



**HAL**  
open science

# Dynamic opportunistic maintenance planning for multi-component redundant systems with various types of opportunities

Hai Canh Vu, Phuc Do Van, Mitra Fouladirad, Antoine Grall

► **To cite this version:**

Hai Canh Vu, Phuc Do Van, Mitra Fouladirad, Antoine Grall. Dynamic opportunistic maintenance planning for multi-component redundant systems with various types of opportunities. Reliability Engineering and System Safety, 2020, 198, pp.106854. 10.1016/j.ress.2020.106854 . hal-02465514

**HAL Id: hal-02465514**

**<https://hal.science/hal-02465514v1>**

Submitted on 22 Aug 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial 4.0 International License

# Dynamic opportunistic maintenance planning for multi-component redundant systems with various types of opportunities

Hai Canh Vu\*, Phuc Do\*\*, Mitra Fouladirad\*, Antoine Grall\*

\*ICD, ROSAS, LM2S, Université de Technologie de Troyes, UMR 6281, CNRS, Troyes, France

\*\* Université de Lorraine, CRAN, UMR 7039, Campus Sciences, BP 70239, Vandoeuvre-les-Nancy, France

**Abstract:** Opportunistic maintenance (OM) is a key solution to reduce the maintenance costs or/and to improve the system dependability/performance. However, the existing OM models are mainly developed to specific classes of systems (series structures) with specific types of maintenance opportunities (MOs) such as component failure. The objective of this paper is to develop a dynamic OM approach for multi-component redundant systems such as parallel, parallel-series, series-parallel, and k-out-of-n, etc. Various types of MOs are also considered. To this purpose, a generalized MO model is firstly proposed for the modeling and formulation of existing MOs and new ones. A dynamic OM model with flexible decision rules allowing to consider various types of MOs in maintenance decision-making is then introduced. In addition, a multi-mode logistic support model is also proposed to better incorporate different types of MOs with associated logistic support requirements. To find the best OM scenario, an efficient optimization algorithm using Genetic algorithm with memory is developed. The proposed optimization algorithm allows also updating online an OM maintenance plan in presence of new MOs which may occur with time. The uses and advantages of the proposed OM approach are illustrated through a six-component redundant system.

**Keywords:** Opportunistic maintenance, maintenance opportunity, maintenance optimization, multi-component systems, redundant systems, Genetic algorithm with memory.

## 1 Introduction

In the maintenance optimization framework, opportunistic maintenance (OM) is one of the most popular approaches, that has drawn a lot of attention from academic researchers and industrial applications [1, 2, 3]. According to the OM, maintenance of a component may be carried out at some specific opportunities. A maintenance opportunity (MO) is defined as any period of time in which one or several favor maintenance conditions are present, e.g. system downtimes or discount periods of maintenance resources. In reality, there exist various types of maintenance opportunities (MOs) with different characteristics. Each MO type requires a specific OM approach. In the next subsection, we then analyze the existing OM approaches with respect to the different classifications of the maintenance opportunities.

### 1.1 Overview on opportunistic maintenance

The maintenance opportunities can be classified into two main classes: external and internal MOs [4]. The external MOs are the system downtimes triggered by several factors such as production buffers, inactivity period, raw material shortage, harsh environmental conditions [1, 5]. Note that maintenance does not influence the arrival of the MOs. The papers working with the external MOs usually consider mono-component systems and focus on the modeling of the maintenance opportunities. Only two characteristics (arrival time and duration) of the MOs are mainly modeled. For more detail, the MOs are assumed to occur according to homogeneous Poisson process in [5, 6, 7] and non-homogeneous process in [8]. Ba et al. [9] consider the external MOs with non-homogeneous

occurrences and stochastic duration. Recently, a MO concept based on inactivity periods with random occurrences has been introduced in [1]. To take into account this type of OM in maintenance decision-making, an adaptive decision rule and an efficient algorithm are proposed. However, the proposed approach is only applicable to the series systems.

Actually, the internal MOs usually exist in multi-component systems with dependencies. According to Robin P. Nicolai and Rommert Dekker [10], the dependencies can be classified into three groups: (a) *economic dependence* which represents the sharing of preparation tasks (machine opening, spare part transportation, etc.) and/or system downtime between different maintenance activities; (b) *stochastic dependence* implying that the failure/degradation state of a component can influence that of the others; and (c) *structural dependence* which means that some operating components have to be replaced, or at least dismantled, before failed components can be replaced or repaired. These dependencies promote the OM application to take advantage of the internal MOs but they bring many difficulties to the maintenance modeling and optimization [11, 12]. Among the three dependence types, the economic and structural dependencies have been studied more often, because they have direct impacts on the OM performance [13]. The internal MOs are triggered by the maintenance of a component in the system. According to the nature of a maintenance activity (preventive or corrective), the internal MOs are divided into two classes: Preventive maintenance (PM) based opportunity and corrective maintenance (CM) based opportunity.

The PM based opportunity is triggered by PM activities of components. Since the PM date may be planned, the support (preparation tasks) for the OM at a PM based opportunity can be done proactively. The papers dealing with the PM based MOs and/or the internal based MOs mainly focus on how to take their advantage rather than on their modeling. To this purpose, different control limits have been proposed to select which components should be maintained together. The control limits may be defined based on the component's failure rate [14, 15], the component's degradation level [16], the component's conditional reliability [17], or the system scheduled downtime losses [18]. Xia et al. [19] propose MTW (Maintenance Time Window) approach for a series-parallel hydraulic steering factory. The MTW is defined as a criterion to separate the PM activities in parallel subsystems, while it is used to combine the PM activities in series subsystems together. In [20], Iung et al. consider all possible combinations of PM activities in a short interval. The optimal combination is found by Genetic algorithm.

The CM based opportunity is triggered by CM activities or failures of components. Since the CM date, as well as, the failure time are random, the support (preparation tasks) for the OM at a CM based opportunity is more complicated when compared to that of the PM based opportunity. At a component/system failure, to select the components that should be opportunistically maintained, several control limits have been proposed based on the component's age in [21, 22], the component's mean time to failure (MTTF) in [23, 24]. Shafiee et al. [25] and Pham et al. [26] proposed several OM models in which all components are replaced at a major failure of the components/system. These papers consider that the condition monitoring of some components/systems is difficult and only possible when they are replaced or even impossible, while their failures are self-announcing. This is the reason why only the CM based MOs is considered in the papers.

There exist also a number of papers that consider both the PM and CM based opportunities, i.e. any maintenance activity (both CM and PM) of a component/system is considered as an opportunity to maintain other components in the system. Given the occurrence of a maintenance opportunity, the components are selected thanks to the different control limits such as component's age limit [27, 28], component's reliability limit [29, 30, 31], failure risk limit [32].

## 1.2 Problem statement

Despite the numerous research works published already, there still exist some limitations. The first one relating to the MO classification scheme. It seems to be not detailed enough and not exhaustive to cover various maintenance opportunities that engineering systems are experiencing in real applications, more precisely:

- For redundant systems (such as series-parallel, parallel-series, or  $k$ -out-of- $n$  systems), components can be

critical or non-critical. While the maintenance of a critical component leads the system to shut down, the system can be still operating during the maintenance of some non-critical component. In such a way, the OM triggered by the maintenance of a critical component should be separated from that of a non-critical one. In addition, according to our knowledge, when a non-critical component fails, it is either left in its failed state or separately repaired. The failure or PM of non-critical components has been not considered as the opportunities to maintain other components;

- As mentioned above, the external MOs are the system downtimes triggered by several factors such as production buffers, raw material shortage, harsh environmental conditions [5]. In reality, the external MOs may be also triggered by other factors such as a discount of maintenance costs (spare part cost, support cost, etc.);
- From a system management point of view, the maintenance planning and production/mission planning are two important items which are strongly interdependent. The system downtimes due to the production plan may be modified in a such way that the OM at the downtimes is the most efficient. On the contrary, external factors such as maintenance cost discounts or weather conditions cannot be modified by the system management. Therefore, it seems to be more adequate to separate the production-based OM from the external factor based OM.

Another limitation of the existing OM models is that they have been developed for specific types of maintenance opportunities with a particular system configuration (see Table 1).

Table 1: The most popular MOs considered in the existing OM models

Maintenance opportunity		Application
External factors	Production and other factors	Components [5, 6, 7, 8, 9] Series systems [1]
Internal factors	PM based opportunity	Series systems [16, 18, 20] Series-parallel systems [14, 15, 19] $k$ -out-of- $n$ systems [17]
	CM based opportunity	Series systems [21, 24, 23, 25] Parallel-series systems [22] $k$ -out-of- $n$ systems [26]
	Both PM and CM based opportunities	Series systems [27, 28, 29, 30, 32] Parallel-series systems [31]

### 1.3 Paper contributions and organization

To overcome these limitations, the objective of this paper is to develop a flexible OM approach for multi-component redundant systems considering various types of MOs. To this purpose, the paper addresses three original contributions:

- A generalized MO model is firstly proposed for the modeling and formulation of existing MOs and new ones. In addition, a multi-mode logistic support model is also proposed to better incorporate different types of MOs with associated logistic support requirements;
- An OM model with flexible decision rules allowing to take into account the various types of MOs in maintenance decision-making is then introduced;

- To find the best OM scenario, an efficient optimization algorithm using Genetic Algorithm with Memory (GAM) is developed. The developed algorithm allows updating online OM maintenance plan in the presence of new MOs which may occur with time.

The rest of the paper is organized as follows. Section 2 is devoted to describe in more detail the problem statement and some general definitions and assumptions. The new generalized OM model is introduced in Section 3. The flexible OM model, which allows taking into account various types of maintenance opportunities, is developed in Section 4. Section 5 is devoted to present an efficient optimization algorithm based on GAM to find the best OM scenario. To validate the flexibility and performance of the proposed models, numerical analyses are carried out in Section 6 for a 6-component redundant system. Different sensitivity analyses are also herein investigated. Finally, the last section presents some conclusions drawn from this work.

## Acronyms & Abbreviations

OM	Opportunistic maintenance	$x_i(t)$	Age of $i$ at $t$
MO	Maintenance opportunity	$O$	Type of MO (A, B, C, D, E, F)
MOs	Maintenance opportunities	$O_{ij}$	$j^{\text{th}}$ MO of $i$
PM	Preventive maintenance	$T_{O_{ij}}$	Available time of $O_{ij}$ information
CM	Corrective maintenance	$t_{O_{ij}}$	Occurrence time of $O_{ij}$
GA	Generic algorithm	$\omega_{O_{ij}}$	Duration of $O_{ij}$
GAM	Generic algorithm with memory	$BE_{O_{ij}}$	List of benefits of $O_{ij}$
RBD	Reliability block diagram	$\Delta O_{ij}$	Penalties related to $O_{ij}$ modification
IM	Individual maintenance	$t_i^f$	Failure time of component $i$
GM	Grouping maintenance	$\omega_i^c, \omega_i^p$	CM and PM duration of $i$
SDOM	System-downtime-based OM	$C_i^c, C_i^p$	CM and PM cost of $i$
CDOM	Component-downtime-based OM	$T_i^*$	PM period of $i$
$i$	Index of components, with $i = 1, \dots, N$	$N_i(a, b)$	Mean failure number of $i$ in $(a, b)$
$N$	Number of components	$X$	An OM scenario in $I_k$
$C_i^{**}$	Cost of a maintenance activity of $i$	$X^*$	The optimal OM scenario in $I_k$
$c_i^{**}$	Specific maintenance cost of $i$	$G^z$	A group of MOs of $X$
$S_i^{**}$	Support cost for the maintenance of $i$	$z$	Index of groups of MOs, $z = 1, \dots, Z$
(**)	$= mr : \text{minimal repair}$ $= re : \text{replacement}$	$Z$	Number of groups of MOs in $X$
$t_m$	Maintenance time	$t_{G^z}$	Opportunistic time of $G^z$
$\eta_i$	Criticality of $i$	$t_{G^z}^*$	Optimal opportunistic time of $G^z$
$r_0$	Downtime cost rate of the system	$C_{sys}^k(X)$	Maintenance cost of $X$
$r_i$	Downtime cost rate of $i$	$C_{G^z}$	Maintenance cost in interval $[t_{G^z}, t_{G^z} + \omega_{G^z}]$
$\omega_i^{**}$	Maintenance duration of $i$	$C_{z-1}^z$	Maintenance cost in $[t_{G^{z-1}} + \omega_{G^{z-1}}, t_{G^z}]$
$ca_i$	Additional support cost of $i$	$S_{G^z}$	Support cost of $G^z$
$ca_i^{ex}$	Additional support cost of express mode	$cd_{G^z}$	Downtime cost of $G^z$
$ca_i^{em}$	Additional support cost of emergency mode	$ca_{G^z}$	Additional support cost of $G^z$
$t_p$	Preparation time for a maintenance action	$\lambda_i$	Failure rate of component $i$
$T_{no}$	Preparation time of normal mode	$\beta_i$	Shape parameter of Weibull distribution
$T_{ex}$	Preparation time of express mode	$\eta_i$	Scale parameter of Weibull distribution
$T_{em}$	Preparation time of emergency mode	$I_k$	Planning horizon between $a_k$ and $b_k$
$CR$	Maintenance cost rate	$a_k$	Start time of $I_k$
		$b_k$	End time of $I_k$

## 2 System assumptions and problem description

This section is devoted to describe in more detail the maintenance problem of multi-component redundant systems in the presence of various types of maintenance opportunities. The description is started with some definitions about the multi-component redundant systems (Subsection 2.1), their maintenance activities, maintenance costs (Subsection 2.2), and maintenance supports (Subsection 2.3). Maintenance opportunistic, maintenance opportunities and the considered maintenance problem are then introduced in Subsection 2.4.

### 2.1 Multi-component redundant system

Consider a system consisting of  $N$  non-identical components. The components are connected in a redundant configuration to increase the system's availability and prevent the system's downtime [33]. The redundant system can operate even when some components fail. According to this property, two kinds of components are considered:

- **Critical components:** a component is said critical one if its shutdown leads to a system shutdown;
- **Non-critical components:** a component is non-critical if the system can operate when the component is stopped.

As an example, Figure 1 represents the Reliability Block Diagram (RBD) of a power plant composed of 6 main components. Pump A, Pump B and Pulverizer A, Pulverizer B are non-critical components since the power plant can still operate when one of the components shuts down. Otherwise, the boiler and the generator are critical to the operating of the power plant. The power plant can not operate without the boiler or the generator. It should be noted that the component criticality, as well as the system configuration, may be changed over time. For example, when all components of the system are operating, Pump A and Pump B are non-critical components. Otherwise, when one of the two pumps is stopped, the other becomes a critical one. In this case, the power plant can still operate but with a higher failure frequency. From a maintenance point of view, the maintenance of non-critical components is more flexible than that of critical ones. Indeed, when a non-critical component fails, it may be separately or jointly maintained with other components at the failure time or even left in its failed state and wait for the next maintenance opportunity. Otherwise, the postponement of the maintenance of critical components is not recommended because it causes a lot of problems relating to the system unavailability. To face this issue, flexible decision rules adapted to the dynamic criticality of components are then needed.

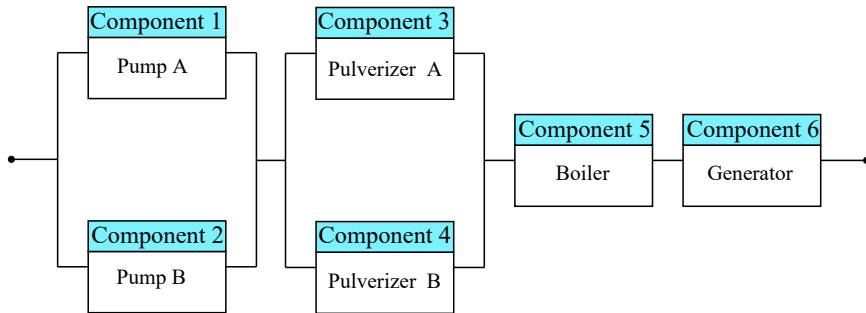


Figure 1: Reliability block diagram of a power plant

### 2.2 Maintenance activities and costs

**Maintenance activities.** Two types of maintenance activities are here considered: corrective maintenance (CM) and preventive maintenance (PM). While the CM is carried out at component failures to restore the component to its operational state, the PM is carried out on a functioning component to prevent it from failures.

When a CM activity is applied to a component, there are two possible maintenance levels: minimal repair and replacement. The minimal repair indicates the fact that if only a small part of the component is repaired or replaced upon failure, this would not affect considerably the overall component health state (failure rate/degradation level/age). The component is in “As Bad As Old” state after the minimal repair. Since the word “component” in our paper may refer to subsystems/equipments (e.g. pumps, generator, boiler presented in Fig. 1) that contain several parts, the definition of minimal repair at the component level is acceptable [34, 35]. The replacement can be applied to both failed or functioning components and restores them to the “As Good As New” state.

Consequently, a PM of a functioning component is a replacement; otherwise, a CM of a failed component may be a minimal repair or a replacement. The choice of CM level is also addressed in our work.

**Maintenance cost structure.** The cost of a maintenance activity depends on its maintenance level (minimal repair or replacement). It can be calculated as:

$$C_i^{**} = c_i^{**} + S_i^{**} + \vartheta_i(t_m) \cdot r_0 \cdot \omega_i^{**} + [1 - \vartheta_i(t_m)] \cdot r_i \cdot \omega_i^{**}, \quad (1)$$

where,

- $(**) = mr$  (minimal repair) or  $(**) = re$  (replacement) indicates that the level of maintenance action: minimal repair or replacement activity respectively;
- $c_i^{**}$  is the specific cost (containing spare part cost, labor cost, etc.) depending on the component characteristics;
- $S_i^{**}$  is the support cost paid for different maintenance preparation tasks such as scaffolding erecting, machine opening, maintenance team traveling, spare part ordering and transportation;
- $\vartheta_i(t_m)$  is the criticality of component  $i$  at the maintenance time  $t_m$ .

$$\vartheta_i(t_m) = \begin{cases} 1 & \text{if } i \text{ is a critical component} \\ 0 & \text{if } i \text{ is a non-critical component} \end{cases} \quad (2)$$

If  $\vartheta_i(t_m) = 1$ , the maintenance of the **critical** component  $i$  leads the system to shut down with a shutdown cost  $r_0 \cdot \omega_i^{**}$ , where  $r_0$  is the system downtime cost rate, and  $\omega_i^{**}$  is the maintenance duration of component  $i$ . Otherwise, if  $\vartheta_i(t_m) = 0$ , the system is still operating during the maintenance of the component. A cost  $r_i \cdot \omega_i^{**}$  has to be paid for negative impacts of the component shutdown on its own and on the system such as production rate and production quality decreasing.  $r_i$  is the component downtime cost rate.

The proposed maintenance cost model is suitable for redundant systems since it takes into account the component criticality ( $\vartheta_i$ ). In addition, the presence of the support and downtime costs are useful for the evaluation of the economic dependence between components and the performance of the opportunistic maintenance in Section 5.

### 2.3 Maintenance logistic support model

The maintenance support contains all preparation tasks that ensure a maintenance activity can be correctly carried out, such as transportation of maintenance resources (spare part, maintenance team, repair tools), preparation of the maintenance site (machine opening, scaffolding erecting), organization of maintenance tasks, etc. We assume that the failure of each component is immediately removed by a minimal repair action. The repair action can be realized by the local maintenance team using local maintenance resources. Otherwise, to perform a replacement action, external maintenance resources (spare parts, specific maintenance tools, or high skill workers) are required and need to be transported to the maintenance site. The support of the replacement actions is more complex and needs to be modeled.

In reality, a number of maintenance support modes may exist at different costs and at different preparation time (the time needed to complete all the preparations tasks). For example, with sufficient preparation time, the spare part can be provided by a normal producer and transported to the maintenance site by using transportation means such as a boat or train. Otherwise, when the preparation time is short, more expensive and faster means of transport (such as an airplane) and producers have to be selected. Thanks to these above discussions, a multi-mode logistic support model is herein proposed and presented in Figure 2. The proposed model contains three

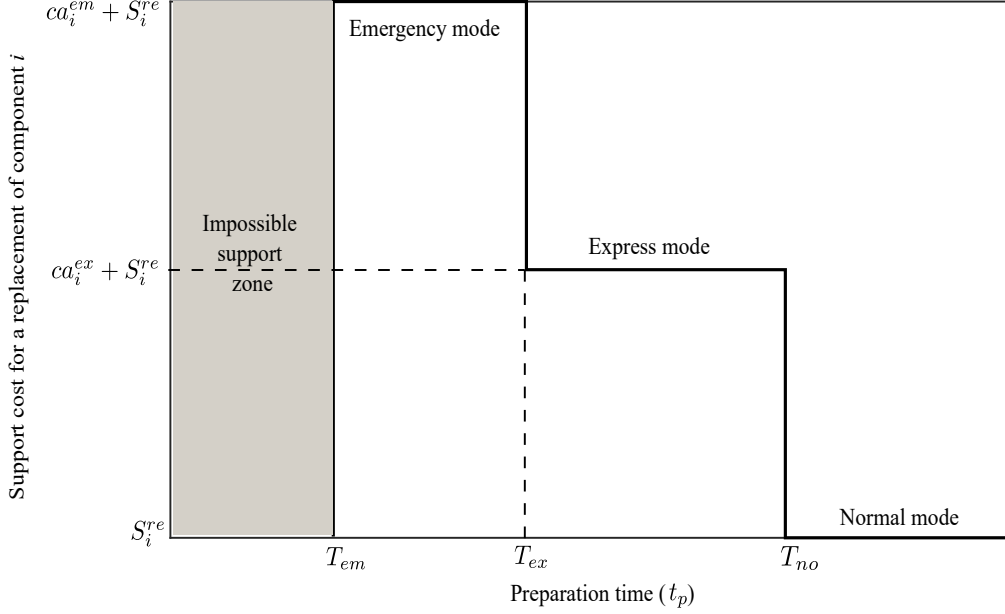


Figure 2: Multi-mode logistic support model

support modes: normal, express, and emergency. It is reasonable to assume that the shorter the preparation time of a support mode is, the higher the support cost is. If the maintenance is not urgent, the preparation time is enough, i.e.  $t_p \geq T_{no}$ , the normal maintenance support mode will be used. The cost of the normal mode is equal to  $S_i^{re}$ . If the replacement is performed sooner than its planned date, more expensive maintenance support modes such as express and emergency modes with shorter preparation time are required. For more detail, the emergency and express modes can guarantee that all the preparation tasks are finished after  $T_{em}$  and  $T_{ex}$  units of time and require additional costs  $ca_i^{em}$  and  $ca_i^{ex}$  respectively. In addition, no support mode can guarantee that the preparation time is smaller than  $T_{em}$ . The interval  $[0, T_{em}]$  is named impossible support zone. The additional support cost in this zone is considered to be infinity, i.e.  $ca_i = +\infty$ . Note that the parameters such as  $T_{em}$ ,  $T_{ex}$ ,  $T_{no}$ ,  $ca_i^{em}$  and  $ca_i^{ex}$  may vary for different components and systems.

Given the existence of these above support modes, the selection of the most appropriate support mode for the replacement of a component/group of components is an interesting issue that will be further investigated in the paper.

## 2.4 Maintenance opportunities and opportunistic maintenance

In this paper, a maintenance opportunity is defined as any period of time in which one or several favor maintenance conditions are present, allowing [doing](#) maintenance at lower costs. It may be a system downtime period according to the production plan, an inactivity period, a discount period of maintenance costs, etc. In the literature, three main characteristics of a maintenance opportunity are considered:

- MO nature: the opportunity comes from internal or external factors;
- MO occurrence time: homogeneous or non-homogeneous Poisson process are usually used to model the occurrence time of a MO [in simulations](#);



- MO duration: the duration time of a MO may be random or deterministic.

In addition, two other important characteristics, which may have significant impacts on the performance of the opportunistic maintenance, are underlining in this paper:

- The benefits that can be obtained if a component is maintained at the MO;
- The ability to change the occurrence time of the MO, e.g. postponing the CM of a failed component.

To deal with the MO characteristics, new classification and modeling of maintenance opportunities should be developed, especially, in the framework of multi-component redundant systems. The MOs triggered by the maintenance of a critical component and that of a non-critical one should be distinguished. Indeed, by taking into account the MO triggered by the maintenance of a critical component, both the support and downtime costs can be saved. Otherwise, only the support costs can be saved when a component is maintained at a MO triggered by the maintenance of a non-critical component. In addition, the advancing or postponing the maintenance of a non-critical component is **simpler** than that of a critical one, since the system can be still operating when some non-critical components do not work. Finally, regarding the ability to change the occurrence time, while the production plan can be modified, most of the external factors such as weather conditions cannot be modified by the system manager. The production-based OM, therefore, should **also be separated** from the external factor based OM (see Section 3 for more detail).

When a maintenance opportunity occurs, the OM decision-making helps to decide which components will be opportunistically maintained and at what time. It should be noted that the opportunistic time is usually fixed at the occurrence time of the maintenance opportunity for most of the papers in the literature. In the paper, we consider the possibility that the occurrence time of the MO may be modified (postponed). The postponement can help to reduce the support costs of the maintenance at the MO by using the support modes with low costs (Subsection 2.3). To take into account the various maintenance opportunities with both random or deterministic occurrence time and the dynamic criticality of components in a redundant system (subsection 2.1), a dynamic OM decision-making approach will be developed. The dynamic approach considers short-term intervals. The best OM plan of each interval can be updated anytime when a new maintenance opportunity occurs or the criticality of components/system configuration is changed (see Section 4).

To find the best OM plan in a short-term interval, the OM performance, measured by the total maintenance cost of the system in the interval, should be first evaluated. This evaluation is not evident due to the existence of the economic dependence between components, and that between components and their opportunities. The economic dependence indicates that the maintenance cost of a component/a group of components may be cheaper (positive economic dependence) or more expensive (negative economic dependence) when they are maintained together at a **MO**. Given the existence of the various MOs and system redundancy, economic dependence is more complex and may be both positive and negative. For example, consider a redundant system in Figure 1, when components 1 and 2 are maintained together, additional costs have to be paid for the system shutdown. Otherwise, when components 5 and 6 maintained together, the system is shut down only once instead of twice when they are separately maintained. The system shutdown costs are then saved. The evaluation of the OM performance by taking into account the complex economic dependence is presented in Subsection 5.1. Finally, given that all maintenance opportunities are considered, the number of OM solutions may be large, a new OM optimization process based on GAM is developed in Subsection 5.2. The process provides not only the best OM solution but also the best solutions for OM postponing and supporting.

In conclusion, the paper aims at developing a flexible opportunistic maintenance approach for redundant systems with various types of maintenance opportunities. The proposed models need to take in the following considerations:

- Redundant systems with dynamic criticality and complex economic dependence;

- Various types of maintenance opportunities;
- Two maintenance levels: minimal repair and replacement;
- Three support modes for a replacement activity: normal, express, and emergency;
- Components have two states: operating or failed. The failure behavior of a component is described by a probability distribution with an increasing failure rate;
- Components are stochastically independent, i.e. the failure behavior of a component does not affect that of the others.

### 3 Classification and modeling of maintenance opportunities

In this section, a new model of maintenance opportunities is firstly developed (Subsection 3.1). The maintenance opportunities are then classified according to the developed model in Subsection 3.2.

#### 3.1 Modeling of MOs

A maintenance opportunity generally may repeatedly occur in a given interval  $I_k = [a_k, b_k]$ , where  $a_k$  and  $b_k$  are the beginning and ending of  $I_k$ . In that way, let  $O_{ij}$  denote the  $j^{\text{th}}$  occurrence of a maintenance opportunity, which is triggered by a maintenance activity/production stop of a component  $i$  ( $i = 1, \dots, N$ ) or external factors ( $i = 0$ ).  $O$  indicates the type of MO (see Subsection 3.2). An MO  $O_{ij}$  can be characterized by the following parameters:

- Occurrence time, denoted  $t_{O_{ij}}$ , is the time at which the maintenance opportunity appears. The occurrence time may be random (e.g. failure time) or deterministic (e.g. planned PM date);
- **Declaration time**, denoted  $T_{O_{ij}}$ , is the instant at which all the information related to  $O_{ij}$  such as its occurrence time, its duration, its advantages, etc., are available. If  $t_{O_{ij}}$  is random, the information related to the maintenance opportunity is not known in advance, we have  $T_{O_{ij}} = t_{O_{ij}}$ . Otherwise, if  $t_{O_{ij}}$  is deterministic, the information may be known in advance  $T_{O_{ij}} < t_{O_{ij}}$ ;
- Duration, denoted  $\omega_{O_{ij}}$ , is the duration of  $O_{ij}$  such as discounted cost periods or component/system downtime;
- Benefit, denoted  $BE_{O_{ij}}$ , is the list of all benefits that can be obtained if a component is maintained at the maintenance opportunity, e.g. the support cost, the downtime cost, etc;
- Modifiability, denoted  $\Delta O_{ij}$ , indicates the level of difficulty in modifying (advancing/postponing) the MO occurrence time. The higher the  $\Delta O_{ij}$  is, the more difficult the modification is. If  $\Delta O_{ij} = 0$ , the modification can be done without any additional charge. Otherwise, if  $\Delta O_{ij} = +\infty$ , the modification is impossible.

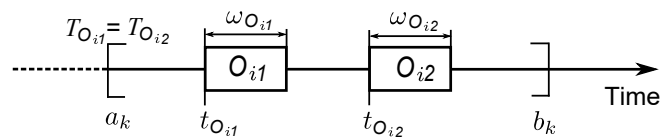


Figure 3: Illustration of the MO parameters

Consider an example in Figure 3, the MO ( $O_{ij}$ ) occurs twice in  $I_k$  at  $t_{O_{i1}}$  and  $t_{O_{i2}}$  with the durations  $\omega_{O_{i1}}$  and  $\omega_{O_{i2}}$  respectively. These two MOs may be triggered by the two PM activities of component  $i$  at  $t_{O_{i1}}$  and  $t_{O_{i2}}$ . The occurrence of these two maintenance activities is known in advance at  $T_{O_{i1}} = T_{O_{i2}} = a_k$ .

### 3.2 Classification of MOs

Based on these above parameters, six types of maintenance opportunities are distinguished in the paper.

**Critical CM-based opportunity (Type A).** When a critical component fails at  $t_i^f$ , the system is shut down. CM with duration  $\omega_i^c$  is often carried out as soon as possible to restore the component as well as the system to their operational state. The  $j^{\text{th}}$  CM activity of component  $i$  is considered as the  $j^{\text{th}}$  maintenance opportunity of type A, denoted  $A_{ij}$ .

Similarly to the failure time, the occurrence time of  $A_{ij}$  is random and therefore unknown in advance, we have then  $T_{A_{ij}} = t_{A_{ij}} = t_i^f$ . The duration of  $A_{ij}$  equals the CM duration of component  $i$ , i.e.  $\omega_{A_{ij}} = \omega_i^c$ . The CM duration  $\omega_i^c$  is equal to  $\omega_i^{mr}$  or  $\omega_i^{re}$  if minimal repair or replacement is applied respectively. Since the system is shut down during the CM, both the system shutdown and the support cost may be saved when  $A_{ij}$  jointly occurs with the other maintenance opportunities to form a group of maintenance opportunities, denoted  $G^z$ , at  $t_{G^z}$ , i.e.  $BE_{A_{ij}} = \{\text{System shutdown cost, Support cost}\}$ . For this purpose, the occurrence time  $t_{A_{ij}}$  has to be moved to  $t_{G^z} \geq T_{A_{ij}}$ , i.e. the CM is postponed until  $t_{G^z}$  (see Figure 4). The CM postponement leads to an additional system downtime cost, which is equal to

$$\Delta A_{ij} = (t_{G^z} - t_{A_{ij}}) \cdot r_0. \quad (3)$$

The modification of type A is then limited, except only in the case where the system downtime cost rate is low.

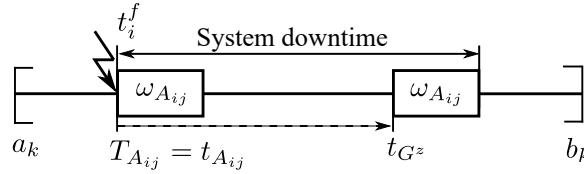


Figure 4: Postponement of the occurrence time of  $A_{ij}$

**Non-critical CM based opportunity (Type B).** A MO of type B is triggered by a CM of a non-critical component. The same as type A, we have  $T_{B_{ij}} = t_{B_{ij}} = t_i^f$  and  $\omega_{B_{ij}} = \omega_i^c$ .

The main difference when compared to type A is that the system can still operate when the non-critical component  $i$  fails. In case where the opportunistic maintenance is applied, only the support cost may be saved, i.e.  $BE_{B_{ij}} = \{\text{Support cost}\}$ . Assume that the CM of the component is postponed until  $t_{G^z} \geq T_{B_{ij}}$  (Figure 5a).

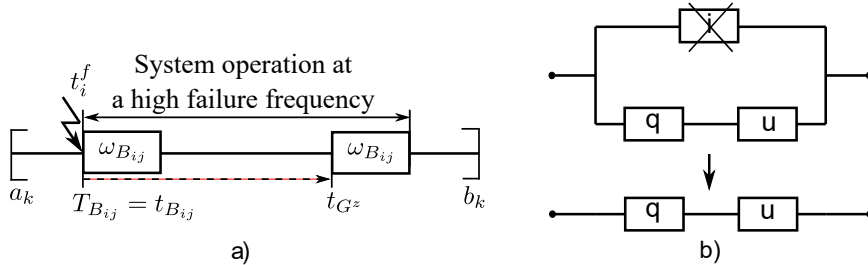


Figure 5: Postponement of the occurrence time of  $B_{ij}$

The postponement of the component does not cause additional system downtime costs. However, it leads to: (a) an additional component downtime cost  $(t_{G^z} - t_{B_{ij}}) \cdot r_i$ , which is normally much smaller than that of the system; (b) an additional cost related to the increase of system failure frequency. In fact, the in-operating state of component  $i$  in  $[t_{B_{ij}}, t_{G^z}]$  leads to an increase of the system failure frequency as well as a set  $L_i$  of new critical components, e.g.  $L_i = \{q, u\}$  in Figure 5b. If a component in  $L_i$  fails, the system downtime cost has to be paid instead of the component one. Consequently, additional costs  $\sum_{l \in L_i} N_l(t_{B_{ij}}, t_{G^z}) \cdot (r_0 - r_j) \cdot \omega_l^c$  are incurred (see

[36] for more detail).  $N_l(t_{B_{ij}}, t_{G^z})$  is the average failure number of component  $l$  in interval  $(t_{B_{ij}}, t_{G^z})$ . We have therefore

$$\Delta B_{ij} = (t_{G^z} - t_{B_{ij}}) \cdot r_i + \sum_{l \in L_i} N_l(t_{B_{ij}}, t_{G^z}) \cdot (r_0 - r_j) \cdot \omega_l^c \quad (4)$$

In most cases,  $\Delta B_{ij}$  is much more smaller than  $\Delta A_{ij}$ . The modification of type B is possible since it is not critical for the system operating.

**Critical PM based opportunity (Type C).** This opportunity is triggered by a PM of a critical component  $i$ . Since the system does not operate during the PM, in the case where the opportunistic maintenance is applied, both the system shutdown and support costs may be saved, i.e.  $BE_{C_{ij}} = \{\text{System shutdown cost, Support cost}\}$ .

The main difference of this type when compared to the CM based opportunities (types A and B) is that its occurrence time is not random. The occurrence time of type C (PM dates) can be planned in advance. If component  $i$  is periodically maintained every  $T_i^*$  units of time. We have  $t_{C_{ij}} = t_{C_{i(j-1)}} + T_i^*$ , where  $t_{C_{i0}}$  denote the last occurrence time before  $a_k$  (Figure 6).

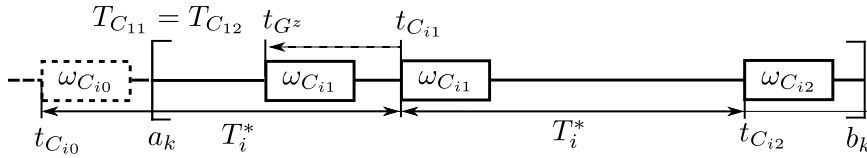


Figure 6: Forward movement of the occurrence time of  $C_{i1}$

Assuming that  $T_i^*$  is known, the occurrence times of type C can be always determined at the beginning of each interval, i.e.  $T_{C_{ij}} = a_k$ . The duration of the maintenance opportunities is  $\omega_{C_{ij}} = \omega_i^p = \omega_i^{re}$ . Since the occurrence times are known in advance, the MO type C may be modified by moving forward ( $t_{G^z} < t_{C_{ij}}$ , Figure 6) or postponing ( $t_{G^z} > t_{C_{ij}}$ ) the occurrence time  $t_{C_{ij}}$ . The change of the occurrence time  $\Delta t_{C_{ij}} = |t_{G^z} - t_{C_{ij}}|$  will lead to the change of the mean number of failures as well as the total expected CM costs of component  $i$ , which is calculated as (see [34] for more detail)

$$\Delta C_{ij} = [N_i(0, t_{C_{ij}} + \Delta t_{C_{ij}}) + N_i(0, t_{C_{ij}} - \Delta t_{C_{ij}}) - 2 \cdot N_i(0, t_{C_{ij}})] \cdot C_i^c \quad (5)$$

**Non-critical PM based opportunity (Type D).** This opportunity occurs when a non-critical component is stopped for a preventive maintenance. The modeling of this type is exactly the same as that of type C, except that the OM based on type C can help to save only the support cost, i.e.  $BE_{D_{ij}} = \{\text{Support cost}\}$ , since the system is still operating. The other parameters are  $t_{D_{ij}} = t_{D_{i(j-1)}} + T_i^*$ ,  $T_{D_{ij}} = a_k$ ,  $\omega_{D_{ij}} = \omega_i^p$ , and

$$\Delta D_{ij} = [N_i(0, t_{D_{ij}} + \Delta t_{D_{ij}}) + N_i(0, t_{D_{ij}} - \Delta t_{D_{ij}}) - 2 \cdot N_i(0, t_{D_{ij}})] \cdot C_i^c \quad (6)$$

where  $\Delta t_{D_{ij}} = |t_{G^z} - t_{D_{ij}}|$ .

**Production based opportunity (Type E).** This opportunity corresponds to a downtime period of components/system according to the production demands (production systems) or the system configuration in a specific mission phase (multi-phase systems). By profiting this maintenance opportunity, the system downtime due to maintenance may be reduced, i.e.  $BE_{E_{ij}} = \{\text{Downtime cost}\}$ . The occurrence time  $t_{E_{ij}}$  and the duration  $\omega_{E_{ij}}$  of type E are usually fixed by the production or mission planning. The information related to the type E is usually available at the beginning of the considered interval  $T_{E_{ij}} = a_k$ . The modification of this type is possible only when the OM planning and the production/mission planning are jointly considered. [In our paper, the production/mission planning will not be considered \( \$\Delta E\_{ij}\$ \).](#)

**External factors based opportunity (Type F).** This type contains all opportunities whose occurrences do not depend on the company/system activities. The occurrence of new technologies/more efficient components, discounts of spare parts, lower labor cost periods, good weather conditions, and system downtimes due to safety reasons can be considered as external factors based opportunities promoting the maintenance of components before their nominal maintenance dates. Most of these above factors occur randomly, i.e.  $T_{F_{ij}} = t_{F_{ij}}$ , and bring the opportunity to reduce different maintenance costs such as support cost, downtime cost, spare part cost, etc. In addition, given that the opportunities are triggered by external factors, the modifications of  $t_{F_{ij}}$  are impossible ( $\Delta F_{ij} = +\infty$ ) from company/system manager point of view. To facilitate further developments, a discount period of support cost [ $t_{F_{ij}}, t_{F_{ij}} + \omega_{F_{ij}}$ ] is considered an example of type F in the following of the paper. If a maintenance activity of component  $i$  is executed in the discount period, its support cost is reduced about  $\mu_{ij}\%$  ( $0 \leq \mu_{ij} < 100$ ). The information related to the discount are assumed to be available at the beginning of the planning horizon  $T_{F_{ij}} = a_k$ .

From these above discussions, the parameters of all the MO types can be summarized as in Table 2.

Table 2: The parameters of the 6 MO types

Type	$t_{O_{ij}}$	$T_{O_{ij}}$	$\omega_{O_{ij}}$	$BE_{O_{ij}}$	$\Delta O_{ij}$
A	$t_i^f$	$t_i^f$	$\omega_i^c$	System shutdown Support cost	Equation 3
B	$t_i^f$	$t_i^f$	$\omega_i^c$	Support cost	Equation 4
C	$t_{C_{i(j-1)}} + T_i^*$	$a_k$	$\omega_i^p$	System shutdown Support costs	Equation 5
D	$t_{D_{i(j-1)}} + T_i^*$	$a_k$	$\omega_i^p$	Support cost	Equation 6
E	$t_{E_{ij}}$	$a_k$	$\omega_{E_{ij}}$	Downtime cost	N/A
F	$t_{F_{ij}}$	$a_k$	$\omega_{F_{ij}}$	Downtime cost Support cost Spare part cost	N/A
N/A: Not Applicable					

### 3.3 On the use of MOs in maintenance decision-making

All the 6 MO types and their characteristics are qualitatively summarized as in Table 3. Among the 6 MO types, PM based (types C and D) and production based (type E) opportunities occur according to PM and production plans respectively. Their occurrence time may be known in advance. Otherwise, the occurrence time of CM based (types A and B) and external factor based (type F) opportunities are random and therefore unknown in advance. The supports for the opportunistic maintenance at the MOs of types A, B and F are then more complex than that of types C, D and E. In reality, if the maintenance support system is able to ensure that the maintenance resources (manpower, spare part, etc.) are available at any time, all these MO types are considered. From an economic point of view, the taking advantage of the maintenance opportunities can help to reduce several maintenance costs such as system downtime cost, support cost and spare part cost. For more detail, both the system shutdown and support costs may be saved with the presence of a critical PM or CM based (type A or C) opportunity. Only support costs can be saved if the occurred opportunity is a non-critical PM or CM based opportunity (type B or D) since the system can be still operating. It should be noted also that these two MO types B and D exist only in a redundant systems. In the literature, most of the papers working on opportunistic maintenance [37, 38] focus on saving the system shutdown costs by considering series systems with opportunities of type A. On the contrary, there are few papers considering the other types of maintenance opportunities, especially types B, D, E and F.

To be opportunistically maintained at a maintenance opportunity, the occurrence time of maintenance op-

Table 3: MO types and their characteristics

Type	Occurrence time	Benefit	Modifiability	Series systems	Redundant systems
A	Random	Shutdown cost Support cost	Limited	Yes	Yes
B	Random	Support cost	Yes	No	Yes
C	Deterministic	Shutdown cost Support cost	Yes	Yes	Yes
D	Deterministic	Support cost	Yes	No	Yes
E	Deterministic	Shutdown cost	Yes*	Yes	Yes
F	Random	Shutdown cost Support cost Spare part cost	No	Yes	Yes
(*) Yes if the production or mission planning is jointly considered					

portunities has to be modified. The modifiability of the MOs has great impacts on the MO performance. It is therefore studied in this paper. Among 6 MO types, the modification of type F is impossible since the maintenance opportunities of this type are triggered by external factors (weather conditions, economic, law and social changes). When a MO of type F occurs, we have to select which components will be maintained at this opportunity. The situation is not the same for the other types, e.g., consider a MO of type E occurring according to a production plan. Its occurrence time may be changed if the production planning is jointly considered with the maintenance planning. In this case, in addition to the selection of components to be maintained at the opportunity, we have to decide if the occurrence time of the opportunity has to be modified or not. The decision will be made based on the balance between the penalty costs paid for the modification of maintenance plan and that of production one. On the contrary to the types E and F, the modification of types A, B, C, and D are quite simple since they concern only the maintenance planning. Among these 4 types, the modification of type A (postponement of critical CM) is the most challenging because it makes the system downtime increase. For this reason, a number of papers in the literature (such as [16, 18, 28, 29, 30]) consider that the modification of type A is impossible, i.e. several components have to be immediately maintained at the system failure. This decision may lead to the use of advanced support modes at higher costs. To overcome this limitation, in the paper, we consider that the modification of type A is possible and depends on the balance between the system downtime costs and the additional support costs.

In the next section, this new way of classification and modeling of maintenance opportunities will be used as the basis for the development of our flexible OM model.

## 4 Flexible opportunistic maintenance approach

This section is devoted to describe step by step our proposed OM approach for multi-component redundant systems (Subsection 4.1). Its flexibility is then studied in Subsection 4.2.

### 4.1 OM approach descriptions

The proposed OM approach is developed based on the rolling horizon approach [1, 34, 36]. According to this approach, the maintenance planning can be divided into two main phases. In the first phase (component level), the preventive maintenance date of each component is individually optimized on a long-term horizon. It means that the interactions (economic dependencies) between components are not considered. In the second phase,

maintenance planning is repeatedly done for each consecutive short-term interval at the system level. For this purpose, all the maintenance activities of components in the considered interval are determined based on the components' individual maintenance date. Several maintenance activities are then grouped to take advantage of the economic dependencies and to reduce the maintenance cost in the interval. Given that the grouping is done in the short-term intervals, it can be updated easily to take into account the different MO types with both deterministic and random occurrence time. In addition, its long-term impacts are also counted since it is done on the long-term optimal maintenance activities. Finally, it should be careful in the selection of the short-term interval because if the interval is too small, the global optimum is not guaranteed. Otherwise, if the interval is too large, many computation resources are required, and the updating of the maintenance plan is difficult when the maintenance opportunities occur. It is shown in [1] that the short-term intervals must be chosen in the way that all components are preventively maintained at least one time in each short-term interval. In addition, the selection of a short-term interval, therefore, must be done by considering a number of factors such as company specifications, the occurrence frequency of maintenance opportunities, global optimum of the maintenance plan, etc.

Our OM approach developed based on the rolling horizon contains the four following phases (see Figure 7):

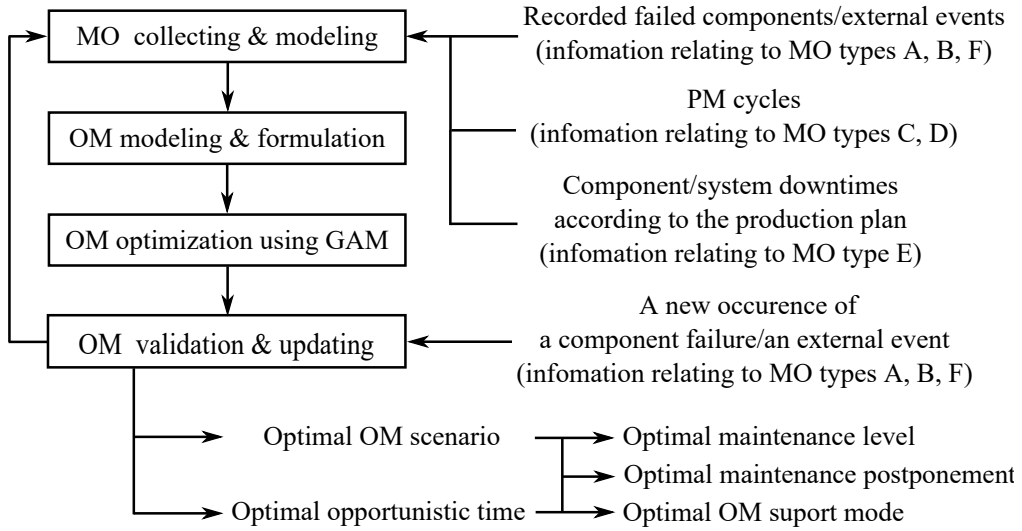


Figure 7: Scheme of the proposed OM approach for multi-component redundant systems

### Phase 1: MO collecting and modeling

In this phase, a short-term interval,  $I_k = [a_k, b_k]$ ,  $k = 1, 2, \dots$  is considered. All available information related to the 6 MO types, which occur in  $I_k$ , are collected and modeled (Section 3). The collected information may be the recorded failed components/external events (information relating to MO types A, B, F); the PM cycles of components (information relating to MO types C, D); the component/system downtimes due to the production plan (information relating to MO type E). At the end of this phase, a plan contains the occurrence time of all the MOs in  $I_k$ , named “initial OM plan”, is determined. Consider an example of a three-component redundant system in Figure 8a. Its initial OM plan in  $I_k$  contains (see Figure 8b):

- Non-critical CM based opportunity  $B_{21}$  at  $t_{B_{21}} = a_k$  since non-critical component 2 is considered to be in the non-operating state at  $a_k$ ;
- Non-critical PM based opportunities  $D_{11}$ ,  $D_{12}$ ,  $D_{21}$  triggered by the PM of non-critical components 1 and 2 at  $t_{D_{11}} = t_{D_{10}} + T_1^*$ ,  $t_{D_{12}} = t_{D_{11}} + T_1^*$ ,  $t_{D_{21}} = t_{D_{20}} + T_2^*$  respectively.  $T_i^*$  is the PM cycle of component  $i$ , which is usually determined based on the manager's experiences, producer's recommendations, or maintenance models developed for mono-component systems. In this paper, the PM cycle is determined based on the periodic replacement with minimal repair model. The detailed calculations are presented in Appendix A;

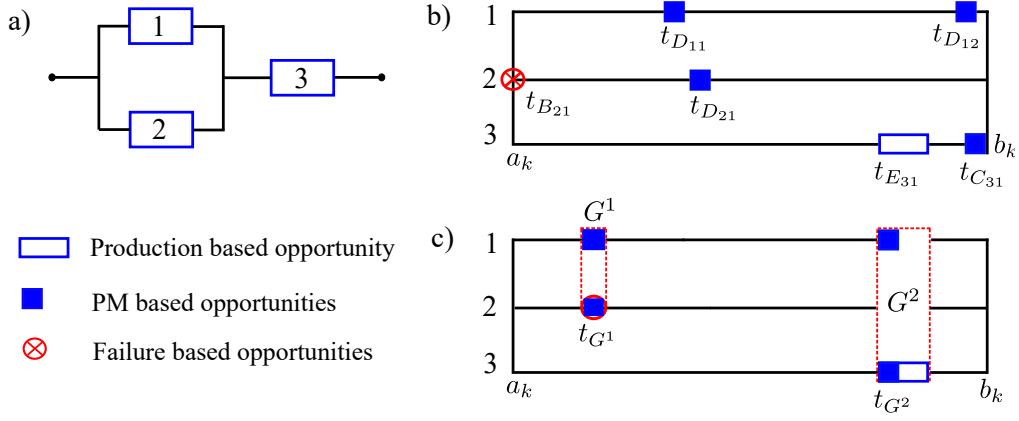


Figure 8: Opportunistic maintenance decisions for a three-component redundant system

- Critical PM based opportunity  $C_{31}$  at  $t_{C_{31}} = t_{C_{30}} + T_3^*$  issued by the PM of critical component 3;
- Production based opportunity  $E_{31}$  starting at  $t_{E_{31}}$ , issued by a downtime period of component 3 according to the production plan.

## Phase 2: OM modeling and formulation

In this phase, these above maintenance opportunities are grouped to reduce the maintenance cost. A group of several MOs, denoted  $G^z$ , is formed by moving the occurrence time of the MOs in the group ( $t_{O_{ij}}$ ) to a common time denoted  $t_{G^z}$  (see Section 3).  $t_{G^z}$  is called the opportunistic time of group  $G^z$ .

An OM scenario, denoted  $X$ , is defined as a collection of  $Z$  mutually exclusive groups  $X = \{G^1, \dots, G^Z\}$  which covers all  $n_{opp}$  maintenance opportunities of  $I_k$ .

$$G^l \cap G^z = \emptyset, \forall l \neq z \text{ and } G^1 \cup G^2 \cup \dots \cup G^Z = Y \quad (7)$$

where  $Y$  is the set of  $n_{opp}$  maintenance opportunities.

An OM scenario  $X$  and the opportunistic time of its groups  $t_{G^z}$  ( $G^z \in X$ ) defines which maintenance opportunities should occur together and at what time. It represents therefore indirectly the opportunistic maintenance plan. Figure 8c represents an example of an OM scenario composed of two groups ( $Z = 2$ ,  $n_{opp} = 6$ ):  $G^1 = \{D_{11}, B_{21}, D_{21}\}$  and  $G^2 = \{D_{12}, E_{31}, C_{31}\}$  with opportunistic time  $t_{G^1}$  and  $t_{G^2}$  respectively.  $G^1$  indicates that component 2 is left in its failed state and then jointly replaced with component 1 at  $t_{G^1}$ . While according to  $G^2$ , components 1 and 3 are jointly replaced at the downtime period of component 3 at  $t_{G^2} = t_{E_{31}}$ .

One of the main objectives of this phase is then to develop a mathematical expression that allows evaluating the performance of an OM plan. The total maintenance cost in  $I_k$ , denoted  $C_{sys}^k(X, t_{G^z})$ , is selected as the evaluation criterion in the paper. Given the existence of the economic dependence between components and the system redundancy, the development of the closed-form of  $C_{sys}^k(X, t_{G^z})$  is challenging and will be described in more detail in Subsection 5.1.

## Phase 3: OM optimization using GAM

With the same set of maintenance opportunities in  $I_k$ , there exist a lot of OM solutions (OM scenario and its opportunistic time) with different maintenance costs. This phase aims at finding the best OM solution, denoted  $(X^*, t_{G^z}^*)$ , which minimizes the total maintenance cost in  $I_k$ . Given the complexity of the optimization problem, Genetic algorithm with memory (GAM) is used and will be described in more detail in Subsection 5.2.



## Phase 4: OM validation and updating

The best OM solution  $(X^*, t_{G^z}^*)$  found in the previous phase will be used as the maintenance plan of the system until the occurrence of a new opportunity. If there are no added opportunities in  $I_k$ , the process returns to the first phase to find the OM solution for the next interval  $I_{k+1}$  with  $a_{k+1} = b_k$ , and so on. Otherwise, if a maintenance opportunity (usually MO types A, B or F with random occurrence time) occurs at instant  $T$  in  $I_k$ , the current OM solution may become a sub-optimal solution, the process returns to the first phase to update the OM plan. For this purpose, a new planning horizon  $I_{k+1}$  has to be established with  $a_{k+1} = T$ .

Finally, thanks to the optimal OM scenario  $X^*$  and its optimal opportunistic time  $t_{G^z}^*$  provided by our OM approach, the following decisions can be also indirectly deduced.

- *Maintenance postponement optimization.* Since the failure of non-critical components does not lead the system to shut down, the repair of these components, therefore, can be postponed to the next maintenance opportunity. The optimal maintenance postponement can be deduced from both  $X^*$  and  $t_{G^z}^*$ . Indeed, let reconsider the example in Figure 8c, as mentioned above, if  $X^*$  contains group  $G^1 = \{D_{11}, B_{21}, D_{21}\}$ , component 2 will be left in its failed state until  $t_{G^1}$ . According to a number of existing works [14, 39, 40], the optimal value  $t_{G^1}$  is usually fixed at one of the three instants  $t_{B_{21}}$ ,  $t_{D_{11}}$  or  $t_{D_{21}}$ . If  $t_{G^1} = t_{B_{21}}$ , group  $G^1$  is maintained at  $a_k$ , advanced support modes at a high cost have to be used since the preparation time is very short. If  $t_{G^1}$  equals  $t_{D_{11}}$  or  $t_{D_{21}}$ , the component may be left in its failed state too long. The risk of system failure and/or component downtime cost may reach their unacceptable levels. In our paper, this limitation is removed by considering that  $t_{G^1}$  may be any instant between  $t_{B_{21}}$  and  $t_{D_{21}}$ . The optimal value  $t_{G^1}^*$  is selected such that the support modes with lower cost may be used and component is not left in its failed state too long. The risk of system failure and/or component downtime cost may increase, but they are always in their acceptable levels.
- *Maintenance level selection.* At failure of a component, if its degradation level or age is high enough, the replacement action may be more appropriate than the minimal repair one. The selection of the maintenance levels is also optimized in our approach. Indeed, if a group contains both the CM and PM of a failed component, the replacement is selected instead of minimal repair. For example, the group  $G^1$  contains  $B_{21}$  and  $D_{21}$ , component 2 is then replaced at  $t_{G^1}^*$ . If the group contains  $B_{21}$ , but not  $D_{21}$ , component 2 will be minimally repaired at  $t_{G^1}^*$ .
- *Support mode selection.* According to the support mode model presented in Subsection 2.3, the support mode can be selected based on the preparation time. When a group of maintenance opportunities  $G^z$  is executed at  $t_{G^z}$ , which is close to  $a_k$ , the preparation time of the group  $t_p(G^z) = t_{G^z} - a_k$  is short. The shorter  $t_p(G^z)$  is, the more advanced support modes should be used at higher costs. The support mode of  $G^z$  is then optimally selected based on the optimal execution time  $t_{G^z}^*$ . For example, the emergency support mode is selected to prepare for the maintenance of group  $G^z$  if

$$T_{em} \leq t_p(G^z) = t_{G^z}^* - a_k < T_{ex} \Rightarrow T_{em} + a_k \leq t_{G^z}^* < T_{ex} + a_k$$

## 4.2 Flexibility of the proposed OM approach

The flexibility of the proposed approach is underlined by the fact that it may cover some maintenance approaches existing in the literature, and provide the same results as them under specific conditions. For example, we consider the following cases:

- *Individual maintenance (IM).* Individual maintenance means that all components are separately maintained. This maintenance approach may be applied when the dependence between components are weak, e.g. the cost of the joint maintenance is quite the same as that of the individual one. The maintenance of components

can then be optimized separately. Our maintenance model can become individual maintenance one if each group  $G^z$  contains only one maintenance opportunity.

- *Grouping maintenance (GM)*. The grouping maintenance considers only the joint execution between the PM activities. Our approach may become a grouping maintenance one when only maintenance opportunities of types C and D, which represent the PM of critical and non-critical components, are considered. In the one hand, this approach can not take advantage of these other MO types such as types A, B, E or F. In the other hand, it can provide maintenance plans with the maintenance dates are known in advance. The support of these maintenance plans is hence more simple than that provided by the opportunistic maintenance. In reality, the choice between grouping maintenance and opportunistic maintenance depends on a number of practical factors such as performance and flexibility of the maintenance support system, system failure frequency, system downtime cost, etc.
- *System-downtime-based opportunistic maintenance (SDOM)*. According to this approach, only system downtimes due to the maintenance (CM or PM) of critical components are considered as opportunities to maintain a selected group of components. The group may contain several non-critical components failed in the previous period. This OM approach was developed for a redundant system such as offshore wind turbines since it limits the number of system interventions and reduces the maintenance support costs. Our approach may provide the same decisions as the considered approach if in each group  $G^z$  contains at least one maintenance opportunity of type A (CM of critical component) or type C (PM of critical component), and the opportunistic time  $t_{G^z}$  is equal to  $t_{A_{ij}}$  or  $t_{C_{ij}}$ .
- *Component-downtime-based opportunistic maintenance (CDOM)*. According to this approach, a maintenance action (both PM or CM) of a component  $i$  (both critical or non-critical) is considered as an opportunity to maintain the other components in the system. However, the opportunistic time is always fixed at the CM or PM time of component  $i$  such as  $t_{A_{ij}}$ ,  $t_{B_{ij}}$ ,  $t_{C_{ij}}$ , or  $t_{D_{ij}}$ . This approach can take advantage of non-critical components, but it requires more effort in condition monitoring at the component level when compared to SDOM. Our approach may provide the same decisions as CDOM if  $t_{G^z}^*$  is simply selected between the occurrence time of the maintenance opportunities in  $G^z$ .

In addition, thanks to the flexibility of the proposed OM approach, it can be applied to a wide range of maintenance problems. For example, if a maintenance opportunity  $O_{ij}$  is only applicable for certain components instead of the whole system, the MO benefit ( $BE_{O_{ij}}$ ) of these components will be set to zero.

## 5 OM performance evaluation and optimization

In this section, the mathematical expression of OM performance is developed by taking into account the economic dependence between maintenance opportunities, the support mode model, the dynamic criticality of components (Subsection 5.1). The expression is then used as a basis for the OM optimization in Subsection 5.2.

### 5.1 OM performance evaluation

**OM performance criterion.** The performance of an OM solution is evaluated by the total maintenance cost in  $I_k$  when the OM solution is applied.

$$C_{sys}^k(X, t_{G^z}) = \sum_{z=1}^Z C_{G^z}(t_{G^z}) + \sum_{z=1}^{Z+1} C_{z-1}^z(t_{G^z}) \quad (8)$$

where,  $C_{G^z}$  is the total maintenance cost of the system during the maintenance of group  $G^z$  in interval  $I_{G^z} = [t_{G^z}, t_{G^z} + \omega_{G^z}]$ ,  $\omega_{G^z}$  is the duration of  $G^z$ ; and  $C_{z-1}^z$  is the total maintenance cost of the system between two

consecutive groups of maintenance opportunities  $G^{z-1}$  and  $G^z$  in  $I_{G^{z-1}}^{G^z} = [t_{G^{z-1}} + \omega_{G^{z-1}}, t_{G^z}]$  with  $t_{G^0} = a_k$  and  $t_{G^{z+1}} = b_k$ . [These costs will be estimated hereunder.](#)

**Economic dependence and total maintenance cost  $C_{G^z}$ .** Due to the existence of the economic dependence between components, components, and their maintenance opportunity, the total maintenance cost of  $G^z$  cannot be calculated directly from that of their elements.

$$C_{G^z} \neq \sum_{i \in G_m^z} C_i \quad (9)$$

where  $G_m^z$  is the set of all the components that are maintained at  $t_{G^z}$ . In the example,  $G^z = \{D_{12}, E_{31}, C_{31}\}$ , there are two components are maintained  $G_m^z = \{1, 3\}$ .

For the above reason, the economic dependence and their impacts on the total maintenance cost have to be analyzed. We consider first the impacts of the economic dependence on the total support costs of a group of components. When several components are jointly maintained, their spare parts can be transported together, the preparation tasks at the maintenance site such as scaffolding erecting, machine opening, has to be done only once. If we consider the support cost of a group of components equals the maximum of their support costs, the support cost paid for group  $G^z$  can be calculated as

$$S_{G^z} = (1 - \mathbb{I}_{\{F_{ij} \in G^z\}} \cdot \mu_{ij} \%) \cdot \max_{i \in G_m^z} S_i^* \quad (10)$$

where,  $\mu_{ij}$  is the support cost reduction (see page 12);  $\mathbb{I}_{(x)}$  is the indicator function.  $\mathbb{I}_{(x)} = 1$  if  $x$  is true; otherwise,  $\mathbb{I}_{(x)} = 0$ . In the above equation,  $\mathbb{I}_{\{F_{ij} \in G^z\}} = 1$  if  $t_{G^z}$  is in a support cost discount period  $F_{ij}$ . Otherwise,  $\mathbb{I}_{\{F_{ij} \in G^z\}} = 0$ .

Besides the support cost, the total downtime costs are also influenced by economic dependence. For instance, if two critical components are maintained together, the system is shut down only once. Otherwise, when the two components are separately maintained, the system is shut down twice. If the number of repairmen is enough to perform all the maintenance activities of  $G_m^z$  at the same time, the time to perform these activities is  $\omega_{G_m^z} = \max_{i \in G_m^z} \omega_i$ . We consider the two following cases:

- $\mathbb{I}_{\{\omega_{E_{ij}} \geq \omega_{G_m^z}\}} = 1$  means that the system is shut down due to the production planning during the maintenance of the components of  $G_m^z$ . The costs relating to the system/component shutdowns have not to be paid,  $C_{G^z}^d = 0$ .
- $\mathbb{I}_{\{\omega_{E_{ij}} < \omega_{G_m^z}\}} = 1$  means that the maintenance duration of components are larger than that of  $E_{ij}$ . The costs relating to the system/component shutdowns in  $I_{G^z} = [t_{G^z} + \omega_{E_{ij}}, t_{G^z} + \omega_{G_m^z}]$  have to be paid. If  $G^z$  does not contain any  $E_{ij}$ ,  $\omega_{E_{ij}}$  is set to be zero, and  $I_{G^z} = [t_{G^z}, t_{G^z} + \omega_{G_m^z}]$ . If  $G_m^z$  is a critical group, the system is shut down during the maintenance of this group, a system shutdown cost, which is equal to  $r_0 \cdot (\omega_{G_m^z} - \omega_{E_{ij}})$ , then has to be paid. Otherwise, if  $G_m^z$  is a non-critical group, the component downtime costs relating to the in-operating state of its components  $(\omega_{G_m^z} - \omega_{E_{ij}}) \cdot \sum_{i \in G_m^z} r_i$  and expected CM costs of the operating components  $\sum_{j \in L_{G_m^z}} N_j(I_{G^z}) \cdot C_j^{mr}$ , have to be paid.  $L_{G_m^z}$  is the set of all operating components during the maintenance of non-critical group  $G_m^z$ .

From these above analyses, the costs relating to the component/system downtimes during the maintenance of components in  $G^z$  are

$$cd_{G^z} = \mathbb{I}_{\{\omega_{E_{ij}} < \omega_{G_m^z}\}} \cdot \left[ \vartheta_{G_m^z}(t_{G^z}) \cdot r_0 \cdot (\omega_{G_m^z} - \omega_{E_{ij}}) \right. \\ \left. + (1 - \vartheta_{G_m^z}(t_{G^z})) \cdot \left( (\omega_{G_m^z} - \omega_{E_{ij}}) \cdot \sum_{i \in G_m^z} r_i + \sum_{j \in L_{G_m^z}} N_j(I_{G^z}) \cdot C_j^{mr} \right) \right] \quad (11)$$

where,  $\vartheta_{G_m^z}(t_{G^z})$  is the criticality of group  $G_m^z$  at time  $t_{G^z}$ .

$$\vartheta_{G_m^z}(t_{G^z}) = \begin{cases} 1 & \text{if } G_m^z \text{ is a critical group at time } t_{G^z} \\ 0 & \text{if } G_m^z \text{ is a non-critical group at time } t_{G^z} \end{cases} \quad (12)$$

In addition, the total specific costs, that has to be paid for the maintenance of the components of  $G_m^z$ , is

$$c_{G^z} = \sum_{i \in G_m^z} c_i^* \quad (13)$$

It should be noted that if  $G^z$  contains both the CM based (types A, B) and PM based (types C, D) maintenance opportunities of a component  $i$ , the component is replaced instead of minimally repaired. The only specific cost for PM of the component  $c_i^{re}$  will be counted.

Finally, if the replacement of components  $G^z$  requires advanced support modes such as express or emergency modes, additional support cost  $ca_{G^z}$  have to be paid.

$$ca_{G^z} = \sum_{C_{ij}, D_{ij} \in G^z} ca_i(t_{G^z} - a_k) \quad (14)$$

where the additional support costs can be expressed as (see Figure 2)

$$ca_i(t_{G^z} - a_k) = \begin{cases} +\infty & \text{if } t_{G^z} - a_k < T_{em} \\ ca_i^{em} & \text{if } T_{em} \leq t_{G^z} - a_k < T_{em} < T_{ex} \\ ca_i^{ex} & \text{if } T_{ex} \leq t_{G^z} - a_k < T_{no} \\ 0 & \text{if } t_{G^z} - a_k \geq T_{no} \end{cases} \quad (15)$$

The total maintenance cost of  $G^z$  with taking into account the economic dependence is then

$$C_{G^z}(t_{G^z}) = S_{G^z} + ca_{G^z}(t_{G^z}) + c_{G^z} + cd_{G^z}(t_{G^z}) \quad (16)$$

**Total maintenance cost between two consecutive groups of maintenance opportunities  $C_{z-1}^z$ .** Given that components may be left in their failed states, the system may be in non-operating state, the partial operating state with some failed components, or full operating state between two consecutive PM periods. To calculate  $C_{z-1}^z$ , a group  $G_{z-1}^z$  of all operating components in  $I_{G_{z-1}^z}$  is first identified. If the group is empty or critical  $\vartheta_{G_{z-1}^z}(t_{G_{z-1}^z} + \omega_{G_{z-1}^z}) = 1$ , the system does not operate in  $I_{G_{z-1}^z}$ , a system shutdown cost  $r_o \cdot [t_{G^z} - (t_{G_{z-1}^z} + \omega_{G_{z-1}^z})]$  has to be paid. Otherwise, if  $G_{z-1}^z$  is not critical, the component downtime costs relating to the in-operating state of its components  $[t_{G^z} - (t_{G_{z-1}^z} + \omega_{G_{z-1}^z})] \cdot \sum_{i \in G_{z-1}^z} r_i$  and expected minimal repair costs of the operating components  $\sum_{j \in L_{G_{z-1}^z}} N_j(I_{G_{z-1}^z}) \cdot C_j^{mr}$ , have to be paid. From the above analyses, we have

$$\begin{aligned} C_{z-1}^z(t_{G^z}) &= \vartheta_{G_{z-1}^z}(t_{G_{z-1}^z} + \omega_{G_{z-1}^z}) \cdot r_o \cdot [t_{G^z} - (t_{G_{z-1}^z} + \omega_{G_{z-1}^z})] \\ &+ [1 - \vartheta_{G_{z-1}^z}(t_{G_{z-1}^z} + \omega_{G_{z-1}^z})] \cdot [t_{G^z} - (t_{G_{z-1}^z} + \omega_{G_{z-1}^z})] \cdot \sum_{i \in G_{z-1}^z} r_i + \sum_{j \in L_{G_{z-1}^z}} N_j(I_{G_{z-1}^z}) \cdot C_j^{mr} \end{aligned} \quad (17)$$

From the equations 8, 16, and 17, the total maintenance cost of an OM solution can be evaluated.  $C_{sys}^k(X)$  is considered as the objective function to select the best OM solution in the next subsection.

## 5.2 OM optimization

The optimal OM solution, denoted  $(X^*, t_{G^z}^*)$ , is found by minimizing the total maintenance cost of the system in the considered interval  $I_k$ .

$$(X^*, t_{G^z}^*) = \arg \min_{X, t_{G^z}} C_{sys}^k(X, t_{G^z}) = \arg \min_{G^z, t_{G^z}} \left( \sum_{z=1}^Z C_{G^z}(t_{G^z}) + \sum_{z=1}^{Z+1} C_{z-1}^z(t_{G^z}) \right) \quad (18)$$

To solve the above optimization problem, an optimization process is proposed in Figure 9. The process contains two mains phases: OM scenario optimization using GAM and opportunistic time optimization.

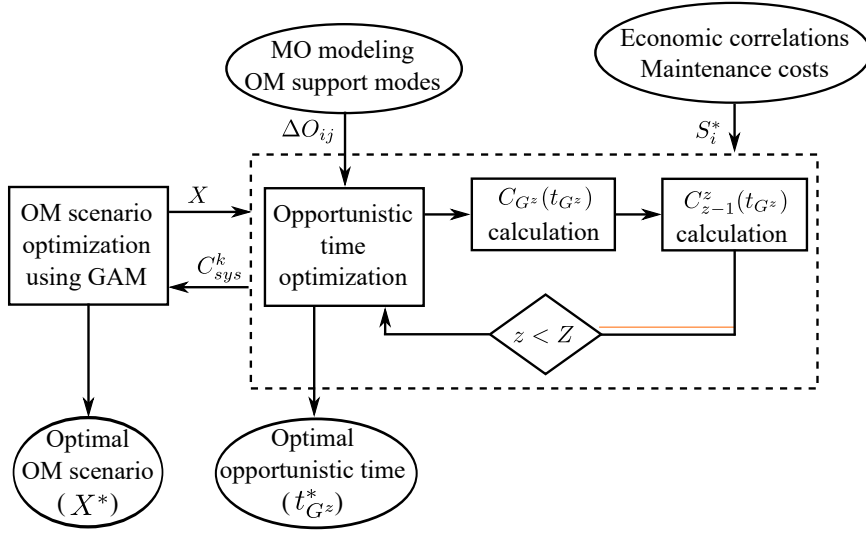


Figure 9: OM optimization approach

**OM scenario optimization.** Given  $n_{opp}$  maintenance opportunities in an initial OM plan, there are  $2^{n_{opp}}$  possible OM scenarios. The use of exact methods, which test all possible scenarios to find the best one, seems to be impossible. In this situation, the Genetic algorithm is a promising solution since it is a powerful global search tool, and has been successfully applied to maintenance optimization problems in [41]. In addition, according to our proposed OM approach, the finding of the optimal OM scenario is repeatedly done for every short-term interval and each time when a new opportunity occurs. We observe that there are many factors that do not change each time when we find the optimal OM scenario such as the system structure, several maintenance costs, etc. Thanks to the existence of these common factors, the optimal OM scenario in one short-term interval or the last run may contain several compositions (a group of MOs) which are also promising for the other intervals or the next runs. From this observation, the Genetic Algorithm with Memory (GAM) proposed in [42] is developed to be applied to our problem.

Genetic Algorithm (GA) starts by creating randomly an initial set of OM scenarios. At each iteration, these OM scenarios will be (a) evaluated to find the potential best solution; (b) modified by the genetic operations such as crossover and mutation. The process is stopped after some predetermined number of iterations. The convergence of GA depends on both the initial set and the number of iterations. To improve the convergence of GA, the previous knowledge about the best OM scenarios is integrated into the initial set generation process of GAM. The process is then not completely random, but guided by the previous knowledge registered in the GAM's memory to explore more quickly the zone of promising solutions. Note that to guarantee the convergence of GA or GAM, they are usually applied several times for the same short-term interval. If we consider the  $i^{\text{th}}$  run of GAM to find the best OM scenario in a short-term interval  $I_k = [a_k, b_k]$ , the previous knowledge registered in the GAM's memory of this run is the promising groups/OM scenarios that have found in the previous intervals  $I_j$  ( $0 < j < k$ ) or in the previous runs  $m$  ( $0 < m < i$ ). The memory is updated each time when a promising solution is found. The detail description of the OM optimization using GAM and GA is given in Appendix B. The performance of the proposed OM optimization using GAM will be validated in Subsection 6.4.

**Opportunistic time optimization.** For a specific group  $G^z$  of an OM scenario, the optimal opportunistic time  $t_{G^z}^*$  has to be determined. We observe that two main factors affect the choice of  $t_{G^z}$ : (a) the penalties related to the movement of the occurrence time of maintenance opportunities; (b) the additional support cost paid for preparation tasks of PM activities. The optimal opportunistic time  $t_{G^z}^*$ , therefore, may be determined as follows

$$t_{G^z}^* = \arg \min_{t_{G^z}} \left[ \sum_{O_{ij} \in G^z} \Delta O_{ij} (t_{G^z} - t_{O_{ij}}) + \sum_{C_{ij}, D_{ij} \in G^z} ca_i (t_{G^z} - a_k) \right] \quad (19)$$

For example, consider group  $G^1 = \{D_{11}, B_{21}, D_{21}\}$  in Figure 8c. The optimal opportunistic time of this group  $t_{G^1}^*$  can be determined by the following expression

$$t_{G^1}^* = \arg \min_{t_{G^z}} \left[ (\Delta D_{11} + \Delta D_{21} + \Delta B_{21}) + (ca_1 + ca_2) \right] \quad (20)$$

The optimality of  $t_{G^1}^*$  is shown in Figure 10. In this example,  $t_{G^1}^* = 10$ . It should be noted that if  $G^z$  contains a maintenance opportunity whose occurrence time can not be modified (types E and F), e.g.  $\Delta O_{ij} = +\infty$ , the opportunistic time  $t_{G^z}^*$  will be set to the occurrence time of this maintenance opportunity  $t_{G^z}^* = t_{O_{ij}}$ .

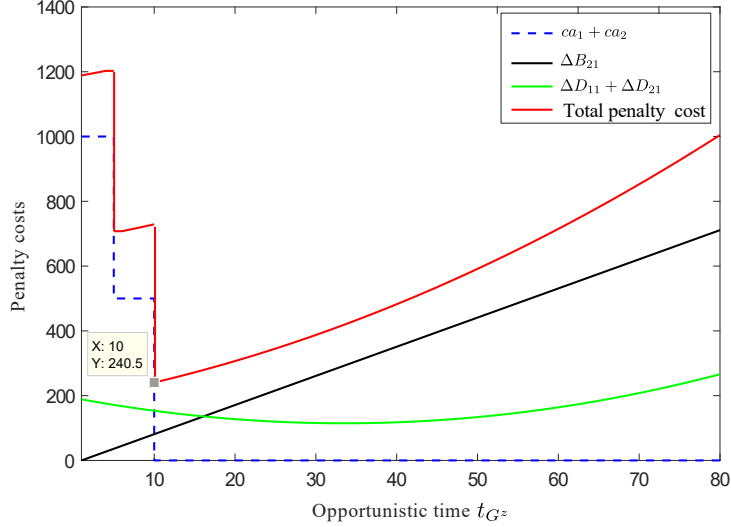


Figure 10: An example of the optimality of  $t_{G^z}^*$

## 6 Numerical examples

In this section, through different numerical examples, we will (a) illustrate how the proposed approach can be applied to the maintenance planning of a redundant system (subsection 6.1); (b) show how the flexibility of the proposed OM approach is by considering different types of maintenance opportunities (subsection 6.2); (c) clarify the advantages of the proposed approach in comparison to the other popular maintenance approaches such as grouping maintenance, system and component-downtime-based opportunistic maintenance (subsection 6.3); (d) validate the performance of the proposed OM optimization using GAM in subsection 6.4. For these purposes, we reconsider the power plant system presented in Figure 1. The structure presented in Figure 1 has been chosen such that the effect of redundancy on opportunistic maintenance optimization can be clearly analyzed. From a methodological point of view, this example is complete enough to illustrate the different steps of the proposed approach, to have meaningful and relevant numerical results and to get more insight into the opportunistic maintenance plan provided by our approach and its optimization process.

The failure behavior of components is modeled by Weibull distributions with an increasing failure rate  $\lambda_i(t)$ .

$$\lambda_i(t) = \frac{\beta_i}{\eta_i} \left[ \frac{x_i(t)}{\eta_i} \right]^{\beta_i - 1} \quad (21)$$

where  $\eta_i > 0$  and  $\beta_i > 1$  are the scale and shape parameters given in Table 4.  $x_i(t)$  denotes the age (cumulative operational time) of component  $i$  at time  $t$ . Due to the operational dependence between components and non-negligible maintenance durations, we have  $x_i(t) \neq t$  in general. If the component is minimally repaired at failures, its mean number of failures in interval  $(a, b)$  is calculated as

$$N_i(a, b) = \int_{x_i(a)}^{x_i(b)} \lambda_i(t) dt = \int_{x_i(a)}^{x_i(b)} \frac{\beta_i}{\eta_i} \left[ \frac{x_i(t)}{\eta_i} \right]^{\beta_i - 1} = \left[ \frac{x_i(b)}{\eta_i} \right]^{\beta_i} - \left[ \frac{x_i(a)}{\eta_i} \right]^{\beta_i} \quad (22)$$

Table 4: Data relating to the failure behaviors and maintenance costs of the components

Components	$\vartheta_i$	$\eta_i$	$\beta_i$	$S_i^{re}$	$S_i^{mr}$	$c_i^{re}$	$c_i^{mr}$	$\omega_i^{re}$	$\omega_i^{mr}$	$r_i$
1	0	190	1.58	220	120	400	15	3	0.25	5
2	0	80	2.05	222	122	550	20	2	0.32	3
3	0	117	1.47	218	118	600	22	2	0.46	2
4	0	117	2.00	217	117	480	18	4	0.42	4
5	1	150	1.65	218	118	350	45	2	0.50	5
6	1	204	1.75	222	122	400	33	3	0.21	4

The maintenance costs and the criticality of all components are reported in Table 4. The other costs including the system downtime cost rate and additional support costs are  $r_0 = 23$  and

$$ca_i(t_p) = \begin{cases} 500 & \text{if } 0 \leq t_p < 3 \quad (\text{Emergency support mode}) \\ 250 & \text{if } 3 \leq t_p < 10 \quad (\text{Express support mode}) \\ 0 & \text{if } t_p \geq 10 \quad (\text{Normal support mode}) \end{cases} \quad (23)$$

It should be noted that, in this study, all parameters are given in arbitrary units, e.g., arbitrary time unit (atu) or arbitrary cost unit (acu).

## 6.1 Opportunistic maintenance planning

This subsection is devoted to show how the proposed OM approach can be applied to find an optimal OM plan of the above system in a short-term interval  $I_k = [a_k, b_k]$ . In this example, we select  $I_1 = [a_1, b_1] = [0, 540.30]$  with  $a_1 = 0$  is the beginning of the first short-term interval, and  $b_1 = t_{D_{11}} + \omega_i^{re} = 540.30$  is the ending of the short-term interval.  $b_1$  is selected such that all the components in the system are maintained at least once in  $I_1$ . We assume that the components 1 and 4 failed at  $a_1$ . For this purpose, two main phases of the proposed approach will be described in more detail: MO identification and modeling, and MO scenario optimization using GAM.

**MO identification and modeling.** The proposed approach starts by collecting all the information related to the occurrence of maintenance opportunities in the considered interval such as PM cycles and failed components at  $a_1$ . The PM cycles are determined by the long-term maintenance planning (see Appendix A) and reported in Table 5. Thanks to the collected information, all maintenance opportunities in  $I_1$  can be identified and modeled (see Table 5). For more detail, in the considered interval, there are 11 opportunities:  $B_{11}$  and  $B_{41}$  (triggered by the failure of components 1 and 4);  $C_{51}$  and  $C_{61}$  (triggered by the PM of critical components 5 and 6);  $D_{11}$ ,  $D_{21}$ ,  $D_{22}$ ,  $D_{23}$ ,  $D_{31}$ ,  $D_{41}$ ,  $D_{42}$  (triggered by the PM of non-critical components 1, 2, 3 and 4). The initial maintenance

Table 5: Identified maintenance opportunities in the considered interval

Components	$T_i^*$	Maintenance opportunities
1	712.30	$B_{11}$ ( $t_{B_{11}} = 0$ ); $D_{11}$ ( $t_{D_{11}} = 537.30$ )
2	181.92	$D_{21}$ ( $t_{D_{21}} = 6.92$ ); $D_{22}$ ( $t_{D_{22}} = 188.84$ ); $D_{23}$ ( $t_{D_{23}} = 370.76$ )
3	658.22	$D_{31}$ ( $t_{D_{31}} = 483.22$ )
4	271.58	$B_{41}$ ( $t_{B_{41}} = 0$ ); $D_{41}$ ( $t_{D_{41}} = 96.58$ ); $D_{42}$ ( $t_{D_{42}} = 368.17$ )
5	421.79	$C_{51}$ ( $t_{C_{51}} = 246.79$ )
6	556.49	$C_{61}$ ( $t_{C_{61}} = 381.49$ )

plan (Figure 11) contains all the above maintenance opportunities when they are not grouped. For more detail, all PM activities are separately executed at their planned dates, and the failed components 1 and 4 are minimally repaired at their failure time  $t_{B_{11}} = t_{B_{41}} = a_1$ . The total maintenance cost of the initial OM plan, denoted  $C_{Initial}$ , is equal to 14578 cost units. The expected maintenance cost rate of the initial OM plan is

$$CR_{Initial} = \frac{C_{Initial}}{b_1 - a_1} = 26.981$$

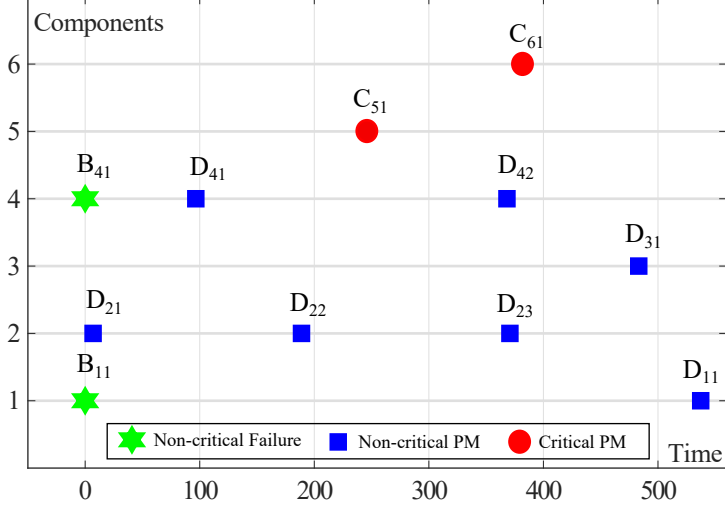


Figure 11: Initial OM plan in  $I_1$

**MO scenario optimization using GAM.** All these above maintenance opportunities are then grouped to form different OM scenarios. Since the number of possible OM scenarios is very high, GAM is applied to search the best OM scenario. To ensure the convergence of the optimization tool, GAM was repeatedly executed 5 times.

The performance of GAM will be studied further in Subsection 6.4. Table 6 and Figure 12 present the best OM scenario (named  $X_1$ ) found by GAM after five runs.

Table 6: The best OM scenarios found by GAM

OM scenario	Group	Opportunistic time	Support mode	Total maintenance cost
$X_1$	$G^1 = \{D_{21}, D_{41}, B_{11}, B_{41}\}$	10.00	Normal	<b>12694</b>
	$G^2 = \{D_{11}, D_{22}, D_{31}, D_{42}, C_{51}, C_{61}\}$	221.19	Normal	
	$G^3 = \{D_{23}\}$	374.76	Normal	
$X_2$	$G^1 = \{D_{21}, D_{41}, B_{11}, B_{41}\}$	3.00	Express	<b>12635</b>
	$G^2 = \{D_{11}, D_{22}, D_{31}, C_{51}, C_{61}\}$	176.74	Normal	
	$G^3 = \{D_{23}, D_{42}\}$	334.12	Normal	

$X_1$  contains three groups of maintenance opportunities. According to the composition of the first group: components 1 ( $B_{11}$ ) and 4 ( $B_{41}$ ) are left in their failed states from  $a_1 = 0$  and then jointly maintained with component 2 ( $D_{21}$ ) at time  $t_{G^1} = 10$ ; PM of component 2 ( $D_{21}$ ) is postponed from  $t_{D_{21}} = 6.92$  to  $t_{G^1}$ ; and PM of component 4 ( $D_{41}$ ) is advanced from  $t_{D_{41}} = 96.58$  to  $t_{G^1}$ . These decisions are completely different from the traditional ones where the postponement of the maintenance opportunities is not allowed. Indeed, the postponement of non-critical components 1 and 4 is reasonable since it does not lead the system to shut down and [makes the application of the normal support mode become possible](#). Moreover, not only the maintenance dates but also the maintenance levels can be deduced from the composition of the group (at time  $t_{G^1} = 10$ , component 1 is minimally repaired; component 2 and 4 are replaced). It should be noted that the failed component 4 is replaced by a new one instead



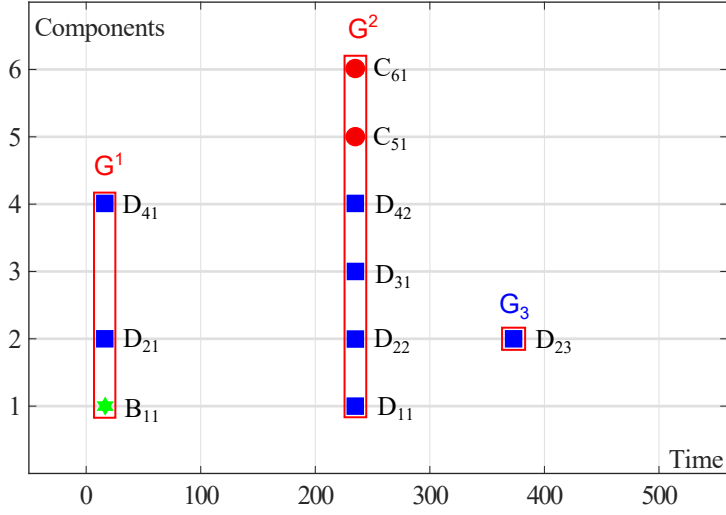


Figure 12: The best OM scenario  $X_1$  provided by GAM

of minimally repaired since the group contains  $B_{41}$  and  $D_{41}$ . The system is renewal at time  $t_{G^2} = 221.19$  since the second group contains the PM of all components. Both the support and downtime costs can be therefore saved. Finally, the composition of group  $G^3$  indicates that component 2 is separately maintained at  $t_{G^3} = 374.76$ . The PM activity  $D_{23}$  cannot be grouped with the second group because a group containing several PM activities of the same component is prohibited.

By considering the difficulties of OM support in maintenance modeling and optimization, the best OM scenario  $X_1$  is configured in such a way that the emergency support cost is reduced to the minimum possible. In reality, Table 6 shows that the normal mode is applied to support the maintenance of all the groups of the best OM scenario, i.e., emergency support cost has not to be paid.

In the case that the best OM scenario  $X_1$  is applied, the expected maintenance cost rate is

$$CR_{OM} = \frac{C(X_1)}{b_1 - a_1} = 23.651$$

The maintenance cost saving is around **13%** when the best OM scenario is applied. Finally, a similar study was carried out with a small emergency support cost, e.g.  $ca_i = 0.5 \cdot ca_i$ . The best OM scenario is  $X_2$  (see Table 6). The maintenance of group  $G^1$  of  $X_2$  is prepared by the express support mode. The use of this mode is reasonable since the emergency support cost is not expensive.

## 6.2 Opportunistic maintenance planning in presence of various MO types

Beside the non-critical CM based opportunity (MO type B) studied in the above subsection, we study here in this subsection other OM types to verify the flexibility of the proposed OM approach. Therefore, critical CM based opportunity (Type A) and external factor based opportunity (Type F) are considered.

**Critical CM based opportunity.** We consider the same system as in subsection 6.1, except that the critical component 5 is assumed to be failed instead of components 1 and 4 at  $a_1$ . The failure of component 5 creates a critical CM based opportunity  $A_{51}$  at  $t_{A_{51}} = a_1$ . The expected maintenance cost rate of the system is 26.145 when the advantage of  $A_{51}$  is not taken. To take advantage of  $A_{51}$ , GAM is executed to update the OM plan in the two following cases: low downtime costs  $r_i$  (the values are reported in Table 4); and high downtime costs  $r_i = r_i + 50$ . Table 7 represents the best OM scenarios  $X_3$  and  $X_4$  found by GAM. When the component downtime costs are small, component 5 is left in its failed state and minimally repaired at time 32.38 (group  $G^1$  of  $X_3$ ). The system does not operate in the interval  $[0, 32.38]$ . Otherwise, when downtime costs are high, component 5 is immediately repaired at its failure time 0 to restore the system to its operational state. In addition, if components 2, 4 and 5 are jointly maintained in group  $G^1$  of  $X_3$  at  $a_1 = 0$ , the emergency costs have to be paid for the PM of components

2 and 4. For this reason, in  $X_4$ , the PM of these two components are separated to the CM of component 5 and jointly executed in group  $G^2$  at  $t_{G^2} = 22.72$ . Another remark is that when the downtime costs are high, the modification of the OM occurrence time is limited since it leads to important additional downtime costs,  $X_4$  is, therefore, contains more groups than  $X_3$ .

Table 7: The best OM scenarios when component 5 fails

Downtime cost	OM scenario	Group	Opportunistic time	Total maintenance cost
$r_i$	$X_3$	$G^1 = \{A_{51}, D_{21}, D_{41}\}$	32.38	<b>12638</b>
		$G^2 = \{D_{11}, D_{22}, D_{31}, C_{51}, C_{61}\}$	191.15	
		$G^3 = \{D_{23}, D_{42}\}$	343.12	
$r_i + 50$	$X_4$	$G^1 = \{A_{51}\}$	0.00	<b>19088</b>
		$G^2 = \{D_{21}, D_{41}\}$	22.72	
		$G^3 = \{D_{11}, D_{22}, D_{31}, C_{51}, C_{61}\}$	185.34	
		$G^4 = \{D_{23}, D_{42}\}$	321.08	
		$G^5 = \{D_{24}\}$	463.60	

**External factor based opportunity.** We reconsider the non-critical CM based opportunity in subsection 6.1, and assume that there is an 80% discount of PM support cost for all components if they are maintained in the interval  $[210, 220]$ . To take advantage of this opportunity, it should be modeled first as  $F_{01}$  with  $t_{F_{01}} = 210$  and  $\omega_{F_{01}} = 10$ .  $i = 0$  means that the opportunity concerns all components of the system. GAM is then run to determine the best OM scenario given the presence of  $F_{01}$ . From the result reported in Table 8, we can notice that the OM scenario  $X_5$  is almost the same as  $X_1$  in Table 6, except that the execution time of group 2 is moved from  $t_{G^2} = 221.19$  to 214.81 to profit the discount of support cost. The support cost of group 2 is then reduced from 222 to 44.4. Consequently, the total support cost of  $X_5$  is only 488.4 instead of 666 in the case of  $X_1$  and 1554 in the case of the initial maintenance plan.

Table 8: The best OM scenario when a 80% discount of support cost is applied

OM scenario	Group	Opportunistic time	Support cost	Total maintenance cost
$X_5$	$G^1 = \{D_{21}, D_{41}, B_{11}, B_{41}\}$	10.00	222	<b>12506</b>
	$G^2 = \{D_{11}, D_{22}, D_{31}, D_{42}, C_{51}, C_{61}, F_{01}\}$	<b>214.81</b>	<b>44.4</b>	
	$G^3 = \{D_{23}\}$	377.76	222	

We consider now the same maintenance opportunity  $F_{01}$ , however the occurrence time is at  $t_{F_{01}} = 110$ . GAM provided the best OM scenario which is the same as  $X_1$ . The obtained result shows that the advantage of opportunity  $F_{01}$  is not taken since its occurrence time is too far from the opportunistic time of the OM groups.

In conclusion, by considering these above examples, the ability to take into account the different OM types of the proposed approach is confirmed. In the next subsection, the performance and the flexibility in maintenance decision making of the proposed approach will be studied and compared to the other popular maintenance approaches developed for multi-component systems.

### 6.3 Performance study of the proposed OM approach

To validate the performance of the proposed OM approach, the three popular maintenance approaches: grouping maintenance (GM), system-downtime-based OM (SDOM) and component-downtime-based OM (CDOM) are

considered (see again subsection 4.2 for more detailed descriptions). These approaches are applied to the same problem in subsection 6.1 (components 2 and 4 are in their failed states at time 0). The optimal OM scenarios provided by these approaches are then reported in Table 9.

Table 9: The best OM scenarios provided by the considered maintenance approaches

Maintenance approach	OM scenario	Group	Opportunistic time	Maintenance cost
SDOM	$X_6$	$G^1 = \{B_{11}, B_{41}\}$	0.00	<b>13199</b>
		$G^2 = \{D_{21}\}$	6.92	
		$G^3 = \{D_{41}\}$	96.58	
		$G^4 = \{D_{21}, D_{22}, D_{31}, D_{42}, C_{51}, C_{61}\}$	246.79	
		$G^5 = \{D_{23}\}$	370.75	
CDOM	$X_7$	$G^1 = \{B_{11}, B_{41}\}$	0.00	<b>12883</b>
		$G^2 = \{D_{21}\}$	6.92	
		$G^3 = \{D_{41}\}$	96.58	
		$G^4 = \{D_{21}, D_{22}, D_{31}, C_{51}, C_{61}\}$	188.84	
		$G^5 = \{D_{23}, D_{42}\}$	368.17	
GM	$X_8$	$G^1 = \{B_{11}, B_{41}\}$	0.00	<b>12716</b>
		$G^2 = \{D_{21}, D_{41}\}$	23.05	
		$G^3 = \{D_{11}, D_{22}, D_{31}, C_{51}, C_{61}\}$	181.15	
		$G^4 = \{D_{23}, D_{42}\}$	338.23	
Proposed approach	$X_1$	$G^1 = \{D_{21}, D_{41}, B_{11}, B_{41}\}$	10.00	<b>12694</b>
		$G^2 = \{D_{11}, D_{22}, D_{31}, D_{42}, C_{51}, C_{61}\}$	221.19	
		$G^3 = \{D_{23}\}$	374.76	

Table 9 shows that the proposed OM approach gives the best results among the considered ones. This approach is more flexible in comparison with the grouping maintenance approach since it allows a group  $G^1$  that contains both PM (Type B) and CM (Type B) activities. When comparing to the SDOM and CDOM, the opportunistic time  $t_{G^z}$  of the proposed approach is more flexible since it is not always fixed at the occurrence time of a specific maintenance opportunity in the group. For more detail, the opportunistic time is fixed at system downtime time according to SDOM, the opportunistic time of group  $G^4$  in  $X_6$  is equal to the PM date of component 5 ( $t_{G^4} = t_{C_{51}} = 246.79$ ). Similarly to SDOM, the opportunistic time of groups  $G^4$  and  $G^5$  in OM scenario  $X_7$  provided CDOM are fixed at the second PM activity of component 2 ( $t_{G^4} = t_{D_{22}} = 188.84$ ) and at the second PM activity of component 4 ( $t_{G^5} = t_{D_{42}} = 368.17$ ) respectively. While the opportunistic time of groups in  $X_1$  provided by our approach does not need to be equal to the occurrence time of a specific maintenance opportunity in the groups. In addition, all the three approaches SDOM, CDOM, and GM do not allow to leave components in their failed states at the failures. Failed components are immediately repaired at  $t_{G^1} = 0$  in  $X_6$ ,  $X_7$ , and  $X_8$ . Otherwise, the failed components are left in their failed states until  $t_{G^1} = 10$  in  $X_1$ . The above flexibility in the construction of the groups and selection of the opportunistic time make the proposed OM approach more efficient and flexible than the other considered approaches.

To get a broader view on the performance of the proposed approach, different sensitivity analyses of the performance of the considered approaches with respect to different maintenance cost settings (different values of the support maintenance cost, downtime costs, and emergency support cost) were carried out.

Figure 13 represents the average maintenance cost provided by the considered approaches with respect to the different values of support costs:  $S_i^p = S_i^p + \Delta S$ , ( $\Delta S = 100, 200, \dots, 800$ );  $S_i^c = S_i^p - 100$ .

When the support costs are low, the performance of the considered approaches is quite the same. Otherwise, the difference between these approaches is more obvious when the support costs increase. The proposed approach

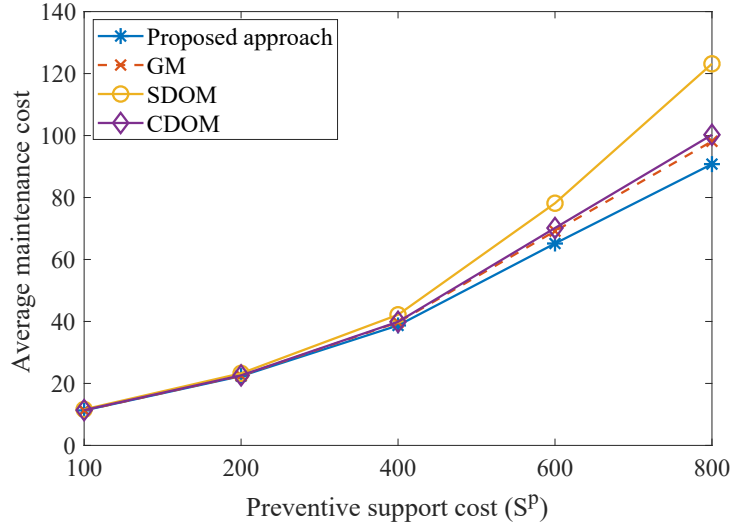


Figure 13: Sensitivity analysis of the considered approaches to the support costs

always provides better results than that provided by the other approaches since it is flexible and allows all the OM scenarios possible. The SDOM provides the worst results because it accepts the joint maintenance only at system failure times, and therefore does not take the maximum possible advantages of the economic dependence between components. From a practical point of view, when SDOM is applied, system monitoring is more simple than when the other approaches are used.

The sensitivity analysis of the four considered approaches to the additional support cost is done by varying the values of  $ca_i^{em}$  (emergency mode) from 0 to 1000 with  $\Delta ca_i^{em} = 125$ , and  $ca_i^{ex} = ca_i^{em}/2$  (express mode). The obtained results are sketched in Figure 14.

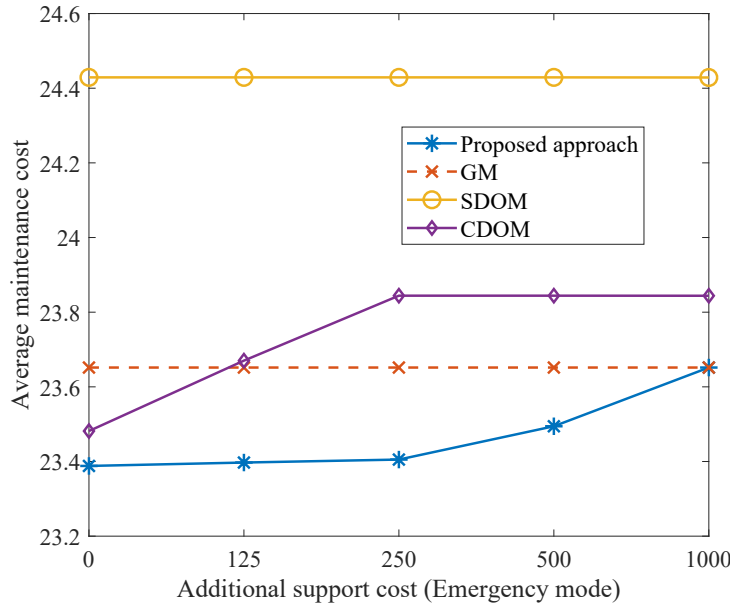


Figure 14: Sensitivity analysis of the considered approaches to the additional support cost

The performance of GM and SDOM are not affected by the value of the additional support costs, since the joint maintenance at  $a_1$  with the two failed components 2 and 4 are prohibited by GM and SDOM. The proposed approach provided the best results when compared to that of the other approaches. Its performance tends toward that of GM when the additional support cost is important because the joint maintenance at  $a_1$  becomes ineffective. In spite of the fact that in this case, all the four considered approaches do not allow the joint maintenance at  $a_1$ , GM and our approach are still provided better results than that of SDOM and CDOM thanks to their flexible opportunistic time. Indeed, according to SDOM and CDOM, the opportunistic time is fixed at component or

system failure times, otherwise, the opportunistic time is more flexible and may be different from the failure times according to GM and our approach. Another remark is that the performance of CDOM and our approach depend on the value of the additional support costs, however, the dependence is not critical thanks to the taking into consideration the additional support cost model in OM decision-making. The situation becomes critical when the additional support costs exist, but they are not taken into consideration (see Figure 15).

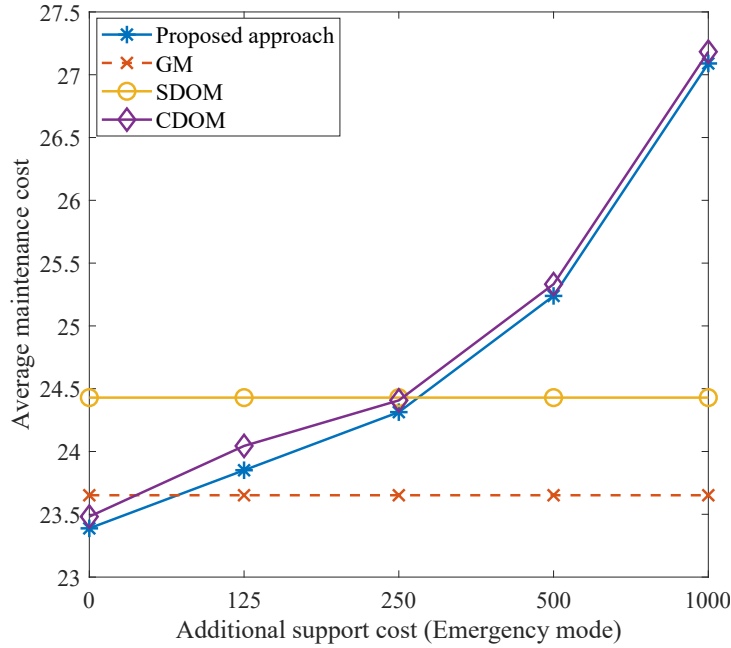


Figure 15: Sensitivity analysis of the considered approaches to the additional support cost

Finally, the sensitivity analysis of the four considered approaches to the component and system downtime cost rates was realized and reported in Figure 16. For this purpose, the component downtime cost rate ( $r_i$ ) was varied from 1 to 50. Note that in this case, we assume that the downtime cost rate is the same for all components.

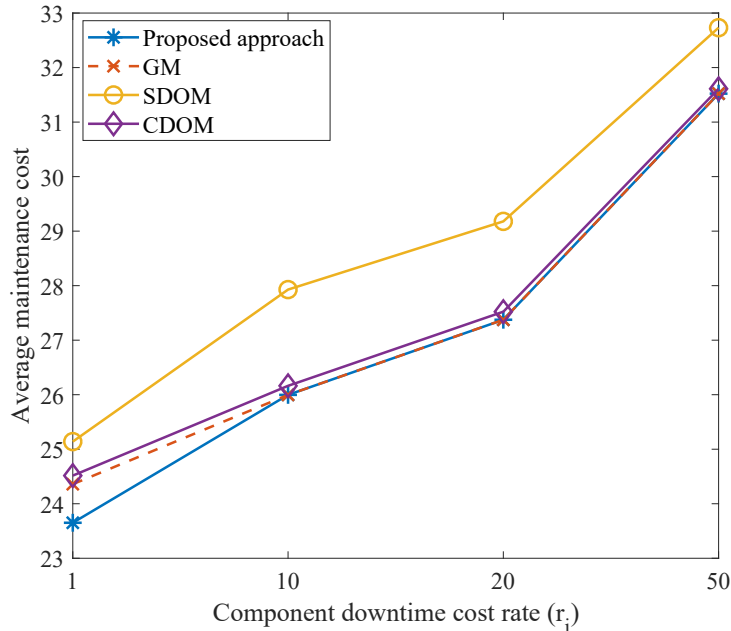


Figure 16: Sensitivity analysis of the considered approaches to the component downtime cost rate

The figure shows that the downtime cost rates have strong impacts on the performance of all the considered approaches. It comes from the fact that to be jointly maintained, the individual maintenance dates have to be modified. The modifications lead to the additional costs which are directly proportional to the component downtime cost rates. In conclusion, our approach provides the best results among the considered approaches,

especially when the downtime cost rates are small, in other words, the modifications of the individual maintenance dates do not cause serious economic consequences.

## 6.4 Performance study of GAM-based OM optimization

To find the optimal OM scenarios in these above studies, GAM was used with the following settings:

Table 10: GAM parameters

Population size	Number of iterations	Crossover probability	Mutation probability
80	100	0.8	0.1 for 30 first iterations 0.05 for 70 last iterations

GAM memorizes automatically the promising groups of maintenance opportunities, which occur most frequently in the recognized OM scenarios ( $X_1, X_2, \dots, X_8$ ). Consequently, the groups such as  $\{D_{21}, D_{22}\}$ ,  $\{D_{23}, D_{42}\}$ ,  $\{B_{11}, B_{41}\}$ ,  $\{C_{51}, C_{61}\}$  are saved in the GAM's memory and will be used as the reference solutions for the next usages of GAM. To study the performance of GAM, the example in subsection 6.1 (components 2 and 4 failed at  $a_k$ ) is considered, but at this time, the support costs are set to  $S_i^p = [20 \ 22 \ 18 \ 17 \ 18 \ 22]$  and  $S_i^c = S_i^p$ . Both GA with memory (GAM) and GA without memory (GA) are applied five times to the considered problem with the same number of iterations (100 iterations). Figure 17 presents the minimum maintenance costs that are found by the GAM and GA at each run.

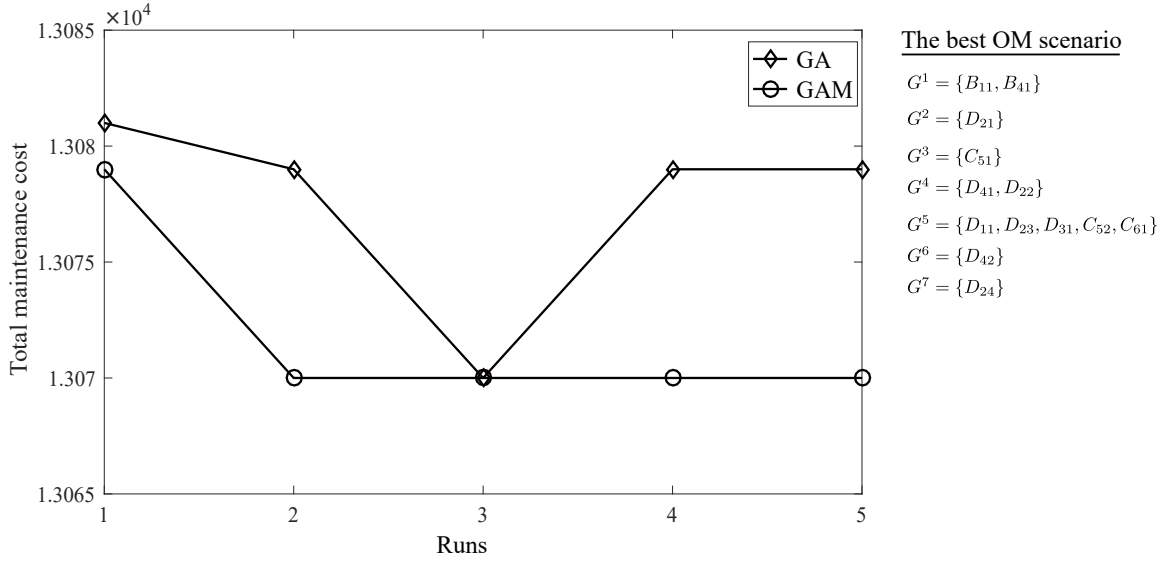


Figure 17: GAM-based OM optimization versus GA-based OM optimization

From the figure, we can see that GAM is able to find the optimal OM scenario after 2 runs, while GA needs 3 runs. In addition, GAM found the optimal solution most of the times (4 times), GAM is, therefore, more reliable than GA. This out-performance of GAM can be explained by the similarities between the groups  $G^1, G^4, G^5$  of the best OM scenario found by GAM (see Figure 17) and the reference groups in the GAM memory.

## 7 Conclusions

In this paper, we develop a flexible opportunity maintenance (OM) approach for multi-component redundant systems. The proposed OM approach allows taking into account various types of maintenance opportunities in the maintenance decision-making process. To this purpose, we firstly propose a generalized maintenance opportunity (MO) model for the modeling and mathematical formulation of various existing MOs and new ones.

MOs are then classified into 6 types. To take into consideration these various MOs in maintenance decision-process, an OM model with flexible decision rules allowing to take into account the various types of MOs in maintenance decision-making is then developed. In addition, a multi-mode logistic support model is also proposed to better incorporate different types of MOs with associated logistic support requirements. Finally, an efficient optimization algorithm using Genetic algorithm with memory is developed to find the optimal OM plan. The flexibility of the proposed OM approach is underlined by the fact that it can be applied to different kinds of a large of system configurations (redundant systems) and to various kinds of MOs. It also covers several maintenance models existing in the literature such as individual maintenance, grouping maintenance, system downtime based OM, and component downtime based OM. In addition, not only the OM solution but also the maintenance level (replacement or minimal repair) and the logistic support modes (emergency, express, normal modes) have been also optimally selected. Different numerical results show that the performance and the flexibility of the proposed OM approach in maintenance optimization of a multi-component system under various maintenance opportunities which may occur with time. It is also highlighted that the system structure, the maintenance opportunity, and its support, as well as the economic dependence, are important issues which need to be considered in maintenance decision-making.

Our future research works will focus on the investigation of the proposed OM maintenance model with consideration of the short-term interval selection, the imperfect maintenance, and [the performance of GAM](#).

## Acknowledgement

We thank the anonymous reviewers for their careful reading of our manuscript and their many insightful comments and suggestions.

## Appendix

### A. Optimization of the PM cycle at component level

Consider a component  $i$  which is periodically replaced according to its PM cycle  $T_i$  and immediately minimally repaired at its failures. According to the renewal theorem, the long-run expected maintenance cost rate of the component  $i$  can be calculated as

$$CR_i(T_i) = \lim_{t \rightarrow +\infty} \frac{\mathbb{E}[C_i(t)]}{t} = \frac{\mathbb{E}[C_i(T_i)]}{\mathbb{E}[T_i]} = \frac{C_i^{re} + C_i^{mr} \cdot N_i(0, T_i)}{T_i} \quad (24)$$

where,  $C_i(t)$  is the total maintenance cost of component  $i$  in  $(0, t)$ ;  $N_i(0, T_i)$  is the mean number of minimal repairs in the renewal cycle  $T_i$ ;

If the failure behavior of component  $i$  follows the Weibull distribution law with parameters  $\eta_i$  and  $\beta_i$ , according to the minimal repair property,  $N_i(0, T_i)$  can be calculated as [34]

$$N_i(0, T_i) = \left( \frac{x_i}{\eta_i} \right)^{\beta_i} \quad (25)$$

where,  $x_i$  is the total operational time of component  $i$  in a renewal cycle. We have

$$T_i = x_i + \omega_i^{re} + \omega_i^{mr} \cdot N_i(0, T_i) \quad (26)$$

From equations 24, 25, 26,  $CR_i(T_i)$  can be rewritten as a function of  $x_i$  as following

$$CR_i(x_i) = \frac{C_i^{re} + C_i^{mr} \cdot \left( \frac{x_i}{\eta_i} \right)^{\beta_i}}{x_i + \omega_i^{re} + \omega_i^{mr} \cdot \left( \frac{x_i}{\eta_i} \right)^{\beta_i}} \quad (27)$$

Let  $x_i^*$  denote the optimal value of  $x_i$  which minimizes  $CR_i(x_i)$ . The optimal PM cycle, denoted  $T_i^*$  can be calculated as

$$T_i^* = x_i^* + \omega_i^{re} + \omega_i^{mr} \cdot \left(\frac{x_i^*}{\eta_i}\right)^{\beta_i} \quad (28)$$

Therefore, we determine first  $x_i^*$  by setting the first derivative of  $CR_i(x_i)$  to zero.

$$\left. \frac{dCR_i(x_i)}{dx_i} \right|_{x_i^*} = 0 \implies C_i^{mr} \cdot (\beta_i - 1) \cdot (x_i^*)^{\beta_i} + (C_i^{mr} \cdot \omega_i^{re} - C_i^{re} \cdot \omega_i^{mr}) \cdot \beta_i \cdot (x_i^*)^{\beta_i - 1} - C_i^{re} \cdot \eta_i^{\beta_i} = 0 \quad (29)$$

Let  $f(x_i)$  be the function defined as the right part of the above equation

$$f(x_i) = C_i^{mr} \cdot (\beta_i - 1) \cdot (x_i)^{\beta_i} + (C_i^{mr} \cdot \omega_i^{re} - C_i^{re} \cdot \omega_i^{mr}) \cdot \beta_i \cdot (x_i)^{\beta_i - 1} - C_i^{re} \cdot \eta_i^{\beta_i} \quad (30)$$

To study the existence of  $x_i^*$  (the solution of the equation  $f(x_i) = 0$ ), we consider the following special values of  $f(x_i)$ :

$$f(0) = -C_i^{re} \cdot \eta_i^{\beta_i} < 0$$

$$f(a) = (C_i^{mr} \omega_i^{re} - C_i^{re} \cdot \omega_i^{mr}) \cdot \beta_i \cdot a^{\beta_i - 1}, \quad \text{with } a = \sqrt[\beta_i]{\frac{C_i^{re} \cdot \eta_i^{\beta_i}}{C_i^{mr} \cdot (\beta_i - 1)}}$$

$$f(b) = C_i^{mr} \cdot (\beta_i - 1) \cdot b^{\beta_i} > 0, \quad \text{with } b = \sqrt[\beta_i - 1]{\frac{C_i^{re} \cdot \eta_i^{\beta_i}}{\beta_i \cdot (C_i^{mr} \omega_i^{re} - C_i^{re} \cdot \omega_i^{mr})}}$$

$$f(c) = -(C_i^{re} \cdot \omega_i^{mr} - C_i^{mr} \omega_i^{re})^{\beta_i} - C_i^{re} \cdot \eta_i^{\beta_i}, \quad \text{with } c = \frac{C_i^{re} \cdot \omega_i^{mr} - C_i^{mr} \omega_i^{re}}{C_i^{mr}}$$

and we calculate

$$f''(x_i) = \frac{(C_i^{mr} \cdot x_i + C_i^{mr} \omega_i^{re} - C_i^{re} \cdot \omega_i^{mr}) \cdot (\beta_i - 1) \cdot \beta_i \cdot x_i^{\beta_i - 2}}{\left(x_i + \omega_i^{mr} \cdot \left(\frac{x_i}{\eta_i}\right)^{\beta_i} + \omega_i^{re}\right)^2}$$

Note that with  $\beta_i > 1$ ,  $f''(x_i) > 0$  if

$$x_i > \frac{C_i^{re} \cdot \omega_i^{mr} - C_i^{mr} \omega_i^{re}}{C_i^{mr}} = c \quad (31)$$

From these above analyses, two following cases are considered:

- If  $C_i^{mr} \omega_i^{re} > C_i^{re} \cdot \omega_i^{mr}$  (case 1), we have  $f(0) < 0$ ,  $f(c) < 0$  with  $c < 0$ , and  $f(a) > 0$ ,  $f(b) > 0$ , and  $f''(x_i) > 0$ . The solution  $x_i^*$  exists and is unique in interval  $(0, \min(a, b))$ ;
- If  $C_i^{mr} \omega_i^{re} \leq C_i^{re} \cdot \omega_i^{mr}$  (case 2),  $f(0) < 0$ ,  $f(a) \leq 0$ ,  $f(c) < 0$ ,  $\lim_{x_i \rightarrow +\infty} f(x_i) = C_i^{mr} \cdot (\beta_i - 1) > 0$  and  $f''(x_i) > 0$  if  $x_i^* > c > 0$ . As a consequence,  $x_i^*$  exists and is unique in interval  $(\max(a, c), +\infty)$ .

We consider an example in which a component  $i$  has the following parameters:  $\beta_i = 3$ ,  $\eta_i = 100$ ,  $C_i^{re} = 500$ ,  $C_i^{mr} = 100$ ,  $\omega_i^{re} = 3$ . The optimal values of  $x_i$  in the two above cases are reported in the following table:

Table 11: Illustration of the  $x_i^*$  determination

$\omega_i^{mr}$	Cases	$a$	$b$	$c$	$x_i^*$
0.5	Case 1	135.72	1825.7	-0.5	135.47
2	Case 2	135.72	488i	7	139.31

With  $x = x_i^*$ , the long-run maintenance cost rate of the component is minimal, and  $f(x_i^*) = 0$  (see Figure 18).



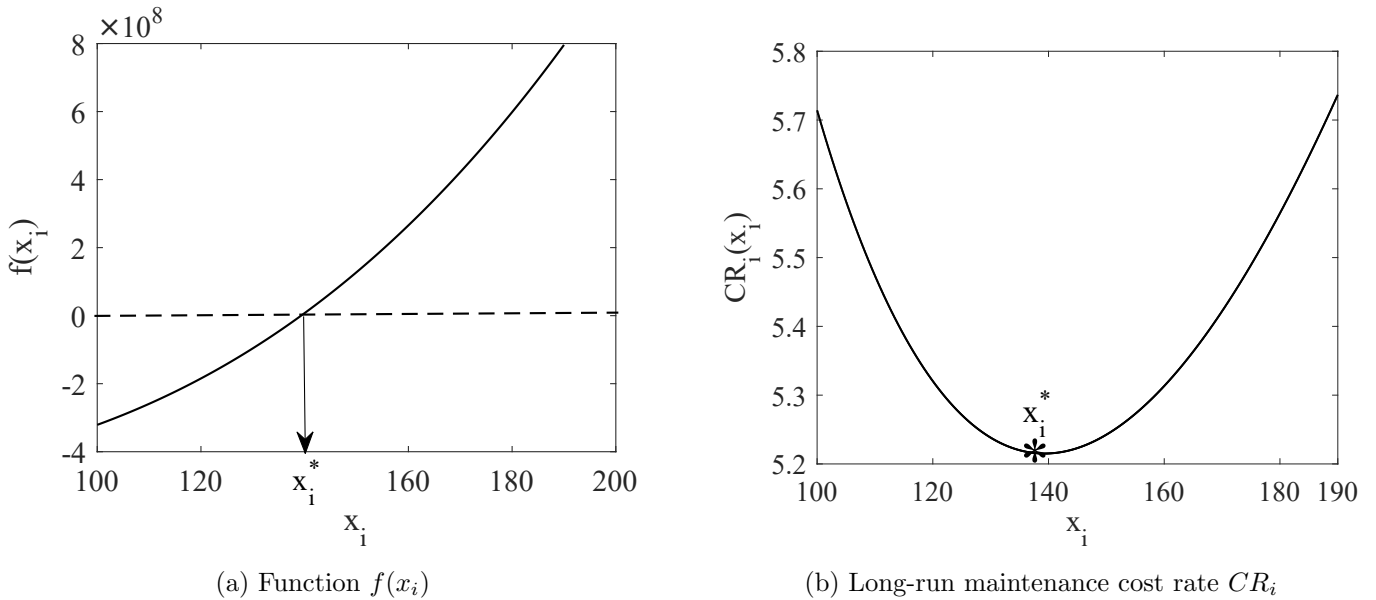


Figure 18: Illustrations of the optimality of  $CR_i$

## B. Implementation of GAM and GA for OM scenario optimization

Genetic Algorithm starts by creating an initial set (population) of OM scenarios based on random generators. At each iteration, each OM scenario is evaluated using an objective function (maintenance cost rate) and is modified by the genetic operations such as crossover and mutation. The process is stopped when one of the final conditions is verified. The GAM has exactly the same principle as that of GA, except that the initial population is generated not only based on the random generators but also based on the promising groups of MOs registered in the GAM's memory. At the end, when a new optimal OM scenario found, the GAM's memory is updated to save new promising groups (Figure 19). In the following paragraphs, we will describe in more detail each step of GA and GAM.

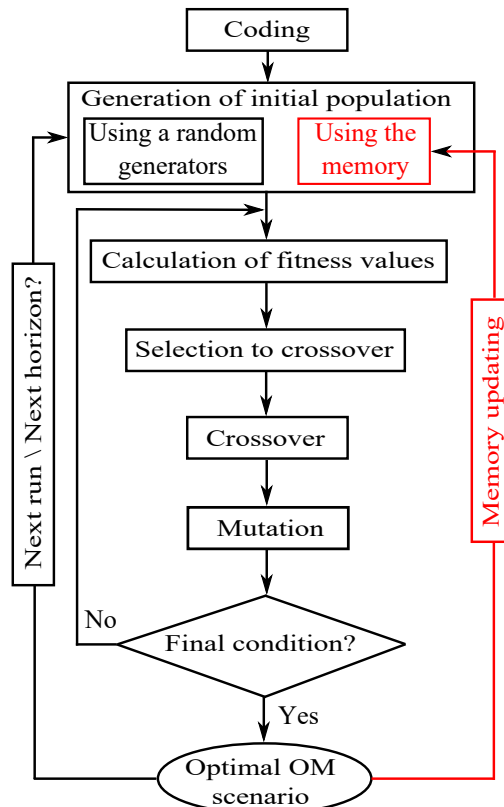


Figure 19: Flowchart of GA and GAM

**Coding** A coding method determines how a solution (an OM scenario) is structured in the algorithm. In this paper, each solution is represented by an array, namely  $X$ , with  $n_{opp}$  elements corresponding to  $n_{opp}$  maintenance opportunities in  $I_k$ . For example,  $X = [1\ 2\ 1\ 2\ 2\ 3]$  means that the OM scenario contains three groups: the first one is composed of the first and the third maintenance opportunities; the second group contains the second, the fourth and the fifth maintenance opportunities; the last group contains only the sixth maintenance opportunity. **This coding method is due to the fact that groups are mutually exclusive and will be well adapted for the GAM.**

**Generation of initial population** GA creates an initial set of OM scenarios. If there are too few scenarios in the population, GA have a few possibilities to perform crossover and only a small part of search space is explored. On the other hand, if there are too many scenarios, GA slows down [42, 43].

The OM scenarios in the initial population are generated by the two following ways:

- Using the recorded groups in the memory. The memory of GAM contains the most promising groups of maintenance opportunities, which are discovered from the previous runs or prior knowledge. One or several recorded groups in the memory can be randomly selected to build a new OM scenario.
- Using a random generator (uniform distribution). To create randomly an OM scenario which is independent from the memory, the number of groups  $m$  in the OM scenario is firstly randomly generated. Next,  $x_i$  with  $i = 1, \dots, n_{opp}$  are randomly selected from 1 to  $m$ .

We introduce a probability  $P_{generation} \in [0, 1]$  to decide which way will be used for the generation of the  $i^{th}$  OM scenario in the initial population. If  $p \leq P_{generation}$ ,  $p \sim U(0, 1)$  (uniform distribution), the OM scenario is created by using the recorded groups in the memory. Otherwise, if  $p > P_{generation}$ , it will be randomly created.

**Calculation of fitness values** The fitness value of an OM scenario  $X$  is defined as

$$CR_X = \frac{C(X)}{b_k - a_k}$$

**Selection to crossover** The goal is to select pairs of parent OM scenarios to Crossover phase. The selection process is based on fitness. Array of parent OM scenarios are sorted according to their fitness values in ascending order. The population is categorized into  $s$  groups. Next, a parent OM scenario is randomly chosen from these  $s$  groups with  $s$  groups probabilities  $p_1 \leq p_2 \leq \dots \leq p_s$  ( $\sum_{i=1}^s p_i = 1$ ) respectively. Thus, the fitter a OM scenario is, the more lucky it is chosen for a parent in the next generation. This combinatorial grouping was found most effective. In this work, the population are divided into 5 groups ( $s = 5$ ) with the groups selection probability are 5%, 10%, 15%, 25% and 45% respectively. Each pair of the selected OM scenarios is now randomly selected for Crossover step with Crossover probability that is introduced to leave some part of population survive to next generation. Crossover probability is usually chosen between [60%, 90%].

**Crossover** The objective is to combine selected OM scenarios to generate the next better generation by preserving their characteristics. Single point crossover is used. This algorithm randomly chooses a locus and exchanges the subsequences before and after that locus between the two selected scenarios to create two children. For example, given the two parents  $X_1 = [1\ 2\ 1\ 2\ 2\ 3]$  and  $X_2 = [1\ 1\ 1\ 1\ 2\ 2]$ , the locus is equal to 3. The two children are then  $X'_1 = [1\ 1\ 1\ 2\ 2\ 3]$  and  $X'_2 = [1\ 2\ 1\ 1\ 2\ 2]$ .

**Mutation** Mutation is made to prevent GA from falling into local optimal solutions, but it should not occur very often, because then GA will in fact change to random search. So, the mutation probability is usually chosen from 0.2% to 2%. For each selected OM scenario  $X$ , a random maintenance opportunity in a group is next moved to another group in order to generate a new scenario. For example, given  $X = [1\ 2\ 1\ 2\ 2\ 3]$  and  $x_2$  (the second maintenance opportunity) which is actually in group 2 is selected to be moved to another group. Assuming that it is moved to group 3, we obtain then the new OM scenario  $X' = [1\ 3\ 1\ 2\ 2\ 3]$ .

**Final condition** Final condition are introduced to stop the algorithm. Herein, the algorithm is stopped when the iteration number reaches its limited value or when the average relative change in the  $CR_X$  value over 10 generations is less than or equal to 0.01.

**Memory updating** All the groups of the best OM scenario are copied to the memory. To maintain the efficiency of the memory, the same number of recorded groups, that have the lowest occurrence frequency, will be removed from the memory.

## References

- [1] Do, Phuc and Barros, Anne and Bérenguer, Christophe and Bouvard, Keomany and Brissaud, Florent. Dynamic grouping maintenance strategy with time limited opportunities. *Reliability Engineering and System Safety*, 120:51–59, 2013.
- [2] Rommert Dekker. Applications of maintenance optimization models: a review and analysis. *Reliability engineering and system safety*, 51(3):229–240, 1996.
- [3] Hongzhou Wang. A survey of maintenance policies of deteriorating systems. *European journal of operational research*, 139(3):469–489, 2002.
- [4] Hasnida Ab-Samat and Shahrul Kamaruddin. Opportunistic maintenance (OM) as a new advancement in maintenance approaches: A review. *Journal of Quality in Maintenance Engineering*, 20(2):98–121, 2014.
- [5] Li Yang, Yu Zhao, Rui Peng, and Xiaobing Ma. Opportunistic maintenance of production systems subject to random wait time and multiple control limits. *Journal of manufacturing systems*, 47:12–34, 2018.
- [6] Ping Li, Wenbin Wang, and Rui Peng. Age-based replacement policy with consideration of production wait time. *IEEE Transactions on Reliability*, 65(1):235–247, 2016.
- [7] Cristiano AV Cavalcante, Rodrigo S Lopes, and Philip A Scarf. A general inspection and opportunistic replacement policy for one-component systems of variable quality. *European Journal of Operational Research*, 266(3):911–919, 2018.
- [8] Li Yang, Yu Zhao, Rui Peng, and Xiaobing Ma. Hybrid preventive maintenance of competing failures under random environment. *Reliability Engineering and System Safety*, 174:130–140, 2018.
- [9] H Truong Ba, Michael E Cholette, Pietro Borghesani, Y Zhou, and L Ma. Opportunistic maintenance considering non-homogenous opportunity arrivals and stochastic opportunity durations. *Reliability Engineering and System Safety*, 160:151–161, 2017.
- [10] Nicolai, Robin P and Dekker, Rommert. Optimal maintenance of multi-component systems: a review. In *Complex system maintenance handbook*, pages 263–286. Springer, 2008.
- [11] Li, Heping and Deloux, Estelle and Dieulle, Laurence. A condition-based maintenance policy for multi-component systems with Lévy copulas dependence. *Reliability Engineering and System Safety*, 149:44–55, 2016.
- [12] Nguyen, Kim Anh and Do, Phuc and Grall, Antoine. Multi-level predictive maintenance for multi-component systems. *Reliability engineering and system safety*, 144:83–94, 2015.
- [13] Chateauneuf, Alaa and Laggoune, Radouane and others. Condition based opportunistic preventive maintenance policy for utility systems with both economic and structural dependencies- application to a gas supply network. *International Journal of Pressure Vessels and Piping*, 165:214–223, 2018.

- [14] Biao Lu and Xiaojun Zhou. Opportunistic preventive maintenance scheduling for serial-parallel multistage manufacturing systems with multiple streams of deterioration. *Reliability Engineering and System Safety*, 168:116–127, 2017.
- [15] Xiaojun Zhou and Kailong Shi. Capacity failure rate based opportunistic maintenance modeling for series-parallel multi-station manufacturing systems. *Reliability Engineering and System Safety*, 181:46–53, 2019.
- [16] Do, Phuc, Roy Assaf, Phil Scarf, and Benoit Iung. Modelling and application of condition-based maintenance for a two-component system with stochastic and economic dependencies. *Reliability Engineering and System Safety*, 182:86 – 97, 2019.
- [17] Huynh, Khac Tuan and Barros, Anne and Bérenguer, Christophe. A reliability-based opportunistic predictive maintenance model for k-out-of-n deteriorating systems. *Chemical Engineering Transactions*, 33:493–498, 2013.
- [18] Wenrui Hou and Zuhua Jiang. An opportunistic maintenance policy of multi-unit series production system with consideration of imperfect maintenance. *Applied Mathematics and Information Sciences*, 7(1L):283–290, 2013.
- [19] Tangbin Xia, Lifeng Xi, Xiaojun Zhou, and Jay Lee. Dynamic maintenance decision-making for series-parallel manufacturing system based on MAM-MTW methodology. *European Journal of Operational Research*, 221(1):231–240, 2012.
- [20] Benoit Iung, Do, Phuc, Eric Levrat, and Alexandre Voisin. Opportunistic maintenance based on multi-dependent components of manufacturing system. *CIRP Annals*, 65(1):401–404, 2016.
- [21] Roy Radner and Dale W Jorgenson. Opportunistic replacement of a single part in the presence of several monitored parts. *Management Science*, 10(1):70–84, 1963.
- [22] Bhaba R Sarker and Tasnim Ibn Faiz. Minimizing maintenance cost for offshore wind turbines following multi-level opportunistic preventive strategy. *Renewable Energy*, 85:104–113, 2016.
- [23] Fangfang Ding and Zhigang Tians. Opportunistic maintenance for wind farms considering multi-level imperfect maintenance thresholds. *Renewable Energy*, 45:175–182, 2012.
- [24] Fangfang Ding and Zhigang Tian. Opportunistic maintenance optimization for wind turbine systems considering imperfect maintenance actions. *International Journal of Reliability, Quality and Safety Engineering*, 18(05):463–481, 2011.
- [25] Mahmood Shafiee, Maxim Finkelstein, and Christophe Bérenguer. An opportunistic condition-based maintenance policy for offshore wind turbine blades subjected to degradation and environmental shocks. *Reliability Engineering and System Safety*, 142:463–471, 2015.
- [26] Hoang Pham and Hongzhou Wang. Optimal  $(\tau, T)$  opportunistic maintenance of a k-out-of-n: G system with imperfect PM and partial failure. *Naval Research Logistics (NRL)*, 47(3):223–239, 2000.
- [27] Menachem Berg. General trigger-off replacement procedures for two-unit systems. *Naval Research Logistics Quarterly*, 25(1):15–29, 1978.
- [28] Xitong Zheng. All opportunity-triggered replacement policy for multiple-unit systems. *IEEE Transactions on Reliability*, 44(4):648–652, 1995.
- [29] Chen Zhang, Wei Gao, Sheng Guo, Youliang Li, and Tao Yang. Opportunistic maintenance for wind turbines considering imperfect, reliability-based maintenance. *Renewable energy*, 103:606–612, 2017.

- [30] Hadi Abdollahzadeh, Karim Atashgar, and Morteza Abbasi. Multi-objective opportunistic maintenance optimization of a wind farm considering limited number of maintenance groups. *Renewable Energy*, 88:247–261, 2016.
- [31] A Erguido, A Crespo Márquez, E Castellano, and JF Gómez Fernández. A dynamic opportunistic maintenance model to maximize energy-based availability while reducing the life cycle cost of wind farms. *Renewable Energy*, 114:843–856, 2017.
- [32] Jinqiu Hu and Laibin Zhang. Risk based opportunistic maintenance model for complex mechanical systems. *Expert Systems with Applications*, 41(6):3105–3115, 2014.
- [33] Heungseob Kim. Optimal reliability design of a system with k-out-of-n subsystems considering redundancy strategies. *Reliability Engineering and System Safety*, 167:572–582, 2017.
- [34] Ralph Edwin Wildeman, Rommert Dekker, and ACJM Smit. A dynamic policy for grouping maintenance activities. *European Journal of Operational Research*, 99(3):530–551, 1997.
- [35] Makis, V and Jiang, X and Cheng, K. Optimal preventive replacement under minimal repair and random repair cost. *Mathematics of operations research*, 25(1):141–156, 2000.
- [36] Hai Canh Vu, Phuc Do, Anne Barros, and Christophe Bérenguer. Maintenance grouping strategy for multi-component systems with dynamic contexts. *Reliability Engineering and System Safety*, 132:233–249, 2014.
- [37] Anil Sharma, GS Yadava, and SG Deshmukh. A literature review and future perspectives on maintenance optimization. *Journal of Quality in Maintenance Engineering*, 17(1):5–25, 2011.
- [38] Rommert Dekker, Ralph E Wildeman, and Frank A Van der Duyn Schouten. A review of multi-component maintenance models with economic dependence. *Mathematical Methods of Operations Research*, 45(3):411–435, 1997.
- [39] Mahmood Shafiee. Maintenance logistics organization for offshore wind energy: current progress and future perspectives. *Renewable Energy*, 77:182–193, 2015.
- [40] Huynh, Khac Tuan and Barros, Anne and Bérenguer, Christophe. Multi-Level Decision-Making for The Predictive Maintenance of  $k$ -Out-of- $n$ : F Deteriorating Systems. *IEEE transactions on Reliability*, 64(1):94–117, 2015.
- [41] Murad S Samhouri. An intelligent opportunistic maintenance (OM) system: a genetic algorithm approach. In *Science and Technology for Humanity (TIC-STH), 2009 IEEE Toronto International Conference*, pages 60–65. IEEE, 2009.
- [42] Shengxiang Yang. Genetic algorithms with memory and elitism-based immigrants in dynamic environments. *Evolutionary Computation*, 16(3):385–416, 2008.
- [43] Arabas, Jaroslaw and Michalewicz, Zbigniew and Mulawka, Jan. GAVaPS-a genetic algorithm with varying population size. In *Proceedings of the First IEEE Conference on Evolutionary Computation. IEEE World Congress on Computational Intelligence*, pages 73–78. IEEE, 1994.