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# Harmonics Prediction Algorithm to Solve Nonlinear Magnetostatic Problems with the Harmonic Balance Method

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The problem of harmonics selection when solving an electromagnetic problem using harmonic balance method combined with finite element method is still a challenge. In this paper, a new technique based on the time residual of the solution is proposed to handle this kind of problem. Numerical results are provided to demonstrate the efficiency of our approach for the case of 2D nonlinear magnetostatic problem.

**Index Terms**—Harmonic balance method, determination of harmonics, finite element method, nonlinear magnetostatic problem.

## I. INTRODUCTION

Modeling electromagnetic problems can be carried out using the finite element method combined either with time stepping method (TS-FEM) or harmonic balance method (HB-FEM) [1], [2]. When the transient phenomenon occurs, TS-FEM leads to a substantial calculation time to reach the steady-state response. To tackle this issue, the harmonic balance method (HBM) may be one of the most suitable approaches. This method relies on the representation of time-periodic functions by a truncated Fourier series with a finite number of harmonics. One specific HBM feature is that the accuracy of the solution depends on the harmonics considered in the summation. Therefore, the most important problem when applying HBM is the selection of harmonics. In electromagnetism, the non-linear behavior as well as the movement of machines yield coupling between the components at different frequencies and so produce harmonics. Since computational cost increases with each harmonic included in the solution, only harmonics with a significant contribution must be taken into account for a given precision. The issue of this work is so to predict these sufficient harmonics. In the literature, some techniques, also known as adaptive HBM are proposed in the field of structural dynamics [3]. Some of these methods are based on the observation of the relative variation for two consecutive harmonics. In this case, the algorithm may stop before saturation which constitutes a limitation of these methods. To avoid this drawback, we propose in this paper, a novel approach based on the time residual when using HBM applied to a nonlinear magnetostatic problem. Up to our knowledge, although the HBM is widely applied in electromagnetic problems, no such approach has been proposed in the literature. In the next section, the FE model of non-linear magnetostatic problem coupled with HBM is presented. Then, the residual approach is described in the third section. The proposed algorithm is applied to a 2D example involving a transformer in the fourth section.

## II. NONLINEAR MAGNETOSTATIC PROBLEM COUPLED WITH HARMONIC BALANCE METHOD

Using the 2D magnetic vector potential, the formulation of the non-linear magnetostatic problem under consideration is

$$\text{curl} (\nu(\mathbf{B}) \text{curl} \mathbf{A}) = \mathbf{J}_s,$$

where  $\nu(\mathbf{B})$  represents the reluctivity depending on the magnetic flux density  $\mathbf{B}$ .  $\mathbf{J}_s(x, t) = \mathbf{N}(x) i(t)$  is the current density with  $\mathbf{N}$  the unit current density and  $i$  the current flowing through the stranded inductor.  $\mathbf{A}$  is the unknown magnetic vector potential. Applying the FEM, the system to be solved can be represented as follows

$$\mathbf{M}(\mathcal{A}(t))\mathcal{A}(t) = \mathbf{F}(t), \quad (1)$$

with  $\mathcal{A}(t) \in \mathbb{R}^{N_x}$  the 2D discretization of  $\mathbf{A}(t)$  on the nodes.  $N_x$  is the number of degrees of freedom.  $\mathbf{F}(t) \in \mathbb{R}^{N_x}$  is the vector corresponding to  $\mathbf{J}_s$  and  $\mathbf{M}(\mathcal{A})$  is the curl-curl matrix. We choose to use a fixed-point algorithm to solve the equation (1). Thus, we rewrite the system as

$$\mathbf{M}_{\nu_{fp}}\mathcal{A}(t) = \mathbf{F}(t) - \mathbf{M}_{fp}(\mathcal{A}(t)), \quad (2)$$

like the classical TS-FEM algorithm.  $\mathbf{M}_{\nu_{fp}}$  is the curl-curl matrix corresponding to a constant  $\nu_{fp}$  and  $\mathbf{M}_{fp}(\mathcal{A}(t))$  is the vector corresponding to the virtual magnetization vector  $\mathbf{H}_{fp}(\mathbf{B}) = (\nu(\mathbf{B}) - \nu_{fp})\mathbf{B}$ . Here, the magnetic field  $\mathbf{H}$  has been expressed as  $\mathbf{H} = \nu_{fp}\mathbf{B} + \mathbf{H}_{fp}(\mathbf{B})$ . Assuming the solution  $\mathcal{A}(t)$  to be periodic in time and of a fundamental frequency  $\omega$ , enables us to use HBM.  $\mathcal{A}(t)$  is so developed as a complex Fourier series, whose expansions must be truncated to a specified number of harmonics  $N$ , i.e.

$$\mathcal{A}(t) \approx \sum_{k=0}^N \mathbf{A}_k e^{j\omega k t}. \quad (3)$$

Then, by substituting  $\mathcal{A}$  in (2) with this summation, multiplying by a set of test functions  $e^{-j\omega l t}$ ,  $0 \leq l \leq N$  and integrating the equation over a period  $T$ , a system of differential algebraic equations is obtained under the form:  $0 \leq l \leq N$

$$\mathbf{M}_{\nu_{fp}}\mathbf{A}_l = \frac{1}{T} \int_T (\mathbf{F}(t) - \mathbf{M}_{fp}(\mathcal{A}(t))) e^{-j\omega l t} dt. \quad (4)$$

As detailed above, this equation is solved using a fixed point algorithm.

The method proposed in the next section focuses on the determination of the predominant harmonics to consider in (3).

### III. HARMONICS PREDICTION APPROACH

The most important question when applying HBM is how to select the harmonics used to approximate the solution (3). In order to answer to this question, we present in this section, a harmonic balance selection technique. This method is based on the analysis of the time residual of the harmonic solution. Indeed, first of all, we solve (4) taking into account only the harmonics occurring in  $i(t)$ . Then, the temporal solution  $(\mathcal{A}(t_j))_{0 \leq j \leq N_T}$  is reconstructed from (3) for each time step such as  $(t_0, \dots, t_{N_T})$  is a subdivision of the interval  $[0, T]$ . The next step is to compute the residual matrix  $\mathbf{R}$  defined by

$$\mathbf{R}(t_j) = \mathbf{F}(t_j) - \mathbf{M}(\mathcal{A}(t))\mathcal{A}(t_j) \quad \forall 0 \leq j \leq N_T, \quad (5)$$

where each row of the matrix represents the temporal evolution over the period  $[0, T]$  of the residual defined in one degree of freedom. Finally, according to the FFT of the residual, one can adjust the harmonics retained when solving (4). It is important to note that one can only compute the FFT of the residual in the non-linear subdomain due to the saturation, which reduces the computational time. This method is summarized in the algorithm below, where  $\epsilon$  is the error criterion threshold chosen by the user.

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#### Algorithm 1: Harmonics prediction algorithm

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1 begin
2    $\mathbf{R}(t_j) = \mathbf{F}(t_j)$  for  $0 \leq j \leq N_T$ .
3   while  $\|\mathbf{R}\| \geq \epsilon$  do
4     Compute the FFT of the time residual  $\mathbf{R}$ .
5     Retain the harmonics with a significant
       contribution, from the FFT of the residual  $\mathbf{R}$ 
       and add them to the set  $S$  of harmonics.
6     Solve the system (4) taking into account the
       harmonics in  $S$ .
7     Reconstruct the solution  $(\mathcal{A}(t_j))_{0 \leq j \leq N_T}$  (3).
8     Compute the residual  $(\mathbf{R}(t_j))_{0 \leq j \leq N_T}$  (5) in the
       domains of interest.
9   end
10  return the solution  $(\mathcal{A}(t_j))_{0 \leq j \leq N_T}$ .
11 end

```

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### IV. NUMERICAL VALIDATION AND COMMENTS

To illustrate the efficiency of our proposed algorithm, a 2D nonlinear magnetostatic example, made of a single phase transformer (Fig. 1) with a sinusoidal current sufficiently high to ensure the saturation, is studied. The numerical results demonstrate that after one iteration of the algorithm, one can predict the predominant harmonics due to the nonlinear effects. Indeed, the FFT of the residual, computed in an element selected inside the nonlinear magnetic subdomain, indicates the harmonics, that one should consider in the system (4) (see Fig. 2). A peak is observed for each harmonic with a significant contribution. On the other hand, the FFT of the residual obtained in the second iteration decreases since one considers the new retained harmonics as illustrated in Fig. 3. To validate the proposed approach, the results are compared with those obtained from TS-FEM. The relative error in terms of the magnetic potential vector obtained with our approach with respect to the one obtained by TS-FEM is about  $10^{-5}$ . These

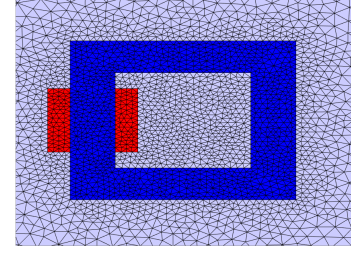


Fig. 1. Mesh of the geometry (3167 nodes, 6252 elements).

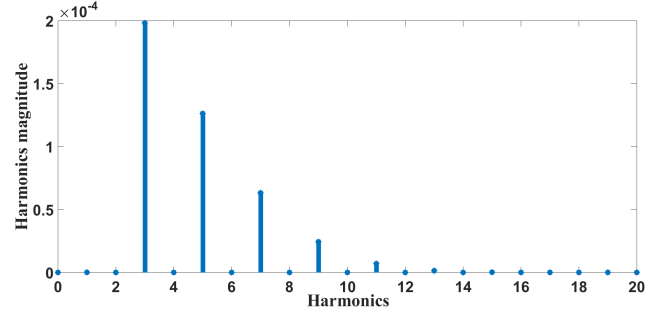


Fig. 2. FFT of the time residual  $\mathbf{R}$  after the first iteration.

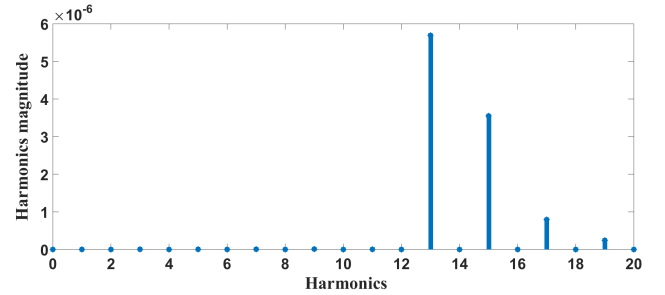


Fig. 3. FFT of the time residual  $\mathbf{R}$  after the second iteration.

numerical results demonstrate therefore the effectiveness of the algorithm proposed, whose implementation is straightforward. The major advantage of the algorithm proposed is that it requires only harmonics occurring in the current source to predict the predominant harmonics. Moreover, the proposed approach can be easily extended to the case of electrical machines in movement.

### V. ACKNOWLEDGMENTS

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