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# Maximal Good Step Graph Methods for Reducing the Generation of the State Space

HAO DOU<sup>1</sup>, KAMEL BARKAOUI<sup>2</sup>, HANIFA BOUCHENEB<sup>3</sup>, XIAONING JIANG<sup>1</sup>,  
AND SHOUGUANG WANG<sup>1</sup>, (Senior Member, IEEE)

<sup>1</sup>School of Information and Electronic Engineering, Zhejiang Gongshang University, Hangzhou 310018, China

<sup>2</sup>Cedric Laboratory, Computer Science Department, Conservatoire National des Arts et Métiers, 75141 Paris, France

<sup>3</sup>Laboratoire VeriForm, Department of Computer Engineering and Software Engineering, École Polytechnique de Montréal, Montréal, QC H3C 3A7, Canada

Corresponding author: Shouguang Wang (wsg5000@hotmail.com)

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**ABSTRACT** This paper proposes an effective method based on the two main partial order techniques which are persistent sets and covering step graph techniques, to deal with the state explosion problem. First, we introduce a new definition of sound steps, the firing of which enables to extremely reduce the state space. Then, we propose a weaker sufficient condition about how to find the set of sound steps at each current marking. Next, we illustrate the relation between maximal sound steps and persistent sets, and propose a concept of good steps. Based on the maximal sound steps and good steps, a construction algorithm for generating a maximal good step graph (MGSG) of a Petri net (PN) is established. This algorithm first computes the maximal good step at each marking if there exists one, otherwise maximal sound steps are fired at the marking. Furthermore, we have proven that an MGSG can effectively preserve deadlocks of a Petri net. Finally, the change performance evaluation is made to demonstrate the superiority of our proposed method, compared with other related partial order techniques.

**INDEX TERMS** Petri nets, state explosion problem, covering step graph methods, persistent sets.

## I. INTRODUCTION

Concurrent systems [1]–[3] are composed of several subsystems operating in parallel and they are especially difficult to be designed or analyzed in the real world [4]–[9]. Thus, the design correctness of concurrent systems needs to be checked via the verification.

The state space exploration method [10]–[14] is one of the most widely used techniques for the verification of finite-state concurrent systems. However, there exists an obstacle in the application of this technique, which is the state-space explosion. This problem is mainly caused by the interleaving semantics of concurrent systems, i.e., all firing orders of concurrent transitions are explored exhaustively, during the application of such technique. Actually, many researchers have studied strategies fighting for this problem and several different techniques are proposed such as compositional verification, symmetric reduction, abstraction and partial order reduction.

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The partial order reduction [38]–[55] has been proven to be the most successful strategy for alleviating the state-space explosion in practice [54]. It can utilize the independence of concurrent execution to eliminate some intermediate states [56]–[57]. More precisely, there is no need to explore all interleaving semantics possessing identical concurrent execution, when analyzing properties of interest (deadlock freeness [17]–[27], reachability [28]–[32], liveness [33]–[37], or linear properties [38]). Note that partial order reduction techniques, such as stubborn sets [46]–[48], sleep sets [43], ample sets [49]–[50], persistent sets [42]–[43] and covering step graphs [52], preserve deadlocks of Petri nets (PNs) [15]–[16] at least.

The covering step graph methods explore all the transitions of the state space and concurrent ones are put together to constitute an atomic step. They aim to reduce the depth of the marking graph while the purpose of the persistent sets is to reduce its breadth. The persistent sets are introduced in [42], [43], which are particular stubborn sets. Different from the covering steps, persistent sets only explore enabled transitions at each marking. To make full advantage of both methods, Ribet *et al.* [53] present a persistent step

graph (PSG) which can both improve persistent sets and covering step graph methods, and reduce the state space from its breadth and depth. More importantly, it has been proven that the PSG preserves the deadlocks of Petri nets. In order to further reduce the state space, Barkaoui *et al.* [54] propose the maximal persistent step graph (MPSG) method and introduce a new definition of weak-persistent sets. Combining the weak-persistent sets with covering steps, the MPSG achieves a more significant reduction of the state space, compared with the PSG.

In this work, according to the definition of covering step graphs introduced in [52] we first propose a new definition of sound steps, which is an extension of covering steps. Then, from a practical point of view, a weaker sufficient condition about how to build the set of sound steps at each marking is introduced. Based on this condition, we can compute the set of maximal sound steps at each marking more intuitively and quickly. Next, we propose a definition of good steps, combining maximal sound steps and persistent sets. Due to the proposed good steps and maximal sound steps, a maximal good step graph (MGSG) is constructed, which significantly reduces the state space compared with other related partial order reduction methods. In addition, the MGSG preserves deadlock markings of Petri nets. The major contributions of this work are listed as follows:

- 1) We propose a new definition of sound steps, based on a better understanding of the concurrent and conflict relations between transitions of a step. Thus, the firing of a sound step at each marking enables to extremely reduce the state space;
- 2) Based on the definition of sound steps, we propose a weaker sufficient condition about how to find the set of sound step at each marking, which is of practical significance;
- 3) Combining the persistent sets and sound steps, a new definition of good steps is introduced, which plays an important role in computing the maximal good step graph (MGSG)
- 4) The generated maximal good step graph permits to preserve the deadlocks of a Petri net.

The rest of this paper is structured as follows: Basic knowledge used throughout this paper is introduced in Section II and Section III presents the definition of persistent sets, and followed by some previous partial order reduction methods, which are the basic of our proposed method. Section IV first introduces the new definition of sound steps, then proposes a weaker sufficient condition for sound steps, next presents the concept of good steps and exhibits the construction algorithm of the MGSG, finally proves that the MGSG preserves the deadlocks of Petri nets. Section V compares our proposed technique with other related partial order reduction methods and shows the experimental results to demonstrate the superiority of our method. Finally, Section VI concludes this work.

## II. PRELIMINARIES

In the following discussion,  $E^*$  represents the set of all sequences constituted by elements of  $E$  (i.e., including an empty sequence  $\varepsilon$ ) and  $E^+$  denotes the set of sequences without  $\varepsilon$ , such that  $E^* := \{\varepsilon\} \cup E^+$ . For instance,  $E = \{e, f\}$ ,  $E^* = \{\varepsilon, e, f, ee, ef, fe, ff, \dots\}$  and  $E^+ = \{e, f, ee, ef, fe, ff, \dots\}$ .

Fundamental notations related to Petri nets and partial order methods are introduced in this section. A reader may consult more details in [15]–[16], [53]–[54].

A generalized Petri Net (PN) is a 4-tuple  $N = (P, T, F, W)$ , where  $P$  and  $T$  are denoted as non-empty, finite, and disjoint sets.  $P$  characterizes a set of places and  $T$  describes a set of transitions. There is a flow relation  $F$ , which is represented by directed arcs from places to transitions or from transitions to places.  $W: (P \times T) \cup (T \times P) \rightarrow \mathbb{N} = \{0, 1, 2, \dots\}$  is a mapping that assigns a weight to an arc. It satisfies that  $W(x, y) > 0$  if  $(x, y) \in F$ , and  $W(x, y) = 0$ , otherwise, where  $x, y \in P \cup T$ . If  $\forall (x, y) \in F, W(x, y) = 1$ , this net is called an ordinary Petri net, denoted by a 3-tuple  $N = (P, T, F)$ . Given a node  $x \in P \cup T$ , the pre-set of  $x$  is denoted by  $\bullet x$ , where  $\bullet x = \{y \in P \cup T \mid (y, x) \in F\}$ , and the post-set of  $x$  is expressed as  $x^\bullet = \{y \in P \cup T \mid (x, y) \in F\}$ . We can extend this notation to a set of nodes, i.e.,  $\forall X \subseteq P \cup T, \bullet X = \cup_{x \in X} \bullet x$  and  $X^\bullet = \cup_{x \in X} x^\bullet$ .

A marking (or state)  $M$  of  $N$  is a mapping from  $P$  to  $\mathbb{N}$ , where  $\mathbb{N}$  represents nonnegative integers. For the sake of convenience, the multi-set symbol  $\sum_{p \in P} M(p)p$  is utilized to denote vector  $M$ , where  $M(p)$  indicates the number of tokens in  $p$  at  $M$ . For example,  $M = [3, 2, 1, 2]^T$  is denoted by  $M = 3p_1 + 2p_2 + p_3 + 2p_4$ . A place  $p$  is marked by  $M$  if  $M(p) > 0$ . We define  $(N, M_0)$  as a Petri net system with its initial marking  $M_0$ .

The transition  $t \in T$  is enabled at a marking  $M$ , denoted by  $M[t]$ , if  $\forall p \in \bullet t, M(p) \geq W(p, t)$ . The firing of  $t$  at  $M$  yields a marking  $M'$ , i.e.,  $\forall p \in \bullet t, M'(p) = M(p) - W(p, t) + W(t, p)$ , which is denoted as  $M[t]M'$ . The marking  $M'$  is called an immediately reachable marking from  $M$ . The set of all transitions enabled at  $M$  is denoted by  $En(M) = \{t \in T \mid \forall p \in \bullet t, M(p) \geq W(p, t)\}$ . The sequence  $\sigma = t_1 t_2 \dots t_n \in T^*$  is enabled at  $M$ , denoted as  $M[\sigma]$ , if there exists a series of markings  $M_1, M_2, \dots, M_{n-1}$  such that  $M[t_1]M_1 \wedge M_1[t_2]M_2 \wedge \dots \wedge M_{n-1}[t_n]$ . A marking  $M''$  is said to be reachable from  $M$  if the firing of a sequence  $\sigma \in T^*$  at  $M$  yields the marking  $M''$ , which is indicated as  $M[\sigma]M''$ . We use the notation  $R(N, M)$  to denote the set of all markings reachable from  $M$  of  $N$ .

A non-empty subset of transitions is said to be a step  $\tau$  of  $N$  ( $\tau \subseteq T$ ) if the firing of transitions in this step is simultaneously and atomically at a marking  $M$  of  $N$ . To consider a firing step from an interleaving semantic standpoint, it can be viewed as an abstraction of all sequences of its transitions. For example, a step  $\tau = \{t, t', t''\}$  hides six sequences of its transitions:  $tt't'', tt't't'', t't't'', t't't't'', t't't't'$ , and  $t''t't$ . A step  $\tau$  is enabled at a marking  $M$ , denoted as  $M[\tau]$ , if  $\forall p \in \bullet \tau, M(p) \geq \sum_{t \in \tau} W(p, t)$ , which means that there

are enough tokens allowing transitions within the step to fire concurrently. Firing a step  $\tau$  yields a marking  $M'$  such that  $\forall p \in \bullet\tau, M'(p) = M(p) + \sum_{t \in \tau} (W(t, p) - W(p, t))$ , which is denoted by  $M[\tau]M'$ . A marking  $M''$  is said to be reachable in multiple ways from  $M$  if there exists an enabled transition  $t$  at  $M$  and after the firing of  $t$ , there exists a sequence  $\sigma$ , a step  $\tau$  and two intermediate markings  $M_1, M_2$  such that  $M[t]M_1[\sigma]M_2[\tau]M''$ , which is indicated as  $M[t\sigma\tau]M''$ . Note that as long as the enabled conditions are satisfied, the firing order of a transition  $t$ , a sequence  $\sigma$ , and a step  $\tau$  is arbitrary, i.e.,  $M[t\sigma\tau]M'', M[t\tau\sigma]M'', M[\sigma t\tau]M''$ , etc. We use  $EnStep(M)$  to denote the set of all enabled steps at  $M$ . Given an enabled step  $\tau \in EnStep(M)$ ,  $\tau$  is maximal at  $M$  if  $(\tau' \in EnStep(M) \text{ such that } \tau \subset \tau')$ . We denote by  $Step(T)$  the set of all steps of a PN  $N$ .

Transitions  $t_1$  and  $t_2$  are in structural conflict, denoted by  $t_1 \perp t_2$ , if  $\bullet t_1 \cap \bullet t_2 \neq \emptyset$ . Transitions  $t_1$  and  $t_2$  are symmetric structural conflict with each other, denoted as  $t_1 \perp_s t_2$ , if  $t_1 \perp t_2$ , and  $\bullet t_1 \cap \bullet t_2 \cap t_1^\bullet = \bullet t_1 \cap \bullet t_2 \cap t_2^\bullet$ . Two transitions  $t_1$  and  $t_2$  are in conflict, denoted as  $t_1 \# t_2$  if the firing of  $t_1$  (or  $t_2$ ) may disable the transition  $t_2$  (or  $t_1$ ) at a marking  $M$  where  $t_2$  (or  $t_1$ ) should have been enabled. We use  $CFS(t) = \{t_2 \in T | t_1 \# t_2\}$  to denote the set of transitions in conflict with  $t_1$ . The transitive closure of conflict relation is weak conflict. Transitions  $t_1$  and  $t_2$  are in weak conflict, which is denoted by  $t_1 \#_w t_2$ . The set of transitions in weak conflict with  $t_1$  is denoted as  $[CFS](t) = \{t_2 \in T | t_1 \#_w t_2\}$ . Notice that  $t \in T$  and  $CFS(t) \subseteq [CFS](t)$ .

Two sequences of transitions  $\sigma$  and  $\sigma'$  are equivalent, denoted as  $\sigma \equiv \sigma'$  if they are the same or each one can be obtained from the other by successive permutations of transitions. Let  $M$  be a reachable marking,  $\forall M' \in R(N, M)$ , equivalent sequences  $\sigma$  and  $\sigma'$  from  $M$  lead to the same marking  $M'$ , i.e.,  $M[\sigma]M'$  and  $M[\sigma']M'$ , if  $\sigma \equiv \sigma'$ . We use  $[\sigma]$  to denote the set of transitions contained in a sequence  $\sigma$ .

A reachability graph  $RG$  of  $N$  is a finite and labeled directed graph, denoted by  $RG = (R(N, M_0), \rightarrow, T, M_0)$ , where  $\rightarrow$  is a directed edge from a marking  $M$  to another reachable marking  $M'$ , where  $M, M' \in R(N, M_0)$ , and it is labeled by a transition  $t$  of  $T$ . A covering step graph  $CSG$  of  $N$  is defined by  $CSG = (R_S(N, M_0), \rightarrow, Step(T), M_0)$ , where  $R_S(N, M_0) \subseteq R(N, M_0)$  is a subset of reachable markings, and  $\rightarrow$  is labeled by an enabled step  $\tau \in Step(T)$  at each marking  $M \in R_S(N, M_0)$ .

Given a PN  $(N, M_0)$ , a transition  $t \in T$  is live at the initial marking  $M_0$  if  $\forall M \in R(N, M_0), \exists M' \in R(N, M), M'[t]$ . This net  $(N, M_0)$  is live if each transition  $t$  of  $N$  is live at  $M_0$ . The reverse case is that a transition  $t$  is dead at  $M_0$  if  $\forall M \in R(N, M_0), \neg M[t]$ .  $(N, M_0)$  is dead if  $\nexists t \in T, M_0[t]$ . A marking  $M \in R(N, M_0)$  is a deadlock marking if  $\forall t \in T, \neg M[t]$ , which can be described as  $En(M) = \emptyset$ . The net  $(N, M_0)$  is called deadlock-free (i.e., not-dead or weak-live) if  $\forall M \in R(N, M_0), \exists t \in T, M[t]$ . A place  $p \in P$  is  $k$ -bounded if given  $M \in R(N, M_0), \exists k \in \mathbb{N}^+ : M(p) \leq k$ , where  $\mathbb{N}^+$  represents positive integers. The PN  $(N, M_0)$  is a  $k$ -bounded

net if each place  $p$  of  $N$  is  $k$ -bounded. Note that the PN  $(N, M_0)$  is said to be safe iff  $k = 1$ .

In this paper, we only focus on the ordinary Petri net  $N = (P, T, F, M_0)$ .

### III. PARTIAL ORDER METHODS

In this section, we introduce the persistent sets and covering step graph methods, and their combination.

#### A. PERSISTENT SETS

At each marking, persistent sets only contain enabled transitions. Note that each transition of a persistent set can not be disabled, by the firing of other transitions not in the same persistent set [43], [53]. In general, the exploration of persistent sets preserves at least deadlocks of Petri nets.

The persistent sets proposed by [43], [53] are called strong-persistent sets. In [54], the authors introduce weak-persistent sets with weaker conditions inspired from stubborn sets of [46]. In the following discussion, the persistent sets used throughout this paper are strong-persistent sets.

*Definition 1* [54]: Let  $M$  be a marking and  $\mu \subseteq En(M)$  a subset of enabled transitions. Formally, the subset  $\mu$  is a persistent set of  $M$  if all the following conditions are satisfied:

- 1)  $En(M) \neq \emptyset \Leftrightarrow \mu \neq \emptyset$ ;
- 2)  $\forall t \in \mu, \forall \omega \in (T - \mu)^+, M[\omega] \Rightarrow M[\omega t]$ ;
- 3)  $\forall t \in \mu, \forall \omega \in (T - \mu)^+, M[\omega t] \Rightarrow M[t\omega]$ .

Condition 1) means that there exists no persistent set at  $M$  iff  $M$  is a deadlock marking. Condition 2) ensures that after the firing of transitions not in  $\mu$  at  $M$ , any transition  $t$  of  $\mu$  can be fired. Condition 3) states that if the firing of any sequence  $\omega$ , which is composed of transitions not in  $\mu$ , does not disable any transition  $t$  of  $\mu$ , and then the firing of  $t$  will not disable  $\omega$ .

Consider the PN1 in Fig. 1(a). The set of enabled transitions at  $M_0$  is  $En(M_0) = \{t_0, t_2\}$ . Let  $\mu_1 = \{t_0\}$  be a subset of  $En(M_0)$ . It is not a persistent set since  $t_0$  is disabled by the firing of a sequence  $t_2 t_3$ , i.e.,  $M_0[t_2 t_3]$  but  $\neg M_0[t_2 t_3 t_0]$ . For the same PN1, the set  $\mu_2 = \{t_2\}$  is persistent as conditions 1, 2 and 3 hold for  $t_2$ .

#### B. STEP GRAPHS COMBINED WITH PERSISTENT SETS

Covering step graphs are proposed in [52]. In a covering step graph, all transitions are visited and concurrent ones are put together to constitute a step. The firing of transitions in a step is simultaneously. The aim of step graph methods is to achieve more reduction of the state space from path depths, and preserve certain global reachability properties such as deadlocks. For example, consider the model PN2 depicted in Fig. 2(a). There exist two steps at the initial marking  $M_0$ ,  $\tau_1 = \{t_0, t_2, t_3, t_4\}$  and  $\tau_2 = \{t_0, t_2, t_3, t_5\}$ . However, the firing of  $\tau_1$  or  $\tau_2$  may disable  $t_1$  that should have been enabled at a marking  $M_1$ , where  $M_0[t_2]M_1$ . Thus,  $t_0$  cannot be fired together with  $t_2$  since some deadlock markings may not be preserved if they are fired simultaneously. The covering step graph (CSG) is shown in Fig. 2(b) and firing steps at

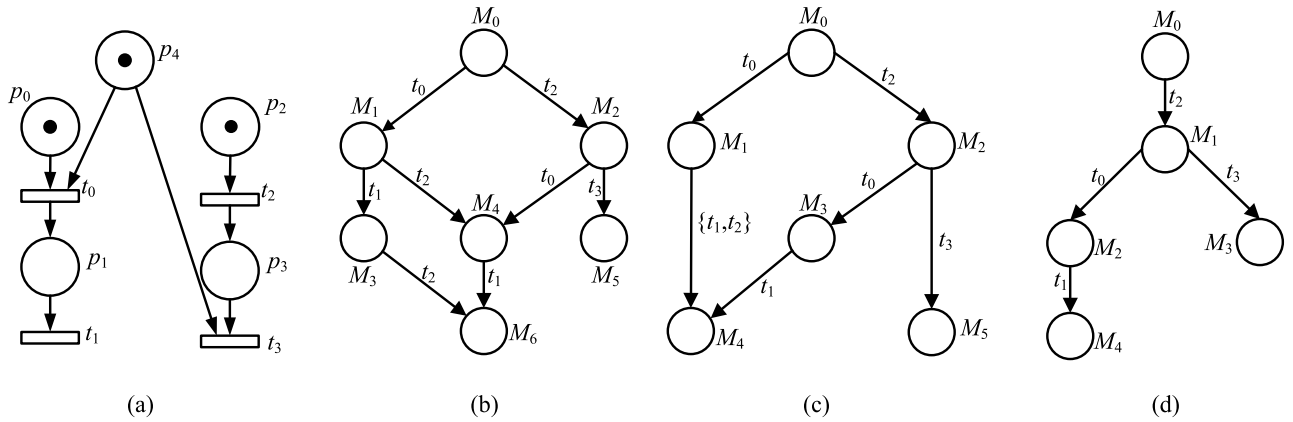


FIGURE 1. (a) A model of PN1, (b) its RG, (c) its CSG, and (d) its MSGG.

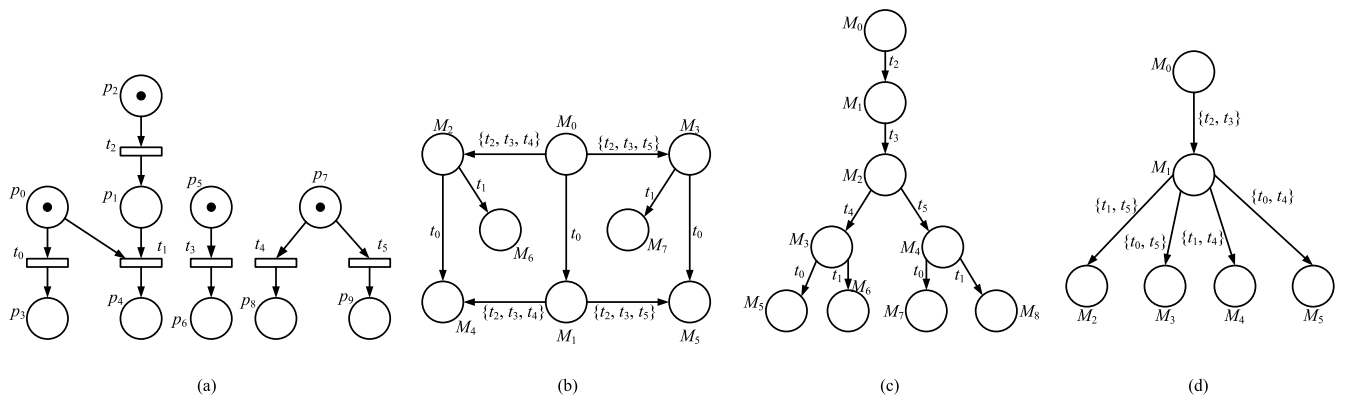


FIGURE 2. (a) A model of PN2, (b) its CSG, (c) its PG, and (d) its MSGG.

$M_0$  are  $\tau_1 = \{t_2, t_3, t_4\}$ ,  $\tau_2 = \{t_2, t_3, t_5\}$  and  $\tau_3 = \{t_0\}$ . Specially, the covering step graph preserves deadlocks of PN2 and reduces the state space from path depths.

Different from covering step graphs, persistent graphs (PG) are to reduce the width of the state space. At each marking, only the enabled transitions of a persistent set are visited and fired individually. For instance, consider the same PN2, the set of enabled transitions at  $M_0$  is  $En(M_0) = \{t_0, t_2, t_3, t_4, t_5\}$  and there are three persistent sets at  $M_0$ :  $\mu_1 = \{t_0, t_2\}$ ,  $\mu_2 = \{t_3\}$  and  $\mu_3 = \{t_4, t_5\}$ . The firing of a transition within different persistent sets may yield different persistent graphs. A minimal persistent graph of PN2 is shown in Fig. 2(c).

To make full use of the advantages of covering step graphs and persistent sets, a hybrid method is proposed in [42]. The main idea of its generation algorithm is to compute persistent sets at each marking firstly, and then determine which transitions in different sets can be combined together as a step. The combination of both methods allows reducing the state space from path depths and the width, and preserves the deadlocks of PNs. As an example, consider the PN2. For the persistent sets  $\{t_0, t_2\}$ ,  $\{t_3\}$  and  $\{t_4, t_5\}$  of the initial marking  $M_0$ , we can build various steps such as  $\{t_0, t_3, t_4\}$ ,  $\{t_0, t_3, t_5\}$ ,  $\{t_2, t_3, t_4\}$ ,  $\{t_2, t_3, t_5\}$ , and  $\{t_2, t_3\}$ . These steps are so-called persistent

steps and the firing of different one at  $M_0$  may lead to different persistent step graphs. According to the algorithm proposed in [41], the maximally reduced persistent step graph (PSG) is depicted in Fig. 2(d).

For these reduced state graphs, some intermediate markings are abstracted and all key markings are preserved. The key markings of a PN can be utilized to explore certain global reachability properties such as deadlocks of a PN.

#### IV. MAXIMAL GOOD STEP METHODS

##### A. SOUND STEP SETS

**Definition 2:** Let  $M$  be a reachable marking of a PN  $N = (P, T, F, M_0)$ ,  $\tau$  an enabled step at  $M$  and  $t \in \tau$ . The transition  $t$  is sound at  $M$  w.r.t  $\tau$  if conditions 1) and 2) hold for all  $t' \in \tau - \{t\}$ :

- 1)  $\forall \sigma \in (T - \{t\})^+, (M[t'\sigma] \wedge M[t'\sigma t]) \Rightarrow$ 
  - a)  $\exists \sigma^* \in (T - \{t\})^+, M[t'\sigma\sigma^*t] \vee$
  - b)  $\exists t_1 \in En(M) - \tau, \exists \sigma_1 \in (T - \{t\})^*, (t_1\sigma_1 \equiv \sigma \wedge M[t_1t'\sigma_1])$
- 2)  $\forall \sigma \in (T - \{t\})^+, M[t'\sigma t] \Rightarrow \exists \sigma' \in (T - \{t\})^+, (\sigma \equiv \sigma' \wedge M[t\sigma'])$

The step  $\tau$  is sound at  $M$  if its transitions are all sound at  $M$  with respect to  $\tau$ . We use the notation  $SS(M)$  to indicate the set of all sound steps at a marking  $M$  of a PN  $N$ .

For a sound transition  $t$  w.r.t.  $\tau$  at  $M$ , Condition a) shows that  $t$  can be re-enabled after the firing of a sequence starting with certain transitions of the same step  $\tau$  and Condition b) means that the disablement of  $t$  cannot be caused by other transitions of the same step  $\tau$ ; and Condition 2) states that if a sequence  $t'\sigma t$  is firable from  $M$ , then there must exist an equivalent sequence  $t'\sigma'$  that is also firable from  $M$ . Specifically, in a case where  $t$  is disabled by an enabled sequence  $t'\sigma$ , i.e.,  $M[t'\sigma] \wedge \neg M[t'\sigma t]$ , if the firing of  $t'\sigma$  satisfies a certain condition (i.e., Condition a) or b)), then we can be sure that  $t$  is sound at  $M$ . Condition a) represents that the sequence  $t'\sigma t$  that is used to be not enabled at  $M$  can be fired s.t.  $M[t'\sigma\sigma^*t]$ . Intuitively, it means that if a transition  $t$  is disabled after the firing of other transitions within the same step  $\tau$ , i.e.,  $\neg M[t'\sigma t]$ , then the sound transition  $t$  can be re-enabled after the firing of other sequences, i.e.,  $M[t'\sigma\sigma^*t]$ . Condition b) shows that the sequence  $t'\sigma$  has an equivalent and enabled sequence at  $M$  that starts with a certain transition outside  $\tau$  s.t.  $t'\sigma \equiv t_1t'\sigma_1$  and  $M[t_1t'\sigma_1]$ , which means that the sequence  $t'\sigma$  leading to the disablement of  $t$  can be fired at  $M$  in another order and then the firing of  $t$  cannot be affected by  $t'\sigma$ . Note that expressions such as “ $t_1\sigma_1 \equiv \sigma$ ”, “ $M[t_1t'\sigma_1]$ ”, “ $\sigma \equiv \sigma'$ ”, and “ $M[t_1t'\sigma_1]$ ” can be considered as boolean expressions. More specifically, the value of “ $t_1\sigma_1 \equiv \sigma$ ” is 1 if the sequence  $t_1\sigma_1$  is equivalent to  $\sigma$ , otherwise its value is 0.

It is obvious that if  $|\tau| = 1$  then  $\tau$  is a sound step at  $M$ . Indeed, in such a case  $t' \in \emptyset$ , the sequence  $t'\sigma$  is not enabled at  $M$ , i.e.,  $\neg M[t'\sigma]$ , since for an enabled sequence, the first transition must belong to  $En(M)$ . Hence, it follows that Conditions 1) and 2) are satisfied.

*Example 1:* As an example of a sound step, consider the PN4 in Fig. 4(a). The enabled step  $\tau = \{t_1, t_2, t_3\}$  of the initial marking  $M_0 = p_1 + p_2 + p_3$  is sound since it satisfies Definition 2. Specifically, an enabled step  $\tau_1 = \{t_2, t_3\}$  of  $M_0$  is also sound. For instance, Condition 1) is satisfied for  $t_2$  w.r.t.  $\tau_1$  since  $M_0[t_2t_4]$  and  $\neg M_0[t_2t_4t_3]$ , there exists an enabled transition  $t_1$  such that  $M_0[t_2t_4t_1t_3]$ ; and Condition 2) also holds for  $t_2$  since  $M_0[t_2t_4t_1t_3]$ , there exists a sequence  $t_1t_4$  equivalent with  $t_4t_1$  s.t.  $M_0[t_3t_2t_1t_4]$ . The set of all sound steps at  $M_0$  of the PN4 is  $SS(M_0) = \{\{t_1\}, \{t_2\}, \{t_3\}, \{t_1, t_2\}, \{t_1, t_3\}, \{t_2, t_3\}, \{t_1, t_2, t_3\}\}$ . As another example of a sound step, consider the PN5 shown in Fig. 6. The enabled step  $\tau = \{t_1, t_4, t_5\}$  is sound at the initial marking  $M_0 = p_1 + p_4 + p_5$  since Conditions 1) and 2) hold for each transition of  $\tau$ . The set of all sound steps at  $M_0$  is  $SS(M_0) = \{\{t_1\}, \{t_4\}, \{t_5\}, \{t_1, t_4\}, \{t_1, t_5\}, \{t_4, t_5\}, \{t_1, t_4, t_5\}\}$ . Taking a sound step  $\tau_1 = \{t_4, t_5\}$  for an example, we can see that Condition 1) is satisfied for  $t_5$  since  $M_0[t_5t_6]$  and  $\neg M_0[t_5t_6t_4]$ , there exists a sequence  $t_1t_2t_3$  such that  $M_0[t_5t_6t_1t_2t_3t_4]$ ; and Condition (2) also holds for  $t_5$  since  $M_0[t_5t_6t_1t_2t_3t_4]$ , there exists a sequence  $t_1t_2t_3t_6$  equivalent with  $t_6t_1t_2t_3$  s.t.  $M_0[t_4t_5t_1t_2t_3t_6]$ . As an example of a non-sound step, consider the PN1 depicted in Fig. 1(a). The

enabled step  $\{t_0, t_2\}$  of the initial marking  $M_0 = p_0 + p_2 + p_4$  is not sound since Condition 1) does not hold for  $t_2$ :  $M_0[t_2t_3]$  and  $\neg M_0[t_2t_3t_0]$ , but we have neither a) nor b) due to there exists no sequence  $\sigma^* \in (T - \{t\})^+$  s.t.  $M[t_2t_3\sigma^*t_0]$  and  $\neg M_0[t_3t_2]$ . Thus, the set of sound steps at  $M_0$  of the PN1 is  $SS(M_0) = \{\{t_0\}, \{t_2\}\}$ . Another example of a non-sound step is PN3 shown in Fig. 3(a). An enabled step  $\tau_1 = \{t_0, t_2\}$  of the initial marking  $M_0 = p_0 + p_1 + p_2 + p_4$  is not sound as Condition (1) does not hold for  $t_2$ . Intuitively, the sequence  $t_2t_3t_1$  is firable at  $M_0$  but  $t_2t_3t_1t_0$  is not firable at  $M_0$ , i.e.,  $M_0[t_2t_3t_1] \wedge \neg M_0[t_2t_3t_1t_0]$ , and we have neither a) nor b) as there exists no sequence  $\sigma^* \in (T - \{t\})^+$  s.t.  $M[t_2t_3t_1\sigma^*t_0]$  and  $\neg M_0[t_1t_2t_3]$ . For the same PN3, the enabled step  $\tau_2 = \{t_1, t_2\}$  of  $M_0$  is also not sound as it does not satisfy Condition(2) since  $M_0[t_2t_3t_1]$  but  $\neg M_0[t_1t_2t_3]$ . Hence, the set of sound steps at  $M_0$  of the PN3 is  $SS(M_0) = \{\{t_1\}, \{t_2\}, \{t_3\}\}$ .

## B. PROBLEMS STATEMENT

We first propose two problems used in the following section.

*Problem 1:* Given a PN  $N$  and its initial marking  $M_0$ , how to find the set of sound steps at each reachable marking from a practical point of view?

Essentially, The answer of Problem 1 corresponds to a weaker sufficient condition for sound steps. From a practical point of view, we need to explain clearly the way about how to build the set of sound steps at each marking. Hence, we analyze how to determine which transitions can be constitute as a sound step at each marking, from the aspect of a net structure.

Let  $M$  be a marking of  $N$  and  $\tau \subseteq En(M)$  an enabled step at  $M$  with  $\tau = \{t, t'\}$  (i.e.,  $|\tau| = 2$ ). If  $t$  and  $t'$  satisfy the following conditions, we can know that the step  $\tau$  at  $M$  is a sound step. In other words, we can find which step is sound at  $M$  according to the following conditions:

- 1) Transitions  $t$  and  $t'$  are not conflict with each other. Actually, if  $t$  are in conflict with  $t'$ , then transitions  $t$  and  $t'$  can not be fired together at  $M$ .
- 2) There is a transition  $\mu$  that is conflict with  $t$ . We can distinguish three cases for the type of  $\mu$ .
  - a) The transition  $\mu$  is not enabled at  $M$  and there exists a sequence  $\sigma \in T^*$  s.t.  $M[t'\sigma]M' \wedge M'[\mu]$ . We need to determine whether there exists an enabled transition  $t^*$ , s.t.  $t^* \in En(M) - \tau$  and the preset  $p^*$  of  $t^*$  can give its token to the common place  $p$  of  $t$  and  $\mu$ . If the transition  $t^*$  and the place  $p^*$  satisfy the above condition, then  $t$  and  $t'$  may be constituted as a sound step at  $M$ .
  - b) The transition  $\mu$  is not enabled at  $M$  and the firing of  $t'$  can not yield a marking  $M'$  s.t.  $M'[\mu]$ .
  - c) The transition  $\mu$  is enabled at  $M$ . We continue to distinguish this case into two categories:
    - i. There is a disabled transition  $v$  that is in symmetric structural conflict with  $\mu$ ;
    - ii. There is a disabled transition  $v$  conflict with  $\mu$  and the firing of  $t'$  can not yield  $M'$  s.t.  $M'[v]$ .

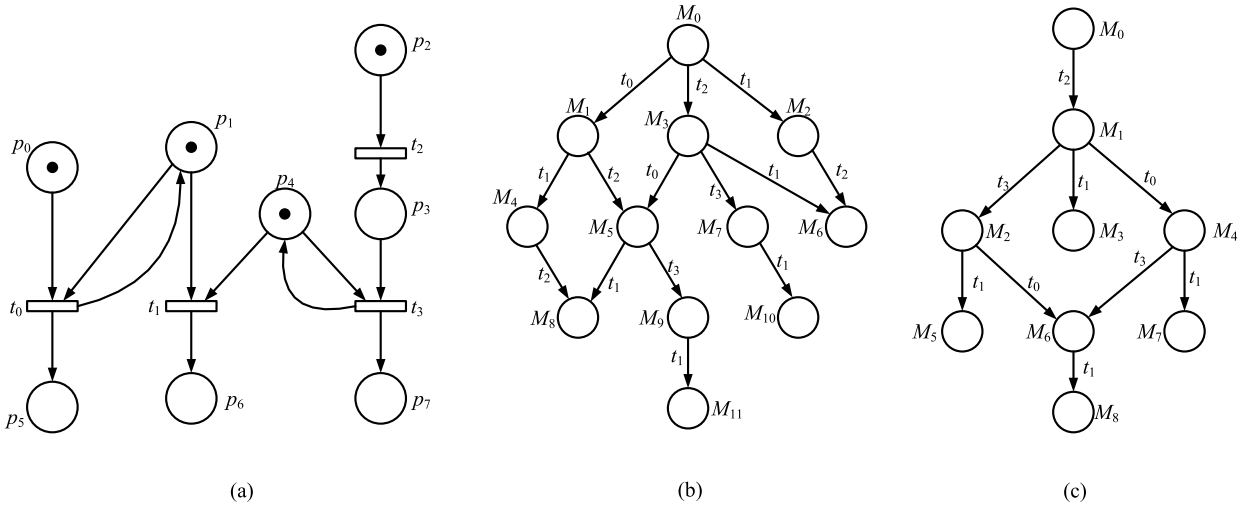


FIGURE 3. (a) A model of PN3, (b) its CSG, and (c) its MSGS.

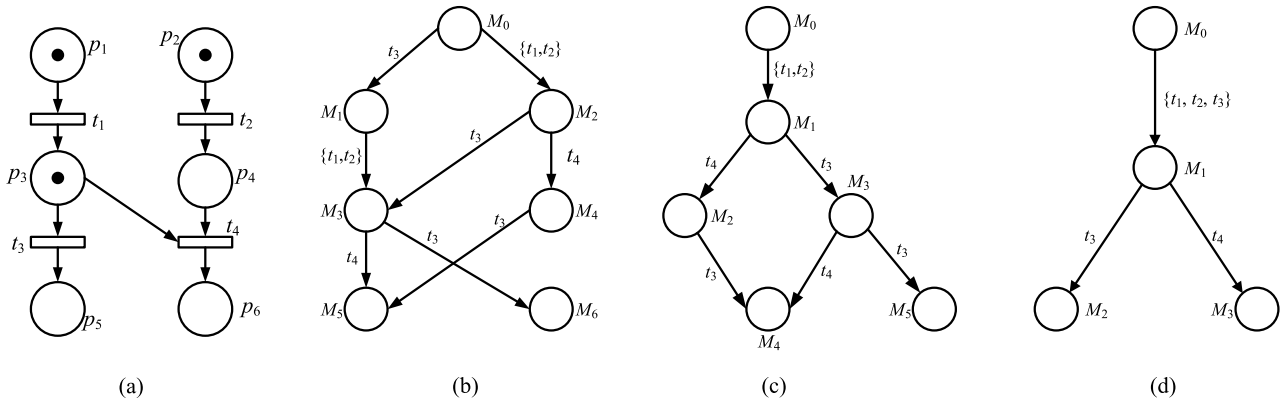


FIGURE 4. (a) A model of PN4, (b) its CSG, (c) its MPSG, and (d) its MSGS.

If two transitions of a step  $\tau$  at  $M$  satisfies the above conditions, we can say that  $\tau$  is sound at  $M$ . For instance,  $\tau = \{t_4, t_5\}$  of the PN5 in Fig. 6 is a sound step at the initial marking  $M_0$  since  $t_4$  is conflict with  $t_6$  and  $M_0[t_5] M' \wedge M'[t_6]$ . We can find that there exists an enabled transition  $t_1$  outside the step  $\tau$  and the preset  $p_1$  of  $t_1$  gives its token to the common place  $p_4$  of  $t_4$  and  $t_6$ . Thus,  $\{t_4, t_5\}$  is a sound step at  $M_0$ . As another example of a sound step, consider the same PN5. The step  $\tau = \{t_1, t_4\}$  is also sound at  $M_0$  since  $t_4$  is conflict with  $t_6$  and there does not exist a sequence  $\sigma \in T^*$  s.t.  $M_0[t_1\sigma]M' \wedge M'[t_6]$ . As an example of a non-sound step, consider the PN3 shown in Fig. 3(a). The step  $\tau = \{t_1, t_2\}$  is not sound at the initial marking  $M_0$  since  $t_1$  is conflict with  $t_3$  and  $M_0[t_2] M' \wedge M'[t_3]$ . However, we cannot find an enabled transition  $t^*$  s.t. the preset  $p^*$  of  $t^*$  can give its token to the common place  $p_4$  of  $t_1$  and  $t_3$ . As another example of a non-sound step, consider the same PN3. The step  $\tau = \{t_0, t_2\}$  is also not sound at  $M_0$  since  $t_0$  is conflict with an enabled transition  $t_1$  of  $M_0$  and  $t_1$  is conflict with another transition  $t_3$  which is not in symmetric

structural conflict with  $t_1$  and the firing of  $t_2$  can lead to a marking  $M'$  s.t.  $M'[t_3]$ .

**Problem 2:** Given a PN  $N$  and a reachable marking  $M$ , how to determine a step  $\tau$  with  $|\tau| > 2$  is a sound step at  $M$ ?

In terms of Problem 1, we know how to find a sound step  $\tau$  at each marking with  $|\tau| = 2$ . It is obvious that  $\tau$  is always a sound step at  $M$  with  $|\tau| = 1$  according to Definition 2. As for a step  $\tau$  at  $M$  with  $|\tau| > 2$ , we first find all transitions of  $\tau$ , and then estimate whether two arbitrary transitions can be fired together as a sound step according to Problem 1. If so, the step  $\tau$  containing these transitions with a bigger range is sound at  $M$ . For example, the step  $\tau = \{t_1, t_4, t_5\}$  is sound at  $M_0$  of the PN5 in Fig. 6 since  $\{t_1, t_4\}$ ,  $\{t_4, t_5\}$  and  $\{t_1, t_5\}$  are all sound at  $M_0$ .

### C. A CONSTRUCTION ALGORITHM FOR AN MSGS

In this section, we first introduce a function named *MaxSoundStep* to compute the set of all maximal sound steps at each marking of a PN  $N$ . Then, we present a relation between maximal sound steps and persistent sets, and followed by the

notion of good steps that combine sound steps with persistent sets. Afterwards, a construction algorithm for a maximal good step graph (MGSG) is established and we prove that such a graph preserves deadlocks of a PN.

We exhibit Function *MaxSoundStep* in the following. Let *IsSound* be a decision function defined by: given a reachable marking  $M$  of a PN  $N$  and an enabled step  $\tau$  of  $M$ ,  $IsSound(M, \tau) = false$ , if transitions of  $\tau$  at  $M$  do not satisfy the sufficient conditions of Problem 1. In other words,  $IsSound(M, \tau) = true$  signifies that  $\tau$  is sound at  $M$ . A sound step  $\tau$  is the maximal one at  $M$ , if there does not exist another sound step  $\tau'$  at  $M$  s.t.  $\tau \subset \tau'$ . Note that Function *MaxSoundStep* will be used in the construction algorithm for an MGSG.

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**Function**  $MSS = MaxSoundStep(M)$

**Input:** A marking  $M$  of a PN  $N$ ;

**Output:** A set  $MSS$  of all maximal sound steps at  $M$ ;

1.  $MSS = \emptyset$ ;
  2.  $MS = \{En(M)\}$ ;
  3.  $S = \emptyset$ ;
  4. **while**  $(\exists \tau \in MS \text{ s.t. } \neg IsSound(M, \tau))$  **do**
  5.      $MS = MS - \tau$ ;
  6.      $S = \{\tau' | \forall t \in \tau, \tau' = \tau - \{t\}\}$ ;
  7.     **for** (each  $\mu \in S$  s.t.  $\forall \pi \in MS, \mu \not\subset \pi$ ) **do**
  8.          $MS = MS \cup \{\mu\}$ ;
  9.     **end for**
  10. **end while**
  11.  $MSS == MS$ ;
  12. **Output:**  $MSS$ .
- 

In brief, Function *MaxSoundStep* is executed as follows: First, let  $MS$  be the set of all enabled transitions set at a marking  $M$ , i.e.,  $MS = \{En(M)\}$ . Then, a step  $\tau$  of  $MS$ , which is not sound according to Problem 1, is deleted. After deleting the step  $\tau$ , we define a symbol  $S$  to compute the set of all maximal subsets of  $\tau$ . Each step  $\mu$  of  $S$ , which does not belong to a step  $\pi$  of the deleted  $MS$ , is added to the set  $MS$  to combine the new set of maximal steps. By repeating the above process, the set of all maximal sound steps at a marking  $M$  is computed.

*Example 2:* Consider the PN3 shown in Fig. 3(a) and its initial marking  $M_0 = p_0 + p_1 + p_2 + p_4$ . In terms of Function *MaxSoundStep*,  $MS = \{En(M_0)\} = \{t_0, t_1, t_2\}$  and  $\tau = \{t_0, t_1, t_2\}$ . By Definition 2, it is obvious that  $\tau$  is not sound at  $M_0$  and thus is deleted from  $MS$ . The set  $MS$  is then replaced with  $MS = \{\emptyset\}$ . The set  $S$  of all maximal steps in  $\tau$  is computed s.t.  $S = \{\{t_0, t_1\}, \{t_1, t_2\}, \{t_0, t_2\}\}$  and  $MS$  is replaced with  $MS = \{\{t_0, t_1\}, \{t_1, t_2\}, \{t_0, t_2\}\}$  since each step of  $S$  does not belong to  $\{\emptyset\}$ . We can see that these steps  $\{t_0, t_1\}$ ,  $\{t_1, t_2\}$  and  $\{t_0, t_2\}$  are all not sound at  $M_0$  and we will repeat the above procedure for each step of  $MS$ . Finally, the set  $MSS$  of all maximal sound steps at  $M_0$  is  $MSS = \{\{t_0\}, \{t_1\}, \{t_2\}\}$ . Consider the PN2 depicted in Fig. 2(a) and its initial marking  $M_0 = p_0 + p_2 + p_5 + p_7$ . Let  $MS = \{En(M_0)\} = \{t_0, t_2, t_3, t_4, t_5\}$  and  $\tau = \{t_0, t_2, t_3, t_4, t_5\}$ . We can see that  $\tau$  is

not sound at  $M_0$  and then  $MS$  is replaced by  $MS = \{\{t_0, t_2, t_3, t_4\}, \{t_0, t_2, t_3, t_5\}, \{t_0, t_2, t_4, t_5\}, \{t_0, t_3, t_4, t_5\}, \{t_2, t_3, t_4, t_5\}\}$ . According to Definition 2, each step of  $MS$  is not sound at  $M_0$ . By repeating the above procedure, the set  $MS$  is then replaced with  $MS = \{\{t_0, t_2, t_3\}, \{t_0, t_2, t_4\}, \{t_0, t_2, t_5\}, \{t_0, t_3, t_4\}, \{t_0, t_3, t_5\}, \{t_2, t_3, t_4\}, \{t_2, t_3, t_5\}, \{t_2, t_4, t_5\}, \{t_3, t_4, t_5\}\}$ . We can note that steps  $\{t_0, t_3, t_4\}$ ,  $\{t_0, t_3, t_5\}$ ,  $\{t_2, t_3, t_4\}$  and  $\{t_2, t_3, t_5\}$  are all sound at  $M_0$ . After deleting non-sound steps of  $MS$  at  $M_0$ , we can obtain the set  $MSS$  of all maximal sound steps at  $M_0$  is  $MSS = \{\{t_0, t_3, t_4\}, \{t_0, t_3, t_5\}, \{t_2, t_3, t_4\}, \{t_2, t_3, t_5\}\}$ .

*Definition 3:* Let  $M$  be a reachable marking of a PN  $N$  and  $\tau$  an enabled step at  $M$ . The step  $\tau$  is a good step at  $M$  if 1)  $\tau$  is a sound step at  $M$ , and 2) the set  $\mu$  of all transitions in the step  $\tau$  is a persistent set at  $M$ . A good step  $\tau$  is maximal at  $M$  if there does not exist a good step  $\tau'$  s.t.  $\tau \subset \tau'$ .

*Example 3:* Consider the PN1 shown in Fig. 1(a). There are two sound steps  $\tau_1$  and  $\tau_2$  at the initial marking  $M_0$ , where  $\tau_1 = \{t_0\}$  and  $\tau_2 = \{t_2\}$ . According to Definition 1, only the step  $\tau_2$  is persistent at  $M$ . Thus,  $\tau_2$  is a maximal good step at  $M_0$ . Consider the PN3 depicted in Fig. 3(a). The set of sound steps at  $M_0$  is  $SS(M_0) = \{\{t_1\}, \{t_2\}, \{t_3\}\}$ . We can see that only the sound step  $\{t_2\}$  is persistent at  $M_0$  via Definition 1. Thus,  $\{t_2\}$  is the only good step at  $M_0$ . Consider the PN4 in Fig. 4(a). There are many sound steps at the initial marking  $M_0$  and the step  $\tau = \{t_1, t_2, t_3\}$  is the only maximal sound step at  $M_0$ . According to Theorem 1,  $\tau$  is persistent at  $M_0$ . Therefore, the step  $\tau$  is a maximal good step at  $M_0$ . Consider the PN2 depicted in Fig. 2(a). There are four maximal sound steps  $\tau_1, \tau_2, \tau_3$ , and  $\tau_4$  at the initial marking  $M_0$ , where  $\tau_1 = \{t_0, t_3, t_4\}$ ,  $\tau_2 = \{t_0, t_3, t_5\}$ ,  $\tau_3 = \{t_2, t_3, t_4\}$ , and  $\tau_4 = \{t_2, t_3, t_5\}$ . According to Definition 1, these transition sets  $\tau_1, \tau_2, \tau_3$  and  $\tau_4$  are all not persistent at  $M_0$ . Actually, there are numerous sound steps at  $M_0$  of the PN2 such as  $\{t_2, t_3, t_4\}$ ,  $\{t_2, t_3\}$ ,  $\{t_2, t_4\}$ ,  $\{t_2, t_5\}$ ,  $\{t_0\}$ ,  $\{t_2\}$  and  $\{t_3\}$ . Sound steps  $\{t_2, t_3\}$ ,  $\{t_2\}$ , and  $\{t_3\}$  are persistent sets of  $M_0$  on the basis of Definition 1. Hence, there are three good steps at  $M_0$ , i.e.,  $\{t_2, t_3\}$ ,  $\{t_2\}$ ,  $\{t_3\}$ , and the step  $\{t_2, t_3\}$  is the maximal good step at  $M_0$ .

*Theorem 1:* Let  $\tau$  be a sound step containing all the enabled transitions at a marking  $M$  of  $N$ . Then,  $\tau$  is a good step at  $M$ .

*Proof:* It has been known that all enabled transitions are in the step  $\tau$ , i.e.,  $\tau = En(M)$ . According to Definition 1, it is obvious that this set  $\tau$  containing all enabled transitions at  $M$ , is a persistent set of  $M$ . Thus,  $\tau$  is a good step at the marking  $M$ .

Algorithm 1 is proposed to generate a maximal good step graph (MGSG). Let  $\lambda(M, \tau)$  define a next-state marking function that returns a reachable marking by firing an enabled step  $\tau$  at a marking  $M$  of  $N$ .

*Remark:* Given a PN  $N$ , its maximal good step graph (MGSG) can be established by Algorithm 1. We briefly explain this algorithm as follows. It can be divided into two stages. The first one is to obtain the set of enabled steps by which directed edges are labeled at each marking, and the second one is to generate the state graph via nodes and



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**Algorithm 1** Construction Algorithm for a Maximal Good Step Graph (MGSG)
 

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**Input** A Petri net  $N = (P, T, F, M_0)$ ;

**Output** An MGSG of  $N$ .
 

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1. Let  $x_0$  be the root node of the MGSG and  $M$  be the marking of node  $x_0$ ;
2. Initialize the stack  $\Lambda := (x_0)$  and the set  $\Sigma := (M)$ ; /\*  $\Lambda$  is a stack allowing to store nodes and  $\Sigma$  is a set consisting of all the markings of nodes that are explored by this algorithm\*/
3.  $\Phi = \emptyset$ ; /\*  $\Phi$  represents a set of enabled steps by which directed edges are labeled at each reachable marking\*/
4. **while**  $\Lambda \neq ()$  **do**
5.    $x := pop(\Lambda)$ ; /\* Remove the last node  $x$  from the stack  $\Lambda$ \*/
6.   Let  $M_x$  be the marking of node  $x$ ;
7.   **if**  $En(M_x) \neq \emptyset$  **then**
8.     **if** there exists a maximal good step  $\tau$  at  $M_x$  **then**
9.        $\Phi = \{\tau\}$ ; /\* Refer to Definition 3\*/
10.    **else**
11.      $\Phi := MaxSoundStep(M_x)$ ; /\*  $MaxSoundStep$  returns the set of all maximal sound steps at  $M_x$ .\*/
12.    **end if**
13.    **for** each step  $\tau \in \Phi$  **do**
14.     Compute a reachable marking  $M_y$  by the next-state marking function  $\lambda(M_x, \tau)$ ;
15.     **if**  $M_y \notin \Sigma$  **then**
16.       Create a new node  $y$ ;
17.       Add a directed edge from  $x$  to  $y$  and this arc is labeled by  $\tau$ ;
18.       Let  $M_y$  be the marking of node  $y$ ;
19.        $\Sigma := \Sigma \cup M_y$ ;
20.        $\Lambda := push(\Lambda, y)$ ; /\* Push node  $y$  into stack  $\Lambda$  as the last node in  $\Lambda$ \*/
21.     **else**
22.       Get the marking  $M'_y$  in  $\Sigma$  that is equal to  $M_y$  and the node of marking  $M'_y$  is named as  $y'$ ;
23.       Add a directed edge from  $x$  to  $y'$  and this arc is labeled by  $\tau$ ;
24.     **end if**
25.    **end for**
26.    **end if**
27. **end while**
28. **end**

---

labeled arcs. First, let  $x_0$  be the root node of an MGSG and  $M$  the marking of  $x_0$ , i.e.,  $\Sigma := (M)$  and  $\Lambda := (x_0)$ , and  $\Phi$  is a set of enabled steps by which each directed edge is labeled at the marking. Second, remove the last node from a stack  $\Lambda$ , and then the current marking  $M_x$  is obtained. Third, for a deadlock-free marking  $M_x$ , if there exists a maximal good step  $\tau$  at  $M_x$ , then the directed edge from  $M_x$  is labeled by  $\tau$  and we denote  $\Phi = \{\tau\}$ , otherwise each directed edge from  $M_x$  is labeled by a maximal sound step at  $M_x$  and we

use  $\Phi := MaxSoundStep(M_x)$  to denote the set of all maximal sound steps at  $M_x$ . What is said above is the first stage. After that, for each  $\tau \in \Phi$ , compute the next-state  $\lambda(M_x, \tau)$  and we can obtain the next-state marking  $M_y$ . If  $M_y$  is an existing marking in the MGSG such that  $M_y \in \Sigma$ , then we find the node  $y'$  of a marking  $M_y$  and add a directed arc from  $x$  to  $y'$ , which is labeled by  $\tau$ , otherwise a new node  $y$  is created and is pushed in to a stack  $\Lambda$ . The second stage is indicated above. Repeat these stages until  $\Lambda$  is empty and an MGSG is hence constructed.

*Example 4:* Consider the PN1 in Fig. 1(a) and its initial marking  $M_0 = p_0 + p_2 + p_4$ . Firstly, Algorithm 1 sets  $x_0$  a root node of the MGSG and  $M_0$  the marking of  $x_0$ , and initializes  $\Lambda := (x_0)$  and  $\Sigma := (M_0)$ . Secondly,  $x_0$  is removed from the stack  $\Lambda$ , which is then called  $x$ . Thirdly, we will compute the next node  $y$  of the MGSG, the marking  $M_y$  of  $y$ , and the directed arc from  $x_0$  to  $y$ . At the initial marking  $M_0$ , there exists the only maximal good step  $\tau$  such that  $\tau = \{t_2\}$ . After firing a step  $\tau$ , we get a reachable marking  $M_1 = p_0 + p_3 + p_4$  by computing the next-state function  $\lambda(M_0, \{t_2\})$ . It is obvious that  $M_1 \notin \Sigma$ . Thus, a new node  $y$  is created and an arc from  $x$  to  $y$  is labeled by  $\{t_2\}$ . Next, Algorithm 1 sets  $\Sigma := (M_0, M_1)$  and  $\Lambda := (y)$ . Repeat stages above until  $\Lambda$  is empty and an MGSG is hence constructed. The MGSG of PN1 is shown in Fig. 1(d).

*Theorem 2:* The MGSG exploration detects deadlock markings of a PN.

*Proof:* Suppose that there exists a deadlock marking  $D$  of a PN  $N$ . Let  $M$  be a reachable marking of the MGSG from  $M_0$  and  $D$  a deadlock marking reachable from  $M$  in the Petri net. Let  $\omega$  be a firing sequence yielding a marking  $D$  from  $M$  in the Petri net s.t.  $M[\omega]D$ . We prove that  $D$  is also a deadlock marking of the MGSG reachable from  $M$ . The proof of this theorem is by induction, focusing on the length of the sequence  $\omega$ :

1) If  $|\omega| = 0$ , then it is obvious that  $M = D$  and  $D$  is a deadlock marking of the MGSG;

2) Assume that  $|\omega| = k$  and  $D$  is a deadlock marking of the MGSG;

3) Let us demonstrate that  $D$  is still a deadlock marking of the MGSG reachable from  $M$  with  $|\omega| = k + 1$ ;

Let  $M'$  be an intermediate marking reachable from  $M$  after the firing of a step  $\tau$  and  $D$  is reached from  $M'$  by a sequence  $\omega_1$  s.t.  $(M[\tau]M'[\omega_1]D) \wedge (\tau \cup \omega_1 = \omega)$ . We distinguish two main cases:

*Case 1:* If there is a maximal good step  $\pi$  at a marking  $M$ , then there are two cases for a step  $\tau$ :

- Case a: If  $\tau = \pi$ , then  $M'$  is a marking of the MGSG according to Algorithm 1. Applying the induction hypothesis on  $\omega_1$ , we can see that  $|\omega_1| \leq k$  and  $M'[\omega_1]D$ . Hence,  $D$  is a deadlock marking of the MGSG.

- Case b: If  $\tau \neq \pi$ , there must exist a sequence  $\omega_2$  such that  $\pi \cup \omega_2 = \omega$ . It is obvious that transitions of  $\pi$  are all in the sequence  $\omega$  since  $\pi$  is persistent and sound at  $M$ , i.e., transitions of  $\pi$  are all enabled at  $M$  and these transitions are all fireable after the firing of transitions not

in  $\pi$ . Then, transitions of  $\pi$  can be shifted to the front of a sequence  $\omega$  to constitute a step firable from  $M$ . The proof of this case is similar to Case a, and  $D$  is proven to be a deadlock marking of the MGSG.

Case 2: Else, there only exist maximal sound steps at  $M$ :

- Case c: If  $\tau$  is a maximal sound step at  $M$ , then  $M'$  is a marking of the MGSG due to Algorithm 1. Applying the induction hypothesis on  $\omega_1$ , we can see that  $|\omega_1| \leq k$  and  $M'[\omega_1]D$ . Thus,  $D$  is a deadlock marking of the MGSG.
- Case d: If  $\tau$  is not, there exists a maximal sound step  $\tau'$  s.t.  $\tau \subset \tau'$ . Let  $\tau' = \tau \cup \{t\}$ . If  $t$  is a transition of  $\omega_1$ , then  $t$  can be shifted to be the first transition of  $\omega_1$  and we can obtain an equivalent sequence of  $\omega_1$  s.t.  $\omega_1 \equiv t\omega'_1$ , where  $\omega'_1$  consists of transitions in  $\omega_1$  without  $t$ . The step  $\tau$  can be fired together with  $t$ , just as the firing of the step  $\tau'$ . Let  $M'_1$  be an intermediate marking reachable from  $M$  s.t.  $M[\tau']M'_1[\omega'_1]D$ . We can see that  $\tau \cup \{\omega_1\} = \tau' \cup \{\omega_1\} = [\omega]$ . The proof is similar to Case c.  $D$  is proven to be a deadlock marking of the MGSG. Else if  $t$  is not a transition of  $\omega_1$ , there may exist a transition  $t'$  of  $\omega_1$  that is in conflict with  $t$  s.t.  $t \# t'$ . In this case, there must exist another maximal sound step  $\tau'' = \tau \cup \{t'\}$ . Similarly, we can easily prove that  $D$  is a deadlock marking of the MGSG.

We have demonstrated that if  $|\omega| = k + 1$ ,  $D$  is a deadlock marking of the MGSG. It is obvious that  $D$  is a deadlock marking of the MGSG reachable from  $M$  for any length of  $\omega$ . The proof for these different cases is shown in Fig. 5.

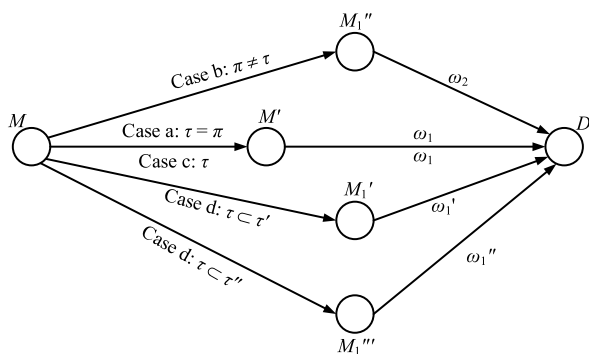


FIGURE 5. Four different cases of the proof.

### V. EXPERIMENTAL RESULTS

Several experimental results for different PNs are shown in this section, to demonstrate the superiority of the algorithm proposed in this paper. Two PNs are shown in Fig. 4(a) and Fig. 5, and the others are models of several different systems taken from the MCC (Model Checking Contest) held within Petri nets conferences: FMS, ClientAndServers (CAS in short), Dining Philosophers (DP in short), and Swimming Pool (SP in short).

First, we consider the PN4 depicted in Fig. 4(a). Its covering step graph (CSG), maximal persistent step graph (MPSG), and maximal good step graph (MGSG) are shown

TABLE 1. Algorithms evaluation on PN4.

PN4	RG	CSG	PG	PSG	MPSG	MGSG
Markings	13	7	7	6	6	4
Edges	20	8	7	6	6	3

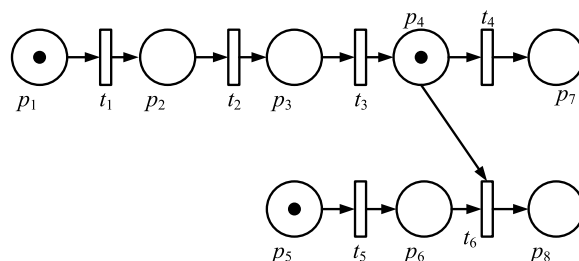


FIGURE 6. A model of the PN5.

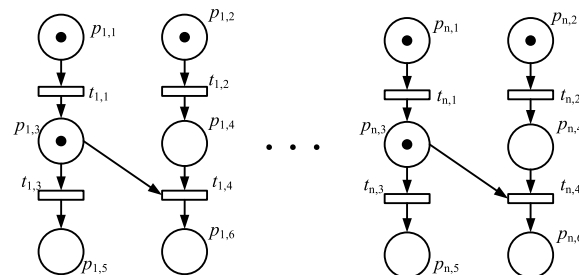


FIGURE 7. Parallel composition of n instances of PN4 ( $||_n$  PN4).

in Fig. 4(b), (c) and (d), respectively. The evaluation of different algorithms on PN4 is summarized in Table 1, focusing on their markings and directed edges labeled by transitions. It is obvious that the size of MGSG is smaller than other state graphs.

The parallel composition of the PN4 is shown in Fig. 7. We exhibit sizes and computing times of various state graphs with respect to these parallel PNs. These state graphs include the reachability graph (RG) computed by the tool TINA, the covering step graph (CSG) proposed in [52], the persistent graph (PG), the persistent step graph (PSG) of [53], the maximal persistent step graph (MPSG) introduced in [54] and the maximal good step graph (MGSG). The experimental results are shown in Table 2.

Actually, the PSG generalizes PG and any PSG is always smaller than the CSG [53]. For this model, the PSG and the MPSG are identical and the MGSG provides a significant reduction on the number of markings and edges compared with the MPSG.

Then, we consider the model of PN5 shown in Fig. 6. According to Definition 2, there exists a maximal sound step  $\tau$  of the initial marking  $M_0 = p_1 + p_4 + p_5$ , i.e.,  $\tau = \{t_1, t_4, t_5\}$ . By Theorem 1, the enabled step  $\tau$  is persistent at  $M_0$  since  $\tau$  is the only maximal sound step at  $M_0$ . More precisely,  $\tau$  is a maximal good step at  $M_0$ . Algorithm 1 establishes the MGSG of PN5 and this graph is shown in Fig. 8.

TABLE 2. Experimental results on parallel composition of Fig. 7.

PN4	RG	CSG	PG	PSG	MPSG	MGSG
<hr/>						
$\parallel_2$ PN4						
Markings	169	23	21	14	14	6
Transitions	520	44	26	19	19	5
CPU(s)	0	0	0	0	0	0
<hr/>						
$\parallel_3$ PN4						
Markings	2197	79	59	36	36	10
Transitions	10140	222	85	62	62	9
CPU(s)	0	0	0	0	0	0
<hr/>						
$\parallel_4$ PN4						
Markings	28561	287	169	98	98	18
Transitions	175760	1088	276	205	205	17
CPU(s)	1	0	0	0	0	0
<hr/>						
$\parallel_5$ PN4						
Markings	371293	1087	495	276	276	34
Transitions	2856100	5266	899	680	680	33
CPU(s)	8	0	0	0	0	0

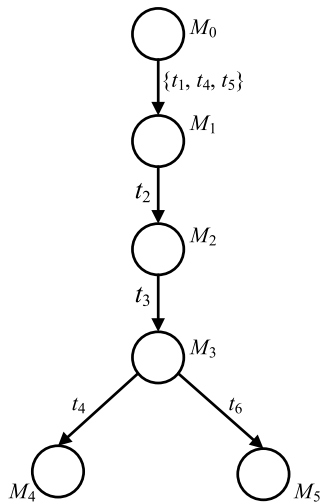


FIGURE 8. The MGSG of the PN5.

For the model of PN5 described in Fig. 6, the evaluation of different algorithms is shown in Table 3. This table summarizes the RG, the CSG, the PG, the PSG, the MPSG and the MGSG of PN5, focusing on their numbers of markings and directed edges of transitions, and computing seconds. Furthermore, the results of different algorithms on the n parallel PN5 are also shown in Table 3. The parallel composition of n instances of PN5 is represented by  $\parallel_n$ PN5.

We can see that the PG is smaller than the CSG, the PSG and the MPSG are identical and the MGSG improves other partial order methods such as the PG, the CSG, and the MPSG.

Next, we consider several PNs of different systems: the FMS (m), the CAS (m), the DP (m), and the SP (m), where m represents the parameter values of their initial markings. The model FMS (m) is a strongly-connected ordinary PN. The model CAS (m) is a strongly-connected loop-free ordinary PN, where there exists no transition whose input places are also output places. The dining philosophers is a famous

TABLE 3. Experimental results on parallel composition of the PN5.

PN5	RG	CSG	PG	PSG	MPSG	MGSG
<hr/>						
$\parallel_1$ PN5						
Markings	23	10	9	8	8	6
Transitions	40	11	9	8	8	5
CPU(s)	0	0	0	0	0	0
<hr/>						
$\parallel_2$ PN5						
Markings	529	26	25	16	16	10
Transitions	1840	41	30	21	21	9
CPU(s)	0	0	0	0	0	0
<hr/>						
$\parallel_3$ PN5						
Markings	12167	70	65	38	38	18
Transitions	63480	153	91	64	64	17
CPU(s)	0	0	0	0	0	0
<hr/>						
$\parallel_4$ PN5						
Markings	279841	194	177	100	100	34
Transitions	1496720	569	284	207	207	33
CPU(s)	4.83	0	0	0	0	0
<hr/>						
$\parallel_5$ PN5						
Markings	>30000	550	505	278	278	66
Transitions	>30000	2109	909	682	682	65
CPU(s)	>233	0	0	0	0	0

model that states an inappropriate use of shared resources generating deadlocks. This DP (m) is a strongly-connected loop-free ordinary PN. The SP (m) is a strongly-connected ordinary PN.

Indeed, MGSG provides a significant reduction on the number of markings and directed edges, and even the computing times compared with the RG for each model. In order to illustrate the superiority of the MGSG compared with other reduced graphs obtained by several partial order reduction methods, Table 4 shows the experimental results, including the state graphs computed by TINA such as the RG, the CSG, the PG, and the PSG, and the MGSG established via Algorithm 1. For some PNs such as the FMS (m), the PG provides a smaller graph than CSG and PSG since the firing of steps leads to more intermediate states compared with ones computed after the firing of transitions within persistent sets. Applying the MGSG method, we first obtain the maximal good step, which is a persistent set in essence. Thus, the firing of maximal good steps yields smaller intermediate states than ones obtained by the firing of transitions in persistent sets. It is obvious that the MGSG provides a significant reduction than the PG. For some PNs such as the CAS (m), the parameter of the initial marking influences the size of reduced graphs. Intuitively, the PSG is smaller than the PG when m is less than 8 and the PG is smaller than the PSG when m is higher than 16. For the model of Dining Philosophers, the PG, PSG and the MGSG are identical. The PSG is smaller than the PG and CSG for the model of Swimming Pool. More importantly, the size of CSG, PG, PSG and MGSG does not change, with increasing of parameters of the initial marking. For all these examples, MGSG builds a smaller graph than CSG, PG and PSG. Note that the MGSG technique improves or equals other partial order reduction methods.

TABLE 4. Experimental results on several PNs of different systems.

PN	RG	CSG	PG	PSG	MGSG	PN	RG	CSG	PG	PSG	MGSG
FMS(2)						FMS(3)					
Markings	3444	1538	48	101	32	Markings	48600	6440	62	171	46
Transitions	16311	3339	55	132	39	Transitions	297420	15510	71	222	55
CPU(s)	0	0	0	0	0	CPU(s)	1.43	0	0	0	0
FMS(4)						FMS(5)					
Markings	439185	17528	76	241	60	Markings	2895018	31126	90	311	74
Transitions	3170390	47784	87	312	71	Transitions	23527185	91438	103	402	87
CPU(s)	9.51	0.95	0	0	0	CPU(s)	95.68	1.46	0	0	0
CAS(4)						CAS(8)					
Markings	27576	756	163	158	158	Markings	7081638	62380	845	811	811
Transitions	113316	1187	177	172	172	Transitions	44030250	126372	941	936	936
CPU(s)	0	0	0	0	0	CPU(s)	251.35	1.57	0	0	0
CAS(16)						CAS(20)					
Markings	12462173	85522	1434	1436	1434	Markings	>39919315	>108664	6371	13178	6371
Transitions	77859168	173353	1595	1647	1595	Transitions	>240893998	>220334	7145	15691	7145
CPU(s)	>1000	1.92	0	0	0	CPU(s)	>1000	>233	0	0	0
DP(5)						DP(10)					
Markings	1223	6	6	6	6	Markings	21031	6	6	6	6
Transitions	8520	30	10	10	10	Transitions	173900	30	10	10	10
CPU(s)	0	0	0	0	0	CPU(s)	0.83	0	0	0	0
DP(15)						DP(20)					
Markings	127628	6	6	6	6	Markings	479061	6	6	6	6
Transitions	1121920	30	10	10	10	Transitions	4345550	30	10	10	10
CPU(s)	3.86	0	0	0	0	CPU(s)	11.07	0	0	0	0
SP(10)						SP(20)					
Markings	89621	4166	140	130	130	Markings	3408031	35006	280	260	260
Transitions	450003	8187	159	149	149	Transitions	19929811	69697	319	299	299
CPU(s)	1.79	0	0	0	0	CPU(s)	80.48	0.75	0	0	0

VI. CONCLUSION AND FUTURE WORK

Although the partial order reduction methods have made some success in alleviating the state space explosion problem of concurrent systems, further improvement is still needed in the state space exploration of large systems. In this paper, we have proposed a new partial order reduction method namely a maximal good step graph (MGSG), based on notions of sound and good steps. Firstly, we present a new definition of sound steps, which breaks the previous requirement of covering steps. Then, a weaker sufficient condition with practical significance is proposed, to find the set of sound steps at each marking. Next, combining with persistent sets and sound steps, the definition of good steps is established and it plays an important role in generating the maximal good step graph (MGSG). The MGSG generation algorithm first determine whether there exists a maximal good step at a marking  $M$ . If there exists one, the maximal good step is fired at  $M$ , otherwise the set of maximal sound steps is fired at  $M$ . Later, we have proven that the generated maximal good step graph preserves the deadlocks of Petri nets. Finally, the effectiveness of our proposed method (MGSG) have been shown via the performed

experimental results. Compared with the covering step graphs, persistent graphs and other reduced reachability graphs combing the above two graphs, the MGSG is more advantageous in state space reduction. In the future, we intend to focus on combining step graphs with any partial order technique as a mean to obtain a more reduction of the state space which is of very great interest for model-checking.

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**HANIFA BOUCHENE** is currently a Professor with the École Polytechnique de Montréal. Her research interests include formal verification techniques of real-time and infinite complex systems. She has published more than 100 research articles in international journals, conferences, workshops, and books. She has served as a member and as a chair in several program committees of conferences and workshops.



**XIAONING JIANG** received the M.E. degree in electronic engineering from Hangzhou Dianzi University, Hangzhou, China, in 1993, and the Ph.D. degree in computer science and technology from Zhejiang University, Hangzhou, China, in 2000. He is currently an Associate Professor, a Senior Engineer with Zhejiang Gongshang University, and the Vice Dean of the IoT Research Institute, Zhejiang Gongshang University. His research interests include applied information system, network and information security, industrial IoT, visual analytic, and Fin-tech. He has published over 30 research articles and 10 invention patents.



**SHOUGUANG WANG** (M'10–SM'12) received the B.S. degree in computer science from the Changsha University of Science and Technology, Changsha, China, in 2000, and the Ph.D. degree in electrical engineering from Zhejiang University, Hangzhou, China, in 2005.

He joined Zhejiang Gongshang University, in 2005, where he is currently a Professor with the School of Information and Electronic Engineering, the Director of the Discrete-Event Systems Group, and the Dean of the System Modeling and Control Research Institute, Zhejiang Gongshang University. He was a Visiting Professor with the Department of Electrical and Computer Engineering, New Jersey Institute of Technology, Newark, NJ, USA, from 2011 to 2012. He was the Dean of the Department of Measuring and Control Technology and Instrument, from 2011 to 2014. He is currently a Visiting Professor with the Electrical and Electronic Engineering Department, University of Cagliari, Cagliari, Italy, from 2014 to 2015. He is also an Associate Editor of IEEE Access and the IEEE/CAA JOURNAL OF AUTOMATICA SINICA.

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**HAO DOU** received the B.S. degree from the School of Information and Electronic Engineering, Zhejiang Gongshang University, China, in 2017, where she is currently pursuing the M.S. degree. Her current research interests include supervisory control of discrete event systems, and Petri nets theory and application.



**KAMEL BARKAOUI** received the Ph.D. degree and the Habilitation à Diriger des Recherches in computer science from Université Paris 6, in 1988 and 1998, respectively. He is currently a Professor of computer science with the Conservatoire National des Arts et Métiers, Paris. He has published more than 100 refereed international journal articles and conference papers. His research interests include formal methods for verification, control, and performance evaluation of concurrent and distributed systems. He received the IEEE International Conference on System, Man, and Cybernetics Outstanding Paper Award, in 1995. He has served as the program committee chair and organizing chairs for numerous international workshops and conferences. He was a Guest Editor of the *Journal of Systems and Software* and the *International Journal of Critical Computer-Based Systems*.