



HAL
open science

M/M/1 Retrial Queue with Collisions and Transmission Errors

Lamia Lakaour, Djamil Aissani, Karima Adel-Aissanou, Kamel Barkaoui

► **To cite this version:**

Lamia Lakaour, Djamil Aissani, Karima Adel-Aissanou, Kamel Barkaoui. M/M/1 Retrial Queue with Collisions and Transmission Errors. *Methodology and Computing in Applied Probability*, 2019, 21 (4), pp.1395-1406. 10.1007/s11009-018-9680-x . hal-02475784

HAL Id: hal-02475784

<https://hal.science/hal-02475784>

Submitted on 7 Mar 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

M/M/1 retrial queue with collisions and transmission errors

Lamia Lakaour · Djamil Aïssani · Karima
Adel-Aissanou · Kamel Barkaoui

the date of receipt and acceptance should be inserted later

Abstract In this paper, an M/M/1 retrial queue with collisions and transmission errors is considered. The collision may occur when a primary arriving customer finds the server busy while the transmission errors usually occur due to an erroneous packet or due to a non-ideal channel condition. We apply the generating function method to derive the joint distribution of the server state and the orbit length in steady state and we obtain important system characteristics. Finally, we present numerical examples to show the applicability of the model.

Keywords Retrial queue · Collisions · Transmission errors · Classical retrial rate · Generating function.

1 Introduction

Queues with repeated attempts are characterized by the feature that arriving customers who find the server busy join a virtual group of blocked customers, called orbit, to try again for their requests in a random order and at random time intervals. Queues in which customers are allowed to conduct retrials have been widely used to model many practical problems in telephone switching systems, telecommunication networks and computers competing.

For the retrial queues, it is necessary to define the mechanism of retrials. The most usual mechanism described in the classical theory of retrial queues is the so-called classical retrial policy in which each source (packet, call, etc.) in orbit seeks service independently of each other after an exponentially distributed time with mean $\frac{1}{\eta}$. Thus, in this policy the intervals between successive repeated attempts are exponentially distributed with rate $j\eta$, when the orbit size is j [1]. Another type of retrial policy, well known for the modeling of *ALOHA* protocol in communication networks, in which the time between two successive repeated attempts is controlled by an electronic device and consequently is independent of the number of customers applying for service. This type of retrial is named constant retrial policy, whose rate is $(1 - \delta_{j,0})v$, where $\delta_{j,0}$ denotes Kronecker's delta and j the number of repeated customers [11]. In [5], Artalejo and Gómez-Corral treat both models in a unified way by defining a linear retrial policy with rate $(1 - \delta_{j,0})v + j\eta$. The detailed overviews for retrial queue can be found in the

L. LAKAOUR

Research Unit LaMOS (Modeling and Optimization of Systems), Faculty of Exact Sciences, Bejaia University, Bejaia 06000, Algeria. E-mail: lakaour.lamia@gmail.com

D. AÏSSANI

Research Unit LaMOS (Modeling and Optimization of Systems), Bejaia, 06000 Bejaia, Algeria. E-mail: lamos_bejaia@hotmail.com

K. ADEL-AISSANOU

Research Unit LaMOS (Modeling and Optimization of Systems), Bejaia, 06000 Bejaia, Algeria. E-mail: ak_adel@yahoo.fr

K. BARKAOUI

CEDRIC, Conservatoire National des Arts et Métiers, Paris, France.
E-mail: kamel.barkaoui@cnam.fr

bibliographies of Artalejo [2]-[4], the surveys paper of Yang and Templeton [18], Falin [9], Choi and Chang [7] and Kim and Kim [14], the books of Falin and Templeton [10] and Artalejo and Gómez-Corral [6].

Retrial queues are widely discussed by researchers for modeling communication problems. However there are few who are interested in the problem of collisions. This phenomenon very widespread in telecommunications and whose management resolves many problems linked to the quality of service (QoS). Concretely, a collision occurs when two (or more) stations try to emit at the same time on the same channel. In the terminology of retrial queue, this phenomenon is translated by the fact that a customer arrives and finds the server busy, then the two customers (the arriving customer and the customer in service) join the orbit with a certain probability.

Queues with repeated attempts is found specifically in communication protocols modeling, such as *CSMA/CD*. Choi et al [8] have discussed a retrial queuing model with collision and constant retrial rate arising from unslotted *CSMA/CD* (Carrier Sense Multiple Access with Collision Detection) protocol. They have derived the generating function of the limiting distribution of the number of customers in the retrial group when the channel is available. In [8], the service time of the customer consists of two consecutive phases and the collision occurs when the arriving customer finds the channel busy with the first phase of a customer's service time. Kim [13] has considered the $M/M/1$ retrial queue with collision and impatience. Kumar et al [15] have analyzed the feedback retrial queuing system with collisions and linear retrial policy. Kvach and Nazarov [16] have considered the $M/M/1//N$ retrial queue with collision. In [12], a performance analysis of an $M/G/1$ retrial queue with general retrial time, modified M-vacations and collision has been considered. For those systems [12][13][15][16], if an arriving customer finds the server busy, then the collision occurs between the arriving customer and the customer in service resulting in both being shifted to the retrial group. In [15], the authors have considered that the probability that primary customers accede to the service when it is either free or busy is the same. However, this is not realistic. This shows the necessity of taking two different probabilities in our work and more details will be given in the rest of the paper.

Several results have been reported on retrial queues with collision by considering a specified retrial policy depending on each particular application. However, the study of retrial queueing systems taking into consideration both the collision and transmissions errors is interesting, and we do not find much work in this respect in the literature. Based on this observation, we have investigated retrial queueing system with collisions, transmissions errors and classical retrial rate. In this paper, we consider the case where probabilities of joining the server by primary customer when the server is idle and busy respectively are different. It is equal to p when the server is free and $(1 - \theta)$ when the server is busy.

In this paper:

- At the customer's arrival from outside the system, if the channel is free, this customer accedes to the server with probability p or joins the orbit with probability $(1 - p)$. In practice, for example, in the distributed coordination function (DCF) mode of IEEE 802.11, even if the channel is idle, the station doesn't transmit immediately, but transmits only if the server is idle for a period of time equal to DIFS (Distributed InterFrame Space). So, we have considered that the customer arriving from outside the system and finding an idle server accedes to the server with probability p ;
- If the server is busy on the arrival of the primary customer, this customer collides with the customer in service and both will join the orbit with probability $(1 - \theta)$, or joins alone the orbit with probability θ ;
- In addition to collisions, transmission errors are also taken into account. We have considered that the customer in service gets served without transmission error and leaves the system with probability γ or joins the retrial group for another attempt with probability $(1 - \gamma)$.

In this study, we consider a retrial queue with collisions and transmission errors. The collision may occur when a primary arriving customer finds the server busy. In this case, both the primary customer and the customer in service will join the orbit, while transmission errors usually occur due to an erroneous packet or due to a non-ideal channel condition. This retrial

queueing system can be applied for the modeling of the distributed coordination function (DCF) mode of IEEE 802.11 basic access scheme, where the loss and the re-transmission of packets can be due to either collisions or transmission errors. In IEEE 802.11, a station can not detect collisions or transmission errors while transmitting a packet, so the station can not interrupt its transmission. In order to notify the sender that the packet has been received successfully, a positive acknowledgment ACK is sent after receiving the packet.

The remainder of the paper is organized as follows. Section 2 presents the mathematical model description. Section 3 provides the steady-state probabilities of the orbit size and the server status, and the stability condition of the system. In Section 4, we obtain interesting performances using steady-state analysis, while in Section 5, we give numerical examples to illustrate the theoretical results. Section 6 gives concluding remarks .

2 Model description

We consider an $M/M/1$ retrial queue with collisions and transmission errors. Primary customers arrive from outside the system according to a Poisson process with rate λ . Upon the arrival of the primary customer, if the server is free, this customer accedes to the server with probability p or joins the orbit with probability $(1-p)$. The inter-retrial time of each customer in the orbit is exponentially distributed with parameter η , i.e. we assume that the repeated attempts follow the classical retrial policy. The customer that enters in service (the primary customer/the customer from the orbit) gets served successfully (without transmission error) and leaves the system with probability γ ($0 < \gamma \leq 1$) or joins the retrial group due to transmission error with probability $(1-\gamma)$. If the server is busy, the arriving customer (primary customer) collides with the customer in service resulting in both being shifted to the orbit with probability $(1-\theta)$ or joins immediately alone the orbit with probability θ .

We assume that the service times of customers are independent, follow an exponential distribution with mean $1/\mu$ and the stochastic processes involved in the system are independent.

The state of the system at time t can be represented by the pair $(C(t), X(t))$, where $X(t)$ denotes the number of customers in the orbit and $C(t)$ equals 1 if the server is busy and 0 if the server is free . It is clear that the process $\{C(t), X(t); t \geq 0\}$ is a continuous time Markov chain, irreducible, aperiodic and time-homogeneous with state space $S = \{(0, j), (1, j)/j = 0, 1, \dots\}$.

3 Analysis of the steady-state distribution

Let $P_j = \lim_{t \rightarrow \infty} P\{C(t) = 0, X(t) = j\}$ and $Q_j = \lim_{t \rightarrow \infty} P\{C(t) = 1, X(t) = j\}$, $j \geq 0$. They represent the joint distributions of the server state and the orbit length (i.e., the number of customers in the orbit) at steady state.

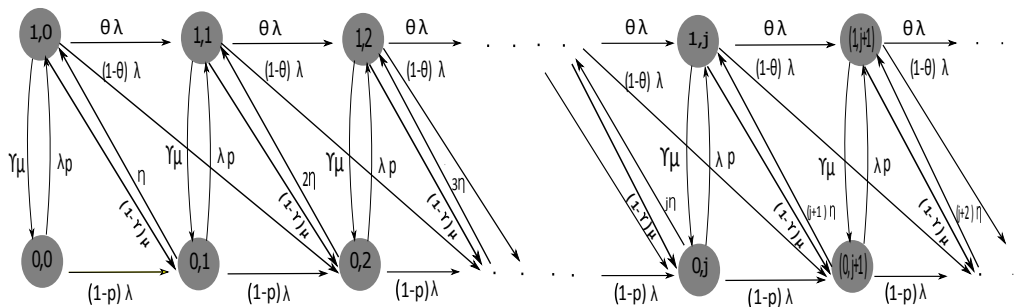


Fig. 1 Transition diagram of an $M/M/1$ retrial queue with collisions, transmission errors and classical retrial policy.

Using the balance equations of the Markovian system illustrated in Fig. 1, we obtain

$$\lambda P_0 = \gamma\mu Q_0, \quad (1)$$

$$(\lambda + \eta)P_1 = \lambda(1-p)P_0 + (1-\gamma)\mu Q_0 + \gamma\mu Q_1, \quad (2)$$

$$(\lambda + j\eta)P_j = (1-p)\lambda P_{j-1} + (1-\theta)\lambda Q_{j-2} + (1-\gamma)\mu Q_{j-1} + \gamma\mu Q_j, \quad j = 2, 3, \dots, \quad (3)$$

$$(\lambda + \mu)Q_0 = \lambda p P_0 + \eta P_1, \quad (4)$$

$$(\lambda + \mu)Q_j = \lambda p P_j + (j+1)\eta P_{j+1} + \lambda\theta Q_{j-1}, \quad j = 1, 2, 3, \dots \quad (5)$$

The condition for the system stability is

$$\lambda < \gamma\mu. \quad (6)$$

We use the partial generating functions method defined below to solve Eqs. (1)-(5)

$$P(z) = \sum_{j=0}^{\infty} P_j z^j \quad \text{and} \quad Q(z) = \sum_{j=0}^{\infty} Q_j z^j, \quad z \in [0, 1], \quad (7)$$

where $P(z)$ (respectively $Q(z)$) is the *PGF* (Probability Generating Function) of the number of customers in the orbit when the channel is idle (respectively busy).

By multiplying both sides of Eqs. (1)-(5) by z^j and summing, we obtain these following two first order linear differential equations

$$\eta z P'(z) + (\lambda - (1-p)\lambda z)P(z) = \gamma\mu Q(z) + (1-\gamma)\mu z Q(z) + (1-\theta)\lambda z^2 Q(z), \quad (8)$$

$$\eta P'(z) + \lambda p P(z) = \lambda Q(z) + u Q(z) - \lambda\theta z Q(z). \quad (9)$$

After multiplying both sides of Eq. (9) by z and subtracting from Eq. (8), we obtain

$$\lambda P(z) = (\gamma\mu - \lambda z)Q(z). \quad (10)$$

For $z = 1$, Eq. (10) becomes

$$\lambda P(1) = (\gamma\mu - \lambda)Q(1).$$

Under the stability condition (6), then $P(1) + Q(1) = 1$, it follows that

the probability that the server is busy is

$$Q(1) = \frac{\lambda}{\gamma\mu}, \quad (11)$$

the probability that the server is idle is

$$P(1) = 1 - \frac{\lambda}{\gamma\mu}. \quad (12)$$

By differentiating Eq. (10) with respect to z and substituting into Eq. (8), we obtain the following differential equation

$$Q'(z) + \frac{[(\theta\lambda^2 - p\lambda^2)z^2 + (p\lambda\gamma\mu - \lambda\eta - \mu\lambda - \lambda^2)z]}{-\lambda\eta z^2 + \eta\gamma\mu z} Q(z) = 0. \quad (13)$$

Resolving (13) for $Q(z)$ and after some computations, for $z \in (0, 1]$, we get

$$Q(z) = \frac{\lambda}{\gamma\mu} (\gamma\mu - \lambda)^{\frac{\eta + \mu + \lambda - \theta\gamma\mu}{\eta}} e^{\frac{(p-\theta)\lambda}{\eta}(1-z)} (\gamma\mu - \lambda z)^{-\frac{\eta + \mu + \lambda - \theta\gamma\mu}{\eta}}. \quad (14)$$

As $z \rightarrow 0+$, $Q(0) = Q_0$. Thus, from (14) and (1), we obtain

$$P_0 = (\gamma\mu - \lambda)^{\frac{\eta+\mu+\lambda-\theta\gamma\mu}{\eta}} e^{\frac{(p-\theta)\lambda}{\eta}} (\gamma\mu)^{-\frac{\eta+\mu+\lambda-\theta\gamma\mu}{\eta}}. \quad (15)$$

By substituting (14) into (10), we obtain the probability generating function $P(z)$ of the orbit size when the server is free

$$P(z) = \frac{\gamma\mu - \lambda z}{\gamma\mu} (\gamma\mu - \lambda)^{\frac{\eta+\mu+\lambda-\theta\gamma\mu}{\eta}} e^{\frac{(p-\theta)\lambda}{\eta}(1-z)} (\gamma\mu - \lambda z)^{-\frac{\eta+\mu+\lambda-\theta\gamma\mu}{\eta}}. \quad (16)$$

The probability generating function for the number of customers in the orbit denoted $K(z)$, is defined as $P(z)+Q(z)$, then

$$K(z) = \frac{\gamma\mu - \lambda z + \lambda}{\gamma\mu} (\gamma\mu - \lambda)^{\frac{\eta+\mu+\lambda-\theta\gamma\mu}{\eta}} e^{\frac{(p-\theta)\lambda}{\eta}(1-z)} (\gamma\mu - \lambda z)^{-\frac{\eta+\mu+\lambda-\theta\gamma\mu}{\eta}}. \quad (17)$$

The probability generating function for the number of customers in the system $H(z)$, defined as $H(z) = P(z) + zQ(z)$ is given in Eq. (18)

$$H(z) = (\gamma\mu - \lambda)^{\frac{\eta+\mu+\lambda-\theta\gamma\mu}{\eta}} e^{\frac{(p-\theta)\lambda}{\eta}(1-z)} (\gamma\mu - \lambda z)^{-\frac{\eta+\mu+\lambda-\theta\gamma\mu}{\eta}}. \quad (18)$$

4 System characteristics

The mean number of customers L_q (respectively L_s) in the orbit (respectively in the system) under steady-state condition are obtained by differentiating (17) (respectively (18)) with respect to z and then evaluating at $z = 1$

$$L_q = -\frac{\lambda}{\gamma\mu} - \frac{(p-\theta)\lambda}{\eta} + \frac{\lambda\eta + \lambda\mu + \lambda^2 - \lambda\theta\gamma\mu}{\eta(\gamma\mu - \lambda)}, \quad (19)$$

$$L_s = -\frac{(p-\theta)\lambda}{\eta} + \frac{\lambda\eta + \lambda\mu + \lambda^2 - \lambda\theta\gamma\mu}{\eta(\gamma\mu - \lambda)}. \quad (20)$$

The mean number of customers L_s and L_q in the system and the orbit respectively at the steady state are related by the following formula

$$L_s = L_q + Q(1),$$

where $Q(1)$ is given in Eq. (11).

The probability R of the orbit being empty is obtained by summing the two probabilities P_0 and Q_0 given by (1) and (15),

$$R = \frac{\lambda + \gamma\mu}{\gamma\mu} (\eta\gamma\mu - \lambda\eta)^{\frac{\eta+\mu+\lambda-\theta\gamma\mu}{\eta}} e^{\frac{(p-\theta)\lambda}{\eta}} (\eta\gamma\mu)^{-\frac{\eta+\mu+\lambda-\theta\gamma\mu}{\eta}}. \quad (21)$$

The mean waiting time W_q (respectively W_s) in the orbit (respectively in the system) is related to the mean number of customers L_q (respectively L_s) in the orbit (respectively in the system) by the Little formula [17], $L_q = \lambda W_q$ and $L_s = \lambda W_s$

$$W_q = -\frac{1}{\gamma\mu} - \frac{p-\theta}{\eta} + \frac{\eta + \mu + \lambda - \theta\gamma\mu}{\eta(\gamma\mu - \lambda)}, \quad (22)$$

$$W_s = -\frac{(p-\theta)}{\eta} + \frac{\eta + \mu + \lambda - \theta\gamma\mu}{\eta(\gamma\mu - \lambda)}. \quad (23)$$

The mean number of repeated attempts \bar{d} by a customer given in Eq. (24) can be determined as follows

$$\bar{d} = \eta W_q,$$

$$\bar{d} = -\frac{\eta}{\gamma\mu} - p + \theta + \frac{\eta + \mu + \lambda - \theta\gamma\mu}{\gamma\mu - \lambda}. \quad (24)$$

Another interesting system performance measure in this model of retrial queue is the steady-state interrupted frequency N of the service caused by collisions. This performance measure, given in Eq. (25), represents the collisions frequency caused when the arriving customer from outside the system (primary customer) accedes directly to the busy server,

$$N = (1 - \theta)\lambda Q(1). \quad (25)$$

By replacing $Q(1)$, by (11), we obtain

$$N = \frac{(1 - \theta)\lambda^2}{\gamma\mu}. \quad (26)$$

The busy period of the system L , starts with the arrival of a primary customer who finds the system empty and ends at the first departure epoch in which the system becomes empty again. From the theory of regenerative processes, it is easy to get the following formula for the expectation

$$E(L) = \frac{P_0^{-1} - 1}{\lambda},$$

using (15), $E(L)$ is obtained as

$$E(L) = \frac{1}{\lambda} \left((\eta\gamma\mu - \lambda\eta)^{-\frac{\eta+\mu+\lambda-\theta\gamma\mu}{\eta}} e^{-\frac{(p-\theta)\lambda}{\eta}} (\eta\gamma\mu)^{\frac{\eta+\mu+\lambda-\theta\gamma\mu}{\eta}} - 1 \right). \quad (27)$$

5 Numerical illustrations

In this section, we present some numerical examples to study the effect of the input parameters on the system characteristics.

In **Figs. 2-4**, to show the impact of θ ($(1 - \theta)$ is the collision probability), we plot P_0 , against η and that for three values of θ ($\theta = 0.2$, $\theta = 0.6$ and $\theta = 1$). For this, we choose $\lambda = 0.5$, $\mu = 1$ and $\gamma = 1$ and we take different values of p ($p = 0$, $p = 0.5$ and $p = 1$). We observe that P_0 increases as the value of the retrial rate η increases for any value of θ , but we observe that P_0 increases further as the probability θ increases. We report the same observation about the evolution of P_0 when we increase the value of p . These results are close to the real behavior of the system. In **Fig. 5**, we plot P_0 , against retrial rate η for three values of γ ($\gamma = 0.5$, $\gamma = 0.7$ and $\gamma = 0.9$); whereas the other parameters are fixed. When the probability γ becomes small, the number of customers (packets) that need retransmission due

to transmission errors becomes more important, which generates a large mass of customers in orbit, which leads to a low probability of having an empty orbit.

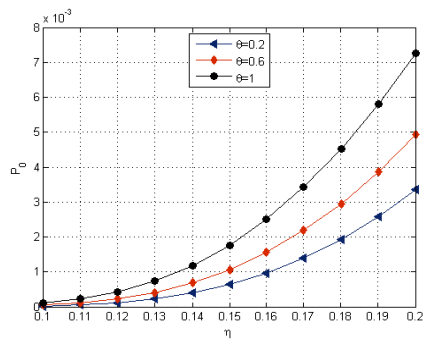


Fig. 2 P_0 as function of η for $p = 0$.

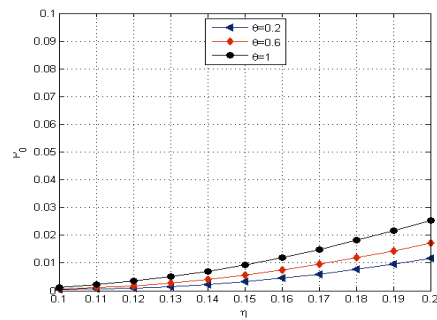


Fig. 3 P_0 as function of η for $p = 0.5$.

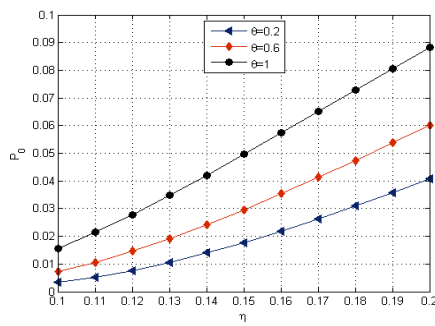


Fig. 4 P_0 as function of η for $p = 1$.

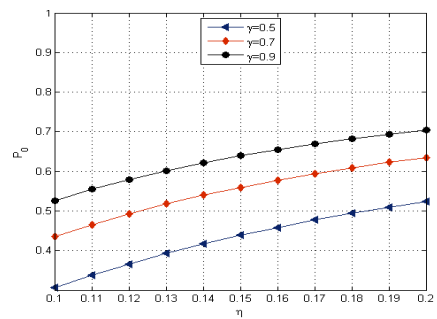


Fig. 5 P_0 as function of η for $\lambda = 0.05$
 $p = 0$, $\theta = 1$, $\mu = 1$.

For $p = 1$, $\lambda = 0.5$ and $\mu = 1$, **Figs. 6-8**, show how the number of customers L_q in the orbit changes when the value of η increases. For this, we represent this impact for different values of γ ($\gamma = 0.6$, $\gamma = 0.8$ and $\gamma = 1$). In each figure, we fixe θ ($\theta = 0$, $\theta = 0.5$ and $\theta = 1$). L_q is a decreasing function of the the parameters η , γ and θ .

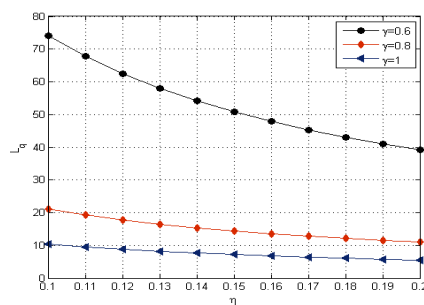


Fig. 6 L_q versus η for $\theta = 0$.

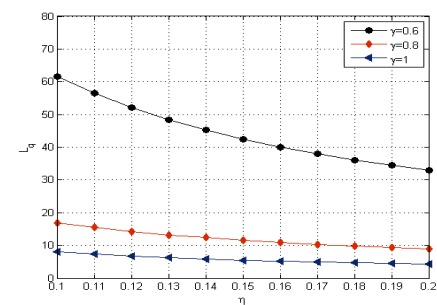


Fig. 7 L_q versus η for $\theta = 0.5$.

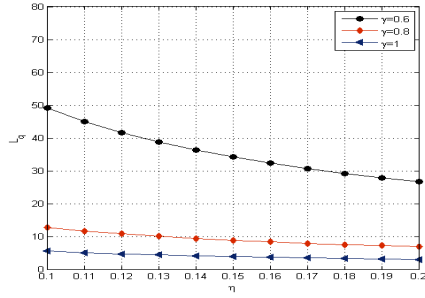


Fig. 8 L_q versus η for $\theta = 1$.

In **Fig. 9**, we want to illustrate how the steady-state interrupted frequency N of the service caused by collisions vary when the value of θ increases. We do this illustration, for two values of λ ($\lambda = 0.5$ and $\lambda = 0.9$). In **Fig. 10**, we give an overview of how the mean busy period $E(L)$ of the system changes when the value of η increases. This for different values of the probability θ .

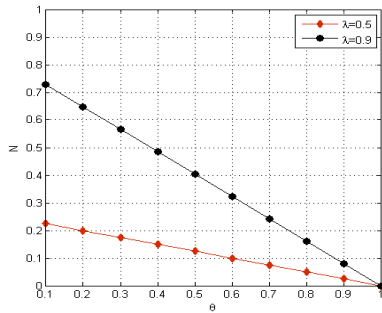


Fig. 9 N versus θ for $\eta = 0,01$
 $\mu = 1, p = 1, \gamma = 1$.

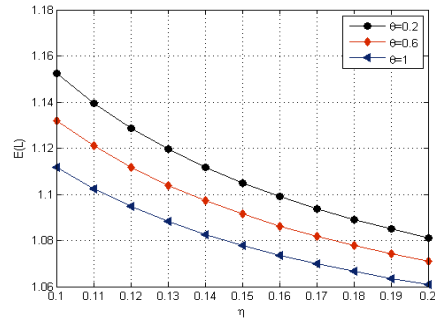


Fig. 10 $E(L)$ versus μ for $\lambda = 0,01$
 $\mu = 0,1, p = 1, \gamma = 1$.

It can be seen from **Fig. 9** that the steady-state interrupted frequency N of the service caused by collisions decreases considerably with respect to the probability θ . However, we observe that $N = 0$ when $\theta = 1$. This is evident and consistent with our results. It is observed also in **Fig. 10** that $E(L)$ decreases as η and θ increase.

6 Conclusion

In this work, a Markovian single server retrial queue with collisions and transmission errors has been investigated. Using the Markov process theory, we have derived the steady-state equations. We have obtained the joint steady-state probability generating functions of the server state and the orbit length, and we have computed some important characteristics. Finally, to validate our model, we did some numerical illustrations. The numerical results obtained confirm our analytical investigations. This retrial queue can be used for the modeling and performance analysis of the Distributed Coordination Function (DCF) of IEEE 802.11 basic broadcast mode (basic access scheme). Our model is different from those studied in the literature in the fact that the probability to join the orbit when the server is free is different when the server is busy. The collision in this study is limited to primary customers that finds the server busy. In future work, we plan to study both collisions due to retrial of customers from the orbit and from outside the system.

Acknowledgements

The authors would like to thank the referee for valuable suggestions and comments that help to improve the presentation of the paper.

References

1. Aissani, A. (2008). Optimal control of an $M/G/1$ retrial queue with vacations. *Journal of Systems Science and Systems Engineering*, vol. 17, pp. 487–502.
2. Artalejo, J. R. (1999). Accessible bibliography on retrial queues. *Mathematical and computer modelling*, vol. 30, pp. 1–6.
3. Artalejo, J. R. (1999). A classified bibliography of research on retrial queues: progress in 1990-1999. *Sociedad de Estadística e Investigación Operativa*, Top, 7, pp. 187–211.
4. Artalejo, J. R. (2010). Accessible bibliography on retrial queues: Progress in 2000-2009. *Mathematical and Computer Modelling*, vol. 51, pp. 1071–1081.
5. Artalejo, J. R., & Gómez-Corral, A. (1997). Steady-state solution of a single-server queue with linear repeated requests. *Journal of Applied Probability*, vol. 34, pp. 223–233.
6. Artalejo, J. R., & Gómez-Corral, A. (2008). *Retrial Queueing Systems: A Computational Approach*. Springer-verlag Berlin.
7. Choi, B. D., & Chang, Y. (1999). Single server retrial queues with priority calls. *Mathematical and Computer Modelling*, vol. 30, pp. 7–32.
8. Choi, B. D., Shin, Y. W., & Ahn, W. C. (1992). Retrial queues with collision arising from unslotted CSMA/CD protocol. *Queueing Systems*, vol. 11, pp. 335–356.
9. Falin, G. (1990). A survey of retrial queues. *Queueing Systems*, vol. 7, pp. 127–167.
10. Falin, G. I., & Templeton, J. G. C. (1997). *Retrial queues*. Chapman and Hall, London.
11. Fayolle, G. (1986, December). A simple telephone exchange with delayed feedbacks. In O. J. Bozma, J. W. Cohen and M. C. Tijms (eds.), *Teletraffic Analysis and Computer Performance Evaluation*, Elsevier Science, Amsterdam, pp. 245–253.
12. Jailaxmi, V., Arumuganathan, R., & Kumar, M. S. (2017). Performance analysis of an $M/G/1$ retrial queue with general retrial time, modified M-vacations and collision. *Operational Research*, vol. 17, pp. 649–667.
13. Kim, J. S. (2010). Retrial queueing system with collision and impatience. *Communications of the Korean Mathematical Society*, vol. 25, pp. 647–653.
14. Kim, J. and Kim, B. (2016), A survey of retrial queueing systems. *Annals of Operations Research*, vol. 247, pp. 3–36.
15. Kumar, B. K., Vijayalakshmi, G., Krishnamoorthy, A., & Basha, S. S. (2010). A single server feedback retrial queue with collisions. *Computers and Operations Research*, vol. 37, pp. 1247–1255.
16. Kvach, A., & Nazarov, A. (2015). Sojourn Time Analysis of Finite Source Markov Retrial Queueing System with Collision, Proceedings of the 14th International Scientific Conference, ITMM 2015, November 18–22; Terpigov, A. F., Anzhero-Sudzhensk, Russia.
17. Little, J. D. C. (1961). A proof for the queuing formula: $L = \lambda W$. *Operations Research*, vol. 9, pp. 383–387.
18. Yang, T. & Templeton, J. G. C. (1987). A survey on retrial queues. *Queueing Systems*, vol. 2, pp. 201–233.