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Scaling arguments to experimentally model deep oceans trapped between icy layers on Ganymede

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Abstract - The potential habitability in icy giant moons such as Ganymede, Callisto (moons of Jupiter), and Titan (moon of Saturn) has been recently studied, where some basic conditions of life may be abundant. Namely, the conditions include the presence of liquid water, a stable source of energy and supply of nutrients. A systematic understanding of the physical processes taking place on these giant moons may be a key to addressing the fundamental question of habitability of these moons. A common feature of such giant moons relates to the presence of a liquid water layer sandwiched in between two layers of ice: a surface layer and a deep layer with a different polymorphism triggered by the high pressure conditions. The study of the exchange processes that occur in the deep layers of giant icy moons and water-rich worlds is a highly non-trivial one. The origin of the natural phenomena cannot be easily reproduced in laboratory experiments particularly because the high pressures acting on the inner ice layers of the moons are virtually impossible to reproduce in table top laboratory experiments. However, based on the fundamental rule of thermodynamic phases stated by Gibbs, we attempt herein to replace in the table top experiments the pressure differences existing on the moons by another pair of (thermodynamically) conjugated variables: an external shear and a stress. To support this, we present preliminary scaling arguments and design a table-top experimental setup able to capture the main physics taking place within the giant moons. The similarity is mainly based on the similarity between the convective and thermo-diffusive time scales. Additionally, we pay attention that the flows at laboratory scale remain laminar at all times which is the case of the convective flows existing on the moons. As an appropriate phase change material that models the ice we chose a paraffin wax. The thermophysical and rheological properties of the paraffin wax have been investigated which allows us to estimate the suitable dimensions of the table top experimental setup that may capture most physics taking place on the giant moons.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_p$</td>
<td>heat capacity, J/kg.K</td>
</tr>
<tr>
<td>$D$</td>
<td>rheometer plate diameter, mm</td>
</tr>
<tr>
<td>$D_e$</td>
<td>depth of the setup, m</td>
</tr>
<tr>
<td>$g$</td>
<td>gravity, m/s$^2$</td>
</tr>
<tr>
<td>$H$</td>
<td>height of the HP ice mantle / HP ice model, m</td>
</tr>
<tr>
<td>$h$</td>
<td>rheometer gap, mm</td>
</tr>
<tr>
<td>$L$</td>
<td>latent heat, J/kg</td>
</tr>
<tr>
<td>$P$</td>
<td>pressure, Pa</td>
</tr>
<tr>
<td>$Q$</td>
<td>molar heat, J/mol</td>
</tr>
<tr>
<td>$R$</td>
<td>gas constant, J/molK</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature, K</td>
</tr>
<tr>
<td>$t$</td>
<td>time scale, s</td>
</tr>
<tr>
<td>$V$</td>
<td>molar volume, m$^3$/mol</td>
</tr>
<tr>
<td>$v$</td>
<td>average velocity, m/s</td>
</tr>
<tr>
<td>$W$</td>
<td>width of the setup, m</td>
</tr>
</tbody>
</table>

$x$ similarity criterion ratio

Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>density, kg/m$^3$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>thermal diffusivity, m$^2$/s</td>
</tr>
<tr>
<td>$\mu$</td>
<td>kinematic viscosity, Pa.s</td>
</tr>
<tr>
<td>$\dot{\gamma}$</td>
<td>shear rate, 1/s</td>
</tr>
</tbody>
</table>

Index and exponent

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>bottom</td>
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<tr>
<td>$f$</td>
<td>final</td>
</tr>
<tr>
<td>$i$</td>
<td>initial</td>
</tr>
<tr>
<td>$m$</td>
<td>melt</td>
</tr>
<tr>
<td>$o$</td>
<td>original</td>
</tr>
<tr>
<td>$t$</td>
<td>top</td>
</tr>
</tbody>
</table>
1. Introduction

The question of habitability in other planets have long been of interest for scientists. A fundamental condition for life sustainability relates to the presence of water, a stable source of energy and supply of nutrients, [1]. Recently, the attention has been redirected from our nearest neighbour Mars (which holds some ice but only sediments of previously existing liquid water [2]), to giant icy moons such as Europa, Ganymede and Callisto (moons of Jupiter) and Titan (moon of Saturn).

This has been prompted by the observations performed during Galileo’s mission in the 90’s which demonstrated the presence of saline water in Europa and in Ganymede [3, 4]. Further observations by NASA’s Hubble Space Telescope in 2015, showed the aurora of Ganymede being affected by its saline water reacting to its internal magnetic field [5].

Even more recently, the Cassini-Huygens mission that ended in 2017 made a surprising discovery from measurements during close flybys near the south pole of the Enceladus moon. Not only the presence of liquid water was demonstrated, but some other basic requirements of habitability related to the thermal energy and the chemical composition were also hinted [6, 7].

On icy moons, underneath an external ice crust (referred to as ice I), the existence of subsurface layers of water above the silicate mantle has been demonstrated. According to the volume of these inner water layers, two types of moons may be distinguished. Corresponding to the first type, the volume of water is relatively small and, consequently, submitted to a moderate difference of the hydrostatic pressure. In this case the inner water solely exists in a liquid phase. This is the case of the Enceladus and Europa moons.

The second type of moons contain much larger volumes of water which, consequently, span much larger depths measured along the radial coordinate of the moon. In this case, the water is subjected to much larger hydrostatic pressures which, according to the thermodynamic phase diagram of the water, is consistent with the emergence of an extra icy phase of the water generally referred to as High Pressure (HP) Ice IV [8]. This is the case of the Ganymede moon of Jupiter where an ocean is sandwiched between an ice I crust and an HP ice or of the Titan moon of Saturn where the ocean is sandwiched between ice I crust and an yet an extra water phase, ice VI crust, [9, 10].

A schematic configuration of ice/water layers on Ganymede is depicted in Fig. 1 where the different aggregation states of the water are shown as a function of the two conjugate thermodynamic control parameters: the temperature $T$ and the pressure $P$.

![Figure 1: A schematic of Ganymede’s mantles (left) with the corresponding water phase diagram (right)](image)

As the second class of moons is concerned the presence of the HP ice layer may, in principle,
hinder the transport of chemical species from the deep silicate core to the liquid water layer which, in turn, could make the life sustainability on such moons questionable. One way of overcoming this natural transport barrier would be the possibility of triggering a slow convective motion within the HP ice layer. Indeed, rheological studies have shown that the viscosity of the HP ice mantle is of the order of $10^{14}$ to $10^{18}$ Pa.s. [11, 12]. This means that, in principle, over sufficiently long time scales, a convective motion might occur. Whether this is indeed the case or not remains, to our best knowledge, elusive and sets the main goal of the present project. A new space mission funded by the European Space Agency (ESA) that is mainly targeting Ganymede [4] which will be launched in 2022 and would reach its target in about seven years. Until then, we aim both setting more solid theoretical grounds for the dynamics of icy layers on Ganymede and, on a parallel track, to build a table top experiment in order to study such dynamics on a model experimental system judiciously designed to “mimic” most of the physical phenomena expected to occur on the giant icy moon Ganymede.

A key element in designing such an experiment is choosing a model system via a basic scaling analysis which, over laboratory time and space scales, would exhibit a behaviour physically similar to that expected over scales of the giant moon. This sets the main scope of this short communication.

2. Scaling arguments

One important issue with experiments like the one foreseen for this project is to ensure that all physical dimensions are scaled in the same way. Convective processes taking place in the deep icy layers of icy moons, are characterised by huge space and time scales (thousands of kilometres and millions of years, respectively). This imposes stringent requirements on the design of a table top experimental setup, which should be properly scaled down. This is commonly addressed as the similarity criteria, [13]. The problem includes the delicate issue of a phase transition, which evolves in space and through time. From an experimental perspective, this means that the constraint on the similarity criteria will be fixed by the ratio [13]:

$$x = \left[ \frac{(1/\mu_o)^m}{(1/\mu_m)^m} \right]$$  

Here $L$ stands for the latent heat (J/Kg), $C_p$ for the heat capacity (J/Kg K) and $T_m$ is the melting temperature. The indices (o), (m) refer to the planetary object and the lab experiments, respectively.

As a model phase change material for the table top experiments we chose to work with a paraffin wax. Depending on the type of paraffin, the ratio $x$ given by Eq. 1 might slightly change but, since the ranges of the thermodynamic parameters of ice and paraffin are comparable, one can safely assume that we will generally have $x \approx 1$. Consequently, all the relevant ratios of the physical parameters needed for the problem should also have a value close to unity. In deep icy planetary mantles, the heat flows from the core of the moon towards its free surface. A number of previous studies dealt with a characterisation of the convective motion in the HP ice layers. Grasset has proposed in Ref. [14] for the averaged velocity through the mantle the following relationship:

$$v_o \approx \frac{(\mu_o)^{-0.4}}{38}$$  

where $\mu_o$ is the scale of the viscosity of the HP ice layer. Considering that the viscosity of
high pressure ice is in the range of \((10^{14}, 10^{18})\) Pa.s. Eq. 2 gives a velocity range: \(\nu^0 \in (6.6 \times 10^{-8}, 1.6 \times 10^{-9})\) m/s. Having an estimate for the convective velocity scale \(\nu^0\) on the icy moon allows one to estimate a time scale for a full convective cycle to complete \(t^o = \frac{H^o}{\nu^0}\), where \(H^o \in (100, 400)\) km is the characteristic size of the HP ice layer. Computing the time scale we obtain \(t^o \in (1.5 \times 10^{12}, 2.5 \times 10^{14})\) s.

Similarly, the convective time scale for the table top experiment may be written:

\[
\tau^m = \frac{H^m}{\nu^m}
\]  

(3)

Via basic dimensional analysis one can readily show that the ratio of the convective time scales \(t^o\) and \(t^m\) is given by:

\[
\frac{t^o}{t^m} = \frac{\mu^o}{\rho^o H^o g^o} \cdot \frac{\mu^m}{\rho^m H^m g^m}
\]

(4)

Here \(\rho\) is the density, \(g\) the acceleration of the gravity and \(\mu\) the viscosity.

A similarity between the thermo-diffusive time scales requires:

\[
\frac{H^o \kappa^o}{\kappa^o t^o} = \frac{H^m \kappa^m}{\kappa^m t^m}
\]

(5)

where \(\kappa\) stands for the thermal diffusivity coefficient.

Once the modeling fluid is chosen and its properties are known, one could substitute the ratio of heights in Eq. 4 and 5 to obtain \(t^m \in (0.1, 38)\) s the table top experiment time scale. Subsequently, using Eq. 5 one could compute the height of characteristic length in the experimental setup \(H^m \in (0.22, 0.88)\) m.

The problem to be solved at a planetary scale is one of convective motions driven by a thermal gradient [13] which is usually controlled by the Rayleigh number. However, considering the high pressure effect that is present on Ganymede but absent in our experiment, the Rayleigh number no longer describes the convection in both cases in a similar manner.

From an experimental standpoint, such pressures are practically impossible to generate (and control) in a laboratory scale (table top) experiment. But an alternative method can be proposed. Bearing in mind that the phase transition is equivalent to a modification of the free energy surface of the system, one can alternatively reach melting conditions in a phase diagram by introducing an external control parameter that could substitute the pressure effect.

To understand how this could be done, we refer to the Clapeyron-Clausius relationship along an equilibrium:

\[
\frac{dT}{dP} = T \frac{\Delta V}{\Delta Q}
\]

(6)

where \(dP\) is the pressure difference, \(dT\) is the change of the temperature around the phase transition, \(\Delta V\) is the change of the molar volume at the phase transition point and \(\Delta Q\) is the molar heat associated to the phase transition. The presence of an additional field may shift either the pressure difference or the temperature corresponding to the phase transition. This general thermodynamic effect is particularly useful for rheological studies where it is known that the presence of an external shear shifts the crystallization point of various polymers [15]. In our case, the effect of the large pressure difference occurring at phase transition should be replaced with an external shear field \(\dot{\gamma}\) such as:

\[
\frac{\Delta P^o - \Delta P^m}{T^m_M} = \frac{R}{\Delta Q} f (\dot{\gamma})
\]

(7)
with $\Delta P^o$ the pressure difference existing in the planetary system, $T^m_M$ the melting temperature of the modeling fluid, $\Delta P^m \leq \Delta P^o$ the pressure difference we would expect in a table top experimental setup, and $f(\dot{\gamma})$ a function of the externally applied shear which will be determined experimentally in this work package. For the paraffin one can estimate $\frac{RT^m_M}{\Delta H} \approx 0.066$. Bearing in mind that $f(\dot{\gamma})$ has the dimensions of a stress and that around the melt point one can reach viscosities of the order of $10^{10} Pa.s$, it is clear from Eq. 7 that the trick we propose could compensate the pressure difference between planetary and lab scales.

3. Modeling fluid and experimental setup

3.1. Choice of the modeling fluid

As previously mentioned, we chose paraffin as a model phase-change material and focus in the following on its thermo-rheological properties. The rheological characterisation was performed using a controlled stress rotational rheometer (Mars III, Thermofischer Scientific) equipped with a nano-torque module and a parallel plate geometry with a diameter $D = 20\text{mm}$ and a gap $h = 0.5\text{mm}$. The measurements have been performed at a constant shear rate of $\dot{\gamma} = 10 \text{ 1/s}$ during a decreasing temperature ramp between $T_i = 70^\circ C$ and $T_f = 45^\circ C$ with increments of $\Delta T = 0.1^\circ C$. The temperature dependence of the viscosity is illustrated in Fig. 2.

![Figure 2](image)

**Figure 2**: Temperature dependence of the shear viscosity of the paraffin. The dash-dotted line is a fit by the Arrhenius law.

For $T > 57^\circ C$, the entire material is in liquid phase and its shear viscosity may be described by the classical Arrhenius correlation, Fig. 2. When the temperature is gradually decreased below this point, the viscosity increases dramatically and a solid-fluid coexistence regime that departs from the Arrhenius law is observed. Upon a further decrease of the temperature, the entire material becomes a “soft” solid with an apparent viscosity $\mu^m \approx 50 \text{Pa.s}$. The data presented in Fig. 2 reveals two main advantages of using the paraffin as a model fluid for the HP ice layer. First, its phase transition occurs at a temperature that is relatively easy to control in a table top experiment. Secondly, the apparent viscosity of the solid phase has an order of magnitude consistent with the results of our scaling arguments.

The thermophysical properties of HP ice and paraffin wax are shown in Table 1 [14]. The
existing set of parameters[14] with their corresponding calculated ranges (in **bold**) based on the scaling arguments in section 2, are shown in Table 2.

<table>
<thead>
<tr>
<th>Material</th>
<th>High Pressure Ice</th>
<th>Paraffin Wax</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_p ) (J/kgK)</td>
<td>2850</td>
<td>2195</td>
</tr>
<tr>
<td>( \rho ) (kg/m³)</td>
<td>1390</td>
<td>900</td>
</tr>
<tr>
<td>( \alpha \times 10^4 ) (1/K)</td>
<td>1.46</td>
<td>1</td>
</tr>
<tr>
<td>( \kappa \times 10^6 ) (m²/s)</td>
<td>6</td>
<td>1.17</td>
</tr>
<tr>
<td>( k ) (W/mK)</td>
<td>1.58</td>
<td>0.232</td>
</tr>
<tr>
<td>( g ) (m/s²)</td>
<td>1.6</td>
<td>9.81</td>
</tr>
<tr>
<td>( L ) (J/kg)</td>
<td>360,000</td>
<td>180,000</td>
</tr>
<tr>
<td>( T_m ) (K)</td>
<td>332</td>
<td>54</td>
</tr>
<tr>
<td>( \mu ) (Pa.s)</td>
<td>( 10^{14} - 10^{18} )</td>
<td>10 – 50</td>
</tr>
</tbody>
</table>

Table 1: Thermophysical properties of High Pressure Ice and Paraffin wax [14]

<table>
<thead>
<tr>
<th>Ganymede</th>
<th>Experimental setup</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g ) (m/s²)</td>
<td>1.6</td>
</tr>
<tr>
<td>( H ) (m)</td>
<td>((100 - 400) \times 10^2)</td>
</tr>
<tr>
<td>( v ) (m/s)</td>
<td>(1^{-9} - 1^{-8})</td>
</tr>
</tbody>
</table>

Table 2: Planetology givens with their corresponding lab data [14]

With the aforementioned values in Table 1, the ratio \( x \) defined in section 2, becomes equal to 1.52. Given that the ratio \( x \) is in the order of a unity, the choice of the fluid confirms the similarity criterion of the phase transition [13].

It is important to acknowledge that there exists a discrepancy in the similarity from the inequality of Reynolds number. Ice mantle convections have Reynolds numbers of \( 10^{-13} \), whereas the proposed scaled experimental model reaches 0.01. With that being said, the regime of both flows remain laminar, and this condition could be considered as partially satisfied.

\[
\left[ \frac{\rho^0 v^0 H^0}{\mu^0} \right]_{\text{laminar}} \approx 10^{-13} \neq \left[ \frac{\rho^m v^m H^m}{\mu^m} \right]_{\text{laminar}} \approx 0.01 \tag{8}
\]

3.2. The experimental setup

The experimental setup we propose is schematically illustrated in Fig. 3. This setup is easy to characterize experimentally and to implement in numerical simulations. Its main advantage is that it provides a homogeneous shear. The homogeneous shear in the bulk flow is ensured by imposing a large width to depth aspect ratio, \( W/De \approx 4 \). The paraffin wax is contained in a rectangular container with optically transparent walls. Based on the scaling arguments presented in the previous section, the height of the container would be in the range of \( H_\epsilon(20,40) \text{cm} \). Two additional containers placed at the bottom and at the top of the fluid container allow one to generate a precisely controlled vertical temperature gradient (with an accuracy of 0.1°C) by circulating water at different temperatures \( T_t, T_b \). Two vertical conveyor belts CB (1-2) driven by two separate AC-servo motors will be used for forcing the convective motions in the solid domain, and for partially substituting the pressure difference. The servo motors/controllers are
justified by the need of having a smooth driving in order to minimize external perturbations, which around a phase change critical point may alter the phenomena.

Within this context, the main experimental strategy would be the following. First, the system should be brought to a state of thermal equilibrium by imposing a steady temperature gradient that reproduces the situation sketched in Fig. 3. By a judicious choice of the temperatures \( T_1, T_b \) in relation to the melt temperature \( T_M \), the position of the melting front can be controlled. Precise point-wise measurements will be conducted at well defined positions within the container by using an array of thermocouples \( T_{1-6} \) inserted through the lateral wall of the container. An alternative technique will also be employed to monitor the temperature distribution within the system. A mixture of thermo-chromic liquid crystals (Hallcrest Inc.) will be used to visualize both the temperature distribution within the setup and the flow field. The liquid crystals will be chosen in correlation with the melt temperature of the paraffin, \( T_M \).

The position of the liquid-solid interface will be gradually shifted by modifying the upper thermal constraints, as indicated in Fig. 3. The route to the new equilibrium state will be characterized in detail by systematic measurements of the space-time temperature variations over extended periods. The emergence of a phase transition front within the system will be accurately detected by a space-time characterization of the temperature fields. The correlation between flow field transients and temperature field transients would be monitored to accurately describe the route to the new thermodynamic equilibrium point.

To measure the flow field generated by the conveyor belts a vertical laser sheet LS will be cast in the plane ZX through the transparent top fluid container. Long time series of the flow images will be acquired with the digital camera CCD. Flow fields will be obtained using a Digital Particle Image Velocimetry technique developed in LTeN. Speeds as small as \( 0.5 \mu m/s \) can be accurately measured with this technique.

![Figure 3: Front and side views of the suggested experimental setup](image)

**4. Conclusion and perspectives**

Based on dimensional analysis, an experimental modeling of the high pressure (HP) ice mantle in Ganymede was proposed in this paper. The next step is to build a experimental setup according to similarity criteria and to simulate at lab scale the convective motions in the solid and liquid phase. In the recent future, the effect of shear rate in the flow will be studied on the crystallization of the fluid; thereby upgrading this study to comprise complex thermodynamics.
References


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