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A Method to Simulate the Skew Normal Distribution

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Abstract

A new method is developed to simulate the skew normal distribution. The result is interesting from a practical as well as a theoretical viewpoint. The new method is simple to program and is more efficient than the standard method of simulation by acceptance-rejection method.

Keywords

Normal Distribution, Skew Normal Distribution

1. Introduction

In this paper, we denote by $SN(\theta)$ the skew normal distribution of parameter θ and density :

$$f(x) = 2\varphi(x)\Phi(\theta x) \tag{1}$$

where φ and Φ denote the standard normal $\mathcal{N}(0,1)$ probability density function and cumulative distribution function, respectively.

The skew normal distribution, due to its mathematical tractability and inclusion of the standard normal distribution, has attracted a lot of attention in the literature. Azzalini [1], Azzalini [2], Chiogna [3], Genton & Liu [4] and Henze [5] discussed basic mathematical and probabilistic properties of the skew normal family. The multivariate skew normal distribution is studied by Azzalini & Capitanio [6] and Azzalini & Dalla Valle [7]. For additional references and a review on related literature, see Azzalini & Capitanio [8] and Pewsey [9] for a collection of papers on the subject. Henze [5], in his paper showed that if U_1 and U_2 are identically and independently distributed $\mathcal{N}(0,1)$ random variables, then $\frac{\theta|U_1|+U_2}{\sqrt{1+\theta^2}}$ has the skew normal distribution.

For the simulation of the skew normal distribution, we propose a combinations of maximum and minimum of

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the independent and identically distributed $\mathcal{N}(0,1)$ random variables.

2. Method

Let U_1 and U_2 two independent and identically distributed $\mathcal{N}(0,1)$ random variables and $U=\max(U_1,U_2)$ and $V=\min(U_1,U_2)$. For simulation of the random variable $X \sim SN(\theta)$, we take the combination of U and V. First note that:

- if $\theta=0$, the density (1) becomes: $f(x)=\varphi(x)$, simply simulate $X \sim \mathcal{N}(0,1)$.
- if $\theta=-1$, the density (1) becomes: $f(x)=2\varphi(x)(1-\Phi(x))$, we take $X=V$.
- if $\theta=1$, the density (1) becomes: $f(x)=2\varphi(x)\Phi(x)$, we take $X=U$.

For $\theta \notin \{-1,0,1\}$, note :

$$\lambda_1=\frac{1+\theta}{\sqrt{2(1+\theta^2)}}, \quad \lambda_2=\frac{1-\theta}{\sqrt{2(1+\theta^2)}} \tag{2}$$

We note that: $\lambda_1^2+\lambda_2^2=1$. For simulation of the random variable $X \sim SN(\theta)$, we take the combination of U and V in the form:

$$X=\lambda_1 U+\lambda_2 V \tag{3}$$

Proposition *The random variable X defined in the Equation* (3) *has the skew normal distribution* $SN(\theta)$.

Proof. The pair (U,V) has density: $f_{U,V}(u,v)=2\varphi(u)\varphi(v)\mathbb{1}_{\{v\le u\}}(u,v)$, where $\mathbb{1}$ is the indicator function. Consider the transformation: $x=\lambda_1 u+\lambda_2 v, y=\lambda_1 u$. The inverse transform is defined by: $u=\frac{y}{\lambda_1}, v=\frac{x-y}{\lambda_2}$ and the corresponding Jacobian is: $J=\frac{1}{\lambda_1\lambda_2}$. X density is defined by:

$$f(x)=\frac{2}{|\lambda_1\lambda_2|}\int_\Delta \varphi\left(\frac{y}{\lambda_1}\right)\varphi\left(\frac{x-y}{\lambda_2}\right)\mathrm{d}y \tag{4}$$

where $\Delta=\left\{\frac{x-y}{\lambda_2}\le\frac{y}{\lambda_1}\right\}$. Taking into account $\lambda_1^2+\lambda_2^2=1$, we can write:

$$\varphi\left(\frac{y}{\lambda_1}\right)\varphi\left(\frac{x-y}{\lambda_2}\right)=\varphi(x)\varphi\left(\frac{y-\lambda_1^2 x}{|\lambda_1\lambda_2|}\right) \tag{5}$$

Equation (4) becomes,

$$f(x)=\frac{2\varphi(x)}{|\lambda_1\lambda_2|}\int_\Delta \varphi\left(\frac{y-\lambda_1^2 x}{|\lambda_1\lambda_2|}\right)\mathrm{d}y \tag{6}$$

For the domain Δ, we have the following three cases:

Case 1: $\theta\in(-1,0)\cup(0,1)$, we have: $|\lambda_1\lambda_2|=\frac{1-\theta^2}{2(1+\theta^2)}$ and $\Delta=\left\{y\ge\frac{\lambda_1 x}{\lambda_1+\lambda_2}\right\}$

Case 2: $\theta<-1$, we have: $|\lambda_1\lambda_2|=\frac{\theta^2-1}{2(1+\theta^2)}$ and $\Delta=\left\{y\ge\frac{\lambda_1 x}{\lambda_1+\lambda_2}\right\}$

Case 3: $\theta>1$, we have: $|\lambda_1\lambda_2|=\frac{\theta^2-1}{2(1+\theta^2)}$ and $\Delta=\left\{y\le\frac{\lambda_1 x}{\lambda_1+\lambda_2}\right\}$

Using Equation (6) and the three cases above, we get the result.

3. Simulation Results

We simulated a sample of size 500,000 for the values $\theta=-7$ and $\theta=13$, The following **Figure 1**, **Figure 2** and **Table 1**, **Table 2** show the results obtained by our method and the method Henze [5].

The results of **Table 1** and **Table 2** show that our method provides results close to the theoretical values and secondly, we obtain results similar to those obtained by Henze [5] results.

4. Conclusion

In this article we propose a very simple method to simulate skew normal family distribution. The obtained

Table 1. Simulation results for a sample size of 500,000 and $\theta = -7$.

	$\theta = -7$		
	Theoretical value	Simulated value[a]	Simulated value[b]
Mean	−0.789865	−0.789651	−0.790496
Variance	0.376113	0.375669	0.376671
Skewness	−0.916950	−0.915649	−0.924019
Kurtosis	0.779197	0.768095	0.796781

[a]by our method; [b]by the method of Henze [5].

Table 2. Simulation results for a sample size of 500,000 and $\theta = 13$.

	$\theta = 13$		
	Theoretical value	Simulated value[a]	Simulated value[b]
Mean	0.795534	0.795174	0.795930
Variance	0.367125	0.366461	0.367464
Skewness	0.971447	0.969756	0.970813
Kurtosis	0.841547	0.836937	0.831219

[a]by our method; [b]by the method of Henze [5].

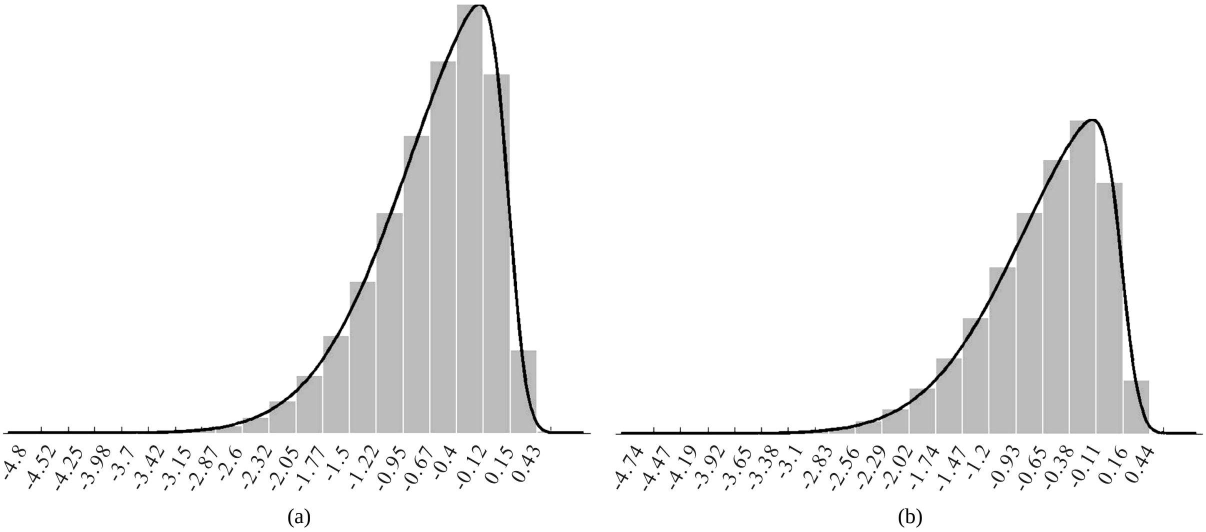

Figure 1. Histogram of simulations for a sample size of 500,000 and $\theta = -7$. (a) histogram of simulations using our method; (b) histogram of simulations using the method of Henze [5].

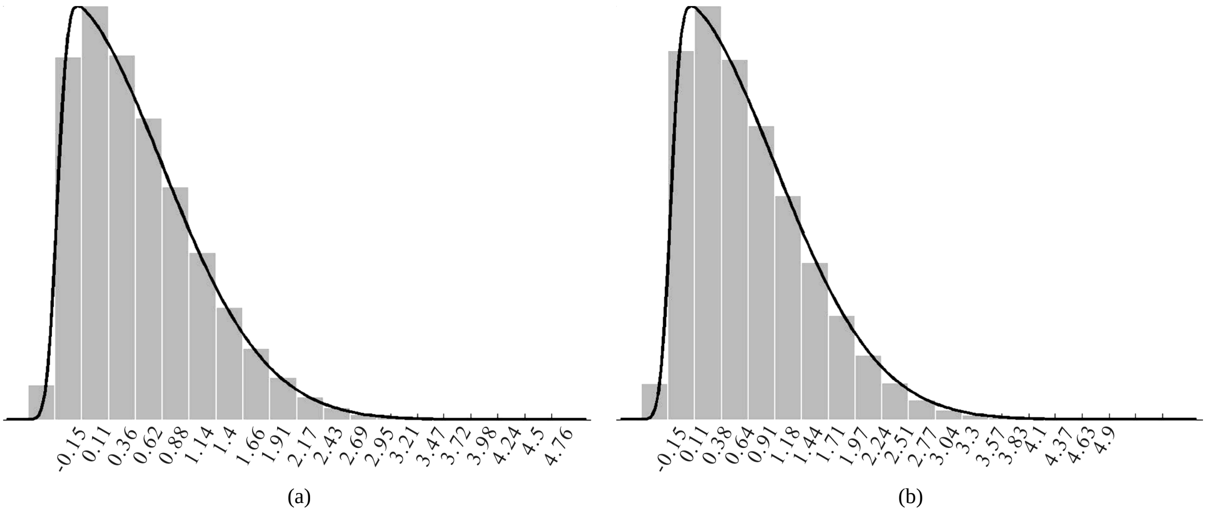

Figure 2. Histogram of simulations for a sample size of 500,000 and $\theta = 13$. (a) histogram of simulations using our method; (b) histogram of simulations using the method of Henze [5].

results are very close to theoretical values and the method is more efficient than the standard one. The method is simple to program and exploit for practical applications.

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