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# Model-based prognosis using an explicit degradation model and Inverse FORM for uncertainty propagation

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**Abstract:** In this paper, an analytical method issued from the field of reliability analysis is used for prognosis. The inverse first-order reliability method (Inverse FORM) is an uncertainty propagation method that can be adapted to remaining useful life (RUL) calculation. An extended Kalman filter (EKF) is first applied to estimate the current degradation state of the system, then the Inverse FORM allows to compute the probability density function (pdf) of the RUL. In the proposed Inverse FORM methodology, an analytical or numerical solution to the differential equation that describes the evolution of the system degradation is required to calculate the RUL model. In this work, the method is applied to a Paris fatigue crack growth model, and then compared to filter-based methods such as EKF and particle filter using performance evaluation metrics (precision, accuracy and timeliness). The main advantage of the Inverse FORM is its ability to compute the pdf of the RUL at a lower computational cost.

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**Keywords:** inverse first-order reliability method (Inverse FORM), remaining useful life (RUL), model-based prognosis, uncertainty propagation, extended Kalman filter, Paris' law.

## 1. INTRODUCTION

The failure of complex systems such as aircraft or spacecraft can lead to human and industrial disasters. To address these safety and cost issues, a prognosis module should be integrated to these systems in order to continuously assess their state of health and estimate their remaining useful life (RUL). The prognosis methods are usually classified into three categories (Liu et al., 2009). In the knowledge-based approaches (Biagetti, 2004), historical and empirical failure data have allowed experts to deduce degradation rules. In data-driven approaches (Si et al., 2011), features from operating data such as current, temperature, or vibration signals are extracted, then statistical and machine learning techniques are employed to estimate and forecast the evolution of the degradation state. The third category gathers the model-based prognosis approaches (Luo et al., 2003) where a dynamic mathematical model of the system is used.

In this paper, emphasis is placed on the model-based prognosis techniques. Although model-based prognosis approaches can be difficult to set up because an accurate degradation model is seldom available, they can outperform knowledge-based and data-driven methods. Indeed, the capacity of model-based techniques to adapt the model to the evolution of the system degradation ensures an accurate prognosis if more information on the degradation becomes available.

Many model-based prognosis approaches have been developed in the literature (Byington et al., 2004), but they do not always include a measure of the uncertainty as-

sociated to RUL prediction. However, it is fundamental to associate a probability density function (pdf) to the predicted RUL to enable risk-based decisions (Baraldi et al., 2013; Sankararaman and Goebel, 2013a). Therefore, greater attention has been paid to the integration of uncertainty quantification, representation and management in prognosis methods (Orchard et al., 2008). Uncertainty quantification consists in finding the different sources of uncertainty so they could be integrated in the models and simulations. The main uncertainty sources are modeling uncertainty, sensor measurement uncertainty and operational uncertainty. Once the various uncertainty sources have been identified, a sensitivity analysis can be carried out to quantify the influence of each uncertainty source and then determine those that have the greatest impact on the prognosis problem. After identifying the significant uncertainty sources, a method for uncertainty representation must be chosen. Usually, model-based prognosis methods represent the uncertainty in a probabilistic framework (Saha and Goebel, 2008). Finally, uncertainty management performs a propagation step after incorporating the relevant uncertainty sources to the models and the simulations with the designated uncertainty representation.

The filter-based techniques are commonly used as prognosis methods that take uncertainty into account (Saha et al., 2009; Daigle and Goebel, 2010). Yet, the Monte-Carlo simulation-based approaches require huge computational efforts and are time consuming. As an alternative to sampling-based methods, an analytical method which is the inverse first-order reliability method (Inverse FORM), originally developed in the reliability analysis field, was

recently adapted for RUL prediction (Sankararaman and Goebel, 2013b; Bressel et al., 2016). This method has the particularity to compute only some points of the RUL pdf and then reconstruct the entire pdf by interpolation if needed. In the cases where only the mean of the RUL pdf and the 95% probability bounds are required, the Inverse FORM is very efficient. In Bressel et al. (2016), the calculation of RUL with the Inverse FORM was applied in the specific case of a proton exchange membrane fuel cell using a linear regression. In this paper, a more general Inverse FORM methodology for RUL prediction is explicitly provided. This methodology can be applied if an expression of the RUL derived from the explicit dynamical degradation model is available. As an example to illustrate the proposed methodology, a fatigue crack growth problem is chosen. Starting from the Paris' law which is a dynamical model of fatigue crack growth, an analytical expression of the RUL is calculated. This analytical expression is then used in the Inverse FORM algorithm to propagate the uncertainty in the model parameters in order to compute the RUL pdf. The proposed methodology has the advantage that it can be applied to various problems as long as an analytical model is available. The efficiency of the method is investigated through a crack growth analysis, and the performance metrics values obtained with the Inverse FORM are compared to those obtained in a previous work (Robinson et al., 2016) where a particle filter and an extended Kalman filter (EKF) were applied for RUL prognosis.

The paper is organized as follows. Section 2 presents the different steps of model-based prognosis. In Section 3, the Inverse FORM methodology is detailed. Section 4 provides the numerical results obtained with a crack growth benchmark model. Finally, Section 5 concludes the paper and presents some perspectives for future work.

## 2. RUL CALCULATION: AN UNCERTAINTY PROPAGATION PROBLEM

This section presents the model-based prognosis process which is conducted in three steps: (i) degradation state estimation, (ii) future degradation state prediction and (iii) RUL calculation. An illustration of the process is given in Fig. 1.

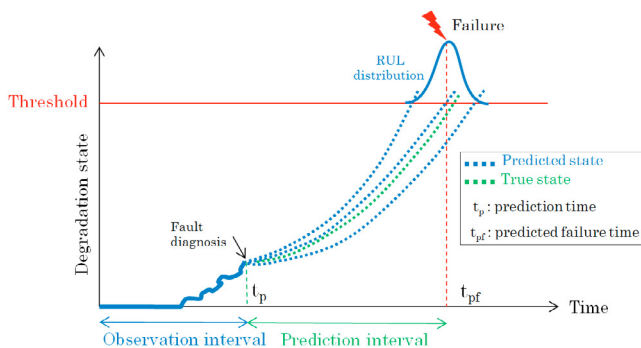


Fig. 1. Scheme of the prognosis process

### 2.1 Degradation state estimation

Usually, a discrete-time state space representation is employed to relate the mathematical model of the degrada-

tion with the data from the different sensors to determine the evolution of the degradation at any time instant:

$$x_k = f(x_{k-1}, \theta_{k-1}, u_k, w_k) \quad (1)$$

$$y_k = h(x_k, \theta_k, u_k, v_k) \quad (2)$$

where  $x \in \mathbb{R}^n$  denotes the state,  $\theta \in \mathbb{R}^q$  represents the unknown model parameter vector,  $y \in \mathbb{R}^p$  is the measured outputs,  $u \in \mathbb{R}^m$  is the vector of system inputs and  $k \in \mathbb{N}$  is a discrete time step. The functions  $f$  and  $h$  describe respectively evolution of the state and the measurements over time. The variables  $w$  and  $v$  are respectively the process and measurement noises which represent the model and measurement uncertainty. The degradation state is estimated at each time step using the measurements until the prediction time  $k_p$ . As the degradation model is often nonlinear, appropriate state estimation techniques should be adopted. Extended Kalman filters and particle filters are commonly used for this purpose.

### 2.2 Future degradation state prediction

During the current state estimation, sensor data are available for a specific observation interval whose size depends on the prediction time  $k_p$ . Then, from this time instant, the forecasting of the degradation state in the future is carried out for time instants  $k > k_p$  without new measurements. The main challenge in this prediction step lies in the fact that future operational conditions of the system are unknown. Therefore, the forecasting of the degradation state must be performed by taking uncertainties into account.

### 2.3 Remaining useful life calculation

The future state is predicted until the failure threshold is reached, giving the predicted failure time  $k_f$ . Finally, the RUL at time  $k_p$  which is denoted by  $R(k_p)$  can be calculated as:

$$R(k_p) = k_f - k_p. \quad (3)$$

For better readability, the dependency in  $k_p$  will be omitted in the following and the RUL simply denoted by  $R$ . However, as the future state prediction is uncertain, the predicted failure time  $k_f$  is uncertain, making the RUL a random variable that depends on:

- Present degradation state:  $x_{k_p}$ ;
- Future operating conditions:  $\{u_{k_p}, u_{k_{p+1}}, \dots, u_{k_f}\}$ ;
- Future parameter values:  $\{\theta_{k_p}, \theta_{k_{p+1}}, \dots, \theta_{k_f}\}$ ;
- Future noises  $\{w_{k_p}, w_{k_{p+1}}, \dots, w_{k_f}\}$  and disturbances  $\{v_{k_p}, v_{k_{p+1}}, \dots, v_{k_f}\}$ .

If the vector  $X$  contains all of the uncertain quantities mentioned earlier *i.e.*  $X = [x_{k_p}, u_{k_p}, u_{k_{p+1}}, \dots] \in \mathbb{R}^s$  where  $s$  is the number of uncertain parameters, and  $\mathcal{M}$  is a function  $\mathcal{M} : \mathbb{R}^s \rightarrow \mathbb{R}$ ,  $X \mapsto \mathcal{M}(X)$  that allows to compute the RUL, then we have:

$$R = \mathcal{M}(X). \quad (4)$$

The model  $\mathcal{M}$  can be a known function defined by a mathematical expression of the RUL, or may be a black-box function such as a complex computer code (e.g. a finite element code) that takes input values and provides a result. The main goal in prognosis is to quantify the uncertainty in  $R$  and to compute its pdf  $f_R(\cdot)$ , which is equivalent to propagate the uncertainty in  $X$  through  $\mathcal{M}$ .

There exist two main categories of probabilistic uncertainty propagation methods: (i) sampling-based methods and (ii) analytical methods. In sampling-based methods such as basic Monte-Carlo simulations (MCS), a large number of random realizations of  $X$  are generated, and the corresponding realizations of  $R$  are calculated to finally construct its pdf. The most commonly used sampling methods in model-based prognosis rely on particle filters and Kalman filters. The ability of these methods to manage uncertainty in RUL calculation have already been proven (Orchard and Vachtsevanos, 2009; Daigle et al., 2012), however they are computationally expensive as the precision of the results depends on the number of simulations. In order to overcome these time consuming issues, analytical methods that are originally from the field of reliability analysis have been used for RUL prediction (Sankararaman et al., 2013). Indeed, with these methods far fewer simulations are needed to quantify the uncertainty in the predicted RUL value. There exists many analytical methods of uncertainty propagation (Sudret, 2007), and the one that was commonly investigated for prognosis is the Inverse FORM that is presented in the next section.

### 3. INVERSE FIRST-ORDER RELIABILITY METHOD FOR RUL CALCULATION

In this section, the general principle of reliability analysis and classical FORM is presented briefly to introduce the notations and definitions that will be used and to illustrate the relationship between reliability analysis and RUL calculation. Complete information about reliability analysis can be found in Lemaire (2009).

#### 3.1 Reliability analysis

Let us consider a general uncertainty propagation problem with a model random response  $Y \in \mathbb{R}$  defined by  $Y = \mathcal{M}(X)$ . The goal is to compute the pdf of  $Y$  using reliability analysis techniques. Reliability analysis aims at calculating the failure probability  $P_f$  of a system, regarding the uncertainty affecting the input vector  $X$ , and with respect to a failure criterion. To characterize this failure criterion, a so-called limit-state function  $G$  and a threshold value  $y_{th}$  of  $Y$  are introduced such that:

$$G(X) = y_{th} - \mathcal{M}(X). \quad (5)$$

Using this definition, the system is considered to be in a failure state if  $\mathcal{M}(X)$  exceeds the prescribed threshold  $y_{th}$ . In this way, if  $x$  denotes the realizations of the uncertain parameter vector  $X$ , this limit-state function  $G$  separates the variable space in two domains:

- $\mathcal{D}_s = \{x : G(x) > 0\}$  defines the safety domain;
- $\mathcal{D}_f = \{x : G(x) \leq 0\}$  defines the failure domain;

Therefore, if  $f_X(\cdot)$  is the joint pdf of  $X$ , then computing the failure probability of the system is equivalent to evaluating the probability that the realizations  $x$  of the input vector  $X$  are in the failure domain:

$$P_f = P(G(X) \leq 0) = \int_{G(X) \leq 0} f_X(x) dx. \quad (6)$$

This integral can be calculated analytically only in some simple academic cases, therefore numerical methods have been developed to compute it.

Approximation methods such as FORM is one of the the most commonly used methods. One needs first to transform the uncertain parameters vector  $X$  into a vector of standard normal variables  $U$  and to work in this new standard normal space. Then, the idea is to proceed to the linearization of the function  $G$  using a first-order Taylor series approximation. This linearization is done around the so-called Most Probable Point (MPP), which is the point on the limit-state surface closest to the origin in the standard normal space. The FORM algorithm aims at identifying the MPP and then computing the reliability index  $\beta$  which is equal to the distance from the origin of the standard normal space to the MPP. The failure probability is finally obtained using:

$$P_{f,FORM} = \Phi(-\beta) \quad (7)$$

where  $\Phi(\cdot)$  denotes the standard normal cumulative distribution function (CDF). If the true limit-state function  $G$  is linear, then this equation is exact. In the case where the true  $G$  function is nonlinear, the failure probability obtained from the above formula is only an approximation. Further details about the FORM algorithm can be found in (Hohenbichler et al., 1987).

#### 3.2 Inverse FORM for RUL calculation

An inverse reliability problem consists in finding the uncertain parameter vector  $X$  such that a prescribed reliability index  $\beta_{target}$  is attained, in other words for which parameter values the system falls into the failure state. In this section, a general methodology to compute the RUL with the Inverse FORM is presented. The Inverse FORM was recently adapted for RUL calculation (Sankararaman et al., 2013; Bressel et al., 2016) as an efficient alternative to computationally expensive MCS-based methods. Indeed, when calculating the RUL at time instant  $k_p$  with sampling-based methods, many trajectories from time  $k_p$  to failure time  $k_f$  of the uncertain parameters vector have to be simulated to obtain the RUL pdf. Whereas with the Inverse FORM, the most probable parameter vector  $X$  at time  $k_p$  associated to a chosen reliability index is computed without simulating the system until failure.

In the problem of RUL prognosis, the Inverse FORM allows to compute the RUL CDF  $F_R(r) = P(r \leq R)$  by calculating the realization  $r$  of the random variable  $R$  such that for a given  $\beta_{target}$ :

$$P(r \leq R) = P_f = \Phi(-\beta_{target}) \quad (8)$$

Therefore, the limit-state function is defined as:

$$G(X) = r_{th} - \mathcal{M}(X) \quad (9)$$

where  $r_{th} \in \mathbb{R}$  is a threshold value of  $R$  and  $\mathcal{M}(X)$  is a function that represents  $R$ . As it was said earlier,  $\mathcal{M}$  can be a black box function (this case is not treated in this work) or an analytical expression of the RUL. Therefore, the first step to perform RUL prediction with the Inverse FORM is to find an explicit expression of the RUL.

In Bressel et al. (2016), the Inverse FORM for RUL prediction has been applied in the particular case of a proton exchange membrane fuel cell (PEMFC). The

evolution of the parameters of the stack voltage model of the PEMFC have been extracted using a Levenberg-Marquardt algorithm, to be able to quantify the effect of aging on these parameters. Then, the parameters whose values are the most affected by aging are chosen to build a degradation model with a linear equation. Finally, the expression of the RUL is a function of the state of health and the degradation speed. In this case, the results of the prognosis with the Inverse FORM are satisfying, however the methodology was only made for a PEMFC.

In this paper, the methodology is more general because it can be applied if any type of solution to calculate the RUL from the dynamical degradation model is found. The solution may be analytical, and in the case where no analytical expression is available, a numerical or an approximate solution can be used.

The first step of the proposed methodology is to solve the differential equation of the dynamical model of the degradation. After obtaining the evolution of the degradation state  $x$  as a function of time, the failure time instant  $k_f$  is deduced by solving the equation  $x(k_p) = x(k_f)$ . The analytical expression of the RUL at time  $k_p$  is obtained with  $R(X) = k_f - k_p$ .

Then, the Inverse FORM algorithm is used to find the parameter vector  $X$  at time  $k_p$  that satisfies a reliability index  $\beta_{target}$ . For the numerical search of the uncertain parameter vector  $X$ , the following constraints must be satisfied:

$$C : \begin{cases} a : P_f = \Phi(-\beta_{target}) \\ b : ||u|| = \beta_{target} \\ c : u + ||u|| \frac{\nabla_u G(u)}{||\nabla_u G(u)||} = 0 \end{cases} \quad (10)$$

where  $u$  is a realization of  $U$  which is the random vector  $X$  expressed in the standard normal space and  $||\cdot||$  is the Euclidean norm. To find the optimum solution to (10.c), a numerical search is required (see Der Kiureghian et al. (1994) for more details). The steps that are followed for each iteration  $j$  of the algorithm are:

- (1) Setting  $j$  to 0 and initial guess of the realizations of the uncertain parameter vector  $x^j = [x_0^j, \dots, x_s^j]$  where  $s$  is the number of uncertain parameters.
- (2) Transformation into the standard normal space. In the case of Gaussian variable we have:

$$u_i^j = \frac{x_i^j - \mu_i}{\sigma_i} \quad (11)$$

where  $\mu$  and  $\sigma$  are respectively the mean and the standard deviation of the uncertain variables. These quantities can be derived from the estimation step, as an output of the filtering algorithm (e.g. a Kalman filter).

- (3) Calculation of the gradient vector of  $G$ :

$$\alpha_i = \frac{\partial G}{\partial u_i} = \frac{\partial G}{\partial x_i} \times \frac{\partial x_i}{\partial u_i} \quad (12)$$

- (4) Calculation of the next point  $u^{j+1}$ :

$$u^{j+1} = -\beta_{target} \frac{\alpha}{||\alpha||} \quad (13)$$

where  $\beta_{target} = -\Phi^{-1}(P_f)$ .

- (5) Transformation into original space to compute  $x^{j+1}$ .

The steps from 3 to 5 are repeated until the following convergence criteria within tolerances  $\delta_1$  and  $\delta_2$  are satisfied:

- (i) The solution belongs to the limit-state surface:

$$|G(x^j) - r_{th}| \leq \delta_1. \quad (14)$$

- (ii) The solution is almost constant between two iterations:

$$|x^{j+1} - x^j| \leq \delta_2. \quad (15)$$

The above procedure can be repeated for different values of  $P_f$ . According to the needs of the user, the entire CDF can be computed with  $P_f = \{0.1, 0.2, \dots, 0.9\}$ , or only the 95% probability bounds and the mean can be computed with  $P_f = \{0.05, 0.5, 0.95\}$ .

#### 4. NUMERICAL RESULTS

In this section, the Inverse FORM algorithm is applied to a fatigue crack growth analysis. The results are compared to those obtained in the previous work by Robinson et al. (2016) where a particle filter and an extended Kalman filter have been used for prognosis.

##### 4.1 Degradation model and RUL expressions

First of all, the degradation model of the system is required. In this work, the Paris' law is used (Paris and Erdogan, 1963):

$$\frac{da}{dN} = C(\Delta K)^m, \quad \Delta K = \Delta\sigma\sqrt{\pi a} \quad (16)$$

where  $a$  is the crack size,  $N$  is the number of cycles,  $\Delta K$  is the range of stress intensity factor and  $\Delta\sigma$  is the stress range.  $C$  and  $m$  are the unknown model parameters to be estimated.

This differential equation is solved to find the expression of the crack length  $a$  with respect to the cycle number  $N$ :

$$\frac{1}{(\sqrt{a})^m} da = C(\Delta\sigma\sqrt{\pi})^m dN$$

$$a^{-\frac{m}{2}} da = C(\Delta\sigma\sqrt{\pi})^m dN$$

$$\int_{a_0}^a a^{-\frac{m}{2}} da = \int_0^N C(\Delta\sigma\sqrt{\pi})^m dN$$

$$\left[ \left( \frac{2}{2-m} \right) a^{\left( \frac{2-m}{2} \right)} \right]_{a_0}^a = C(\Delta\sigma\sqrt{\pi})^m (N - N_0).$$

This yields:

$$a(N) = \left[ a_0^{\left( \frac{2-m}{2} \right)} + \left( \frac{2-m}{2} \right) C(\Delta\sigma\sqrt{\pi})^m (N - N_0) \right]^{\left( \frac{2}{2-m} \right)}. \quad (17)$$

Then, the expression of the failure time  $N_f$  is calculated by solving  $a(N) = a_f$  where  $a_f$  is the crack length threshold:

$$\left[ a_0^{\left( \frac{2-m}{2} \right)} + \left( \frac{2-m}{2} \right) C(\Delta\sigma\sqrt{\pi})^m (N_f - N_0) \right]^{\left( \frac{2}{2-m} \right)} = a_f$$

$$\Leftrightarrow N_f = \left( \frac{2}{2-m} \right) \left( \frac{a_f^{\left( \frac{2-m}{2} \right)} - a_0^{\left( \frac{2-m}{2} \right)}}{C(\Delta\sigma\sqrt{\pi})^m} \right) + N_0.$$

Finally, we obtain the following expression of the RUL calculated at the prediction time  $N_p$ :

$$R(N_p) = N_f(N_p) - N_p \quad (18)$$

$$R(N_p) = \left( \frac{2}{2 - m_p} \right) \left( \frac{a_f^{\left(\frac{2-m_p}{2}\right)} - a_{0p}^{\left(\frac{2-m_p}{2}\right)}}{C_p(\Delta\sigma\sqrt{\pi})^{m_p}} \right) \quad (19)$$

where  $a_{0p}$ ,  $C_p$  and  $m_p$  are the values of the parameters at  $N_p$ .

The Inverse FORM algorithm is used to find for which parameter values the system fails, with reference to a specified failure probability level. In this case, the uncertain parameters vector is  $\mathbf{X} = [a_{0p}, C_p, m_p]$ .

#### 4.2 Simulation results

The first step which consists in estimating the state of health of the system is realized with an EKF, then from the prediction time  $k_p$ , the RUL is computed with the Inverse FORM algorithm.

The real crack size data is generated using the values given in Table 1 and measurements used for the estimation step are obtained by adding a uniform noise distributed in the interval  $[-0.002, 0.002]$ . In the simulation,  $\log(C)$  is used because  $C$  has a very small value.

Table 1. Simulation parameters

$\Delta\sigma$	$dN$	$a_{true}$	$\log(C_{true})$	$m_{true}$
78	50	0.01	-22.33	3.5

During the estimation step with the EKF, 24 measurements were generated every 50 cycles, from cycle 0 to the prediction time  $k_p$  at cycle 1200. The EKF that was used has exactly the same parameters as the one applied in Robinson et al. (2016). Then, from this time instant  $k_p$ , the Inverse FORM algorithm was applied to compute the value of the RUL associated to  $P_f = 0.5$ .

A total of 100 experiments have been repeated to evaluate the numerical performance of Inverse FORM for RUL prognosis in terms of accuracy, precision and timeliness. Accuracy measures the degree of closeness of the predicted RUL to the actual RUL, and its values are between 0 and 1 where 1 gives the best accuracy. Precision evaluates the narrowness of the interval in which the RUL predictions fall, and ranges between 0 and 1 which reflects the highest precision. Finally, timeliness indicates the relative position of the predicted RUL pdf along the time axis with respect to the occurrence of the actual failure event. There are three cases: (i) the failure occurs after the predicted failure time  $t_{pf}$ , (ii) the failure occurs at the same time as the predicted failure time, and finally, (iii) the failure occurs earlier than predicted. This last case must be absolutely avoided, that is why the timeliness function allows to penalize late predictions. Timeliness has positive values and 0 is the best score.

The values of these metrics are compared to those obtained in previous work with a particle filter and an EKF in Table 2 and the RUL pdf obtained within 100 simulations are shown in Fig. 2 and Fig. 3. More details about the PF and the EKF algorithms for prognosis and the performance metrics that were used can be found in Robinson et al. (2016).

The results show that the Inverse FORM outperforms the EKF for RUL computation in terms of accuracy

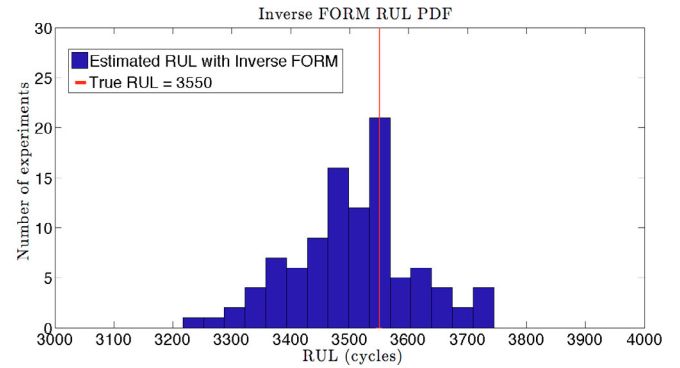


Fig. 2. RUL PDF obtained with the Inverse FORM for 100 experiments

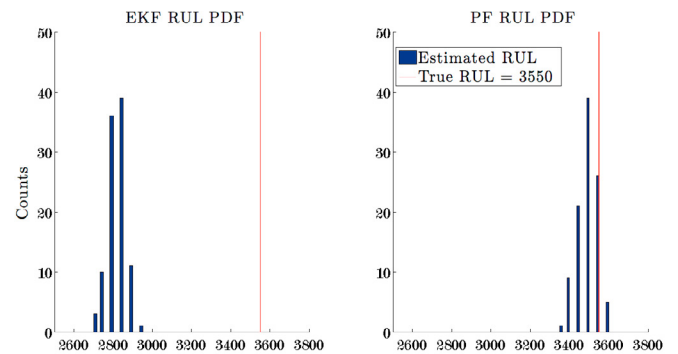


Fig. 3. RUL PDF obtained with the EKF and the particle filter (PF) for 100 experiments

Table 2. Performance evaluation results

Method	Accuracy	Precision	Timeliness
IFORM	0.9759	0.5353	0.1390
EKF	0.8151	0.7501	1.8277
PF	0.9842	0.7283	0.0873

and timeliness. Moreover, the values of the performance metrics obtained with the Inverse FORM are approaching those obtained with the particle filter although the EKF was used for the estimation of the current degradation state. Even if the particle filter has the best results, its main drawback is its complex implementation that is time consuming as the entire pdf of the RUL must be computed. Moreover, as it can be seen in Fig. 4, a large number of the whole trajectories of the degradation state until the threshold is reached have been generated to deduce the failure time and then compute the RUL pdf. The Inverse FORM only needs to compute a few values of RUL associated to a reliability index  $\beta_{target}$ . Usually, three values of  $\beta_{target}$  associated to  $P_f = \{0.05, 0.5, 0.95\}$  are chosen because it allows to obtain the mean value of the RUL pdf with the 95% probability bounds. Therefore, the computational time with the Inverse FORM is much lower, and the performance metrics are satisfying and comparable to the particle filter.

## 5. CONCLUSION

In this paper, a general methodology to compute the RUL with the Inverse FORM was presented, then evaluated and compared to filter-based methods through different performance metrics. The results have highlighted that

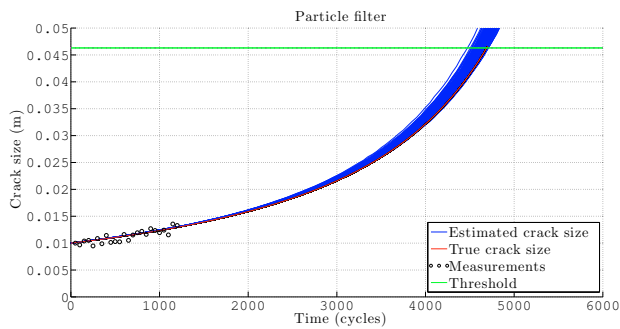


Fig. 4. Predicted trajectories of 100 experiments obtained with the particle filter

in terms of accuracy and timeliness the Inverse FORM gives better results than the EKF. Moreover, the accuracy, precision and timeliness scores of the Inverse FORM are very close to those of the particle filter. Therefore, the Inverse FORM can be an alternative to particle filter-based methods for RUL prognosis in the case where low computational cost algorithms are needed. Indeed, the Inverse FORM algorithm is less time consuming as the computation of the entire pdf of the RUL is not required, and there is no need to propagate the model equation step by step until the threshold is reached. Future work will include the application and validation of these techniques to real data.

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