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# A Method to Simulate the Skew Normal Distribution

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## Abstract

A new method is developed to simulate the skew normal distribution. The result is interesting from a practical as well as a theoretical viewpoint. The new method is simple to program and is more efficient than the standard method of simulation by acceptance-rejection method.

## Keywords

Normal Distribution, Skew Normal Distribution

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## 1. Introduction

In this paper, we denote by  $SN(\theta)$  the skew normal distribution of parameter  $\theta$  and density :

$$f(x) = 2\varphi(x)\Phi(\theta x) \quad (1)$$

where  $\varphi$  and  $\Phi$  denote the standard normal  $\mathcal{N}(0,1)$  probability density function and cumulative distribution function, respectively.

The skew normal distribution, due to its mathematical tractability and inclusion of the standard normal distribution, has attracted a lot of attention in the literature. Azzalini [1], Azzalini [2], Chiogna [3], Genton & Liu [4] and Henze [5] discussed basic mathematical and probabilistic properties of the skew normal family. The multivariate skew normal distribution is studied by Azzalini & Capitanio [6] and Azzalini & Dalla Valle [7]. For additional references and a review on related literature, see Azzalini & Capitanio [8] and Pewsey [9] for a collection of papers on the subject. Henze [5], in his paper showed that if  $U_1$  and  $U_2$  are identically and independently distributed  $\mathcal{N}(0,1)$  random variables, then  $\frac{\theta|U_1|+U_2}{\sqrt{1+\theta^2}}$  has the skew normal distribution.

For the simulation of the skew normal distribution, we propose a combinations of maximum and minimum of

the independent and identically distributed  $\mathcal{N}(0,1)$  random variables.

## 2. Method

Let  $U_1$  and  $U_2$  two independent and identically distributed  $\mathcal{N}(0,1)$  random variables and  $U = \max(U_1, U_2)$  and  $V = \min(U_1, U_2)$ . For simulation of the random variable  $X \sim SN(\theta)$ , we take the combination of  $U$  and  $V$ . First note that:

- if  $\theta = 0$ , the density (1) becomes:  $f(x) = \varphi(x)$ , simply simulate  $X \sim \mathcal{N}(0,1)$ .
  - if  $\theta = -1$ , the density (1) becomes:  $f(x) = 2\varphi(x)(1 - \Phi(x))$ , we take  $X = V$ .
  - if  $\theta = 1$ , the density (1) becomes:  $f(x) = 2\varphi(x)\Phi(x)$ , we take  $X = U$ .
- For  $\theta \notin \{-1, 0, 1\}$ , note :

$$\lambda_1 = \frac{1+\theta}{\sqrt{2(1+\theta^2)}}, \quad \lambda_2 = \frac{1-\theta}{\sqrt{2(1+\theta^2)}} \tag{2}$$

We note that:  $\lambda_1^2 + \lambda_2^2 = 1$ . For simulation of the random variable  $X \sim SN(\theta)$ , we take the combination of  $U$  and  $V$  in the form:

$$X = \lambda_1 U + \lambda_2 V \tag{3}$$

**Proposition** *The random variable  $X$  defined in the Equation (3) has the skew normal distribution  $SN(\theta)$ .*

*Proof.* The pair  $(U, V)$  has density:  $f_{U,V}(u, v) = 2\varphi(u)\varphi(v)\mathbb{I}_{\{v \leq u\}}(u, v)$ , where  $\mathbb{I}$  is the indicator function.

Consider the transformation:  $x = \lambda_1 u + \lambda_2 v, y = \lambda_1 u$ . The inverse transform is defined by:  $u = \frac{y}{\lambda_1}, v = \frac{x-y}{\lambda_2}$

and the corresponding Jacobian is:  $J = \frac{1}{\lambda_1 \lambda_2}$ .  $X$  density is defined by:

$$f(x) = \frac{2}{|\lambda_1 \lambda_2|} \int_{\Delta} \varphi\left(\frac{y}{\lambda_1}\right) \varphi\left(\frac{x-y}{\lambda_2}\right) dy \tag{4}$$

where  $\Delta = \left\{ \frac{x-y}{\lambda_2} \leq \frac{y}{\lambda_1} \right\}$ . Taking into account  $\lambda_1^2 + \lambda_2^2 = 1$ , we can write:

$$\varphi\left(\frac{y}{\lambda_1}\right) \varphi\left(\frac{x-y}{\lambda_2}\right) = \varphi(x) \varphi\left(\frac{y - \lambda_1^2 x}{|\lambda_1 \lambda_2|}\right) \tag{5}$$

Equation (4) becomes,

$$f(x) = \frac{2\varphi(x)}{|\lambda_1 \lambda_2|} \int_{\Delta} \varphi\left(\frac{y - \lambda_1^2 x}{|\lambda_1 \lambda_2|}\right) dy \tag{6}$$

For the domain  $\Delta$ , we have the following three cases:

Case 1:  $\theta \in (-1, 0) \cup (0, 1)$ , we have:  $|\lambda_1 \lambda_2| = \frac{1-\theta^2}{2(1+\theta^2)}$  and  $\Delta = \left\{ y \geq \frac{\lambda_1 x}{\lambda_1 + \lambda_2} \right\}$

Case 2:  $\theta < -1$ , we have:  $|\lambda_1 \lambda_2| = \frac{\theta^2 - 1}{2(1+\theta^2)}$  and  $\Delta = \left\{ y \geq \frac{\lambda_1 x}{\lambda_1 + \lambda_2} \right\}$

Case 3:  $\theta > 1$ , we have:  $|\lambda_1 \lambda_2| = \frac{\theta^2 - 1}{2(1+\theta^2)}$  and  $\Delta = \left\{ y \leq \frac{\lambda_1 x}{\lambda_1 + \lambda_2} \right\}$

Using Equation (6) and the three cases above, we get the result.

## 3. Simulation Results

We simulated a sample of size 500,000 for the values  $\theta = -7$  and  $\theta = 13$ , The following [Figure 1](#), [Figure 2](#)

and **Table 1**, **Table 2** show the results obtained by our method and the method Henze [5].

The results of **Table 1** and **Table 2** show that our method provides results close to the theoretical values and secondly, we obtain results similar to those obtained by Henze [5] results.

#### 4. Conclusion

In this article we propose a very simple method to simulate skew normal family distribution. The obtained

**Table 1.** Simulation results for a sample size of 500,000 and  $\theta = -7$ .

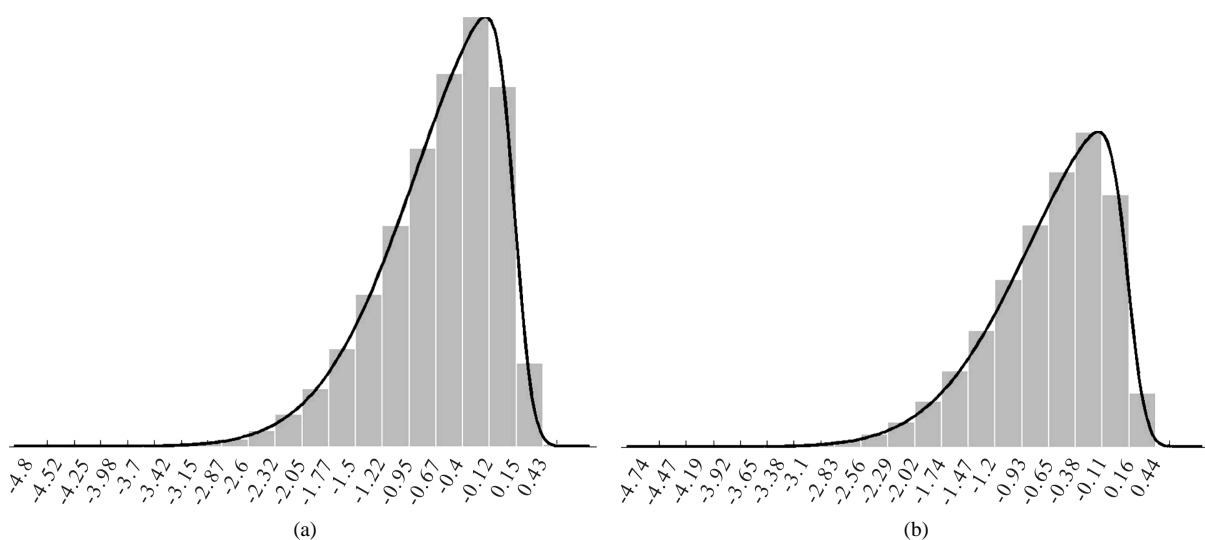
$\theta = -7$			
	Theoretical value	Simulated value <sup>a</sup>	Simulated value <sup>b</sup>
Mean	-0.789865	-0.789651	-0.790496
Variance	0.376113	0.375669	0.376671
Skewness	-0.916950	-0.915649	-0.924019
Kurtosis	0.779197	0.768095	0.796781

<sup>a</sup>by our method; <sup>b</sup>by the method of Henze [5].

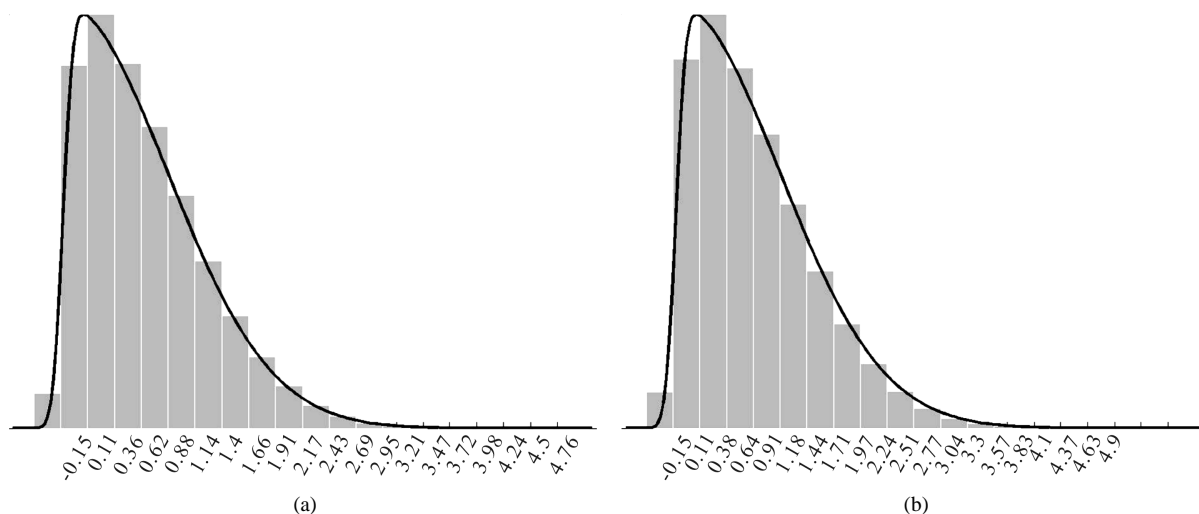
**Table 2.** Simulation results for a sample size of 500,000 and  $\theta = 13$ .

$\theta = 13$			
	Theoretical value	Simulated value <sup>a</sup>	Simulated value <sup>b</sup>
Mean	0.795534	0.795174	0.795930
Variance	0.367125	0.366461	0.367464
Skewness	0.971447	0.969756	0.970813
Kurtosis	0.841547	0.836937	0.831219

<sup>a</sup>by our method; <sup>b</sup>by the method of Henze [5].



**Figure 1.** Histogram of simulations for a sample size of 500,000 and  $\theta = -7$ . (a) histogram of simulations using our method; (b) histogram of simulations using the method of Henze [5].



**Figure 2.** Histogram of simulations for a sample size of 500,000 and  $\theta = 13$ . (a) histogram of simulations using our method; (b) histogram of simulations using the method of Henze [5].

results are very close to theoretical values and the method is more efficient than the standard one. The method is simple to program and exploit for practical applications.

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