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Harmonic Balance Finite Element Method Applied to Electrical Machines with rotor movement : Comparison of Two Potential Formulations

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In the field of computational electromagnetics, taking into account the motion of the rotor in electrical machines requires special attention. In this work, we propose to compare the solutions obtained by the Harmonic Balance Finite Element Method (HB-FEM) coupled, on the one hand, with the \mathbf{A} formulation and, on the other hand, with Ω formulation with those obtained by the reference method namely, the Time Stepping Finite Element Method (TS-FEM). This comparison is carried out in the case of electrical machines while considering the movement of the rotor. To this issue, a permanent magnet machine is considered.

Index Terms— \mathbf{A} formulation, Ω formulation, Harmonic Balance Method, Time Stepping Method, permanent magnet machine.

I. INTRODUCTION

There are two principal methods combined with the Finite Element Method (FEM) currently used to model electromagnetic problems : the Time Stepping Method (TSM) and the Harmonic Balance Method (HBM). The choice between the frequency domain approach and the time one is a frequent question whose answer depends often of applications considered. Indeed, HBM plays an important role, mainly when only the steady state is investigated. It differs from TSM since it directly solves the steady-state response of the electromagnetic field in the frequency domain. Although the HBM is widely applied in electromagnetic problems, this research field remains always very active due to challenges to overcome, depending on the approximation of the solution as well as the formulations under consideration. Indeed, on the one hand, when dealing about electrical machines, the motion of the rotor requires a flexible implementation since it produces harmonics due to slots effect. On the other hand, in order to model in 3D electromagnetic devices, some formulations can be used according to the problem. Indeed, when using the $\mathbf{A}-\varphi$ formulation, a gauge condition must be imposed if one wants to ensure uniqueness of the three-dimensional formulations. However, it is well known that taking into account this condition complexifies the numerical implementation. To avoid this problem, one may use the $\mathbf{T}-\Omega$ formulation. This paper focuses on the comparison of both formulations combined, on the one hand, with the HB-FEM and, on the other hand, with the TS-FEM when considering motion in machines. Since HBM applied to magnetodynamic problems don't induce any difficulty with respect to magnetostatic one, we choose in this paper to present the results for magnetostatic problem focusing on the comparison of potentials when taking into account motion. Moreover, it is important to note that some works related to HBM have already been established for the \mathbf{A} formulation [1] but not yet for Ω one.

In the next section, the \mathbf{A} and Ω formulations are recalled for magnetostatic problems, to model problems taking into account

the movement of machines. Then, the HBM is presented, in the third section, and applied to the problem under consideration. Finally, in the fourth section, we present and discuss our numerical results.

II. THEORETICAL MODEL

The scalar and vector potential formulations constitute a key ingredient to solve a magnetostatic problem. Taking into account the movement of the machine, the linear magnetostatic vector potential and the linear magnetostatic scalar potential are respectively given by

$$\begin{cases} \mathbf{curl} (\nu \mathbf{curl} \mathbf{A}(\theta)) = \mathbf{J}_s & \text{in } D, \\ \mathbf{A} \times \mathbf{n} = 0 & \text{on } \Gamma, \end{cases} \quad (1)$$

and

$$\begin{cases} \text{div} (\mu (\mathbf{H}_s - \mathbf{grad} \Omega(\theta))) = 0 & \text{in } D, \\ \mu \mathbf{grad} \Omega \cdot \mathbf{n} = \mu \mathbf{H}_s \cdot \mathbf{n} & \text{on } \Gamma, \end{cases} \quad (2)$$

where \mathbf{A} respectively Ω represents the unknown vector respectively scalar potential depending of the angular position θ ; ν is the reluctivity; μ is the magnetic permeability and \mathbf{n} is the normal vector. \mathbf{H}_s represents a source magnetic field such that $\mathbf{curl} \mathbf{H}_s = \mathbf{J}_s$ where \mathbf{J}_s is the current density. When applying the FEM to (1) or (2), the system to be solved can be represented as follows

$$\mathbf{M}(\theta(t)) \mathbf{X}(t) = \mathbf{F}(t), \quad (3)$$

with \mathbf{X} the vector solution corresponding to the circulation of \mathbf{A} on all edges of the mesh in the case of \mathbf{A} formulation or the vector solution corresponding to the value of Ω on the nodes in the case of Ω formulation. $\mathbf{F}(t)$ is the source vector.

One of the major tasks that we have to deal with electrical machines is the movement involving in the rotor, which is taken into account in our paper by the locked-step method [2]. This choice is motivated not only by its easy implementation but also by the fact that the mesh remains conform. Therefore, $\mathbf{M}(\theta(t))$ can be rewritten as $\mathbf{M}(\theta(t)) = \mathcal{M} + \mathcal{M}_\theta(t)$. \mathcal{M} respectively $\mathcal{M}_\theta(t)$ is the matrix related to the fixed domains

respectively the domains when movement occurs. To approximate $\mathbf{X}(t)$, one may use HBM.

III. HARMONIC BALANCE METHOD AND ITS APPLICATION TO MACHINES TAKING INTO ACCOUNT THE MOVEMENT

Due to the time periodicity of the solution, HBM can be applied [1]. The main idea of this method is to approximate the solution by Fourier series, truncated at some finite number N such as

$$\mathbf{X}(t) \approx \sum_{k=-N}^N \mathbf{X}_k e^{j\omega k t}, \quad (4)$$

where ω is the fundamental frequency of the solution. Furthermore, applying the HBM to the system (3), one can derive the following algebraic equations : $-N \leq l \leq N$

$$\frac{1}{T} \int_0^T \mathbf{M}(\theta(t)) \sum_{k=-N}^N \mathbf{X}_k e^{j\omega(k-l)t} dt = \frac{1}{T} \int_0^T \mathbf{F}(t) e^{-j\omega l t} dt,$$

which can be rewritten in the following matrix form

$$\left(\begin{bmatrix} \mathcal{M} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathcal{M} \end{bmatrix} + \begin{bmatrix} \mathcal{R}_{-N,-N} & \dots & \mathcal{R}_{-N,N} \\ \vdots & \ddots & \vdots \\ \mathcal{R}_{N,-N} & \dots & \mathcal{R}_{N,N} \end{bmatrix} \right) \begin{bmatrix} \mathbf{x}_{-N} \\ \vdots \\ \mathbf{x}_N \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{-N} \\ \vdots \\ \mathbf{G}_N \end{bmatrix}$$

$$\text{where } \mathcal{R}_{l,k} = \frac{1}{T} \int_0^T \mathcal{M}_\theta(t) e^{j\omega(k-l)t} dt$$

$$\text{and } \mathbf{G}_l = \frac{1}{T} \int_0^T \mathbf{F}(t) e^{-j\omega l t} dt \quad \text{with } T = \frac{1}{\omega}.$$

Once this system is solved, the time solution is reconstructed using the coefficients $(\mathbf{X}_k)_{-N \leq k \leq N}$ given by (4).

IV. NUMERICAL RESULTS AND DISCUSSION

The accuracy of the HBM coupled with **A** or with Ω formulations is evaluated by considering an electrical motor and by comparing results with the reference method (TSM). Let us consider a synchronous machine (Fig. 1) composed of 4 permanent magnets (PM), 2 pole pairs in the rotor and 18 slots in the stator with a three phase connected windings. Only half of the machine is modeled due to its symmetry, as depicted in Fig.1. All the numerical results exposed below are carried out by prescribing $N = 5$ in the equation (4). First of all, the electromotive force has been compared for HBM combined with both formulations. We remark in Fig. 2 that curves match well. Moreover, the relative error of HBM in terms of the electromotive force with respect to TSM is around 0,7 % respectively 0,8 % for **A** formulation respectively Ω formulation. These results demonstrate the efficiency of our approach and the sufficiency of 5 harmonics in this case. Concerning the computational time, no difference has been detected between HBM and TSM since no transient phenomenon occurs. Moreover, in order to study locally the accuracy of HBM, an element below the stator slot has been selected as indicated in Fig. 1 (red cross). We have compared our results with respect to the x-component of the magnetic flux density \mathbf{B} (see Fig. 3). It can be observed that the obtained results with different methods as well as formulations are very close, except for the case of harmonics higher than 5 where

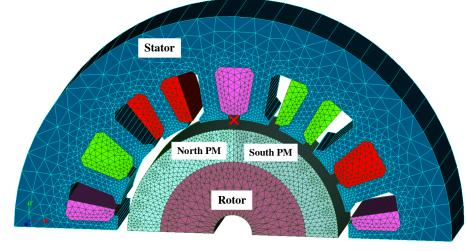


Fig. 1. Permanent magnet machine (6138 prisms).

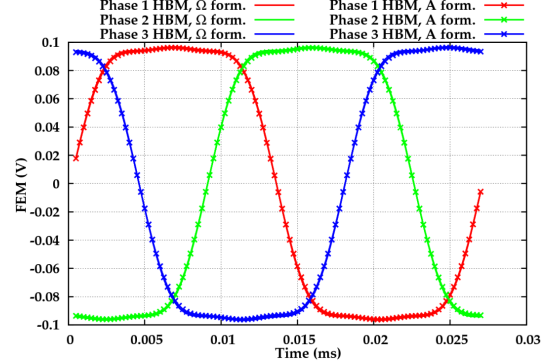


Fig. 2. The FEM for HBM combined with Ω and **A** formulations.

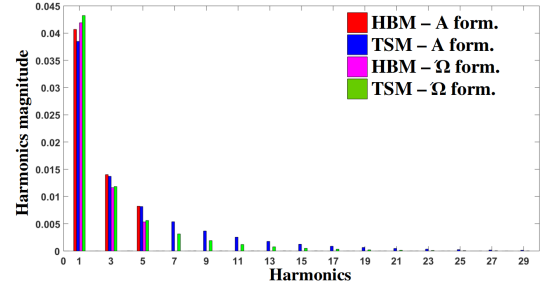


Fig. 3. The x-component of the magnetic flux density \mathbf{B} for both TS-FEM and HB-FEM and both Ω formulation and **A** formulation .

information is missing. Here, we need to take into account more harmonics in order to detect the information concerned the harmonics created due to the slots effect. This is considered as a drawback of the HBM. Some works are in the process to predict the sufficient harmonics to take into account in the solution (4). Another issue of further research might be to involve this comparison to the non-linear electromagnetic problems [3].

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