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# Propellant Sloshing Torque $\mathcal{H}_\infty$ - based Observer Design for Enhanced Attitude Control<sup>★</sup>

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**Abstract:** In this paper a control-oriented LPV model of the sloshing torque arising during attitude maneuvers, supported by Computational Fluid Dynamics results, is presented and used for attitude control design. The proposed strategy essentially relies on the design of a robust LPV-based disturbance torque observer with the help of the structured multi-model  $\mathcal{H}_\infty$  synthesis framework. The estimated torque is then used to improve a satisfying attitude controller initially designed without sloshing. The stability of the parameter-varying closed-loop system is finally proved with parameter-dependent Lyapunov functions.

*Keywords:* Sloshing, LPV observers,  $\mathcal{H}_\infty$  control, attitude control.

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## 1. INTRODUCTION

In any liquid-carrying vehicle, more importantly during acceleration phases, the on-board fluid, as long as it has a free surface, is set in motion by fluid-structure dynamical coupling. This low frequency and badly damped phenomenon that corresponds to the movement of a liquid free surface inside containers [Ibrahim (2005)] is referred to as sloshing. According to the vehicle motion, different types of sloshing dynamics arise, such as surface waves, bulk fluid or vortices. As satellites carry a lifespan-defining quantity of liquid propellant consumed by thrusters to perform orbital maneuvers (station-keeping, relocation and de-orbiting), they are subject to sloshing. Therefore satellites experience disturbing forces and torques that affect their pointing accuracy. Slosh dynamics can then compromise the system performance and stability [Hoffman et al. (1999)], complicating the controller design. For space applications sloshing dynamics become even more complex as surface tension effects must be considered.

The main solution to mitigate sloshing effects is to divide propellant tanks with baffles and bladders [Dodge (1971); Tam et al. (2002)], which increases the sloshing frequency and reduces its amplitude. Mass is added to the satellite, and mission costs raise. Time margins can also be applied between aggressive maneuvers in order to allow the propellant to settle down. But this considerably reduces mission availability. Alternatively, smoothed references can be used instead of usual sharp bang-off-bang angular velocity reference profiles [Somov et al. (2015); Liu et al. (2016)]. The sloshing response will then be smoother, but the whole satellite agility may no longer be exploited. Sloshing influence can also be tempered by notch filters [Preumont

(1997)], which reduce satellites bandwidth (especially as sloshing frequencies are uncertain).

From a control design perspective, the most common way to address sloshing is based on the famous equivalent mechanical model. These models approximate the liquid with a mechanical system and can be treated like flexible modes [Mazzini (2015)]. Numerous models exist and reproduce a specific kind of sloshing dynamics [Abramson et al. (1966); Berry and Tegtart (1975); Vreeburg and Chato (2000); Sopasakis et al. (2015)]. Tuning the mechanical parameters (such as mass, spring stiffness and pendulum length) to accurately reproduce the liquid behavior is of crucial importance. For instance Enright and Wong (1994) uses Computational Fluid Dynamics (CFD) to evaluate sloshing frequencies in the *Cassini* probe tanks. The control of spacecraft with multiple propellant sloshing modes represented with pendulums has been addressed in Reyhanoglu and Hervas (2011) with both linear and Lyapunov-based nonlinear feedback controllers. Hervas and Reyhanoglu (2012) uses spring-masses and take into account consumption of propellant with time-varying parameters. In de Souza and de Souza (2014) pendulum model parameters are identified by a Kalman filter. Uncertainties in pendulum models can be addressed with robust control, as in Yano and Terashima (2001). The main advantage of equivalent mechanical models is their simplicity, which is also their main drawback. These models are based on linearized fluid dynamics models and often valid only for axisymmetric problems (e.g. hemispherical tanks accelerated along their main axis) with small amplitude motion. More importantly, they do not depend on angular speed or acceleration of the spacecraft, which however induce significant large inertial forces acting on the fluid.

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Infinite-dimensional models can also be used to represent sloshing, Ardakani and Bridges (2011) proposed three-dimensional sloshing equations for prescribed motion of tanks, Petit and Rouchon (2002) considered direct actuation of tank speed or acceleration and Cardoso-Ribeiro et al. (2015) used the Port-Hamiltonian formalism [Duin-dam et al. (2009)] to design a controller for slosh mitigation in a fluid-coupled structure. The considered fluid dynamics equations are valid for specific cases, generally shallow waters, i.e. low filling ratio, perfect fluid and negligible tension surface effects. The generalization to Navier-Stokes equations with inertial forces and tension surface is very complex, and far from the Attitude and Orbit Control Systems paradigm, prohibiting the use of controller design and stability analysis tools.

As attitude pointing accuracy and stability requirements are always more stringent, very effective Attitude Control Systems (ACS) are needed and thus the models used for controller design need to evolve. Recent progress have been achieved in the field of Computational Fluid Dynamics and several codes, such as DIVA from *Institut de Mécanique des Fluides de Toulouse* [Lepilliez et al. (2016)] and ComFlo from *University of Groningen* [Veldman et al. (2007)], are now able to accurately compute the fluid behavior in maneuvering spacecraft. Interestingly, CFD can also be used in order to improve simplified models suitable for controller design. Inspired by equivalent mechanical models, a generalization to nonlinear parameter-varying sloshing models along with a CFD-based parameter identification procedure was proposed in [Bourdelle et al. (2019)]. Based on this work, a reformulation of the slosh effects as an uncertain Linear Parameter Varying (LPV) model is proposed in this paper. This model will then serve as a basis for the robust design of an LPV state observer capable of taking into account model uncertainties to provide a reliable estimate of the sloshing torque. The latter is then used to decouple the satellite dynamics from sloshing and ensure an accurate attitude control with reduced settling time.

The remainder of the paper is organized as follows. First, section 2 recalls some key ingredients on slosh modeling and introduces the aforementioned uncertain LPV model of the liquid-filled satellite. Next, a robust  $\mathcal{H}_\infty$ -based observer design procedure is detailed in section 3. Then, in section 4, a time-varying implementation of our enhanced attitude control system is presented together with simulation results and a thorough parameter-varying stability analysis. Finally, concluding comments and perspectives are proposed in section 5

## 2. NONLINEAR SLOSHING TORQUE MODEL

Using recent developments in Computational Fluid Dynamics solvers, supported by *in-situ* experiments (such as SloshSat-FLEVO [Prins (2000)] and Fluidics [Mignot et al. (2017)]) , we propose a new approach by directly modeling the sloshing disruptive efforts instead of the fluid behavior. In the following sections we will consider a satellite attitude maneuver around a single axis, though our reasoning can be generalized to any maneuver.

### 2.1 Sloshing Torque Modeling and Identification

Sloshing dynamics depends on :

- tank filling ratio
- gravitational acceleration effects
- propellant properties, e.g. density, surface tension
- tank geometry and position inside the spacecraft
- spacecraft angular speed and acceleration

Reaction wheels or Control Moment Gyros are preferred for attitude control, propellant is saved for orbit control which uses thrusters. In the context of this paper we can thus consider a constant filling ratio. Gravitational effects are assumed negligible in front of inertial and capillary forces. The tank is considered rigid and still. Hence, only the dependence to the angular velocity  $\Omega(t)$  and acceleration  $\dot{\Omega}(t)$  have to be taken into account by the sloshing torque model. For clarity reasons, the time-dependence will be omitted in the next equations.

In Bourdelle et al. (2019) it was proposed to describe the sloshing torque  $\Gamma_F$  as the output of a nonlinear second order system with varying frequency and damping ratio :

$$\ddot{\Gamma}_F + C_s(\Omega, \dot{\Omega})\dot{\Gamma}_F + K_s(\Omega, \dot{\Omega})\Gamma_F = -A_s(\Omega, \dot{\Omega})\Omega - B_s(\Omega, \dot{\Omega})\dot{\Omega} \quad (1)$$

Due to their similarities, we can consider this nonlinear model as a generalization or an abstraction of equivalent mechanical models. The nonlinear properties of this model result from the dependence of  $A_s$ ,  $B_s$ ,  $C_s$  and  $K_s$  to  $(\Omega, \dot{\Omega})$ . These parameters can be identified using CFD results covering angular speed and acceleration ranges. With numerically computed sloshing torque, it is possible to compute boundaries for the natural frequency  $\omega(\Omega, \dot{\Omega})$  and the damping ratio  $\xi(\Omega, \dot{\Omega})$  in order to constrain  $C_s$  and  $K_s$ , which can be written as :

$$C_s(\Omega, \dot{\Omega}) = 2\xi(\Omega, \dot{\Omega})\omega(\Omega, \dot{\Omega}) \quad (2)$$

$$K_s(\Omega, \dot{\Omega}) = \omega(\Omega, \dot{\Omega})^2 \quad (3)$$

The nonlinear coefficients are identified on a set of  $N$  small time intervals on which both  $\Omega$  and  $\dot{\Omega}$  are assumed constant and the nonlinear model becomes Linear Time Invariant (LTI). By doing so it is possible to use a constrained least squares method. As a result,  $N$  sets  $\{C_{s_i}, K_{s_i}, A_{s_i}, B_{s_i}\}_{i \leq N}$  associated to  $\{\Omega_i, \dot{\Omega}_i\}_{i \leq N}$  are obtained. Standard curve-fitting techniques may then be used to rewrite each parameters as multivariate polynomials of  $\Omega$  and  $\dot{\Omega}$  for example.

This identification procedure has been applied on DIVA data sets courtesy of *Institut de Mécanique des Fluides de Toulouse*. These data correspond to the Z-axis rotation of a spherical tank with a diameter of 0.585 m and half-filled with hydrazine. The tank is off-center to the axis of rotation, the resulting lever arm is 0.4 m. The rotation follows a bang-off maneuver, i.e. full acceleration for a certain time, then constant speed (square shape acceleration profile). As stated in Bourdelle et al. (2019), better results are obtained by proceeding on two different sub-models, for each side of the acceleration discontinuity.

## 2.2 Towards an uncertain LPV model

From Equation 1, a state-space representation of the sloshing torque is:

$$\underbrace{\begin{pmatrix} \dot{\Gamma}_F \\ \ddot{\Gamma}_F \end{pmatrix}}_{\dot{x}_F} = \underbrace{\begin{pmatrix} 0 & 1 \\ -K_S & -C_S \end{pmatrix}}_{A_F(K_S, C_S)} \underbrace{\begin{pmatrix} \Gamma_F \\ \dot{\Gamma}_F \end{pmatrix}}_{x_F} + \underbrace{\begin{pmatrix} 0 & 0 \\ -A_S & -B_S \end{pmatrix}}_{B_F(A_S, B_S)} \begin{pmatrix} \Omega \\ \dot{\Omega} \end{pmatrix} \quad (4)$$

$$\Gamma_F = (0 \ 1) x_F \quad (5)$$

where the dependencies to  $(\Omega, \dot{\Omega})$  have been omitted for simplicity. The parameters  $A_S$ ,  $B_S$ ,  $C_S$  and  $K_S$  are affected by uncertainties, which arise from :

- numerical simulations errors
- identification and modelling errors

Since such uncertainties are unfortunately poorly known, it is useless to develop any accurate model (based on the Linear Fractional Transformation for example) to take them into account. Then, a basic model that consists of introducing a bounded disturbance  $w$  such that  $\|w\|_2 \leq \bar{w}$  will be used next. Equation 4 thus becomes:

$$\dot{x}_F = A_F(K_S, C_S)x_F + B_F(A_S, B_S) \begin{pmatrix} \Omega \\ \dot{\Omega} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} w \quad (6)$$

Let us consider the LTI state-space representation of the single-axis dynamics of an actuated satellite :

$$\dot{x}_{SAT} = A_{SAT}x_{SAT} + B_{SAT}(\Gamma_F + \Gamma_P + \Gamma_C) \quad (7)$$

$$\theta = C_\theta x_{SAT} \quad (8)$$

where  $\Gamma_P$  is a non-sloshing disturbing torque,  $\Gamma_C$  is the control torque and  $\theta$  is the satellite attitude. In order to also estimate  $\Gamma_P$  we extend the state vector and, without any information, we consider that :

$$\dot{\Gamma}_P = 0 \quad (9)$$

By further analyzing the identification results, it appeared that  $B_S$  and  $C_S$  can be approximated by linear functions respectively of  $A_S$  and  $K_S$  :

$$B_S = \alpha_{AB}A_S + \beta_{AB} \quad (10)$$

$$C_S = \alpha_{KC}K_S + \beta_{KC} \quad (11)$$

Using Equations 5-11 the liquid-filled satellite can be described by the following uncertain LPV system :

$$\dot{x} = A(\alpha(t))x + B_u\Gamma_C + \underbrace{[0 \ 1 \ 0 \ \dots \ 0]^T}_{B_w} w \quad (12)$$

$$\begin{pmatrix} \theta \\ \Gamma_D \end{pmatrix} = \begin{bmatrix} C_m \\ C_z \end{bmatrix} x \quad (13)$$

where :

$$\begin{aligned} \alpha(t) &= (\alpha_A(t), \alpha_K(t)) \\ &= (A_S[\Omega(t), \dot{\Omega}(t)], K_S[\Omega(t), \dot{\Omega}(t)]) \end{aligned} \quad (14)$$

$$\Gamma_D = \Gamma_F + \Gamma_P \quad (15)$$

$$x = [x_F \ x_{SAT} \ \Gamma_P]^T \quad (16)$$

Because of reaction wheels limitations which bound the control torque capacity, restricted variations of  $(\Omega(t), \dot{\Omega}(t))$  are necessarily considered :

$$|\Omega(t)| \leq S_C \bar{\Omega} \quad (17)$$

$$|\dot{\Omega}(t)| \leq S_C \bar{\dot{\Omega}} \quad (18)$$

where  $\bar{\Omega}$  and  $\bar{\dot{\Omega}}$  are the spacecraft maneuver capabilities, and  $S_c$  a security coefficient (equal to 1.5 for this study). This permits to characterize a narrowed definition domain for  $A_S$  and  $K_S$ . Thus, the time-varying vector  $\alpha(t)$  takes its values in a polytope  $\mathcal{P}$  of 15 vertices  $\mathcal{P}_i$ ,  $i \in \{1, 2, \dots, 15\}$  :

$$\alpha(t) \in \mathcal{P} := \text{Co}\{\mathcal{P}_1, \mathcal{P}_2, \dots, \mathcal{P}_9\} \quad (19)$$

The vertices are the extreme values of  $\alpha(t)$ .

*Remark 1.* Due to the filtering effect of the low-pass actuators, the parametric variations appear only in the  $A$  matrix of the system. Note also that the use of  $\alpha$  as the system parameter, instead of  $(\Omega, \dot{\Omega})$ , has the following advantages :

- $A(\alpha)$  is a linear function of  $\alpha$  which emphasizes the obvious LPV nature of the system and will simplify both the observer design and stability analysis,
- $A_S$ ,  $B_S$ ,  $C_S$  and  $K_S$  do not need to be explicitly written as functions of  $(\Omega, \dot{\Omega})$

## 3. A ROBUST $\mathcal{H}_\infty$ BASED OBSERVER DESIGN

The LPV model of the liquid-filled satellite, given by (12) and (13), is now used to design a sloshing torque observer robust to model disturbances.

### 3.1 Problem statement

The aim is to enhance attitude control using the estimate of disturbing torques to decouple the satellite from the sloshing dynamics and other perturbations, independently of any existing controller. An LPV observer has then to be designed in order to provide a reliable estimation of the disturbing torque than can next be easily canceled from the control input. This observer is described by :

$$\dot{\hat{x}} = A(\alpha(t))\hat{x} + B_u\Gamma_C + L(\alpha(t))(\theta - \hat{\theta}) \quad (20)$$

$$= \underbrace{(A(\alpha) - L(\alpha)C_m)}_{A_{Obs}} \hat{x} + \underbrace{[B_u \ L(\alpha)]}_{B_{Obs}} [\Gamma_C \ \theta]^T \quad (21)$$

$$\hat{\Gamma}_D = C_z x + \underbrace{[0 \ 0]}_{D_{Obs}} [\Gamma_C \ \theta]^T \quad (22)$$

where  $\hat{x}$  and  $\hat{\Gamma}_D$  respectively denote the estimation of  $x$  and  $\Gamma_D$ , and  $L(\alpha)$  is the observer gain to be computed. The dynamics of the state error is represented by the system :

$$(\mathcal{S}) \begin{cases} \dot{\epsilon} = A_{Obs}x + B_w w, \ \epsilon = x - \hat{x} \end{cases} \quad (23)$$

$$z = C_z \epsilon \quad (24)$$

$$= \Gamma_D - \hat{\Gamma}_D \quad (25)$$

To ensure an efficient compensation, the estimated disruptive torque needs to be accurate in spite of model disturbances captured by  $w$ . It should also take into account the fact that the compensation is realized by actuators whose dynamics introduce rather small but not negligible delays. As is clarified next, this can be achieved by the introduction of a small derivative term in the signal to be estimated.

### 3.2 Tuning process and resolution aspects

Because  $A(\alpha)$  is a linear function of  $\alpha$ , we can write :

$$A(\alpha) = A_0 + \alpha_A A_A + \alpha_K A_K \quad (26)$$

We then propose to search a structured observer gain :

$$L(\alpha) = L_0 + \alpha_A L_A + \alpha_K L_K \quad (27)$$

The system ( $\mathcal{S}$ ) has an affine LPV structure given by (26), hence a polytopic model with 15 vertices can be easily deduced :

$$\alpha = \sum_{i=1}^{15} \beta_i \mathcal{P}_i, \quad \beta_i \geq 0 \text{ and } \sum_{i=1}^{15} \beta_i = 1 \quad (28)$$

$$\mathcal{S}(\alpha) = \sum_{i=1}^{15} \beta_i \mathcal{S}(\mathcal{P}_i) \quad (29)$$

As stated by Biannic and Apkarian (1999), this approach is suitable to be addressed by a  $\mathcal{H}_\infty$  multi-model robust design techniques on the 15 LTI models ( $\mathcal{S}_{i \leq 15}$ ), that corresponds to the LPV system frozen at the vertices  $\mathcal{P}_{i \leq 15}$ . The problem is then to find the gain matrices  $L_0$ ,  $L_A$  and  $L_K$  such that the observer/error dynamics is quadratically stable over  $\mathcal{P}$  and  $\|z\|_2 < \|w\|_2$  to ensure robustness to model disturbances.

Thanks to the flexibility of nonsmooth optimization algorithms implemented in the `sysune` routine of the Matlab<sup>TM</sup> Robust Control Toolbox (see Apkarian and Noll (2006) Apkarian et al. (2014)) it is possible to compute bounded gains  $L_0$ ,  $L_A$  and  $L_K$  so as to minimize the estimation error as further detailed and to constrain the observer/error dynamics. For implementation purposes, the following constraints have been defined :

- Minimum decay rate : 0.001 rad/s
- Minimum damping ratio : 0.7
- Maximum observer frequency : 10 rad/s
- Absolute value of gains  $< 2$

In order to compensate delays introduced by the actuators, the output torque  $z$  to be estimated is augmented by a derivative term as follows :

$$z = (\Gamma_D - \hat{\Gamma}_D) + E(\dot{\Gamma}_F - \hat{\Gamma}_F) \quad (30)$$

where the gain  $E$  is tuned according to the characteristics of the actuator.

In order to minimize the steady-state estimated torque error, the error signal and the model disturbance are weighted respectively by a low-pass transfer function  $W_z(s)$  and a constant filter  $W_w(s) = 0.01$  :

$$W_z(s) = W_{z,0} \frac{0.1}{s + 0.1}, \quad W_{z,0} = 2.79 \quad (31)$$

The filter  $W_z(s)$  is designed such that the transfer between the model disturbance  $w$  and the output  $z$  is as small as possible given the observer gains constraints, mainly in low-frequency (assuming that the model disturbance is a low-frequency signal). The filter  $W_w(s)$  is equal to the maximum amplitude of  $w$ , note that the value 0.01 is almost ten times higher than the sloshing torque maximum amplitude, which ensures an effective rejection of the model disturbance.

## 4. IMPLEMENTATION AND RESULTS

The required attitude control performances are inspired by Pittet and Arzelier (2006), that describes a benchmark corresponding to the DEMETER satellite bus :

- Pointing steady-state error  $< 0.04$  deg
- Pointing rate steady-state error  $< 0.1$  deg/s
- Angular momentum  $< 0.12$  Nms
- Control torque  $< 0.005$  Nm

We consider a satellite with an inertia  $I_z = 30$  kg.m<sup>2</sup>, controlled by a satisfying Proportional-Derivative controller that respects these constraints in the absence of sloshing :

$$\Gamma_C = 0.3553\delta_\theta + 6.2845\delta_\Omega \quad (32)$$

where  $\delta_\theta$  is the attitude error and  $\delta_\Omega$  is the angular velocity error. In the case of the DEMETER benchmark the velocity is pseudo-derived from the attitude measurement provided by a star tracker. Here, it is given as an additional output of the state observer designed above.

The actuator is a reaction wheel modeled by the following transfer function :

$$RWS(s) = \frac{1.2s + 0.76}{s^2 + 2.4s + 0.76} \quad (33)$$

To get faster responses we also add the torque guidance profile  $\Gamma_d$  in a feed-forward path. The closed-loop system is illustrated by Figure 1.

### 4.1 Quadratic stability of the parameter-varying observer

As stated above, the proposed approach, based on LTI design tools, offers great flexibility but no theoretical guarantee regarding time-varying stability. The latter should then be checked *a posteriori*. This is achieved here with the help of quadratic Lyapunov functions.

We focus beforehand on the parameter-varying stability analysis of the observer dynamics. This is easily verified, independently of the rate of variation of the parameters, if a symmetric positive definite matrix  $P_{Obs} > 0$  can be found such that :

$$A_{Obs}(\alpha)^T P_{Obs} + P_{Obs} A_{Obs}(\alpha) < 0, \quad \forall \alpha \in \mathcal{P} \quad (34)$$

which, in our context, using (28) and (29) reduces to 15 Linear Matrix Inequalities (LMI) :

$$A_{Obs}(\mathcal{P}_i)^T P_{Obs} + P_{Obs} A_{Obs}(\mathcal{P}_i) < 0, \quad i = 1, \dots, 15 \quad (35)$$

Since the observer has 7 states, the above LMI problem exhibits  $7 \times 8/2 = 28$  decision variables and was easily solved using the standard `feasp` LMI solver available with the Matlab Robust Control Toolbox. It can then be concluded that the observer remains stable for arbitrarily fast variations of the parameters inside the polytope  $\mathcal{P}$ .

### 4.2 Quadratic stability of the parameter-varying closed-loop

Let us now focus on the parameter-varying closed-loop plant described in Figure 1. As it results from a linear interconnexion of LTI and LPV systems that affinely depends on  $\alpha$ , the closed-loop dynamics are described by a matrix  $A_{CL}(\alpha) \in \mathbb{R}^{13 \times 13}$  with the same properties.



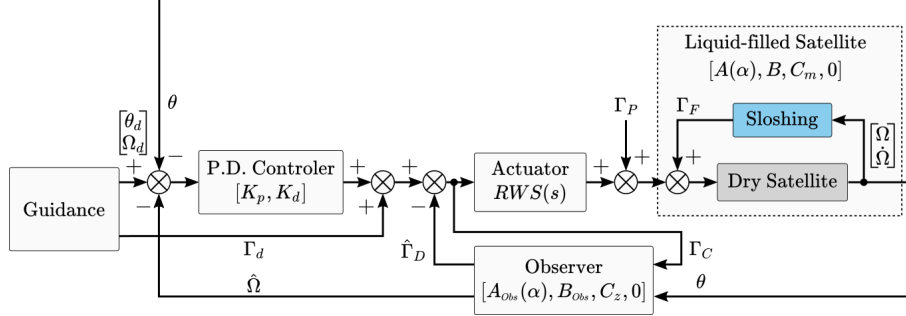


Fig. 1. Parameter-varying closed-loop model block diagram

Thus  $A_{CL}(\alpha) \in \mathcal{Co}\{A_{CL}(\mathcal{P}_1), \dots, A_{CL}(\mathcal{P}_9)\}$ . The stability analysis of the closed-loop dynamics follows the same strategy than the observer stability analysis. Since the closed-loop has 13 states, the LMI problem now exhibits  $13 \times 14/2 = 91$  decision variables. As the problem has been solved, we can conclude that the closed-loop with sloshing torque compensation remains stable for arbitrarily fast variations of the parameters.

*Remark 2.* When the security coefficient  $S_C$  exceeds a threshold value, quadratic stability cannot be proved anymore. We then have to rely on Parameter-Dependent Lyapunov Functions (PDLF) and a  $\mu$ -test such as presented in Biannic et al. (2011).

#### 4.3 Simulation results

The parameter-varying observer performances are compared to an observer with a static gain and an observer designed on a rigid model of the spacecraft (that assume  $\dot{\Gamma}_D = 0$ , as we did for  $\Gamma_P$ ). The design procedure for these alternative observers is also based on a  $\mathcal{H}_\infty$  multi-model design technique with the same constraints and filter  $W_w(s)$ . The value of  $W_{z,0}$  in the filter  $W_z(s)$  (31) is taken such that the  $\mathcal{H}_\infty$  norm of the weighted transfer from  $w$  to  $z$  is equal to 1. Thus  $W_{z,0} = 2.41$  for the static gain observer and  $W_{z,0} = 99$  for the rigid model observer.

*Remark 3.* The Simulink model used for the simulations does not use the linear approximations (11) and (10), hence the parameters used by the observers and the ones used by the sloshing torque model are slightly different.

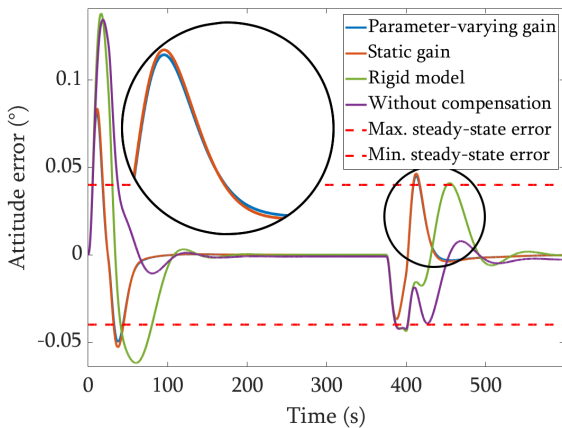


Fig. 2. Performances - attitude error (°)

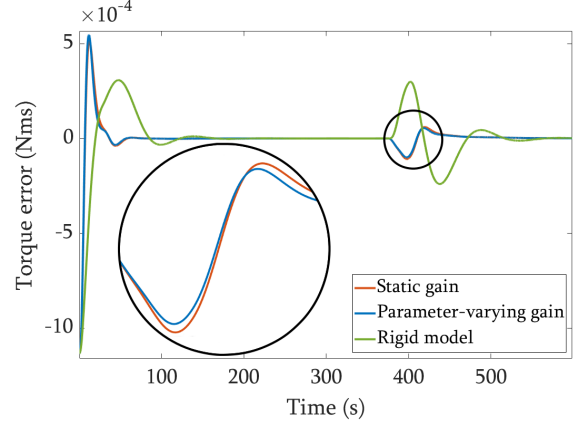


Fig. 3. Performances - torque error (Nms)

Figure 3 shows the torque error, defined as the difference between the estimated torque filtered by the actuator and the actual torque; and figure 2 shows the attitude error. Both figures reveal that the static gain observer and the parameter-varying observer have very good performances, despite the limiting dynamics of the actuator and design constraints. By using these observers to mitigate sloshing effects the attitude error requirement is respected sooner and for a longer time. Without compensation the attitude error is much higher. The rigid model observer, despite a lower torque error overshoot, has lower performances and is even less efficient than the P.D. controller alone.

*Remark 4.* The angular velocity error requirement is respected by all of the presented observers, yet the static gain observer and the parameter-varying observer are the most efficient.

*Remark 5.* Despite similar performances, the parameter-varying observer is preferred to the static gain observer as it is more robust, indeed its  $W_{z,0}$  and  $S_C$  threshold values (at which the quadratic stability cannot be verified anymore) are higher.

## 5. CONCLUSION AND FUTURE WORK

In this paper we have presented a new way to model sloshing disturbing torque as an LPV system. This model has been successfully exploited to design an LPV torque observer that allows perturbation compensation in order to enhance an already existing satisfying controller, which has been designed without sloshing. This observer has been proven, along with the parameter-varying closed-loop, to be quadratically stable over the parametric domain.

It should be emphasized that the sloshing model used in this paper was identified from data that correspond to an half-filled (worst case scenario) wall less tank which is moreover slightly larger than the one usually fitted to DEMETER satellite. Despite such adverse conditions, the proposed compensation technique succeeded in considerably reducing the attitude error to acceptable values. It is therefore likely that the use of such attitude control systems with enhanced slosh disturbance rejection capacity could permit to reduce tank complexity and mass. However, it should also be noted that the control torque and angular momentum maximum values of the reaction wheels are sometimes exceeded, depending on the sloshing torque model initial conditions. Hence future work will address this issue by using *reference governors* technique [Kolmanovsky et al. (2014)] in order to adapt, when necessary, the reference to be tracked. Motivated by interesting simulation results, future work will also focus on stability analysis in the case where the observer parameter  $\alpha_{Obs}$  is different from the sloshing torque model parameter  $\alpha$ .

## REFERENCES

- Abramson, H.N., Bauer, H.F., Brooks, G.W., and Chu, W.H. (1966). The dynamic behavior of liquids in moving containers, with applications to space vehicle technology (nasa-sp-106). Technical report.
- Apkarian, P. and Noll, D. (2006). Nonsmooth  $\mathcal{H}_\infty$  synthesis. *IEEE Trans. on Automatic Control*, 51(1), 71–86.
- Apkarian, P., Gahinet, P., and Buhr, C. (2014). Multi-model, multi-objective tuning of fixed-structure controllers. In *Proceedings of ECC 2014*, 856–861.
- Ardakani, H.A. and Bridges, T.J. (2011). Shallow-water sloshing in vessels undergoing prescribed rigid-body motion in three dimensions. *Journal of Fluid Mechanics*, 667, 474–519.
- Berry, R.L. and Tegart, J.R. (1975). Experimental study of transient liquid motion in orbiting spacecraft (nasa-cr-144003). Technical report.
- Biannic, J.M. and Apkarian, P. (1999). Missile autopilot design via a modified lqv synthesis technique. *Aerospace Science and Technology*, 3(3), 153–160.
- Biannic, J.M., Roos, C., and Pittet, C. (2011). Linear parameter varying analysis of switched controllers for attitude control systems. *Journal of Guidance, Control, and Dynamics*, 34(5), 1561–1567.
- Bourdelle, A., Burlion, L., Biannic, J.M., et al. (2019). Towards new control design oriented models for fuel sloshing in observation spacecraft. In *AIAA Scitech 2019 Forum*, 0115.
- Cardoso-Ribeiro, F.L., Matignon, D., and Pommier-Budinger, V. (2015). Control design for a coupled fluid-structure system with piezoelectric actuators. *Proceedings of the 3rd CEAS EuroGNC*, 13–15.
- de Souza, L.C.G. and de Souza, A.G. (2014). Satellite attitude control system design considering the fuel slosh dynamics. *Shock and Vibration*, 2014.
- Dodge, F.T. (1971). Engineering study of flexible baffles for slosh suppression (nasa cr-1880). Technical report.
- Duindam, V., Macchelli, A., Stramigioli, S., and Bruyninckx, H. (2009). *Modeling and control of complex physical systems: the port-Hamiltonian approach*. Springer.
- Enright, P.J. and Wong, E.C.. (1994). Propellant slosh models for the cassini spacecraft. Technical report, Jet Propulsion Laboratory, Caltech.
- Hervas, J.R. and Reyhanoglu, M. (2012). Control of a spacecraft with time-varying propellant slosh parameters. In *ICCAS 2012*, 1621–1626. IEEE.
- Hoffman, E.J., Ebert, W., Femiano, M., Freeman, H., Gay, C., Jones, C., Luers, P., and Palmer, J. (1999). The near rendezvous burn anomaly of december 1998. *Applied Physics Laboratory, Johns Hopkins University*.
- Ibrahim, R.A. (2005). *Liquid sloshing dynamics: theory and applications*. Cambridge University Press.
- Kolmanovsky, I., Garone, E., and Di Cairano, S. (2014). Reference and command governors: A tutorial on their theory and automotive applications. In *American Control Conference (ACC), 2014*, 226–241. IEEE.
- Lepilliez, M., Popescu, E.R., Gibou, F., and Tanguy, S. (2016). On two-phase flow solvers in irregular domains with contact line. *Journal of Computational Physics*, 321, 1217–1251.
- Liu, X., Xin, X., Li, Z., Chen, Z., and Sheng, Y. (2016). Near minimum-time feedback attitude control with multiple saturation constraints for agile satellites. *Chinese Journal of Aeronautics*, 29(3), 722–737.
- Mazzini, L. (2015). *Flexible Spacecraft Dynamics, Control and Guidance*. Springer.
- Mignot, J. et al. (2017). Fluid dynamics in space experiment. IAC.
- Petit, N. and Rouchon, P. (2002). Dynamics and solutions to some control problems for water-tank systems. *IEEE Transactions on Automatic Control*, 47(4), 594–609.
- Pittet, C. and Arzelier, D. (2006). Demeter: A benchmark for robust analysis and control of the attitude of flexible micro satellites. *IFAC Proceedings Vol.*, 39(9), 661–666.
- Preumont, A. (1997). *Vibration control of active structures*, volume 2. Springer.
- Prins, J. (2000). *SLOSHSAT FLEVO - Description of the mini satellite*. Citeseer.
- Reyhanoglu, M. and Hervas, J.R. (2011). Nonlinear control of a spacecraft with multiple fuel slosh modes. In *CDC-ECC*, 6192–6197. IEEE.
- Somov, Y., Butyrin, S., Somov, S., and Hajiye, C. (2015). Attitude guidance, navigation and control of land-survey mini-satellites. *IFAC OnLine*, 48(9), 222–227.
- Sopasakis, P., Bernardini, D., Strauch, H., Bennani, S., and Bemporad, A. (2015). Sloshing-aware attitude control of impulsively actuated spacecraft. In *2015 European Control Conference (ECC)*, 1376–1381. IEEE.
- Tam, W., Dommer, K., Wiley, S., Mosher, L., and Persons, D. (2002). Design and manufacture of the messenger propellant tank assembly. In *38th AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit*, 4139.
- Veldman, A.E., Gerrits, J., Luppens, R., Helder, J.A., and Vreeburg, J. (2007). The numerical simulation of liquid sloshing on board spacecraft. *Journal of Computational Physics*, 224(1), 82–99.
- Vreeburg, J. and Chato, D. (2000). Models for liquid impact onboard sloshsat flevo. In *Space 2000 Conference and Exposition*, 5152.
- Yano, K. and Terashima, K. (2001). Robust liquid container transfer control for complete sloshing suppression. *IEEE Trans. on Control Systems Tech.*, 9(3), 483–493.