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Comments on ‘Distributed observers design for leader–following control of multi-agent networks, *Automatica*, 44:846–850, 2008.’^{*}

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Abstract

In this note it is pointed out that the proof of Theorem 1 of [1] is incorrect for the theorem as stated. A proof for a modified version of the theorem is also given¹.

Key words: Multi-agent system; Active leader; Distributed control; Distributed observer; Common Lyapunov function

The seminal contribution of [1] was the extension of the conventional observer design to a distributed observers’ design and in particular for a scenario of a multi-agent system, where an active leader with unknown velocity has to be followed. This work has been extremely popular and, as a result, has been cited by an increasing number of publications on multi-agent systems. However, the proof of Theorem 1 of [1] is incorrect for the theorem as stated. In this note we give a proof for a modified version of the theorem. This yields a different set of sufficient conditions which establish the stability proof and highlight the restrictive conditions on the acceptable topologies. In what follows we adopt the same notation as in the original work [1].

The dynamic models for the leader and the agents consist of double integrators as in equations (1)-(2) of [1]. Then,

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¹ While the paper was under review, we became aware that another way to address the error in [1] is to simply replace $1/k$ with $1/k^2$ in equation (4) of that paper, as was done in [2]. With such change the proof of [1] is valid. We now also understand that the simulations in [1] were carried with the corrected $1/k^2$ in equation (4) of [1].

the authors proposed the neighbor-based control for agent i , given in (3) of [1], to track the leader while also obtaining an estimate of the (unavailable) leader velocity by the observer in equation (4) of [1]. We first note that the expression of the closed-loop dynamics in equation (6) in the proof of Theorem 1 of [1] is incorrect since the k coefficient in the last term of $\dot{\eta}$ was missing. This puts into question the correctness of the statement of Theorem 1 in [1], and to this end we propose an alternative stabilization theorem for coordination rule (3)-(4) of [1].

Theorem 1 *Consider the closed-loop multi-agent system. In each time interval $[t_i, t_{i+1})$, if the entire graph is connected and satisfies*

$$\underline{\lambda} > \sqrt{\frac{8}{9}\bar{\lambda}}, \quad (1)$$

then, there exist gains $k > 0$ and $l > 0$ satisfying the conditions

$$k > \max\left(2, \frac{1}{2}\left(1 + \sqrt{1 + \frac{8\bar{\lambda}^2}{9\underline{\lambda}^2 - 8\bar{\lambda}^2}}\right)\right) \quad (2)$$

$$l_1 \leq l \leq l_2, \quad (3)$$

with

$$l_{1,2} := \frac{12k^2(k-1)\underline{\lambda} \pm \sqrt{\Delta}}{2(2k-1)^2\bar{\lambda}^2}$$

$$\Delta := -16k^3(k-1)\left(8\bar{\lambda}^2 k^2 - (9\underline{\lambda}^2 + 8\bar{\lambda}^2)k + 2\bar{\lambda}^2\right)$$

such that $\lim_{t \rightarrow \infty} |x_i(t) - x_0(t)| = 0$, $\lim_{t \rightarrow \infty} |v_i(t) - v_0(t)| = 0$, namely, the agents can follow the leader (in the sense of both position and velocity).

PROOF. We will provide the stability proof for one time interval as in the original work. Following the notation in [1] we express the closed-loop dynamics in the compact form of equation (6) of [1] but with the matrix

$$F_p := \begin{bmatrix} 0 & I & 0 \\ -lH_p & -kI & kI \\ -\frac{l}{k}H_p & 0 & 0 \end{bmatrix}. \quad (4)$$

Now, define the candidate Lyapunov function V as in equation (8), of [1], that is positive-definite for $k > 1$. The time-derivative of V along z -trajectories gives (9) of [1] but with

$$Q_p := \begin{bmatrix} lH_p & \frac{l}{2k}(2k-1)H_p & -kI \\ \frac{l}{2k}(2k-1)H_p & 2(k-1)I & -kI \\ -kI & -kI & kI \end{bmatrix}.$$

To check positive definiteness of Q_p , and ensure global exponential stability of the origin, we use the Schur's lemma with

$$A := \begin{bmatrix} lH_p & \frac{l}{2k}(2k-1)H_p \\ \frac{l}{2k}(2k-1)H_p & 2(k-1)I \end{bmatrix},$$

$$E := \begin{bmatrix} -kI \\ -kI \end{bmatrix}, C := kI \text{ to obtain the inequalities}$$

$$D := A - EC^{-1}E^T$$

$$= \begin{bmatrix} lH_p - kI & \frac{l(2k-1)}{2k}H_p - kI \\ \frac{l(2k-1)}{2k}H_p - kI & (k-2)I \end{bmatrix}$$

$$=: \begin{bmatrix} D_1 & D_2 \\ D_2 & D_3 \end{bmatrix} > 0, \quad k > 0.$$

While the second one is trivially satisfied, for the positive-definiteness of matrix D we invoke, again, the Schur's lemma to obtain the conditions $D_3 > 0 \iff k > 2$ and $D_1 - D_2 D_3^{-1} D_2^T > 0 \iff (2k-1)^2 l^2 H_p^2 - 12k^2(k-1)lH_p + 8k^3(k-1)I < 0$. In addition, using the bounds on the matrix H_p yields $(2k-1)^2 \bar{\lambda}^2 l^2 - 12k^2(k-1)\underline{\lambda}l + 8k^3(k-1) < 0$.

This last condition can be seen as a polynomial of second degree in l , expressed as $f(l) := \alpha l^2 + \beta l + \gamma$, $\alpha > 0$, $\beta < 0$, $\gamma > 0$, that needs to be negative, i.e. $f(l) < 0$. We will now derive the general conditions for which set of values of k , l this is true. We calculate first the discriminant $\Delta = 16k^3(k-1)(9\underline{\lambda}^2 - 8\bar{\lambda}^2)\left(k^2 - k - \frac{2\bar{\lambda}^2}{9\underline{\lambda}^2 - 8\bar{\lambda}^2}\right)$, which

needs to be positive to provide us with acceptable real solutions. For $\Delta > 0$, and taking into account also that $k > 2$, we have two cases.

(1) $9\underline{\lambda}^2 - 8\bar{\lambda}^2 < 0$: This condition is always satisfied since $\underline{\lambda}^2 < \frac{8}{9}\bar{\lambda}^2 < \bar{\lambda}^2$. For positivity of Δ the following condition should hold $g(k) := k^2 - k - \frac{2\bar{\lambda}^2}{9\underline{\lambda}^2 - 8\bar{\lambda}^2} < 0$. Calculating the discriminant for $g(k) = 0$ yields $\Delta_k = \frac{1}{9\underline{\lambda}^2 - 8\bar{\lambda}^2} 9\underline{\lambda}^2$ that can never be positive since $9\underline{\lambda}^2 - 8\bar{\lambda}^2 < 0$.

(2) $9\underline{\lambda}^2 - 8\bar{\lambda}^2 > 0$: In this case, the topologies are constrained to the configurations where $\underline{\lambda}^2 > \frac{8}{9}\bar{\lambda}^2$. Now, for positivity of Δ one requires $g(k) := k^2 - k - \frac{2\bar{\lambda}^2}{9\underline{\lambda}^2 - 8\bar{\lambda}^2} > 0$. The discriminant for $g(k) = 0$ reads $\Delta_k = \frac{1}{9\underline{\lambda}^2 - 8\bar{\lambda}^2} 9\underline{\lambda}^2$ and is always positive. We can deduce that $g(k) = 0$ has two roots, one negative k_1 and one positive k_2 , with $|k_1| < k_2$, with k_2 explicitly defined as $k_2 := \frac{1}{2}\left(1 + \sqrt{1 + \frac{8\bar{\lambda}^2}{9\underline{\lambda}^2 - 8\bar{\lambda}^2}}\right)$. Hence, we obtain that, for $g(k) > 0$, k should satisfy the condition $k > \frac{1}{2}\left(1 + \sqrt{1 + \frac{8\bar{\lambda}^2}{9\underline{\lambda}^2 - 8\bar{\lambda}^2}}\right)$. Now for the polynomial $f(l)$, due to the fact that its coefficients are such that $\alpha > 0, \beta < 0, \gamma > 0$ and the non-negativity of Δ , the gain l should satisfy $l_1 \leq l \leq l_2$ with l_1, l_2 the positive roots of $f(l)$, $l_{1,2} := \frac{12k^2(k-1)\underline{\lambda} \pm \sqrt{\Delta}}{2(2k-1)^2 \bar{\lambda}^2}$ with $\Delta = -16k^3(k-1)\left(8\bar{\lambda}^2 k^2 - (9\underline{\lambda}^2 + 8\bar{\lambda}^2)k + 2\bar{\lambda}^2\right)$. The proof can be concluded similarly to the last part of the proof of Theorem 1 in [1].

Remark 1 We should note that there exist topologies and values of the gains k, l , that do not satisfy the conditions of our theorem but still ensure the consensus objective. Such scenario is, e.g. the one considered in the simulations of [1]. This is intrinsically related to the selection of the Lyapunov function and, as such, it implies that another Lyapunov function can provide a better set of conditions.

References

- [1] Y. Hong, G. Chen and L. Bushnell, "Distributed observers design for leader-following control of multi-agent networks", *Automatica*, 44:846–850, 2008.
- [2] X. Wang and Y. Hong, "Distributed observers for tracking a moving target by cooperative multiple agents with time delays," *2009 ICCAS-SICE*, Fukuoka, pp. 982-987, 2009.