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A deterministic approach for shape and topology optimization under uncertain loads

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Summary

The present work aims at handling uncertain loads in shape and topology optimization. More specifically, we minimize objective functions combining mean values and variances of standard cost functions and assume that uncertainties are small and generated by a finite number N of random variables. A deterministic approach that relies on a second-order Taylor expansion of the cost function has been proposed by Allaire & Dapogny.¹ That method requires a computational effort comparable to the one for an N -load problem. This work presents a general framework to handle uncertainties on arbitrary static load cases where perturbations on both surface forces and body forces are considered. We demonstrate the effectiveness of the approach in the context of level-set-based topology optimization for the robust compliance minimization on three-dimensional test cases.

Keywords: *shape and topology optimization, level set, random uncertainties, deterministic approach.*

1 Introduction

Shape and topology optimization is routinely used in industry in the design process in mechanical parts. Most if not all commercial grade tools assume that exact functional specifications are available whereas in practice these can only be determined approximately, especially during early stages of design, when topology optimization provides the most benefits. Moreover, the feasibility and optimality of the solution can be very dependent on a slight change on the actual requirements.¹ Unless uncertainty has been taken into account, the chosen design may turn out to be suboptimal, or even worse, defective.² Therefore, the

development of procedures that improve the robustness of the solution should increase the added value of shape and topology optimization in the industrial design process.

Several strategies have been proposed to account for the effects of small uncertainties. On the one hand, when no statistical information is available, a worst-case design approach can be followed : The idea is simply to minimize the maximal value of the objective function for any possible perturbation. The computation is made tractable by assuming that it is possible to linearize the criterion with respect to the perturbation.³ While easy to implement, this strategy can lead to structures with poor nominal performances. On the other hand, when the user

has some knowledge on the statistical distribution of the perturbation, a probabilistic description can be introduced. Then it becomes possible to consider the mean value and the variance of the objective function in the optimization problem, leading to solutions with better performance than in the previous case.

In this work, the latter approach is followed. The paper is organized as follows. First, the theoretical framework, mostly based on Allaire & Dapogny's work,¹ allows a general treatment of uncertain loads. Then, the special case of the robust compliance is derived and further developed. Finally, the method is implemented in the context of level-set-based topology optimization and illustrated on a three-dimensional test-case.

2 Shape and topology optimisation via level set

Shape and topology optimisation

The goal of shape and topology optimization is to find a shape that is a solution of a problem of the form:

$$\min_{\Omega \in \mathcal{O}_{ad}} \mathcal{J}(\Omega), \quad (1)$$

where $\mathcal{J}(\Omega)$ is a cost function that depends on the domain $\Omega \in \mathbb{R}^d$ and \mathcal{O}_{ad} is the set of admissible shapes.

Relying on Hadamard's notion of shape derivative, we evaluate the cost function sensitivity with respect to a certain class of domain perturbations in order to implement an iterative continuous optimization algorithm. A variation of a domain Ω is given by:

$$\Omega_\theta = (I + \theta)(\Omega), \quad (2)$$

where $\theta \in W^{1,\infty}(\mathbb{R}^d, \mathbb{R}^d)$ and for θ sufficiently small, $(I + \theta)$ is diffeomorphism in \mathbb{R}^d .

A function $\mathcal{J}(\Omega)$ admits a shape derivative if the mapping $\theta \rightarrow \mathcal{J}(\Omega_\theta)$ from $W^{1,\infty}(\mathbb{R}^d, \mathbb{R}^d)$ to \mathbb{R} is Fréchet differentiable at $\theta = 0$. The Fréchet derivative (or shape derivative) of $\mathcal{J}(\Omega)$ is denoted by $\theta \rightarrow \mathcal{J}'(\Omega)(\theta)$ and defined as:

$$\mathcal{J}(\Omega_\theta) = \mathcal{J}(\Omega) + \mathcal{J}'(\Omega)(\theta) + o(\theta), \quad (3)$$

where

$$\frac{|o(\theta)|}{\|\theta\|_{W^{1,\infty}(\mathbb{R}^3, \mathbb{R}^3)}} \xrightarrow{\theta \rightarrow 0} 0$$

and $\mathcal{J}'(\Omega)$ is a continuous linear form on $W^{1,\infty}(\mathbb{R}^d, \mathbb{R}^d)$.

The shape derivative depends on the normal trace of the perturbation θ on Ω .⁴ In many relevant cases, it can be written as:⁵

$$\forall \theta \in W^{1,\infty}(\mathbb{R}^3, \mathbb{R}^3), \quad \mathcal{J}'(\Omega)(\theta) = \int_{\partial\Omega} v_\Omega(s) \theta n_\Omega ds, \quad (4)$$

where n_Ω is outward unit normal to the boundary $\partial\Omega$ and $v_\Omega(s)$ depends on $\mathcal{J}(\Omega)$.

Level set method

The level set method introduced by Osher and Sethian⁶ is used for the numerical implementation of the shape and topology optimization problem. In this framework, a shape $\Omega \subset \mathbb{R}^d$ is represented by the negative subdomain of an auxiliary level set function $\phi : \mathbb{R}^d \rightarrow \mathbb{R}$:

$$\forall x \in \mathbb{R}^d, \quad \begin{cases} \phi(x) < 0 & \text{if } x \in \Omega \\ \phi(x) = 0 & \text{if } x \in \partial\Omega \\ \phi(x) > 0 & \text{if } x \in \overline{\Omega}^c \end{cases} \quad (5)$$

The motion of a domain $\Omega(t)$ for $t \in [0, T]$ according to a normal velocity field $V(x, t)$ translates into an Hamilton-Jacobi equation for the associated level set function $\phi(t, \cdot)$:

$$\frac{\partial \phi}{\partial t} + V|\nabla \phi| = 0, t \in (0, T), x \in \mathbb{R}^d, \quad (6)$$

where t is a pseudo-time, the upper bound T is analogous to the step size of a gradient descent and the advection field V derives from the shape derivative \mathcal{D}_Ω of the objective function $\mathcal{J}(\Omega)$. Instead of taking the direction of steepest descent $V = -\mathcal{D}_\Omega$, a regularisation and an extension process to compute V from \mathcal{D}_Ω is implemented.⁷

3 Random perturbations on mechanical loads in shape and topology optimization

3.1 A model problem

Let us consider a linear elastic solid domain $\Omega \subset \mathbb{R}^d$. The Dirichlet boundary condition is imposed on $\Gamma_D \subset \partial\Omega$. Body and surface forces are respectively denoted by $f \in L^2(\Omega)^d$ and $g \in L^2(\Gamma_N)^d$ with $\Gamma_N \subset \partial\Omega$ and $\Gamma_0 = \partial\Omega \setminus (\Gamma_N \cup \Gamma_D)$. The mechanical problem obeys the system of equations:

$$\begin{cases} -\text{div}(\sigma(u_\Omega)) = f & \text{in } \Omega \\ u_\Omega = 0 & \text{on } \Gamma_D \\ \sigma(u_\Omega)n = g & \text{on } \Gamma_N \\ \sigma(u_\Omega)n = 0 & \text{on } \Gamma_0 \end{cases} \quad (7)$$

We introduce small perturbations \hat{f} to the body forces $f = f_0 + \hat{f}$ such that $f_0 \in L^2(\Omega)^d$, $\hat{f} \in L^2(\Omega)^d$ and \hat{g} to the surface forces $g = g_0 + \hat{g}$ such that $g_0 \in L^2(\Gamma_N)^d$, $\hat{g} \in L^2(\Gamma_N)^d$.

A second order Taylor expansion of a cost function

$$\mathcal{C}(\Omega, f, g) = \int_{\Omega} j(f, u_\Omega) dx + \int_{\Gamma_N} k(g, u_\Omega) ds \quad (8)$$

around $f_0 \in L^2(\Omega)^d$ and $g_0 \in L^2(\Gamma_N)^d$ yields :

$$\begin{aligned}
& \mathcal{C}(\Omega, f_0 + \hat{f}, g_0 + \hat{g}) \\
&= \int_{\Omega} j(f_0 + \hat{f}, u_{\Omega}) dx + \int_{\Gamma_N} k(g_0 + \hat{g}, u_{\Omega}) ds \\
&= \int_{\Omega} j(f_0, u_{\Omega}) dx + \int_{\Omega} (\nabla_f j(f_0, u_{\Omega}) \cdot \hat{f} \\
&+ \nabla_{u_{\Omega}} j(f_0, u_{\Omega}) \cdot u_{\Omega}^1(\hat{f})) dx \\
&+ \frac{1}{2} \int_{\Omega} (\nabla_f^2 j(f_0, u_{\Omega})(\hat{f}, \hat{f}) + 2 \nabla_f \nabla_{u_{\Omega}} j(f_0, u_{\Omega})(\hat{f}, u_{\Omega}^1(\hat{f})) \\
&+ \nabla_{u_{\Omega}}^2 j(f_0, u_{\Omega})(u_{\Omega}^1(\hat{f}), u_{\Omega}^1(\hat{f}))) dx + R_f(\hat{f}) \\
&+ \int_{\Gamma_N} k(g_0, u_{\Omega}) ds + \int_{\Gamma_N} (\nabla_g k(g_0, u_{\Omega}) \cdot \hat{g} \\
&+ \nabla_{u_{\Omega}} k(g_0, u_{\Omega}) \cdot u_{\Omega}^1(\hat{g})) ds \\
&+ \frac{1}{2} \int_{\Gamma_N} (\nabla_g^2 k(g_0, u_{\Omega})(\hat{g}, \hat{g}) + 2 \nabla_g \nabla_{u_{\Omega}} j(g_0, u_{\Omega})(\hat{g}, u_{\Omega}^1(\hat{g})) \\
&+ \nabla_{u_{\Omega}}^2 j(g_0, u_{\Omega})(u_{\Omega}^1(\hat{g}), u_{\Omega}^1(\hat{g}))) ds + R_g(\hat{g})
\end{aligned} \tag{9}$$

where

$$R_f(\hat{f}) = \int_{\Omega} \int_0^1 \frac{\partial^3 \mathcal{C}}{\partial f^d}(\hat{f}, \hat{f}, \hat{f}) dt dx \tag{10}$$

and

$$R_g(\hat{g}) = \int_{\Gamma_N} \int_0^1 \frac{\partial^3 \mathcal{C}}{\partial g^d}(\hat{g}, \hat{g}, \hat{g}) dt ds \tag{11}$$

$R(\hat{f}, \hat{g}) = R_f(\hat{f}) + R_g(\hat{g})$ is the residual of the expansion.

We assume that the perturbations are random and accordingly we set $\hat{f} \equiv \hat{f}(x, \omega)$, $\hat{g} \equiv \hat{g}(x, \omega)$, for $x \in \Omega$ and $\omega \in \mathcal{O}$, where $(\mathcal{O}, \mathcal{F}, \mathbf{P})$ is a probability space.

3.2 Mean and variance of the cost function

Mean of the cost function

Let :

$$\mathcal{M}(\Omega) = \int_{\mathcal{O}} \mathcal{C}(\Omega, f(\cdot, \omega), g(\cdot, \omega)) \mathbf{P}(d\omega) \tag{12}$$

denote the mean of the cost function $\mathcal{C}(\Omega, f(\cdot, \omega), g(\cdot, \omega))$

It is assumed that $\hat{f} \in L^2(\mathcal{O}, L^2(\Omega)^d)$ (respectively \hat{g}) is a finite sum of deterministic functions $f_i \in L^2(\Omega)^d$, $i = 1, \dots, N$ (respectively g_i) weighted by independant Gaussian random variables ξ_{f_i} (respectively ξ_{g_i}):

$$\hat{f}(x, \omega) = \sum_{i=1}^N f_i(x) \xi_{f_i}(\omega) \chi_{f_i} \tag{13}$$

$$\hat{g}(x, \omega) = \sum_{i=1}^N g_i(x) \xi_{g_i}(\omega) \chi_{g_i} \tag{14}$$

where $\chi_{f_i} : \Omega \rightarrow \{0, 1\}$ (respectively χ_{g_i}) is an indicator function that identifies the region affected by the perturbation f_i (respectively g_i).

Then it comes:

$$\begin{aligned}
\mathcal{M}(\Omega) &= \int_{\Omega} j(f_0, u_{\Omega}) \chi_{f_0} dx \\
&+ \frac{1}{2} \int_{\Omega} (\sum_{i=1}^N \nabla_f^2 j(f_0, u_{\Omega})(f_i, f_i)) \chi_{f_i} dx \\
&+ \sum_{i=1}^N \int_{\Omega} (\nabla_f \nabla_{u_{\Omega}} j(f_0, u_{\Omega})(f_i, u_{\Omega}^1)) \chi_{f_i} dx \\
&+ \frac{1}{2} \int_{\Omega} (\sum_{i=1}^N \nabla_{u_{\Omega}}^2 j(f_0, u_{\Omega})(u_{\Omega}^1, u_{\Omega}^1)) \chi_{f_i} dx \\
&+ \int_{\Gamma_N} k(g_0, u_{\Omega}) \chi_{g_0} ds \\
&+ \frac{1}{2} \int_{\Gamma_N} (\sum_{i=1}^N \nabla_g^2 k(g_0, u_{\Omega})(g_i, g_i)) \chi_{g_i} ds \\
&+ \sum_{i=1}^N \int_{\Gamma_N} (\nabla_g \nabla_{u_{\Omega}} k(g_0, u_{\Omega})(g_i, u_{\Omega}^1)) \chi_{g_i} ds \\
&+ \frac{1}{2} \int_{\Gamma_N} (\sum_{i=1}^N \nabla_{u_{\Omega}}^2 k(g_0, u_{\Omega})(u_{\Omega}^1, u_{\Omega}^1)) \chi_{g_i} ds
\end{aligned} \tag{15}$$

such that $u_{\Omega,i}^1 := u_{\Omega}^1(f_i, g_i)$ is the unique solution in $H_{\Gamma_D}^1(\Omega)^d$ of the problem :

$$\begin{cases} -\operatorname{div}(\sigma(u_{\Omega,i}^1)) &= f_i \chi_{f_i} & \text{dans } \Omega \\ u_{\Omega,i}^1 &= 0 & \text{sur } \Gamma_D \\ \sigma(u_{\Omega,i}^1)n &= g_i \chi_{g_i} & \text{sur } \Gamma_N \end{cases} \tag{16}$$

Variance of the cost function

The variance $\mathcal{V}(\Omega)$ of the cost function $\mathcal{C}(\Omega)$ is defined as:

$$\mathcal{V}(\Omega) = \int_{\mathcal{O}} (\mathcal{C}(\Omega, f(\cdot, \omega)) - \mathcal{M}(\Omega))^2 \mathbf{P}(d\omega) \tag{17}$$

Following Allaire & Dapogny,¹ we obtain the expression:

$$\tilde{\mathcal{V}}(\Omega) = \sum_{i=1}^N (a_{\Omega,i}^2 + b_{\Gamma_N,i}^2) \tag{18}$$

where

$$a_{\Omega,i} := \int_{\Omega} (\nabla_f j(f_0, u_{\Omega}) \cdot f_i + \nabla_{u_{\Omega}} j(f_0, u_{\Omega}) \cdot u_{\Omega,i}^1) \chi_{f_i} dx$$

and

$$b_{\Gamma_N,i} := \int_{\Gamma_N} (\nabla_g k(g_0, u_{\Omega}) \cdot g_i + \nabla_{u_{\Omega}} k(g_0, u_{\Omega}) \cdot u_{\Omega,i}^1) \chi_{g_i} dx$$

3.3 Shape derivatives of mean and variance of the cost function

Shape derivative of the mean of the cost function

According to Theorem 14 in Allaire & Dapogny,¹ the Fréchet derivative of the mean of the cost function is given by :

$$\begin{aligned}
\forall \theta \in \mathcal{O}_{ad}, \mathcal{M}'(\Omega)(\theta) &= \\
&+ \int_{\Gamma_N} (j(f_0, u_{\Omega}) \chi_{f_0} + \frac{1}{2} \sum_{i=1}^N \nabla_f^2 j(f_0, u_{\Omega})(f_i, f_i) \chi_{f_i} \\
&+ \sum_{i=1}^N \nabla_f \nabla_{u_{\Omega}} j(f_0, u_{\Omega})(f_i, u_{\Omega}^1) \chi_{f_i} \\
&+ \frac{1}{2} \sum_{i=1}^N \nabla_{u_{\Omega}}^2 j(f_0, u_{\Omega})(u_{\Omega}^1, u_{\Omega}^1) \chi_{f_i} + \sigma(u_{\Omega}) : e(p_{\Omega}^0) \chi_{f_0} \\
&+ \sum_{i=1}^N \sigma(u_{\Omega,i}^1) : e(p_{\Omega,i}^1) \chi_{f_i} \\
&- f_0 \cdot p_{\Omega}^0 \chi_{f_0} - \sum_{i=1}^N f_i \cdot p_{\Omega,i}^1 \chi_{f_i}) \theta n ds + \int_{\Gamma_N} (\frac{\partial}{\partial n} k(g_0, u_{\Omega}) \chi_{g_0} \\
&+ Hk(g_0, u_{\Omega}) \chi_{g_0} \\
&+ \frac{1}{2} \sum_{i=1}^N \nabla_g^2 k(g_0, u_{\Omega})(g_i, g_i) \chi_{g_i} \\
&+ \sum_{i=1}^N \nabla_g \nabla_{u_{\Omega}} k(g_0, u_{\Omega})(g_i, u_{\Omega}^1) \chi_{g_i} \\
&+ \frac{1}{2} \sum_{i=1}^N \nabla_{u_{\Omega}}^2 k(g_0, u_{\Omega})(u_{\Omega}^1, u_{\Omega}^1) \chi_{g_i} - \frac{\partial}{\partial n} k(g_0, p_{\Omega}^0) \chi_{g_0} \\
&- Hk(g_0, p_{\Omega}^0) \chi_{g_0}) \theta n ds,
\end{aligned} \tag{19}$$

p_{Ω}^0 and $p_{\Omega,i}^1$ for $i = 1, \dots, N$ are the $(N+1)$ adjoint states, defined as the unique solutions of $H_{\Gamma_D}^1(\Omega)^d$ of the following variational problems :

$$\begin{aligned}
\forall v \in H_{\Gamma_D}^1(\Omega)^3, \int_{\Omega} (\sigma(p_{\Omega}^0) : e(v)) dx = & \\
& - \int_{\Omega} (\nabla_u j(f_0, u_{\Omega}) v) dx \\
& - \frac{1}{2} \int_{\Omega} (\sum_{i=1}^N \nabla_f^2 j(f_0, u_{\Omega})(f_i, f_i, v)) dx \\
& - \sum_{i=1}^N \int_{\Omega} (\nabla_f \nabla_u^2 j(f_0, u_{\Omega})(f_i, u_{\Omega,i}^1, v)) dx \\
& + \frac{1}{2} \int_{\Omega} (\nabla_u^3 j(f_0, u_{\Omega})(u_{\Omega,i}^1, u_{\Omega,i}^1, v)) dx \\
& - \int_{\Gamma_N} (\nabla_u k(g_0, u_{\Omega}) v) ds \\
& - \frac{1}{2} \int_{\Gamma_N} (\sum_{i=1}^N \nabla_g^2 \nabla_u k(g_0, u_{\Omega})(g_i, g_i, v)) ds \\
& - \sum_{i=1}^N \int_{\Gamma_N} (\nabla_g \nabla_u^2 k(g_0, u_{\Omega})(g_i, u_{\Omega,i}^1, v)) ds \\
& + \frac{1}{2} \int_{\Gamma_N} (\nabla_g^3 k(g_0, u_{\Omega})(u_{\Omega,i}^1, u_{\Omega,i}^1, v)) ds
\end{aligned} \quad (20)$$

$$\begin{aligned}
\forall v \in H_{\Gamma_D}^1(\Omega)^3, \int_{\Omega} (\sigma(p_{\Omega,i}^1) : e(v)) dx & \\
= - \int_{\Omega} (\nabla_f \nabla_u j(f_0, u_{\Omega})(f_i, v) + \nabla_u^2 j(f_0, u_{\Omega})(u_{\Omega,i}^1, v)) dx & \\
- \int_{\Gamma_N} (\nabla_g \nabla_u k(g_0, u_{\Omega})(g_i, v) + \nabla_u^2 k(g_0, u_{\Omega})(u_{\Omega,i}^1, v)) ds &
\end{aligned} \quad (21)$$

Shape derivative of variance of the cost function

The Fréchet derivative of the variance of the cost function is given by :

$$\begin{aligned}
\forall \theta \in \mathcal{O}_{ad}, \tilde{\mathcal{V}}'(\Omega)(\theta) = \int_{\Gamma_N} (\sigma(u_{\Omega}) : e(p_{\Omega}^0)) & \\
+ 2 \sum_{i=1}^N (a_{\Omega,i} + b_{\Gamma_N,i}) \sigma(u_{\Omega,i}^1) : e(p_{\Omega,i}^1) \theta n ds &
\end{aligned} \quad (22)$$

4 Application for robust compliance

The so-called compliance function is defined as :

$$\mathcal{C}(\Omega) = \int_{\Omega} \sigma : e(u_{\Omega}) dx = \int_{\Gamma_N} g \cdot u_{\Omega} ds + \int_{\Omega} f \cdot u_{\Omega} dx \quad (23)$$

It is a special case of (8) with $j(f, u_{\Omega}) = f \cdot u_{\Omega}$ and $k(g, u_{\Omega}) = g \cdot u_{\Omega}$. The expressions for the mean and the variance of the cost function as well as the associated shape derivatives are then :

$$\begin{aligned}
\tilde{\mathcal{M}}(\Omega) &= \int_{\Omega} (f_0 \cdot u_{\Omega} \chi_{f_0} + \sum_{i=1}^N f_i \cdot u_{\Omega,i} \chi_{f_i}) dx \\
&+ \int_{\Gamma_N} (g_0 \cdot u_{\Omega} \chi_{g_0} + \sum_{i=1}^N g_i \cdot u_{\Omega,i} \chi_{g_i}) ds \\
&= \int_{\Omega} (\sigma(u_{\Omega}) : e(u_{\Omega})) \\
&+ \sum_{i=1}^N \int_{\Omega} (\sigma(u_{\Omega,i}^1) : e(u_{\Omega,i}^1)) dx
\end{aligned} \quad (24)$$

$$\begin{aligned}
\forall \theta \in \mathcal{O}_{ad}, \tilde{\mathcal{M}}'(\Omega) = & \\
\int_{\Gamma_N} (\sigma(u_{\Omega}) : e(u_{\Omega}) + \sum_{i=1}^N \sigma(u_{\Omega,i}^1) : e(u_{\Omega,i}^1)) n \theta ds &
\end{aligned} \quad (25)$$

$$\tilde{\mathcal{V}}(\Omega) = \sum_{i=1}^N (a_{\Omega,i}^2 + b_{\Gamma_N,i}^2) \quad (26)$$

$$\begin{aligned}
\forall \theta \in \mathcal{O}_{ad}, \tilde{\mathcal{V}}'(\Omega) = & \\
-4 \sum_{i=1}^N (a_{\Omega,i} + b_{\Gamma_N,i}) \int_{\Gamma_N} \sigma(u_{\Omega}) : e(u_{\Omega,i}^1) n \theta ds, &
\end{aligned} \quad (27)$$

where

$$a_{\Omega,i} = \int_{\Omega} (f_i \cdot u_{\Omega} + f_0 \cdot u_{\Omega,i}) \chi_{f_i} dx$$

and

$$b_{\Gamma_N,i} = \int_{\Gamma_N} (g_i \cdot u_{\Omega} + g_0 \cdot u_{\Omega,i}) \chi_{g_i} ds$$

5 Numerical Example

Compliance minimization with uncertain loads

In the following example, we consider the volume-constrained minimization of the weighted sum

$$\mathcal{J}(\Omega) := \tilde{\mathcal{M}}(\Omega) + \delta \sqrt{\tilde{\mathcal{V}}(\Omega)} \quad (28)$$

of the approximate mean value and standard deviation of the compliance (23) where $\delta \geq 0$.

Let us consider the test case of a three-dimensional chair. The design domain is a subset of a $1.2 \times 1 \times 2$ box, as shown on Figure 1. At the bottom of the chair, four small height cylindrical non-designed zones are clamped. The seat and the backrest are also non-design zones, denoted respectively by χ_1 and χ_2 .

The vertical nominal force applied on the seat surface is equal to $f_{10} = -0.025e_3$, and the horizontal nominal force applied on the backrest surface is equal to $f_{20} = -0.003e_1$. We introduce the perturbations $\hat{f}_{11} = m_1 \xi_1 e_1$ on the seat surface, and $\hat{f}_{21} = m_2 \xi_2 e_3$ on the backrest surface, where ξ_1 and ξ_2 denote two independant identically distributed Gaussian random variables with distribution $\mathcal{N}(\mu, s)$.

We can write the applied surface forces as :

$$f = (f_{10} + \hat{f}_{11}) \chi_1 + (f_{20} + \hat{f}_{21}) \chi_2,$$

In the example below, the numerical values for the mean and the variance are set to $\mu = 1$. and $s = 0.01$.

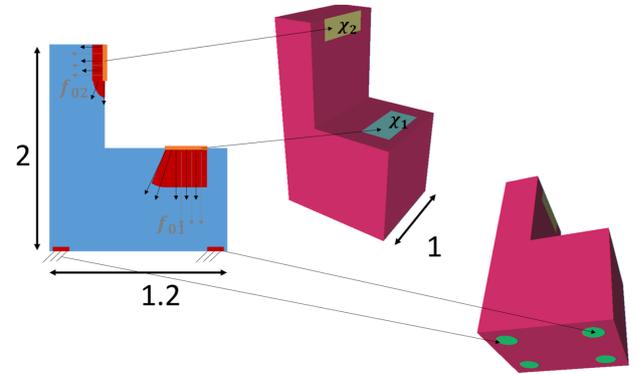


Figure 1: Geometry and boundary conditions for the 3D chair test case.

The optimized shapes obtained at iteration 120 for $(m_1, m_2) \in \{(0, 0), (0.005, 0.0006), (0.008, 0.001)\}$ are displayed on Figure 2. This result clearly shows that the optimized solution changes significantly when uncertainties are taken into account. The convergence histories of compliance and volume for the three cases are shown on Figure 3.

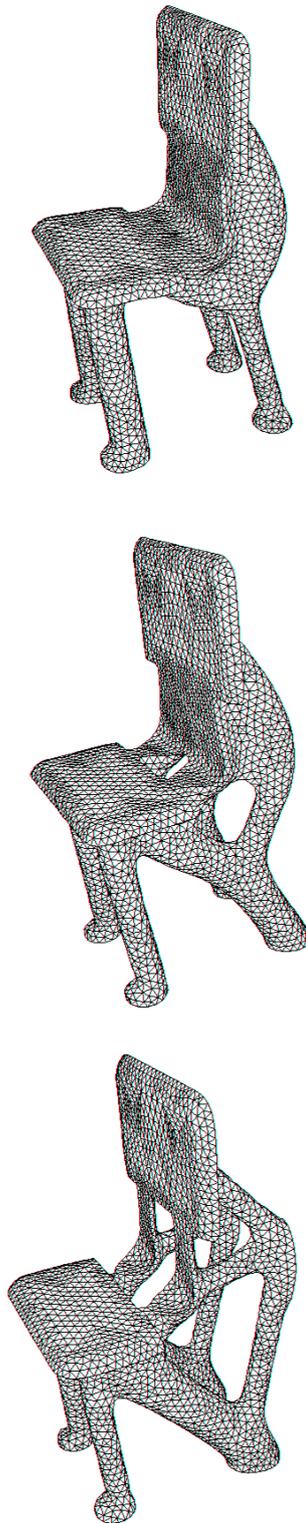


Figure 2: Optimal shape at iteration 120 for $(m_1 = 0, m_2 = 0)$ (top), $(m_1 = 0.005, m_2 = 0.0006)$ (middle) and $(m_1 = 0.008, m_2 = 0.001)$ (bottom).

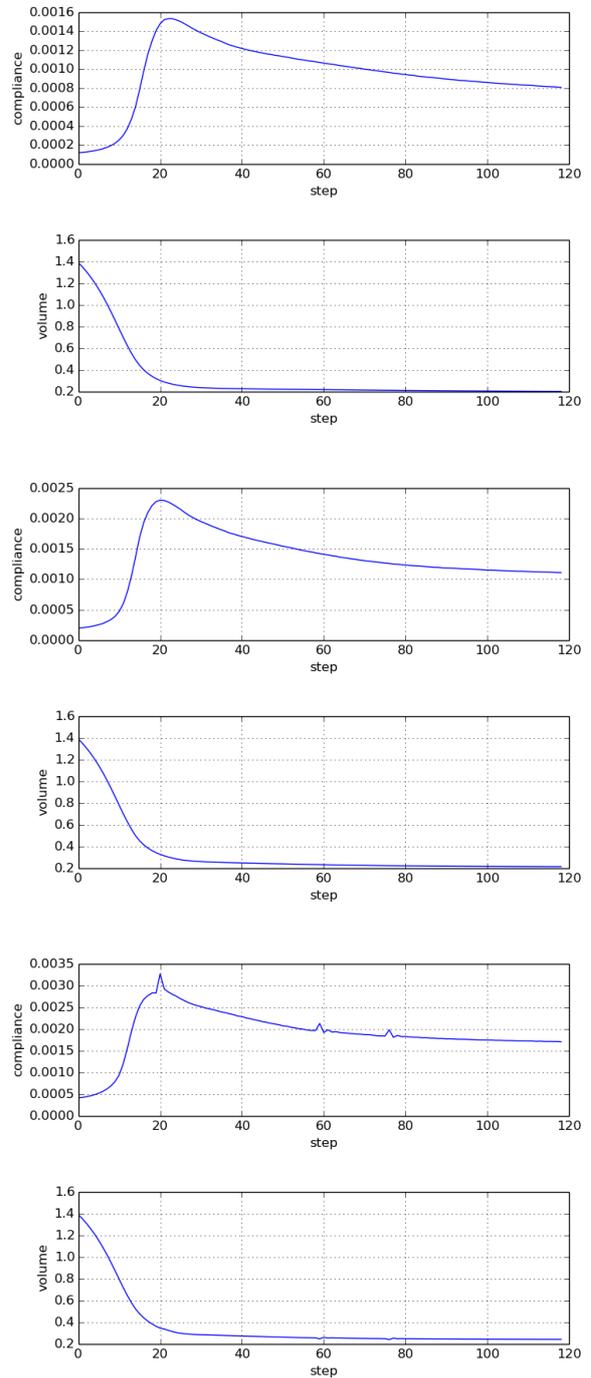


Figure 3: Convergence histories for compliance (upper) and volume (lower) for $(m_1 = 0, m_2 = 0)$ (top), $(m_1 = 0.005, m_2 = 0.0006)$ (middle) and $(m_1 = 0.008, m_2 = 0.001)$ (bottom).

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