Analytical modeling of a coil in a ferromagnetic circuit including a superconductor pellet

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***Abstract*— In the design phase of the inductor required for the magnetization of High Temperature Superconductor (HTS) bulks, modeling and simulation are necessary for the design of this inductor. In this paper we have developed an analytical model to calculate the inductance for a coil of a ferromagnetic circuit with gap including a superconducting pellet. This model is based on the determination of the magnetic vector potential from solving the Laplace's and Poisson's equations in different regions of interest, using the separation of variables method in which the Cartesian coordinates are used. The boundary and continuity conditions between regions are considered to determine the global solution. This analytical resolution is carried out using a computation code developed under MATLAB. The results obtained are compared with those obtained by a numerical simulation based on the finite element method implemented under COMSOL. A remarkable concordance is observed between both approaches.**

Keywords—Analytical Model, Electromagnets, Inductance, Superconducting pellet.

I. Introduction

Superconductivity has been the subject of several theories and research works to unravel the mystery of this phenomenon. The magnetization of superconducting pellets is an interesting way to investigate the trapped field in superconducting machines. Several researches have been devoted to studying the possibility of taking advantage of the significant magnetic induction produced by magnetized pellets for applications in Electrical Engineering [1], [2]. The most widely used method for magnetizing superconductors bulk's is to apply a pulsed magnetic field (PFM method). Since a long time, analytical models have been proposed to study the behavior of the magnetic field created by ironless circular coils [3], [4] and more recently for an axisymmetric coil in a ferromagnetic circuit [5].

In this paper, we extend a previous work [5] by adding a superconducting pellet in the gap area where a Cartesian coordinates are used to make the problem close to the real. The analytical solutions obtained using MATLAB are compared and validated with those obtained numerically through a finite element method implemented under COMSOL.

II. Problem Description and Assumptions

The geometry of the studied problem is shown on Fig. 1. The regions *I*, *III*, *IV*, *V*, and *VI* correspond to the air and the region II corresponds to a circular coil of N turns with rectangular cross-section. The whole system was limited so that $y \in [0, y_5]$ and $x \in [0, x_7]$. The superconducting pellet is modeled as a perfect diamagnetic, which corresponds to a zero magnetic vector potential in the pellet and on its edges. However, the ferromagnetic circuit is modeled by an infinite permeability, this implies that the magnetic field is not calculated inside the ferromagnetic circuit.

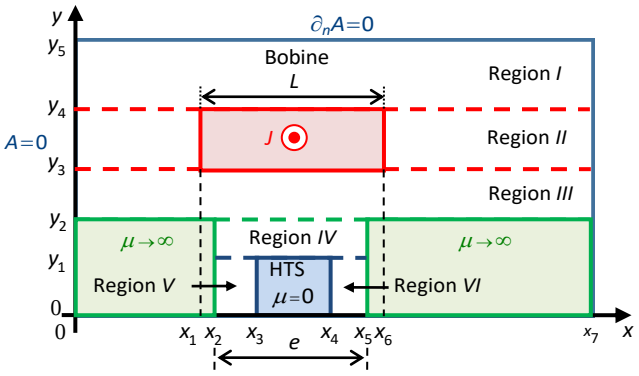

Fig. 1. Geometry of the studied problem: Open magnetic circuit with an air gap of a length *e*, where a superconducting pellet is implemented.

The interface conditions are then used to obtain the tangential component of magnetic field to be null. The homogeneous Dirichlet boundary conditions are considered in regions *I*, *II*, *III*, *V* and *VI*. However, homogeneous Neumann boundary conditions are used in regions *IV*, *V* and *VI*. A magnetic vector potential formulation in Cartesian coordinate is used in this problem, where only the *z*-component exists and it depends only on the *x* and *y* coordinates.

The expressions of magnetic vector potential are obtained by resolving a Poisson's equation in the coil region and a Laplace's equation in the other regions, as follows:

$$\nabla^2 A_i = \begin{cases} -\mu_0 J & \text{for } i = II \text{ (Coil region)} \\ 0 & \text{for } i = I, III, IV, V \text{ and } VI \end{cases} \quad (1)$$

Then, the expressions of the magnetic vector potential in the different regions can be expressed as:

$$A_I(x,y) = \sum_{n=1}^{\infty} b_n^I \left(e^{\alpha_n (y-y_5)} + e^{-\alpha_n y}\right) \sin(\alpha_n x) \quad (2)$$

$$A_{II}(x,y) = \sum_{n=1}^{\infty} \begin{pmatrix} a_n^{II} e^{\alpha_n y} + b_n^{II} e^{-\alpha_n y} \\ +\mu_0 J_{2n} \alpha_n^{-2} \end{pmatrix} \sin(\alpha_n x) \quad (3)$$

$$A_{III}(x,y) = \sum_{n=1}^{\infty} \left(a_n^{III} e^{\alpha_n y} + b_n^{III} e^{-\alpha_n y}\right) \sin(\alpha_n x) \quad (4)$$

$$A_{IV}(x,y) = \begin{pmatrix} A_{01} y \\ +A_{02} \end{pmatrix} + \sum_{n=1}^{\infty} \begin{pmatrix} a_n^{IV} e^{\beta_{1n} y} \\ +b_n^{IV} e^{-\beta_{1n} y} \end{pmatrix} \cos\left(\beta_{1n}(x - x_2)\right) \quad (5)$$

$$A_V(x,y) = \sum_{n=1}^{\infty} 2 a_n^V \sinh(\beta_{2n} y) \cos\left(\beta_{2n}(x - x_2)\right) \quad (6)$$

$$A_{VI}(x,y) = \sum_{n=1}^{\infty} 2 a_n^{VI} \sinh(\beta_{2n} y) \sin\left(\beta_{3n}(x - x_4)\right) \quad (7)$$

where n is a positive integer, and

$$J_{2n} = (2J / n\pi)\left(\cos(\alpha_n x_1) - \cos(\alpha_n x_6)\right),$$

$$\alpha_n = n\pi / x_7, \beta_{1n} = n\pi / (x_5 - x_2), \beta_{2n} = \frac{(2n-1)}{2(x_3 - x_2)},$$

$$\beta_{3n} = (2n-1)\pi / 2(x_5 - x_4).$$

A_{01} et A_{02} are integration constants due to the Neumann boundary conditions in region *IV*. It can be determined by applying the condition of continuity of the magnetic vector potential at the interfaces between regions as

$$A_{IV}(x, y = y_2) = A_{III}(x, y = y_2) \quad x \in [x_2, x_5] \quad (8)$$

$$A_{IV}(y = y_1, x) = \begin{cases} A_V(y = y_1, x) & x \in [x_2, x_3] \\ 0 & x \in [x_3, x_4] \\ A_{VI}(y = y_1, x) & x \in [x_4, x_5] \end{cases} \quad (9)$$

$b_n^I, a_n^{II}, b_n^{II}, a_n^{III}, b_n^{III}, a_n^{IV}, b_n^{IV}, a_n^V$ and a_n^{VI} are the integration constants. The relations between those integration constants are determined by applying the interface conditions between the different regions. Finally, we have to solve here a system of nine linear equations with nine unknowns by rewriting this system in matrix and vectors format.

III. Results and Discussion

Fig. 3 shows the radial and the axial components of the magnetic flux density along the *x*-axis in Region *III* obtained with analytical and numerical models where a true concordance is observed between these two approaches.

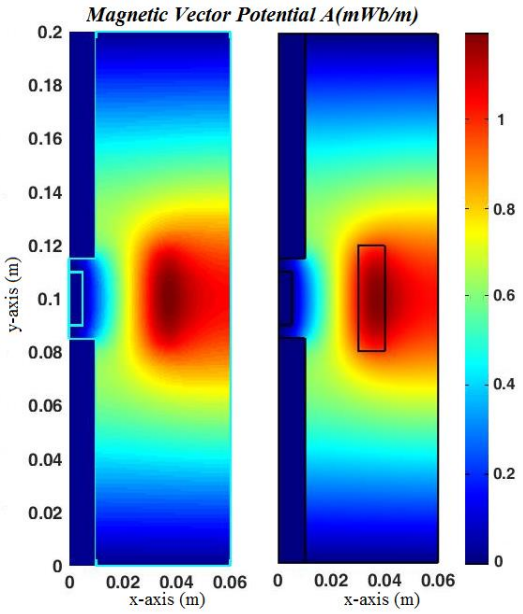

Fig.2. Comparison of the distributions of the magnetic vector potential obtained from the analytical calculation (on the left) and the numerical calculation (on the right).

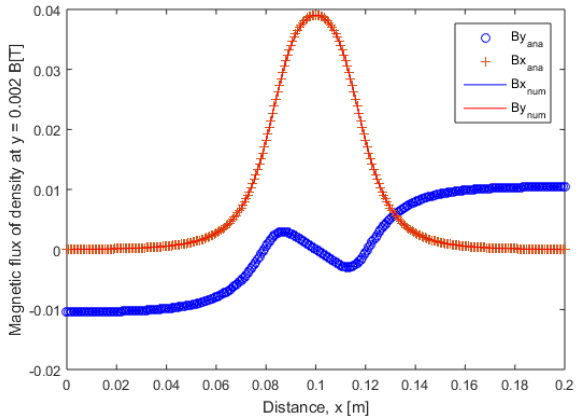

Fig.3. B_x and B_y components of the magnetic flux density along the *x*-axis in the Region *III* at $y = (y1+y2)/2$.

IV. Conclusion

In this work and by using a Cartesian coordinates, we have demonstrate that results provide by the analytical model show a very good match with numerical ones.

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