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**On the relation between the mean compressive strength and the  
characteristic one**

**Short title: Relation between mean and characteristic strengths**

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**ABSTRACT**

Since the beginning of the construction of structures with reinforced concrete, it has been known that concrete presents a variability that should be taken into account. In modern codes, this variability implies the use of a characteristic strength corresponding to a 5% fractile of the distribution of strength.

The actual relation in Eurocode 2 between the characteristic and the mean strength  $f_{ck} = f_{cm} - 8 \text{ MPa}$  was introduced several years ago by Rüsch and is integrated in CEB or fib model codes

since 1978. In this paper, it is presented how the relation was obtained and it is discussed if this relation is still valid considering the fact that the range of concrete strengths is now larger. Considering the scatter of the standard deviation, the relation proposed by Rüsç could still be used but engineers should keep in mind that the standard deviation on site could be very different from the one predicted by means of the relation between  $f_{ck}$  and  $f_{cm}$ .

## INTRODUCTION

The variability of the compressive strength concrete is well known. Figure 1 presents the results obtained during the production of a high performance concrete for the Millau viaduct. This variability has several sources: the variability of the constituents (cement and aggregates, including recycled concrete aggregates - RCA), the variation of the dosage of the constituents (water to cement ratio), the effect of the batching and the compaction processes (energy, duration), the curing method and the variation of the environmental conditions (temperature, relative humidity) and the variation due to sampling and testing (geometry of the samples for instance). Placement of concrete is also a source of variability that can be measured by means of cores or NDT. This last effect is not considered here: all the results presented in this paper result

from tests on samples specially prepared for the measurement of the strength.

How this variability has been taken into account in codes has evolved over time. In the first French code (1906), the compressive stress under serviceability conditions was limited to 28% of the mean compressive strength. Only minor changes can be noted until the notion of characteristic strength was introduced by CEB-FIP in 1970 for prestressed concrete [1]. Since that time, this approach is the basis for the design in all CEB/FIP and also fib model codes, respectively, and in European concrete standards [2]. Now, as proposed by CEB-FIP [3], in EN 1992-1-1 (Eurocode 2 - EC2) [4], the characteristic strength corresponds to a 5% fractile.

For practical reasons, it is nevertheless required to have a relation between this characteristic strength and the mean strength. At the design phase, when a specified characteristic strength needs to be defined, this relation is used to obtain properties that are related to the mean strength (and not to the characteristic one like e.g. the modulus of elasticity or the development of the strength before 28 days). For the construction, because the relations used to determine the mix design of concrete are based on the mean strength, the relation is used (normally with a safety margin added in order to avoid problems on site) to prepare a concrete with a 5%

fractile of the strength distribution larger than the specified characteristic strength [5, 6]

The actual relation in EN 1992-1-1 between the characteristic and the mean strength is:

$$f_{ck} = f_{cm} - 8 \text{ MPa} \quad (\text{Eq. 1})$$

It was introduced several years ago by Rüsch [7] as well as by Rüsch, Sell and Rackwitz [8] and is integrated in CEB-FIP or fib model codes since 1978 [3]. In this paper, it is shown how the relation was obtained and it is discussed if this relation is still valid considering the fact that the range of concrete strengths is larger nowadays.

## **RÜSCH'S PIONEERING WORK**

The equation 1 was proposed by Rüsch on the basis of results (pair of values of the standard deviation and the mean strength) obtained on several construction sites in different countries [7,8]. For this paper, only the 368 results measured at 28 days have been regarded. For each result, the mean strength and the standard deviation is available. The main points of this study were the following: the assumption of a normal distribution for test results from one site is satisfying (even if for the lowest strength a lognormal distribution could be better [9]), the standard deviation varies from 1 to 10 MPa depending on the constitutive materials and on the general conditions of the concrete production, the mean standard deviation

increases parabolically until a mean value of 30 MPa and is almost constant and equal to 5MPa after. If the distribution is Gaussian, the 5% fractile, i.e. the characteristic strength, corresponds to 1,64 times the standard deviation which is almost 8 MPa. Figure 2 presents the results corresponding to Rüsch's analysis. On the basis of it, the constant relation between the mean strength and the characteristic strength was introduced.

## **EXTENDED RESULTS**

Of course, since the studies performed by Rüsch, Sell and Rackwitz[8], the mix design of concrete has evolved: high and very high performance concretes are nowadays used, for instance in the case of bridges where a long durability is needed. Admixtures, additions, recycled concrete aggregates are now widely used. Therefore, it is interesting to extend the database to verify if the relation proposed by Rüsch is still valid. This was done using the available published results. Table 1 presents the results that are used to complete the database. All the results concern samples tested at an age of 28 days with a cylindrical geometry and a large number of samples, depending on the different references. When cubes were used, the mean strength was corrected using the relation proposed in EN1992-1 [4].

All the results are presented in figure 3. The results are compared to relations using equations 2 and 3 where the parameters

a, b, n and m are fitted by means of the minimization of the mean square error:

$$SD_{fit} = 1/(m + \frac{n}{f_{cm}^2}) \quad (\text{Eq. 2})$$

$$SD_{fit} = a f_{cm}^b \quad (\text{Eq. 3})$$

where  $SD_{fit}$  is the estimated standard deviation. The equation 2 was originally proposed by Rüsch. With this equation, an horizontal asymptote is imposed and equal to  $1/m$ . The fit for parameter m is 0.23 so  $1/m=4.35$ . Equation 2 has also a horizontal tangent line at the origin which is not physical. That is why equation 3 is proposed. The best fit for this equation is  $b=1/3$ . With this equation, the coefficient of variation could be estimated (equation 4). Considering the best fit, the COV evolves with a power  $-2/3$  with the mean strength. From these results, a constant COV is not the best fit (indicating that a linear relation for the standard deviation is not the best fit).

$$COV = a f_{cm}^{b-1} \quad (\text{Eq. 4})$$

Compared to EN 1992-1-1, the difference with the constant value given in this standard is not very large in front of the scatter of the standard deviation for a given value of  $f_{cm}$ .

## DISCUSSION

The results presented in figure 3 show that the proposed constant relation between the mean strength and the characteristic strength is

still valid. Indeed, even if there is a slight tendency for an increase of the standard deviation with the mean strength, the variability of the standard deviation is very high. It is of course possible to introduce an equation taking into account this tendency but the results are strongly depending on the used relation. Considering the interval between 20 and 60 MPa where the number of results is the largest, it can be seen that the differences between equation 2 and equation 3 is very small. So, it is difficult from the experimental results to choose between the possible equations.

It is important to note that the used results were obtained for 28 days old concrete. A constant value for the standard deviation is not valid when very early age concrete is considered. In this case, it could be interesting to use a relation where the standard deviation tends to zero when the mean strength tends to zero like the relation proposed in equation 3.

## **CONCLUSION**

The relation between the characteristic strength and the mean strength which is used in CEB-FIP model codes since 1970 was originally defined by Rüsch on the basis of test results obtained on building sites and ready-mixed concrete plants in Germany in the 1960's. It is a simplification because the standard deviation

slightly increases with the mean strength but, considering the scatter of the values of the standard deviation, it is an acceptable simplification. This simplification is still valid nowadays with a range of concrete strengths which is now larger and with very different mix designs for concrete. Finally, engineers should keep in mind that the standard deviation on site could be very different from the one predicted by means of the relation between  $f_{ck}$  and  $f_{cm}$ .

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Figure 1. Distribution of the compressive strength during the construction of the Millau viaduct. The total number of samples is 415 (source Eiffage company)

Figure 2. Evolution of the standard deviation with the mean strength; Rüschi results [7] compared to EC2.

Figure 3: Evolution of the standard deviation with the mean strength; all results (Rüschi + table 1) are compared to the fit obtained using equation 2 with  $a=1.14$  and  $b=0.3$  and equation 3 with  $m=0.23$  and  $n=40.8$

**Table 1. Source of the additional results.**

	Strength range [MPa]	Application	Remarks
German data [10]	25 - 114	Buildings and bridges	
Chmieliewski [11]	44 - 55	N/A	Ready mixed concrete
Precast concrete	25 - 90	Precast products	Personal com.
Tabsch [9]	74 - 96	Buildings and bridges	
French data [12, 13, 14, 15]	55 - 125	Buildings and bridges	Millau viaduct: personal com.
RCA [16,17]	20 - 47	Buildings	
ACI214 [18]	30 - 116	N/A	
Portuguese RMC [6]	25 - 50	N/A	Ready mixed concrete
Moksnes [19]	54 - 66	Sea platforms	