industrial contexts are often too large to be directly solved by standard optimization solvers. To this purpose, parallelization approaches are very useful to transform the single resolution of a complex problem into the resolution of smaller independent sub-problems, leading to much smaller computational times. We first motivate and propose new practically relevant instances for the Capacitated Lot-Sizing Problem (CLSP) with lost sales, setup times and target ending inventory. Two ways of parallelizing the problem are then introduced. A first approach is based on a Lagrangian relaxation of the item binding capacity constraints. We show that the problem linked to each item can be solved in polynomial time. A second approach uses a parallelized version of Relax-And-Fix and Fix-and-Optimize heuristics where, instead of optimizing the time intervals in chronological order at each iteration, different time intervals are optimized in parallel. The best interval is picked according to various strategies and is fixed for the following iterations.

1 Industrial context and motivations

A classical dynamic lot-sizing problem consists in planning quantities to be produced in each period of a finite horizon, discretized in periods, to satisfy demands, while optimizing the trade-off between setup and holding costs. The well-known Capacitated Lot-Sizing Problem (CLSP) considers multiple products and capacity constraints. For most instances of the literature, solutions of the CLSP have no ending inventory, because there are no incentives to store products at the end of the planning horizon. However, as shown for instance in the survey of [5], real-life planning processes are often conducted in a rolling horizon. Hence, to be practically relevant, plans should keep enough inventory of some products at the end of the planning horizon, and should have inventories of some products at the beginning of the planning horizon.

In this work, we focus on a specific and well-know lot-sizing problem: The CLSP with setup times. Trigeiro et al. [6] created a set of instances for this problem, that are commonly used as benchmarks or to create benchmarks to validate solution approaches ([1], [2], [3], [4]). Yet, as discussed earlier, these instances are not realistic because they consider neither initial nor ending inventories, which leads to start-of-horizon and end-of-horizon effects (see Figure 1). We are also allowing lost sales as the satisfaction of all demands cannot be guaranteed in many industrial contexts.

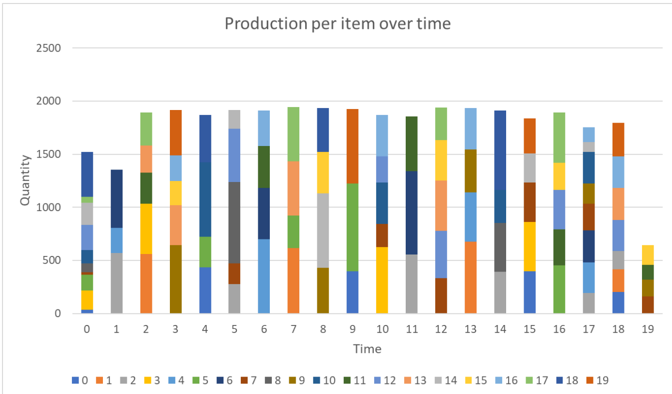

Figure 1: Optimal solution for an instance of Trigeiro et al. [6]

2 Creation of new instances

Following the characteristics of the instances of Trigeiro et al. [6], our assumption is that the demand variability over time and between products is small. We propose new instances where initial inventories and maximum ending inventories are added for each product, as well as a common target ending inventory for all products. The objective is to create instances that make sense in a rolling horizon setting. Indeed, it is unrealistic to define a target ending inventory for each single product in a given period. Using the information on cost components and the capacity, a MIP is solved to create new consistent data to complement and modify the instances of Trigeiro et al. [6]. The MIP still guarantees that there exists a feasible plan without lost sales.

3 Parallelization approaches for the CLSP with setup times, lost sales and target ending inventory

In this section, we introduce parallelization approaches for the CLSP with setup times, lost sales and target ending inventory. The latter characteristic is the main difference with previous research on related problems. First, a Lagrangian relaxation heuristic (used for instance in [3]) is considered, which can "naturally" be parallelized. Then, different ways to parallelize Relax-and-Fix and the Fix-and-Optimize heuristics (used for instance in [1]) are proposed.

3.1 Lagrangian relaxation heuristic

As in [6] and [3], the capacity constraints are relaxed using Lagrangian multipliers. When the maximum ending inventory is the same for each product, we show that the multi-item uncapacitated lot-sizing problem with target ending inventory and lost sales is polynomially solvable by decomposing into several uncapacitated single-item lot-sizing problems with ending inventory. At each iteration of the Lagrangian heuristic, a reconstruction method, following the same principle than in [6], is used to build a feasible solution. A first production plan is thus determined, that can be improved by various methods (for instance a parallelized Fix-and-Optimize heuristic).

3.2 Parallelized Relax-and-Fix and Fix-and-Optimize heuristics

Our approach works in two main steps. An initial solution is first obtained using a Relax-and-Fix heuristic. This solution is then improved with a Fix-And-Optimize heuristic.

In a classical Relax-and-Fix heuristic, sub-problems are optimized chronologically for each time interval. Instead, in our parallelized approach, sub-problems are optimized in parallel in order to select the best time interval to fix in the following iterations. The setups are then fixed for the "best" sub-problem at each iteration in an iterative process (see illustration in Figure 2).

To evaluate the quality of each partial solution and define the best interval, several score functions are defined. One of the score functions consists in reconstructing a feasible solution from the partially optimized solution. We also consider different ways to define the time intervals, leading to different resolutions strategies. The same approach can be applied to the Fix-and-Optimize heuristic, where the Boolean variables outside of the optimized time interval are fixed to a given value rather than relaxed.

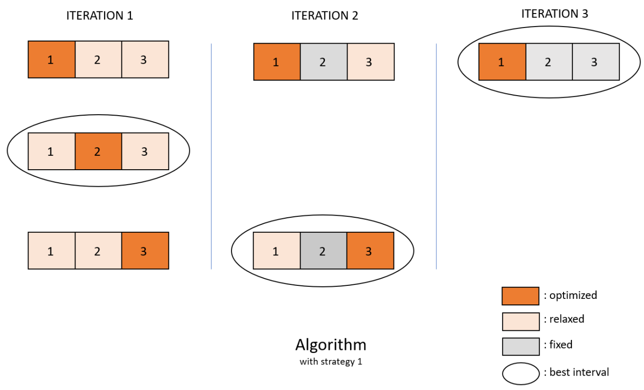

Figure 2: Illustration of parallelized Relax-and-Fix heuristic

3.3 Numerical results

Computational experiments were performed on the modified instances of Trigeiro et al. [6]. In the workshop, we will present and discuss the results of our parallelized approaches compared to the standard solver IBM ILOG CPLEX and the "classical" Relax-And-Fix and Lagrangian relaxation heuristics. The different strategies of our Parallelized Relax-and-Fix and Fix-and-Optimize heuristics will also be compared and analyzed.

References

[1] Absi, N., Kedad-Sidhoum, S. *MIP-based heuristics for multi-item capacitated lot-sizing problem with setup times and shortage costs.* RAIRO Operations Research, 41, 2007, 171192.

[2] Absi, N., Kedad-Sidhoum, S. *The multi-item capacitated lot-sizing problem with setup times and shortage costs.* European Journal of Operational Research, 185 (3), 2008, 1351-1374.

[3] Absi, N., Detienne, B., Dauzère-Pérès, S. *Heuristics for the multi-item capacitated lot-sizing problem with lost sales.* Computers & Operations Research 40 (1), 2013, 264-272.

[4] De Araujo, S.A., De Reyck, B., Degraeve, Z., Fragkos, I., Jans, R. *Period Decompositions for the Capacitated Lot Sizing Problem with Setup Times.* INFORMS Journal on Computing, 27(3), 2015, 431448.

[5] Jans, R., Degraeve, Z. *Modeling industrial lot sizing problems: a review.* International Journal of Production Research, 46(6), 2008, 1619-1643.

[6] Trigeiro, W.W., Thomas, L.J., McClain, J.O. *Capacitated lot-sizing with setup times.* Management science, 35(3), 1989, 353-366.